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KINEMATIC ANALYSIS OF A 5 DOF OVERCONSTRAINED MANIPULATOR FOR REHABILITATION OF UPPER EXTREMITE

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Abstract

This study deals with one of the applications of parallel manipulator as a rehabilitation robot. This device is an over –constrained parallel manipulator 5 degree of freedom with 3 legs. This manipulator consists of a moving platform which is connected to a fixed base via three legs. Each leg is made of RRR(RR) (revolute) joints where the first three joint in all legs are parallel and the recent two joint are intersecting .Inverse kinematics of this device is solved by dividing manipulator into two sub-manipulators with the help of three imaginary joints placed at the intersection of platform joints with a direction parallel to base joints.

ÜST EXTREMİTENİN REHABİLİTASYONU İÇİN KULLANILACAK 5 SERBESTLİK DERECELİ KISITLI MANİPULATÖRÜN KİNEMATİK ANALİZİ

Özetçe

Bu tezde paralel bir manipülatörün rehabilitasyon amacı ile tasarımı ve analizi gerçekleştirilmiştir. Önerilen sistem beş serbestlikte ve yere 3 bacaktan bağlanmaktadır. Her bir bacak RRR(RR) yapılandırmasına sahiptir. Bütün bacaklardaki ilk üç mafsal birbirine paralel iken son iki mafsallar eksenleri çakışacak şekilde yerleştirilmiştir. Sistemin ters kinematic analizi işlemleri 3 hayali mafsalın yardımıyla sistemin iki parçaya ayrılmasıyla gerçekleştirilmiştir.

Keywords: *Overconstrained Manipulators, Rehabilitation, Inverse Kinematic Analysis .*
Anahtar Kelimeler: *Kısıtlı Manipulatörler, Rehabilitasyon, Ters Kinematik Analiz.*

1. INTRODUCTION

The field of rehabilitation robotics is one of technology-based solutions that use to assist people who become unable to do major life activities and have disability to interact physically with environment. Rehabilitation robotics also used for functional neural stimulation, artificial limb development, and technology for the diagnosis and monitoring of people during ADLs and movement therapy for the upper extremity after neurologic injury.

In the literature several studies include the application of robotics to rehabilitation. One of the most famous one is MIT- MANUS [1] which is an arm therapy robotic system developed by Hogan, Krebs, and colleagues at the Massachusetts Institute of Technology lead to several new studies for the effect of these types of parallel robots to rehabilitation. Machiel [2]provide descriptions of the main achievements of rehabilitation robotics field and domains of these robotics with short history also describes both of therapy robots for physical therapy and training and assistance robots for people with disabilities. For the hand rehabilitation device, it is important to provide some criterion to guide the design or evaluate the performance .Li-Chieh[3]proposed the functional workspace of palmar opposition of with respect to maximal workspace of the other fingers with two-dimensional to characterize the motor capability. Zheng [4] proposed an exoskeleton of hand rehabilitation assistive device . The workspace and kinematics of this device for thumb and index finger rehabilitation are also considered in this study. To provide passive movements in the frequency of the repetitions of the movements in robot-aided therapy, a pre-determined trajectory should be selected [5-7]. Also the safety requirement plays important role in design of rehabilitation robotics. As Kang [8] studied an adaptive control strategy for exoskeleton robotic of 5 dof upper limb and try to improve the safety of robotic by considering the architecture of robot, unknown variances and actuator faults

Investigations show that patients who received arm movement by robot therapy make more improvement of movement ability than who received arm movement by conventional therapy, in additional the robotic therapy do without any adverse effects. Also, the robot group patients received more therapy by a robotic device for an hour each day, five days per week, after several weeks, the robot group have recovery of a brain and arm movement ability. To improve the movement ability and movement practice stimulating, the robotic make equivalent to computer mouse. Due to these features of the robotic device, it becomes necessary.

In this study a manipulator for the rehabilitation for human arm is proposed as a 5-DoF 3RRR-(RR) mechanism as shown in Figure (1). The selection of this structure has some advantages to the other similar parallel manipulators such as;

- Its subspace includes both planar and spherical movements which are needed for the rehabilitation of both hand and wrist.
- Parallel structure can carry more loads and don’t need to carry all motors.
- 3 legs are optimum for this kind of subspace so it is simpler in structure and less in potential interference than same 5-DOF PM with 5 or 4 active legs.
- Its workspace is larger than that of 5-DOF with 5 active legs PM.

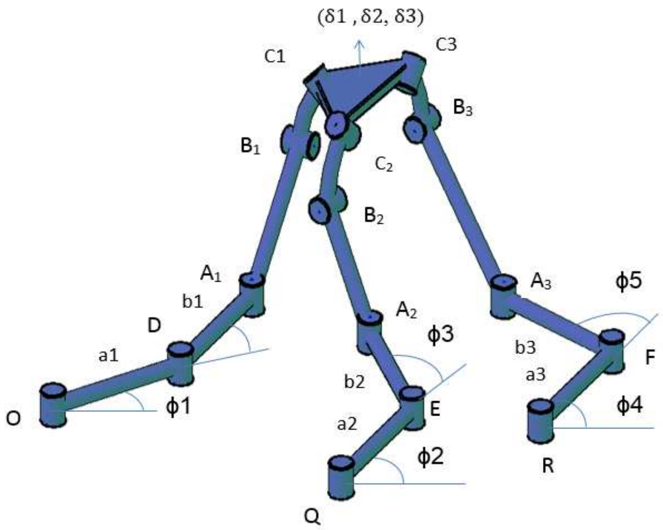

Figure 1. 5 DoF over –constrained parallel manipulator

2. GEOMETRY OF THE MANIPULATOR

The selected parallel manipulator is shown in Figure (1). This manipulator is composed of a moving platform attached to the base by three kinematic independent chains. The point O represented the center of the reference coordinate system O _xyz. Each limb contains five joints where first three from the ground have parallel axes and later two joints axes are intersecting in a point P. The mechanism needs five actuators thus one motor is attached to the base revolute joint of the first leg (φ1) and two motors are attached to first two revolute joints from the ground on the second leg (φ2, φ3). For the third leg same procedure is carried with two motors (φ4, φ5). Manipulator is designed symmetrical. To supply symmetry the distances between base joints are selected equal and also angle between platforms joints are selected equal. All three legs have identical architectures, first two link lengths are defined as a_i and b_i where i is the index for the limb. Connection between 3rd and 4th joint is described by distance A_iP as r_i and angle between the axis of the 4th joint and A_iP is $\alpha_{1,i}$. Angle between 4th and 5th joint is described as $\alpha_{2,i}$. orientation of the platform joint is described by $_i$.

3. METHODOLOGY (INVERSE KINEMATICS)

The kinematics model investigates the analytical relations between the values of the actuated joints (input variables) and the location of the moving platform (output variables). This section dealt with finding the inverse kinematics to solve the values of the actuated joints with respect to the end-effector pose. The inverse kinematics model consists in finding the value of the joints displacements with respect to configuration of the platform. The inverse kinematics is essential for control of location of the platform of parallel robots and an urge for the analysis or synthesis of the workspace of the manipulator. For designed manipulator 5 degree of freedom with 3 legs the desired position and orientation of the output link (x, y, $\delta 1$, $\delta 2$, $\delta 3$) is given and the problem is finding the required values of the input actuated joints (φ1, φ2, φ3, φ4, φ5).

To solve inverse kinematics, the manipulator will be divided in to two sub-manipulators in subspace λ=3 with the help of three imaginary joints placed at the intersection of platform joints with a direction parallel to base joints as shown in figure 2. The upper part will be a 3 Dof -3(RRR) spherical manipulator where input axes are coaxial as shown in figure 3a. The lower part wil be a redundant 5 Dof - 3 RRRR manipulator as shown in figure 3b.

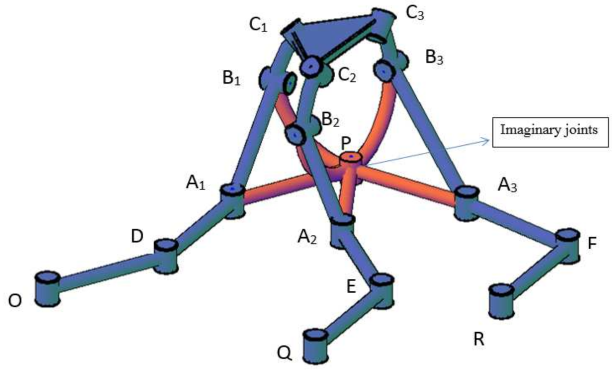

Figure 2. The geometry of the manipulator with imaginary joints

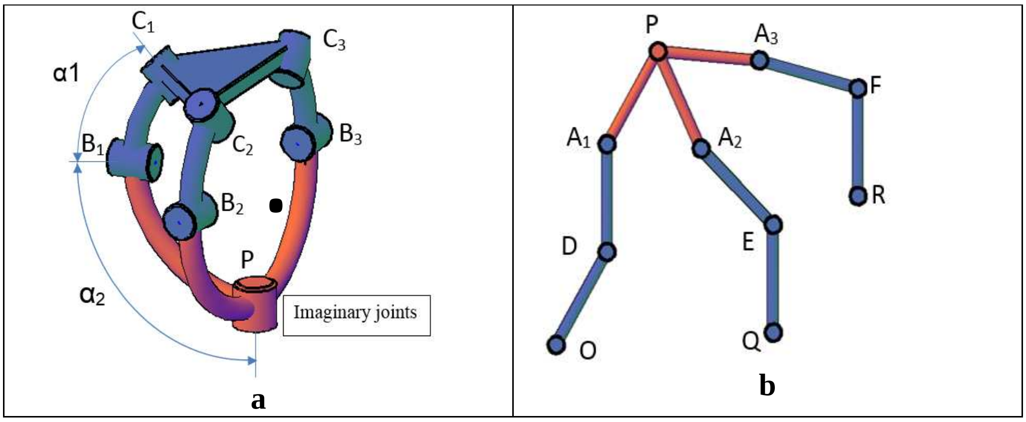

Figure 3. (a) The upper spherical mechanism (b) The lower planar mechanism

The orientation of the platform is either given as a 3x3 matrix or calculated using angles δz, δy and δx as the rotation matrices around z, y, x in Euler angles with angles δ1, δ2, δ3 respectively as $\delta = \delta\mathbf{z} \cdot \boldsymbol{\delta}\mathbf{y} \cdot \boldsymbol{\delta}\mathbf{x}$.

As the orientation of the platform is known orientation of the axes of each joint on the platform as shown in figure 4 which are defined by unit vectors $w_i = \{w_{x,i}, w_{y,i}, w_{z,i}\}$ can be calculated by using equation (1).

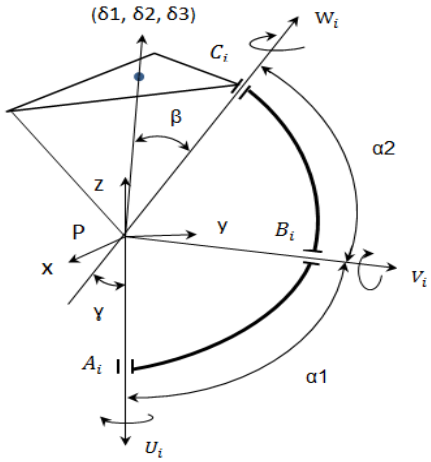

Figure 4. The vectors of spherical part

$$W_i = \begin{pmatrix} w\mathbf{x}_i \\ w\mathbf{y}_i \\ w\mathbf{z}_i \end{pmatrix} = \begin{pmatrix} \mathrm{Cos}[\gamma_i] & -\mathrm{Sin}[\gamma_i] & 0 \\ \mathrm{Sin}[\gamma_i] & \mathrm{Cos}[\gamma_i] & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 0 & 0 \\ 0 & \mathrm{Cos}[\beta_i] & -\mathrm{Sin}[\beta_i] \\ 0 & \mathrm{Sin}[\beta_i] & \mathrm{Cos}[\beta_i] \end{pmatrix} . \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (1)$$

Where β_i is the orientation with respect to z axis for each joint around x axis and γ_i is the orientation around z axis for each joint for i=1,2,3.Now for each leg, orientation of each joint on the platform $\mathbf{W}_i$ ={ , , } with respect to ground can be calculated with respect to rotations ($\delta 1$, $\delta 2$, $\delta 3$) as shown in equation (2) for i=1,2,3

$$\mathbf{W}_{\boldsymbol{i}} = \delta \cdot \boldsymbol{w}_{\boldsymbol{i}} \tag{2}$$

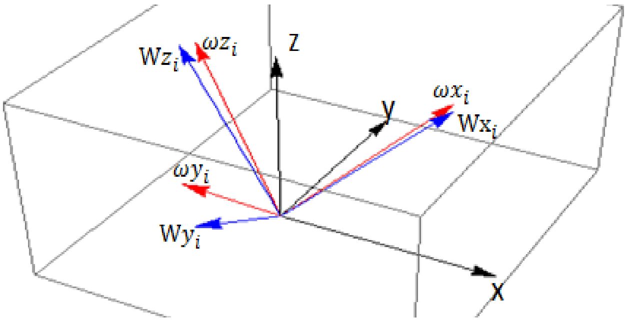

Figure 5. The orientations of the platform and platform joints in space

According to the structure of mechanism in spherical part the orientation of joints on platform $\mathbf{W}_i$ is equal to the orientation of the coordinate system at point P to the $\mathbf{W}_i$ using joint angles θ1,i and θ2,i and link angles α1,i and α2,i.

$$\mathbf{W}_i = \mathbf{R}_{\theta_{1,i},z} \cdot \mathbf{R}_{\alpha_{1,i},z} \cdot \mathbf{R}_{\theta_{2,i},z} \cdot \mathbf{R}_{\alpha_{2,i},z} . [\mathbf{0}, \mathbf{0}, \mathbf{1}]^T \tag{3}$$

That leads three equations;

$$\cos[\alpha 2]\sin[\alpha 1]\sin[\theta 1_i] + \sin[\alpha 2]\left(\cos[\alpha 1]\cos[\theta 2_i]\sin[\theta 1_i] + \cos[\theta 1_i]\right. \tag{4}$$

$$-\cos[\alpha 2]\cos[\theta 1_i] + \sin[\alpha 1]\sin[\alpha 2]\left(-\cos[\alpha 1]\cos[\theta 1_i]\cos[\theta 2_i] + \sin[\theta 1_i]\right. \tag{5}$$

$$\cos[\alpha 1]\cos[\alpha 2] - \cos[\theta 2]\sin[\alpha 1]\sin[\alpha 2] = \mathbf{W}z_i \tag{6}$$

Solving equations (4) and (5) for Cos$[\theta 2_i]$ and Sin$[\theta 2_i]$ yields

$$\cos[\theta 2_i] = \csc[\alpha 2]\sec[\alpha 1](\mathrm{Wyi}\,\mathrm{Cos}[\theta 1_i] + \mathrm{W}xi\,\mathrm{Sin}[\theta 1_i]) - \cot[\alpha 2]\tan[\alpha 1] \quad (7)$$

(8)

We can eliminate $\theta 2_i$ by substitute equations (7), (8) in equation (6) we obtain

$$\cos[\alpha 1]\cos[\alpha 2] - \sin[\alpha 1]\sin[\alpha 2](\csc[\alpha 2]\sec[\alpha 1](-\mathrm{Wyi}\cos[\theta 1_i] + \mathrm{W}x \quad (9)$$

using half tangent formulas where t =tan[$[\theta 1]_i/2]$ in equation (9) we obtain:

$$\frac{\mathrm{Sec}[\alpha 1]\left((1+t^2)\mathrm{Cos}[\alpha 2] + (-2t\,\mathrm{Wxi} + \mathrm{Wyi} - t^2\mathrm{Wyi})\mathrm{Sin}[\alpha 1]\right)}{1+t^2} = \mathrm{W}_i \quad (10)$$

We can solve equation (10) and obtain the value of t as

$$t = \frac{\mathrm{Sin}[\alpha 1]\mathrm{Wx_i} \pm \sqrt{\mathrm{Sin}[\alpha 1]^2\mathrm{Wx_i^2} + \mathrm{Sin}[\alpha 1]^2\mathrm{Wy_i^2} - (\mathrm{Cos}[\alpha 2] - 2\mathrm{Cos}[\alpha 1]\mathrm{Wz_i})^2}}{\mathrm{Cos}[\alpha 2] - \mathrm{Sin}[\alpha 1]\mathrm{Wy_i} - 2\mathrm{Cos}[\alpha 1]\mathrm{Wz_i}}$$

Which gives $\theta 1_i =$ 2*Arctan [t] ,results two solution for , for i=1,2,3

In the second step for the inverse kinematics of the planar manipulator the point (P={X,Y}) which is the position of the end-effector is given and $\theta 1_i$ which are found in the previous part represent the orientation for r_i around z axis. Thus the position of A_i can be found.

Where represent the position of third joint in each legs as show in figure for i=1,2,3.

(11)

X$\mathbf{A}_\mathbf{i}$ = *Cos[$\theta 1_i$]+ X

(12)

Y$\mathbf{A}_\mathbf{i}$ = *Sin[$\theta 1_i$]+ Y

Where: $\mathbf{r_i}$ represent the distance of link from point P to $\mathbf{A_i}$, $\theta 1_i$ represent the rotation angle for this link about z axis. From equation (11) and (12), we obtain the position of ($X_{\mathbf{A_i}}, Y_{\mathbf{A_i}}$) for each leg. Finding position ($X_{\mathbf{A_i}}, Y_{\mathbf{A_i}}$) leads to find the input angles (φ1,φ2,φ3,φ4,φ5). For the first leg as shown in figure 7 a.

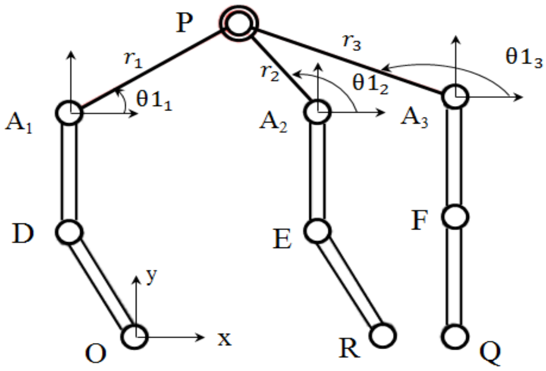

Figure 6. The planar part

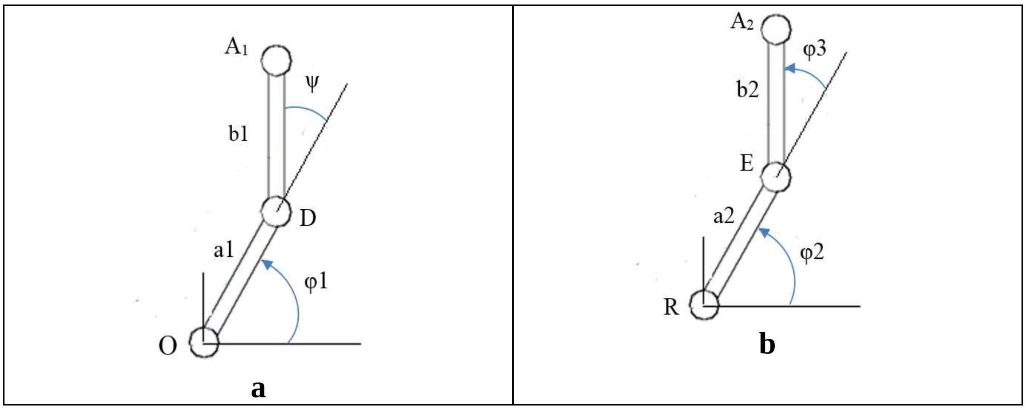

Figure 7. (a)The first leg of manipulator, (b)The second leg of manipulator

From the geometry of the leg we can write a vector- loop equation

$$\mathbf{OA_1 = OD + DA_1} \tag{13}$$

Expressing the vector loop equation in fixed coordinate frame gives

$$\begin{aligned} \mathbf{XA_1} &= \mathbf{a1\cos[\varphi 1] + b1\cos[\varphi 1 + \psi 1]} \\ \mathbf{YA_1} &= \mathbf{a1\sin[\varphi 1] + b1\sin[\varphi 1 + \psi 1]} \end{aligned} \tag{14}$$

Since ψ is passive joint angle, It should be eliminated from the equation above by summing the squares of two equation yields

$$\mathbf{e1\sin[\varphi 1] + e2\cos[\varphi 1] + e3} = 0 \tag{15}$$

where: , ,

Substituting half tangent formulation where t = tan[$\varphi 1/2$] in equation (15), we obtain

$$\mathbf{e2 + e3 + 2e1t + (-e2 + e3)t^2 = 0} \tag{16}$$

By solving equation (16) we get $\mathbf{t = \dfrac{-e1 \pm \sqrt{e1^2 + e2^2 - e3^2}}{e3 - e2}}$ which gives

$$\varphi 1 = 2\mathbf{Arctan}\frac{-\mathbf{e1} \pm \sqrt{\mathbf{e1}^2 + \mathbf{e2}^2 - \mathbf{e3}^2}}{\mathbf{e3} - \mathbf{e2}} \tag{17}$$

From the equation (17), there are two solution of $\varphi\mathbf{1}$ and there for two configurations of leg 1. when equation (15)yields a double root ,the two links OB and BA are in a folded –back or fully stretched out configuration called the singular configuration. when equation (15) yields no real root , the specified location of moving platform in not reachable]. For the leg 2 which is shown in figure 7.b. The location of point $\mathbf{A_2}$ is calculated from the second step and the problem is to find the joint angles (φ2,φ3). From the geometry of Figure (22) we can write a vector- loop equation

$$\mathrm{O}\mathbf{A_2} = \mathrm{OR} + \mathrm{RE} + \mathrm{E}\mathbf{A_2} \tag{18}$$

Expressing the vector loop equation in fixed coordinate frame gives

$$\mathbf{XA_2 - XR = a2} * \cos[\boldsymbol{\varphi}\mathbf{2}] + \mathbf{b2} * \mathbf{cos[\varphi 2 + \varphi 3]} \tag{19}$$

$$\mathbf{YA_2 - YR = a2} * \sin[\boldsymbol{\varphi}\mathbf{2}] + \mathbf{b2} * \mathbf{sin[\varphi 2 + \varphi 3]} \tag{20}$$

Since φ3 is active joint angle and shouldn't be eliminated. From figure 7.b we observe that the distance from point R to $\mathbf{A_2}$ is independent of φ2, hence we can eliminate φ2 by summing the squares of equations (19) and (20) yields:

$$\mathbf{(XA_2 - XR)^2 + ((YA_2 - YR))^2 = a_2{}^2 + b_2{}^2 + 2a2b2Cos[\varphi 3]} \tag{21}$$

Solving equation(21), we obtain

$$\varphi 3 = \mathrm{Cos}^{-1}\, k \qquad (22)$$

where

equation (22) yields (1) one double root if ||=1, (2) two real roots if || and (3)

$< 1 , \varphi 3 = \varphi 3$

no real roots if || . In general, if || is a solution, $\varphi 3 = -\varphi 3$ is also solution where $\pi \geq \varphi 3 \geq 0$. We call $\varphi 3 = \varphi 3$ the elbow-down solution and $\varphi 3 = -\varphi 3$ elbow-up solution. If ||=1, the arm is in a fully stretched or

folded configuration. If || the position is not reachable. Corresponding to each $\varphi 3$,we can find $\varphi 2$ by expanding equations (19) and (20) as follow:

$$\mathbf{XA_2 - XR = a2 + b2}\ \cos[\varphi 3]\cos[\varphi 2] - (\mathbf{b2}\ \sin[\varphi 3])\sin[\varphi 2] \qquad (23)$$

$$\mathbf{YA_2 - YR = b2}\ \sin[\varphi 3]\cos[\varphi 2] + (\mathbf{a2 + b2}\ \cos[\varphi 3])\sin[\varphi 2] \qquad (24)$$

Solving equations (23) and (24) for Cos[φ2] and Sin[φ2], yields:

$$\sin[\varphi 2] = \frac{-(\mathbf{XA2} - \mathbf{XR})\mathbf{b2}\ \sin[\boldsymbol{\varphi}\mathbf{3}] + \ (\mathbf{YA2} - \mathbf{YR})(\mathbf{a2} + \mathbf{b2}\ \cos[\boldsymbol{\varphi}\mathbf{3}])}{\square}\boldsymbol{\rho}$$

$$\cos[\varphi 2] = \frac{(\mathbf{XA2} \ - \mathbf{XR})(\mathbf{a2} + \mathbf{b2}\ \cos[\boldsymbol{\varphi}\mathbf{3}]) + (\ \mathbf{YA2} - \mathbf{YR})\mathbf{b2}\ \sin[\boldsymbol{\varphi}\mathbf{3}]}{\square}\ \boldsymbol{\rho}$$

Where $\rho = \mathbf{b2}^2 + \mathbf{a2}^2 + \mathbf{2b}\ \mathbf{2a2}\ \cos[\boldsymbol{\varphi}\mathbf{3}]$. Hence, corresponding to each φ3, we obtain a unique solution for φ2:

$$\varphi 2 = \mathbf{Atan2}(\mathbf{Sin}[\boldsymbol{\varphi}\mathbf{2}], \mathbf{Cos}[\boldsymbol{\varphi}\mathbf{2}]) \tag{25}$$

From equation (25) we can obtain unique solution for φ2 by using the function Atan2(x,y) In a computer program. However, these equation yields real or complex solution. A complex solution mean the end _effector position that is not reachable. The third leg has same form with the second leg and same procedure can be applied to find $\varphi 4$ **and** $\varphi 5$ for known $\mathbf{A_3}$ position and lengths a3,b3,r3. Now we will describe the orientation and position of platform which are given for 50 sec as shown in figure 8 (a) and (b).

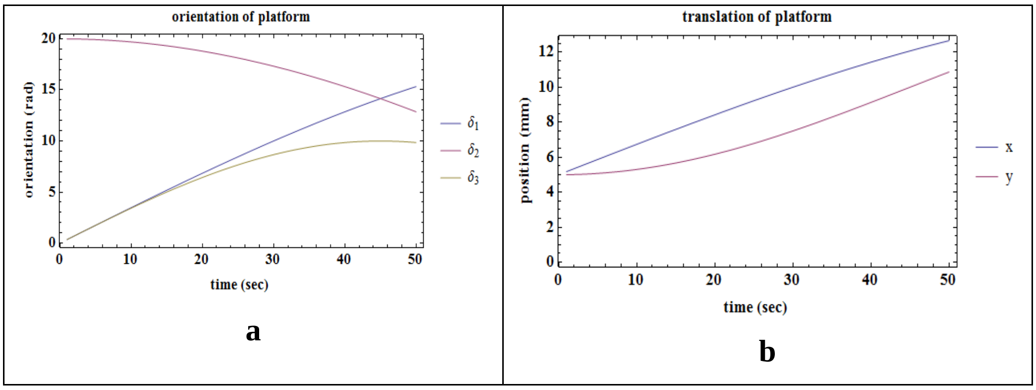

Figure 8. (a) The given orientation of platform with respect to time, (b)The given translation of platform with respect to time

The results of the motion can be found as motor orientations from inverse kinematic equations (18), (23) and (26) result is shown in figure 10.

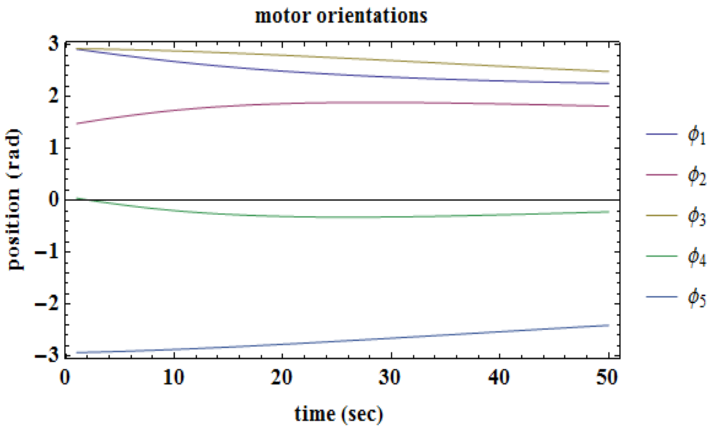

Figure 10. The position of motors orientation with respect to time

4. CONCLUSION

This study deals with one of the possible applications of a parallel manipulator in rehabilitation robotics. The focus is placed on the study of robot therapy to improve physical or cognitive function for persons who becomes unable to move their hands. However, this device can provide the hand by necessary movement. The selection of this structure leads to advantages to the other similar parallel manipulators such as; the subspace including both planar and spherical movements which are needed for the rehabilitation of both hand and wrist, by using parallel structure to carry more load and don't need to carry all motors.3 legs are found to be optimum for this kind of subspace so it is simpler in structure and less in potential interference than same 5-DOF PM with 5 or 4 active legs.

For solving the inverse kinematics and reduce complexity of the manipulator, it was divided in to two sub-manipulators in subspace $\lambda=3$ with the help of three imaginary joints placed at the intersection of platform joints with a direction parallel to base joints. The upper part being a 3 Dof -3(RRR) spherical manipulator where input axes are coaxial and the lower part being a redundant 5 Dof - 3 RRRR manipulator. This division is resulted in the easiness of calculations and then the inverse kinematic analysis is made for the manipulator.The results are a manipulator which can be controlled using these calculations and achieve the necessary motion for the rehabilitation of the human arm. This study is a base for a more detailed study of rehabilitation of the arm and wrist. For the future, the optimization of the design parameters for a given workspace should be done. For the design part by using parallelograms the floating motors can be transferred to ground. A control algorithm should be applied to the system for the defined rehabilitation movements and forces.

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