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A Fractional Model of an Effect of Awareness Programs by Media on Binge Drinking Model with Mittag-Leffler Kernel

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Abstract

Alcoholism is one of the problems that societies have not been able to find a solution for a long time. Today, over 2 billions of people consume alcohol and it is estimated that approximately 76 million of these people are addicted. In this study, an alcoholism model is broadly researched with the help of AB derivative. The existence and uniqueness of the drinking model solutions together with the stability analysis is demonstrated through Banach fixed point theorem. The special solution of the model is investigated using the Sumudu transformation and following that, a set of numeric graphics are given for different fractional orders with the intention of showing the effectiveness of fractional derivative. Hence, it is obtained that while awareness programs driven by media increases, the number of heavy drinkers decreases.

Keywords: *Fractional differential equations; Atangana-Baleanu derivative; mathematical model.*
2010 Mathematics Subject Classification: *Primary 34A34; 34A08; 47H10.*

1. Introduction

Alcohol consumption, especially excessive drinking, is an issue that may lead to great societal and heath problems which has been tried to be solved for so long. One of the organs that alcohol damages the most is brain. The direct effect of alcohol to brain is that it disrupts reasoning, decision making, and mobility. The reason for this is that it damages nerve cells and as a result it causes hand tremors and paralysis. Alcohol also increases the risk of cancer to a great extent. In addition, due to being addictive, it affects domesticity and therefore causes great financial losses. Considering all these negative affects, scientist from great variety of disciplines, have been doing researches to reduce alcohol consumption. In these researches, mathematical modeling plays the most important role. By the help of mathematical models, researchers are able to obtain very crucial information related to process of drinking and precautions to be taken, so a variety of mathematical models of drinking have been studied. For example, Sanchez et al. [1] introduced a simple mathematical model of drinking lapse. Mulone et al. [2] examined a two-stage model for youths with critical drinking problems and their cure. Gomez-Aguilar [3] analyzed an alcoholism model involving the impact of Twitter.

In the last few decades, the concept of fractional calculus has attracted great interest among the researchers. It has been reported by a wide variety of authors that, within this concept, fractional derivative reflects the system behavior in a much more accurate and efficient way than the integer order derivative [4, 5, 6, 7, 8, 9, 10, 11, 12]. For this reason, there are many fractional derivatives in the literature classics such as Riemann-Liouville (RL) and Caputo operators. However, the singularity arising from the kernel functions of these derivatives is considered a weakness. In order to cope with this weakness, Atangana-Baleanu (AB) derivative, a new non-singular derivative with Mittag-Leffler kernel, has been introduced by Atangana and Baleanu. Over the last years, the studies conducted in parallel with this improvement, evidently shows that the new fractional derivative with Mittag-Leffler kernel can be used as an effective mathematical tool in order to model the complex problems of real-life. Benefiting from AB derivative, Inc et al. [13] studied logarithmic-KdV equation, Khan et al.[14] formulated a new fractional order epidemic model for the tuberculosis disease with relapse, Kumar et al. [15] introduced a new fractional extension of regularized long-wave equation. Other outstanding studies can be found in [16, 17, 18, 19, 20, 21, 22, 23, 24, 25].

By the motivated of the above studies, we focus on a drinking model taking into consideration the effect of media with AB derivative

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introduced by Huo et. al [26] of the following integer order form:

$$\begin{aligned}
\frac{dS(t)}{dt} &= a_1 - a_2 S(t) A(t) - a_3 S(t) M(t) + a_4 X(t) - a_1 S(t), \\
\frac{dX(t)}{dt} &= a_3 S(t) M(t) - a_4 X(t) - a_1 X(t), \\
\frac{dA(t)}{dt} &= a_2 S(t) A(t) + a_5 R(t) - a_6 A(t) - a_1 A(t), \\
\frac{dR(t)}{dt} &= a_6 A(t) - a_5 R(t) - a_1 R(t), \\
\frac{dM(t)}{dt} &= a_7 A(t) - a_8 M(t),
\end{aligned}$$

where the susceptible group who do not drink or drink to a certain extent, those who try to avoid drinking due to the awareness of risks they take , those who drink heavily and those who receive treatment, are denoted respectively as $S(t)$, $X(t)$, $A(t)$ and $R(t)$. The cumulative density of awareness programs that are driven by media is also displayed as $M(t)$. The model parameters are: the rate of individuals who are part of the program at a certain time interval is a_1. The death rate is assumed negligible since we are concerned with youth. Therefore, the leaving rate is also considered a_1. a_2 stands for the transmission coefficient for the nondrinkers who are prone to turning into heavy drinkers due to peer pressure. a_3 denotes the spread rate of awareness among people who do not drink due to awareness program. Nondrinkers who avoid interacting with heavy drinkers form a separate class named as $X(t)$. The constant a_4 represents the transition ratio of aware individuals that convert to nondrinker category. The coefficient of proportionality for R which represents people who relapse into drinkers and for A receiving treatment are signified by a_5 and a_6, respectively. The growth ratio in the number of awareness programs is supposed to be commensurate to the population of heavy drinkers. This growth rate is represented by a_7. Finally, a_8 corresponds to the depletion ratio of these programs through issues such as ineffectiveness or social problems.

2. Basic definitions and preliminaries

In this part, we give some fundamental definitions which are useful in the succeeding chapters.

Definition 2.1. *Let $a < b$, $g \in H^1(a,b)$ be a function and $\zeta \in [0,1]$. Then AB derivative in Caputo sense of order ζ of g is given by [29]*

$$ {}_{a}^{ABC}D_t^{\zeta}[g(t)] = \frac{B(\zeta)}{1-\zeta}\int_a^t g'(x) E_{\zeta}\left[-\zeta\frac{(t-x)^{\zeta}}{1-\zeta}\right]dx, \tag{2.1}$$

where $B(\zeta)$ is a normalization function with $B(0) = B(1) = 1$.

Definition 2.2. *Let $a < b$, $g \in H^1(a,b)$ be a function and $\zeta \in [0,1]$. Then AB derivative in RL sense of order ζ of g is given by [29]:*

$$ {}_{a}^{ABR}D_t^{\zeta}[g(t)] = \frac{B(\zeta)}{1-\zeta}\frac{d}{dt}\int_a^t g(x) E_{\zeta}\left[-\zeta\frac{(t-x)^{\zeta}}{1-\zeta}\right]dx. \tag{2.2}$$

Definition 2.3. *The fractional integral associated to the AB fractional derivative is given by [29]:*

$$ {}_{a}^{AB}I_t^{\zeta}[g(t)] = \frac{1-\zeta}{B(\zeta)}g(t) + \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_a^t g(\lambda)(t-\lambda)^{\zeta-1}d\lambda. \tag{2.3}$$

3. Effect of awareness programs by media on binge drinking model with Atangana-Baleanu derivative

Let us modify this model by replacing the integer order derivative by the AB derivative:

$$\begin{aligned}
{}_{0}^{ABC}D_t^{\zeta}(S(t)) &= a_1 - a_2 S(t) A(t) - a_3 S(t) M(t) + a_4 X(t) - a_1 S(t), \\
{}_{0}^{ABC}D_t^{\zeta}(X(t)) &= a_3 S(t) M(t) - a_4 X(t) - a_1 X(t), \\
{}_{0}^{ABC}D_t^{\zeta}(A(t)) &= a_2 S(t) A(t) + a_5 R(t) - a_6 A(t) - a_1 A(t), \\
{}_{0}^{ABC}D_t^{\zeta}(R(t)) &= a_6 A(t) - a_5 R(t) - a_1 R(t), \\
{}_{0}^{ABC}D_t^{\zeta}(M(t)) &= a_7 A(t) - a_8 M(t).
\end{aligned} \tag{3.1}$$

with the initial conditions $S(0) \geq 0$, $X(0) \geq 0$, $A(0) \geq 0$, $R(0) \geq 0$, $M(0) \geq 0$ and where ${}_{0}^{ABC}D_t^{\zeta}$ is AB derivative in Caputo sense.

Applying fractional integral, the model can be given as

$$\begin{aligned}
S(t)-h_1(t) &= \frac{1-\zeta}{B(\zeta)}\{a_1-a_2S(t)A(t)-a_3S(t)M(t)+a_4X(t)-a_1S(t)\}\\
&+ \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-y)^{\zeta-1}\{a_1-a_2S(y)A(y)-a_3S(y)M(y)+a_4X(y)-a_1S(y)\}\,dy,\\
X(t)-h_2(t) &= \frac{1-\zeta}{B(\zeta)}\{a_3S(t)M(t)-a_4X(t)-a_1X(t)\}\\
&+ \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-y)^{\zeta-1}\{a_3S(y)M(y)-a_4X(y)-a_1X(y)\}\,dy,\\
A(t)-h_3(t) &= \frac{1-\zeta}{B(\zeta)}\{a_2S(t)A(t)+a_5R(t)-a_6A(t)-a_1A(t)\}\\
&+ \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-y)^{\zeta-1}\{a_2S(y)A(y)+a_5R(y)-a_6A(y)-a_1A(y)\}\,dy,\\
R(t)-h_4(t) &= \frac{1-\zeta}{B(\zeta)}\{a_6A(t)-a_5R(t)-a_1R(t)\}\\
&+ \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-y)^{\zeta-1}\{a_6A(y)-a_5R(y)-a_1R(y)\}\,dy,\\
M(t)-h_5(t) &= \frac{1-\zeta}{B(\zeta)}\{a_7A(t)-a_8M(t)\}\\
&+ \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-y)^{\zeta-1}\{a_7A(y)-a_8M(y)\}\,dy. \qquad (3.2)
\end{aligned}$$

Iteratively, the above can be given as

$$\begin{aligned}
S_0(t) &= h_1(t),\\
X_0(t) &= h_2(t),\\
A_0(t) &= h_3(t),\\
R_0(t) &= h_4(t),\\
M_0(t) &= h_5(t). \qquad (3.3)
\end{aligned}$$

and

$$\begin{aligned}
S_{n+1}(t) &= \frac{1-\zeta}{B(\zeta)}\times\{a_1-a_2S(t)A(t)-a_3S(t)M(t)+a_4X(t)-a_1S(t)\}\\
&+ \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-y)^{\zeta-1}\{a_1-a_2S(y)A(y)-a_3S(y)M(y)+a_4X(y)-a_1S(y)\}\,dy,\\
X_{n+1}(t) &= \frac{1-\zeta}{B(\zeta)}\times\{a_3S(t)M(t)-a_4X(t)-a_1X(t)\}\\
&+ \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-y)^{\zeta-1}\{a_3S(y)M(y)-a_4X(y)-a_1X(y)\}\,dy,\\
A_{n+1}(t) &= \frac{1-\zeta}{B(\zeta)}\times\{a_2S(t)A(t)+a_5R(t)-a_6A(t)-a_1A(t)\}\\
&+ \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-y)^{\zeta-1}\{a_2S(y)A(y)+a_5R(y)-a_6A(y)-a_1A(y)\}\,dy,\\
R_{n+1}(t) &= \frac{1-\zeta}{B(\zeta)}\times\{a_6A(t)-a_5R(t)-a_1R(t)\}\\
&+ \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-y)^{\zeta-1}\{a_6A(y)-a_5R(y)-a_1R(y)\}\,dy,\\
M_{n+1}(t) &= \frac{1-\zeta}{B(\zeta)}\times\{a_7A(t)-a_8M(t)\}\\
&+ \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-y)^{\zeta-1}\{a_7A(y)-a_8M(y)\}\,dy. \qquad (3.4)
\end{aligned}$$

For a large value of n, if we take the limit we hope to get the exact solution of equation as below:

$$\begin{aligned}
\lim_{n\to\infty} S_n(t) &= S(t), \\
\lim_{n\to\infty} X_n(t) &= X(t), \\
\lim_{n\to\infty} A_n(t) &= A(t), \\
\lim_{n\to\infty} R_n(t) &= R(t), \\
\lim_{n\to\infty} M_n(t) &= M(t).
\end{aligned}$$

4. Existence of solution via Picard-Lindelof approach

Let us consider the below operator in order to show existence of solution:

$$\begin{aligned}
g_1(t,S) &= a_1 - a_2 S(t) A(t) - a_3 S(t) M(t) + a_4 X(t) - a_1 S(t), \\
g_2(t,X) &= a_3 S(t) M(t) - a_4 X(t) - a_1 X(t), \\
g_3(t,A) &= a_2 S(t) A(t) + a_5 R(t) - a_6 A(t) - a_1 A(t), \\
g_4(t,R) &= a_6 A(t) - a_5 R(t) - a_1 R(t), \\
g_5(t,M) &= a_7 A(t) - a_8 M(t).
\end{aligned}$$

It is clear that g_1, g_2, g_3, g_4, g_5 are contraction according to the functions S, X, A, R, M respectively. Let

$$\begin{aligned}
K_1 &= \sup_{C[d,e_1]} \|g_1(t,S)\|, \\
K_2 &= \sup_{C[d,e_2]} \|g_2(t,X)\|, \\
K_3 &= \sup_{C[d,e_3]} \|g_3(t,A)\|, \\
K_4 &= \sup_{C[d,e_4]} \|g_4(t,R)\|, \\
K_5 &= \sup_{C[d,e_5]} \|g_5(t,M)\|. \qquad (4.1)
\end{aligned}$$

where

$$\begin{aligned}
C[d,e_1] &= [t-d,t+d] \times [x-e_1,x+e_1] = D \times E_1, \\
C[d,e_2] &= [t-d,t+d] \times [x-e_2,x+e_2] = D \times E_2, \\
C[d,e_3] &= [t-d,t+d] \times [x-e_3,x+e_3] = D \times E_3, \\
C[d,e_4] &= [t-d,t+d] \times [x-e_4,x+e_4] = D \times E_4, \\
C[d,e_5] &= [t-d,t+d] \times [x-e_5,x+e_5] = D \times E_5. \qquad (4.2)
\end{aligned}$$

Now, we apply Banach-fixed point theorem by means of the metric on $C[d,e_i]$ $(i = 1,2,3,4,5)$ induced by the uniform norm:

$$\|f(t)\| = \sup_{t\in[t-d,t+d]} |f(t)|. \qquad (4.3)$$

Taking into consideration Picard's operator

$\Upsilon : C(D,E_1,E_2,E_3,E_4,E_5) \to C(D,E_1,E_2,E_3,E_4,E_5)$

defined by

$$\Upsilon\vartheta(t) = \vartheta_0(t) + \vartheta(t)\frac{1-\zeta}{B(\zeta)} + \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-y)^{\zeta-1} F(y,\vartheta(y))\,dy,$$

where

$$\begin{aligned}
\vartheta(t) &= \begin{bmatrix} S(t) & X(t) & A(t) & R(t) & M(t) \end{bmatrix}^T, \\
\vartheta_0(t) &= \begin{bmatrix} S(0) & X(0) & A(0) & R(0) & M(0) \end{bmatrix}^T, \\
F(t,\vartheta(t)) &= \begin{bmatrix} g_1(t,S(t)) & g_2(t,X(t)) & g_3(t,A(t)) & g_4(t,R(t)) & g_5(t,M(t)) \end{bmatrix}^T. \qquad (4.4)
\end{aligned}$$

Suppose that the problem under examined satisfies

$$\|\vartheta(t)\| \le \max\{e_1, e_2, e_3, e_4, e_5\}. \qquad (4.5)$$

We consider

$$\begin{aligned}
\|\Upsilon\vartheta(t)-\vartheta_0(t)\| &= \left\|\frac{1-\zeta}{B(\zeta)}F(t,\vartheta(t))+\frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-y)^{\zeta-1}F(y,\vartheta(y))\,dy\right\| \\
&\leq \frac{1-\zeta}{B(\zeta)}\|F(t,\vartheta(t))\|+\frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-y)^{\zeta-1}\|F(y,\vartheta(y))\|\,dy \\
&\leq \frac{1-\zeta}{B(\zeta)}K+\frac{\zeta}{B(\zeta)}Ka^{\zeta}
\end{aligned} \tag{4.6}$$

where $K=\max\{K_1,K_2,K_3,K_4,K_5\}$. Let $d<\frac{e}{K}$ and $e=\max\{e_1,e_2,e_3,e_4,e_5\}$. Then we find

$$\|\Upsilon\vartheta(t)-\Upsilon\vartheta_0(t)\|<dK<e.$$

Also, we have

$$\|\Upsilon\vartheta_1-\Upsilon\vartheta_2\|=\sup_{t\in A}|\vartheta_1-\vartheta_2|.$$

Now, we take into consideration

$$\begin{aligned}
\|\Upsilon\vartheta_1-\Upsilon\vartheta_2\| &= \left\|\frac{1-\zeta}{B(\zeta)}\left(F(t,\vartheta_1(t))-F(t,\vartheta_2(t))\right)\right. \\
&+ \left.\frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-y)^{\zeta-1}\left(F(y,\vartheta_1(y))-F(y,\vartheta_2(y))\right)dy\right\| \\
&\leq \frac{1-\zeta}{B(\zeta)}\|F(t,\vartheta_1(t))-F(t,\vartheta_2(t))\| \\
&+ \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-y)^{\zeta-1}\|F(y,\vartheta_1(y))-F(y,\vartheta_2(y))\|\,dy \\
&\leq \frac{1-\zeta}{B(\zeta)}q\|\vartheta_1(t)-\vartheta_2(t)\| \\
&+ \frac{\zeta q}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-y)^{\zeta-1}\|\vartheta_1(y)-\vartheta_2(y)\|\,dy \\
&\leq \left(\frac{1-\zeta}{B(\zeta)}q+\frac{\zeta q a^{\zeta}}{B(\zeta)\Gamma(\zeta)}\right)\|\vartheta_1(t)-\vartheta_2(t)\| \\
&\leq aq\|\vartheta_1(t)-\vartheta_2(t)\|
\end{aligned}$$

with $q<1$. Benefiting from contraction properties of F, we have $aq<1$. So, Υ is contraction and this implies the model has a unique set of solution.

5. Special solution via iteration approach

Because our fractional model is nonlinear, it can be impossible to reach an exact solution. The goal of this part is to show a special solution of the model by applying Sumudu transform with iterative method. At first, we will give Sumudu transform for AB derivative introduced by Atangana and Koca [27].

Theorem 5.1. *[27]Let $\zeta\in[0,1]$, $b>a$ and $f\in H^1(a,b)$. The Sumudu transform of AB derivative in the Caputo sense is given as*

$$ST\left\{{}_0^{ABC}D_t^{\zeta}(f(t))\right\}=\frac{B(\zeta)}{1-\zeta}\left(\zeta\Gamma(\zeta+1)E_{\zeta}\left(-\frac{1}{1-\zeta}k^{\zeta}\right)\right)\times ST(f(t)-f(0)).$$

In order to solve the model (3.1), the Sumudu transform of AB derivative is applied:

$$\begin{aligned}
&\frac{B(\zeta)}{1-\zeta}\left(\zeta\Gamma(\zeta+1)E_{\zeta}\left(-\frac{1}{1-\zeta}k^{\zeta}\right)\right)ST\left(S(t)-S(0)\right)\\
=\;&ST\left\{a_1-a_2S(t)A(t)-a_3S(t)M(t)+a_4X(t)-a_1S(t)\right\},\\
&\frac{B(\zeta)}{1-\zeta}\left(\zeta\Gamma(\zeta+1)E_{\zeta}\left(-\frac{1}{1-\zeta}k^{\zeta}\right)\right)ST\left(X(t)-X(0)\right)\\
=\;&ST\left\{a_3S(t)M(t)-a_4X(t)-a_1X(t)\right\},\\
&\frac{B(\zeta)}{1-\zeta}\left(\zeta\Gamma(\zeta+1)E_{\zeta}\left(-\frac{1}{1-\zeta}k^{\zeta}\right)\right)ST\left(A(t)-A(0)\right)\\
=\;&ST\left\{a_2S(t)A(t)+a_5R(t)-a_6A(t)-a_1A(t)\right\},\\
&\frac{B(\zeta)}{1-\zeta}\left(\zeta\Gamma(\zeta+1)E_{\zeta}\left(-\frac{1}{1-\zeta}k^{\zeta}\right)\right)ST\left(R(t)-R(0)\right)\\
=\;&ST\left\{a_6A(t)-a_5R(t)-a_1R(t)\right\}\\
&\frac{B(\zeta)}{1-\zeta}\left(\zeta\Gamma(\zeta+1)E_{\zeta}\left(-\frac{1}{1-\zeta}k^{\zeta}\right)\right)ST\left(M(t)-M(0)\right)\\
=\;&ST\left\{a_7A(t)-a_8M(t)\right\}
\end{aligned}\tag{5.1}$$

Rearranging, we have the followings:

$$\begin{aligned}
ST(S(t)) &= S(0)+\overline{\Psi}\times ST\left\{a_1-a_2S(t)A(t)-a_3S(t)M(t)+a_4X(t)-a_1S(t)\right\},\\
ST(X(t)) &= X(0)+\overline{\Psi}\times ST\left\{a_3S(t)M(t)-a_4X(t)-a_1X(t)\right\},\\
ST(A(t)) &= A(0)+\overline{\Psi}\times ST\left\{a_2S(t)A(t)+a_5R(t)-a_6A(t)-a_1A(t)\right\},\\
ST(R(t)) &= R(0)+\overline{\Psi}\times ST\left\{a_6A(t)-a_5R(t)-a_1R(t)\right\},\\
ST(M(t)) &= M(0)+\overline{\Psi}\times ST\left\{a_7A(t)-a_8M(t)\right\},
\end{aligned}$$

where $\overline{\Psi}=\frac{1-\zeta}{B(\zeta)\left(\zeta\Gamma(\zeta+1)E_{\zeta}\left(-\frac{1}{1-\zeta}k^{\zeta}\right)\right)}$. Now, we have the following recursive formula:

$$\begin{aligned}
S_{n+1}(t) &= S_n(0)+ST^{-1}\left\{\overline{\Psi}\times ST\left\{a_1-a_2S(t)A(t)-a_3S(t)M(t)+a_4X(t)-a_1S(t)\right\}\right\},\\
X_{n+1}(t) &= X_n(0)+ST^{-1}\left\{\overline{\Psi}\times ST\left\{a_3S(t)M(t)-a_4X(t)-a_1X(t)\right\}\right\},\\
A_{n+1}(t) &= A_n(0)+ST^{-1}\left\{\overline{\Psi}\times ST\left\{a_2S(t)A(t)+a_5R(t)-a_6A(t)-a_1A(t)\right\}\right\},\\
R_{n+1}(t) &= R_n(0)+ST^{-1}\left\{\overline{\Psi}\times ST\left\{a_6A(t)-a_5R(t)-a_1R(t)\right\}\right\},\\
M_{n+1}(t) &= M_n(0)+ST^{-1}\left\{\overline{\Psi}\times ST\left\{a_7A(t)-a_8M(t)\right\}\right\}.
\end{aligned}\tag{5.2}$$

So, the solution of the Eq.(5.2) is obtained when n tends to infinity

$$\begin{aligned}
S(t) &= \lim_{n\to\infty}S_n(t),\\
X(t) &= \lim_{n\to\infty}X_n(t),\\
A(t) &= \lim_{n\to\infty}A_n(t),\\
R(t) &= \lim_{n\to\infty}R_n(t),\\
M(t) &= \lim_{n\to\infty}M_n(t).
\end{aligned}\tag{5.3}$$

6. Stability analysis of iteration method by means of fixed point theory

Suppose that $(F,\|\|)$ is Banach space, $H:F\to F$ is a map, $g_{n+1}=\varphi(H,g_n)$ is a recursive procedure and $K(H)$ is the fixed point set of H with at least one element and g_n which converges to a point $w\in K(H)$. Also assume that $\{x_n\}\subset E$ and $e_n=\|x_{n+1}-\varphi(H,x_n)\|$. The iteration method $g_{n+1}=\varphi(H,g_n)$ is said to be H-stable if $\lim_{n\to\infty}e_n=0$ gives $\lim_{n\to\infty}x_n=w$.

Theorem 6.1. *Let $(F,\|\|)$ be Banach space and H be a self map of F satisfying*

$\left\|H_x\_H_y\right\|\leq K\|x-H_x\|+k\|x-y\|$

for all $x,y\in F$ where $0\leq K, 0\leq k<1$. Assume that H is Picard H-stable [28].

Theorem 6.2. *Let H be a self map given as*

$$\begin{aligned}
H(S_n(t)) &= S_{n+1}(t)=S_n(t)+ST^{-1}\left\{\overline{\Psi}\times ST\left\{\begin{array}{c}a_1-a_2S_n(t)A_n(t)-a_3S_n(t)M_n(t)\\+a_4X_n(t)-a_1S_n(t)\end{array}\right\}\right\},\\
H(X_n(t)) &= X_{n+1}(t)=X_n(t)+ST^{-1}\left\{\overline{\Psi}\times ST\left\{a_3S_n(t)M_n(t)-a_4X_n(t)-a_1X_n(t)\right\}\right\},\\
H(A_n(t)) &= A_{n+1}(t)=A_n(t)+ST^{-1}\left\{\overline{\Psi}\times ST\left\{\begin{array}{c}a_2S_n(t)A_n(t)+a_5R_n(t)\\-a_6A_n(t)-a_1A_n(t)\end{array}\right\}\right\},\\
H(R_n(t)) &= R_{n+1}(t)=R_n(t)+ST^{-1}\left\{\overline{\Psi}\times ST\left\{a_6A_n(t)-a_5R_n(t)-a_1R_n(t)\right\}\right\},\\
H(M_n(t)) &= M_{n+1}(t)=M_n(t)+ST^{-1}\left\{\overline{\Psi}\times ST\left\{a_7A_n(t)-a_8M_n(t)\right\}\right\}.
\end{aligned}\tag{6.1}$$

Then Eq. (6.1) is H-stable in $L^1(a,b)$ *if*

$$\begin{array}{rcl} 1-a_2(\bar{s}_5+\bar{s}_2)\overline{f}_1(\gamma)-a_3(\bar{s}_9+\bar{s}_2)\overline{f}_2(\gamma)+a_4\overline{f}_3(\gamma)-a_1\overline{f}_4(\gamma) & < & 1, \\ 1+a_3(\bar{s}_1+\bar{s}_{10})\overline{f}_5(\gamma)-a_4\overline{f}_6(\gamma)-a_1\overline{f}_7(\gamma) & < & 1, \\ 1+a_2(\bar{s}_1+\bar{s}_6)\overline{f}_8(\gamma)+a_5\overline{f}_9(\gamma)-a_6\overline{f}_{10}(\gamma)-a_1\overline{f}_{11}(\gamma) & < & 1, \\ 1+a_6\overline{f}_{12}(\gamma)-a_5\overline{f}_{13}(\gamma)-a_1\overline{f}_{14}(\gamma) & < & 1, \\ 1+a_7\overline{f}_{15}(\gamma)-a_8\overline{f}_{16}(\gamma) & < & 1, \end{array}$$

where $\overline{f}_i(\gamma)$ $(i=1,2,...,16)$ *are functions from* $ST^{-1}\left\{\overline{\Psi}\times ST\right\}$.

Proof. At first, we show that H has a fixed point. To do this, for all $(n,m)\in\mathbb{N}\times\mathbb{N}$ we consider the followings:

$$\begin{array}{l} H(S_n(t))-H(S_m(t))=S_n(t)-S_m(t) \\ +ST^{-1}\left\{\overline{\Psi}\times ST\left\{\begin{array}{c} a_1-a_2S_n(t)A_n(t)-a_3S_n(t)M_n(t) \\ +a_4X_n(t)-a_1S_n(t)\end{array}\right\}\right\}, \\ -ST^{-1}\left\{\overline{\Psi}\times ST\left\{\begin{array}{c} a_1-a_2S_m(t)A_m(t)-a_3S_m(t)M_m(t) \\ +a_4X_m(t)-a_1S_m(t)\end{array}\right\}\right\} \end{array} \tag{6.2}$$

Taking norm Eq. (6.2) and using norm properties, we have

$$\begin{array}{l} \|H(S_n(t))-H(S_m(t))\|\le\|S_n(t)-S_m(t)\| \\ +\left\|ST^{-1}\left\{\overline{\Psi}\times ST\left\{\begin{array}{c} -a_2(A_n(S_n-S_m)+S_m(A_n-A_m))-a_3(M_n(S_n-S_m) \\ +S_m(M_n-M_m))+a_4(X_n-X_m)-a_1(S_n-S_m)\end{array}\right\}\right\}\right\| \end{array}$$

and

$$\begin{array}{rcl} \|H(S_n(t))-H(S_m(t))\| & \le & \|S_n(t)-S_m(t)\| \\ & + & \left\|ST^{-1}\left\{\overline{\Psi}\times ST\left\{-a_2(A_n(S_n-S_m)+S_m(A_n-A_m))\right\}\right\}\right\| \\ & + & \left\|ST^{-1}\left\{\overline{\Psi}\times ST\left\{-a_3(M_n(S_n-S_m)+S_m(M_n-M_m))\right\}\right\}\right\| \\ & + & \left\|ST^{-1}\left\{\overline{\Psi}\times ST\left\{a_4(X_n-X_m)\right\}\right\}\right\| \\ & + & \left\|ST^{-1}\left\{\overline{\Psi}\times ST\left\{-a_1(S_n-S_m)\right\}\right\}\right\| \end{array} \tag{6.3}$$

Because the solutions play same role, we suppose that

$$\begin{array}{rcl} \|S_n(t)-S_m(t)\| & \cong & \|X_n(t)-X_m(t)\|, \\ \|S_n(t)-S_m(t)\| & \cong & \|A_n(t)-A_m(t)\|, \\ \|S_n(t)-S_m(t)\| & \cong & \|R_n(t)-R_m(t)\|, \\ \|S_n(t)-S_m(t)\| & \cong & \|M_n(t)-M_m(t)\|. \end{array}$$

Writing these in Eq. (6.3), we find

$$\begin{array}{rcl} \|H(S_n(t))-H(S_m(t))\| & \le & \|S_n(t)-S_m(t)\| \\ & + & ST^{-1}\left\{\overline{\Psi}\times ST\left\{\|-a_2(A_n(S_n-S_m)+S_m(S_n-S_m))\|\right\}\right\} \\ & + & ST^{-1}\left\{\overline{\Psi}\times ST\left\{\|-a_3(M_n(S_n-S_m)+S_m(S_n-S_m))\|\right\}\right\} \\ & + & ST^{-1}\left\{\overline{\Psi}\times ST\left\{\|a_4(S_n-S_m)\|\right\}\right\} \\ & + & ST^{-1}\left\{\overline{\Psi}\times ST\left\{\|-a_1(S_n-S_m)\|\right\}\right\} \end{array} \tag{6.4}$$

Because S_n, X_n, A_n, R_n and M_n are bounded, there are different constants $\bar{s}_1,\bar{s}_2,\bar{s}_3,\bar{s}_4,\bar{s}_5,\bar{s}_6,\bar{s}_7,\bar{s}_8,\bar{s}_9,\bar{s}_{10}$ such that

$$\begin{array}{rcl} \|S_n(t)\| & \le & \bar{s}_1, \|S_m(t)\|\le\bar{s}_2, \\ \|X_n(t)\| & \le & \bar{s}_3, \|X_m(t)\|\le\bar{s}_4, \\ \|A_n(t)\| & \le & \bar{s}_5, \|A_m(t)\|\le\bar{s}_6, \\ \|R_n(t)\| & \le & \bar{s}_7, \|R_m(t)\|\le\bar{s}_8, \\ \|M_n(t)\| & \le & \bar{s}_9, \|M_m(t)\|\le\bar{s}_{10}. \end{array} \tag{6.5}$$

□

Taking into consideration Eqs.(6.4) and (6.5), we find

$$\begin{array}{rcl} \|H(S_n(t))-H(S_m(t))\| & \le & \|S_n(t)-S_m(t)\| \\ & \times & \left\{\begin{array}{c} 1-a_2(\bar{s}_5+\bar{s}_2)\overline{f}_1(\gamma)-a_3(\bar{s}_9+\bar{s}_2)\overline{f}_2(\gamma) \\ +a_4\overline{f}_3(\gamma)-a_1\overline{f}_4(\gamma)\end{array}\right\} \end{array}$$

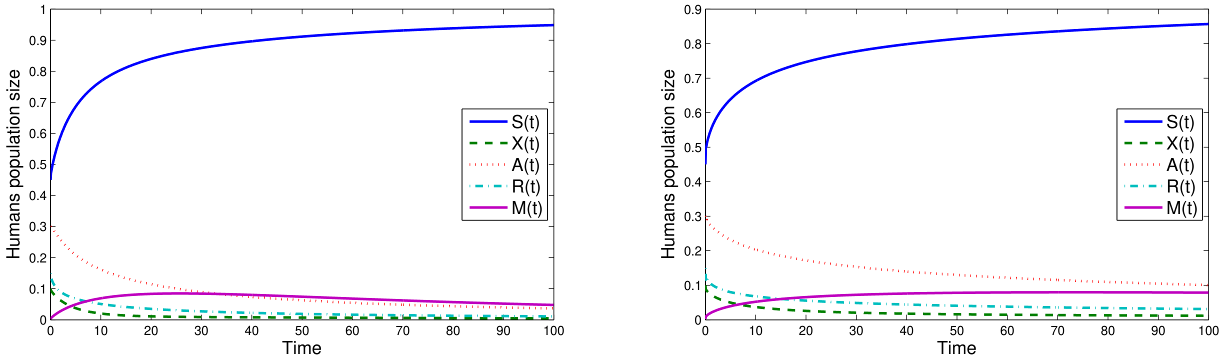

Figure 7.1: Numerical simulations for the Eq. (3.1) at $\zeta = 0.9$ and $\zeta = 0.7$, respectively.

where $\overline{f}_1(\gamma), \overline{f}_2(\gamma), \overline{f}_3(\gamma), \overline{f}_4(\gamma)$ are functions from $ST^{-1}\left\{\overline{\Psi} \times ST\right\}$. Similarly, we obtain

$$\begin{aligned} \|H(X_n(t)) - H(X_m(t))\| &\leq \|X_n(t) - X_m(t)\| \\ &\times \left\{1 + a_3(\overline{s}_1 + \overline{s}_{10})\overline{f}_5(\gamma) - a_4\overline{f}_6(\gamma) - a_1\overline{f}_7(\gamma)\right\}, \end{aligned}$$

$$\begin{aligned} \|H(A_n(t)) - H(A_m(t))\| &\leq \|A_n(t) - A_m(t)\| \\ &\times \left\{\begin{array}{c} 1 + a_2(\overline{s}_1 + \overline{s}_6)\overline{f}_8(\gamma) + a_5\overline{f}_9(\gamma) \\ -a_6\overline{f}_{10}(\gamma) - a_1\overline{f}_{11}(\gamma) \end{array}\right\}, \end{aligned}$$

$$\begin{aligned} \|H(R_n(t)) - H(R_m(t))\| &\leq \|R_n(t) - R_m(t)\| \\ &\times \left\{1 + a_6\overline{f}_{12}(\gamma) - a_5\overline{f}_{13}(\gamma) - a_1\overline{f}_{14}(\gamma)\right\} \end{aligned}$$

and

$$\begin{aligned} \|H(M_n(t)) - H(M_m(t))\| &\leq \|M_n(t) - M_m(t)\| \\ &\times \left\{1 + a_7\overline{f}_{15}(\gamma) - a_8\overline{f}_{16}(\gamma)\right\} \end{aligned}$$

for

$$\begin{aligned} 1 - a_2(\overline{s}_5 + \overline{s}_2)\overline{f}_1(\gamma) - a_3(\overline{s}_9 + \overline{s}_2)\overline{f}_2(\gamma) + a_4\overline{f}_3(\gamma) - a_1\overline{f}_4(\gamma) &< 1, \\ 1 + a_3(\overline{s}_1 + \overline{s}_{10})\overline{f}_5(\gamma) - a_4\overline{f}_6(\gamma) - a_1\overline{f}_7(\gamma) &< 1, \\ 1 + a_2(\overline{s}_1 + \overline{s}_6)\overline{f}_8(\gamma) + a_5\overline{f}_9(\gamma) - a_6\overline{f}_{10}(\gamma) - a_1\overline{f}_{11}(\gamma) &< 1, \\ 1 + a_6\overline{f}_{12}(\gamma) - a_5\overline{f}_{13}(\gamma) - a_1\overline{f}_{14}(\gamma) &< 1, \\ 1 + a_7\overline{f}_{15}(\gamma) - a_8\overline{f}_{16}(\gamma) &< 1. \end{aligned}$$

Then H has a fixed point. Let

$k = (0,0,0,0,0)$,

$$K = \left\{\begin{array}{c} 1 - a_2(\overline{s}_5 + \overline{s}_2)\overline{f}_1(\gamma) - a_3(\overline{s}_9 + \overline{s}_2)\overline{f}_2(\gamma) + a_4\overline{f}_3(\gamma) - a_1\overline{f}_4(\gamma), \\ 1 + a_3(\overline{s}_1 + \overline{s}_{10})\overline{f}_5(\gamma) - a_4\overline{f}_6(\gamma) - a_1\overline{f}_7(\gamma), \\ 1 + a_2(\overline{s}_1 + \overline{s}_6)\overline{f}_8(\gamma) + a_5\overline{f}_9(\gamma) - a_6\overline{f}_{10}(\gamma) - a_1\overline{f}_{11}(\gamma), \\ 1 + a_6\overline{f}_{12}(\gamma) - a_5\overline{f}_{13}(\gamma) - a_1\overline{f}_{14}(\gamma), \\ 1 + a_7\overline{f}_{15}(\gamma) - a_8\overline{f}_{16}(\gamma) \end{array}\right\}.$$

The above equations show Theorem 6.1 holds for the mapping H. Then H is Picard H-stable.

7. Numerical simulations and discussion

In this part, we show numerical simulations which belong to the model (3.1) with the numerical schemes given by [30]. The parameters utilized in these simulations are $a_1 = 0.25$, $a_2 = 0.3$, $a_3 = 0.02$, $a_4 = 0.001$, $a_5 = 0.8$, $a_6 = 0.3$, $a_7 = 0.05$, $a_8 = 0.05$ given in [26]. In Figs. 7.1 and 7.2 , different fractional order fluctuations of $S(t), X(t), A(t), R(t), M(t)$ components can be observed. Figure 7.1 shows that as the fractional order declines, the $S(t)$ component decreases while the $X(t), A(t), R(t)$ components increase. Figure 7.2 displays the the behavioral changes of model components according to varying fractional order. As fractional order falls , $A(t)$ increases and $M(t)$ decreases, which means as the awareness program becomes widespread, the population of heavy drinkers goes into a decline. Therefore, awareness programs driven by media can be an effective method in case of dealing with alcoholism problem.

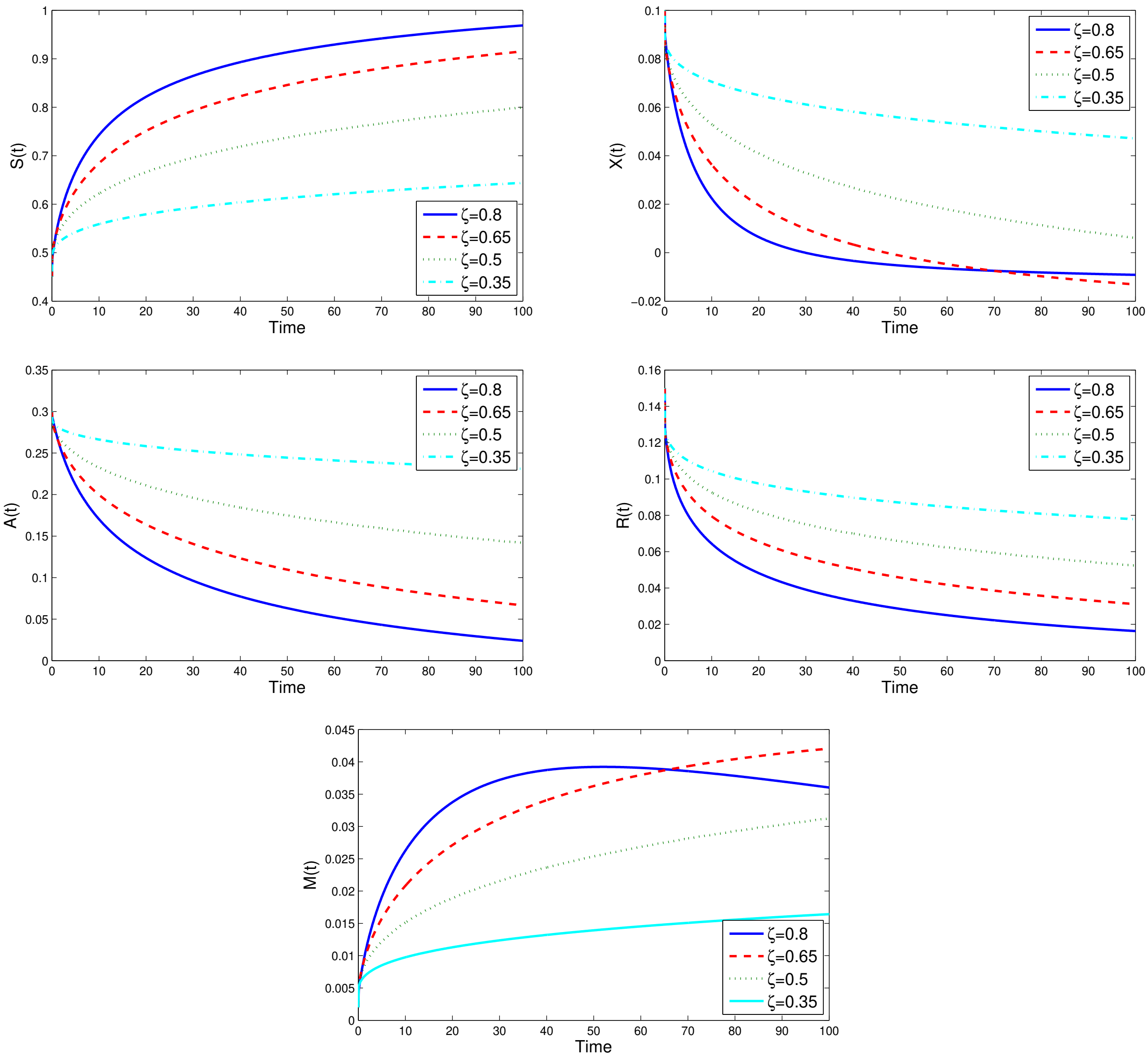

Figure 7.2: The behavior of the fractional model components for distinct values ζ.

8. Concluding remarks

In recent times, Atangana with collaboration Baleanu have presented a new fractional order derivative with Mittag-Leffler kernel. Firstly, the model based on [26] is expanded with AB derivative in order to see further applications of this fractional derivative and to observe alcoholism better. Then, the solutions of this extended model is proved to be unique by using Banach fixed method and special solution was also derived by means of Sumudu transform. Additionally, stability analysis is confirmed via the Picard H-stable approach. In order to understand the effect of media along with fractional order, another set of numerical graphics are given for this fractional model. In these graphics, it is observed that as awareness programs driven by media increases, the number of heavy drinkers decreases. We believe that our results are very helpful in describing and preventing the drinking problem with awareness programs driven by media for both our environment and the world.

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