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Soft Quasilinear Operators

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Abstract
In this paper, we have introduced a new concept, called soft quasilinear operator over soft quasilinear spaces which extends the notion of quasilinear operator. Also, we studied some properties of soft quasilinear operators with illustrating examples. Further, we have defined inverse of a soft quasilinear operator and its some different properties from inverse of soft linear operators are obtained.

Keywords: *Quasilinear space; Soft quasilinear space; Normed quasilinear space; Soft normed quasilinear space; Quasilinear operator; Soft quasilinear operator.*

AMS Subject Classification (2020): *Primary: 06B99 ; Secondary: 03E72; 08A72; 06F99; 04A99; 04B40.*

1. Introduction

In 1986, Aseev [1] introduced the concept of quasilinear spaces, normed quasilinear spaces and quasilinear operators which are generalization of the linear spaces, normed linear spaces and linear operators, respectively. Additionally, in [2], [3], [4], [5], [6], [7], [8], the authors introduced some new concepts and results on quasilinear spaces. Recently, in [9], Yılmaz et all. introduced the notion of inner product quasilinear space and investigated some basic properties of inner product quasilinear spaces. Also, in [10], Levent and Yılmaz deal with bounded quasilinear interval-valued functions and analized the Hahn Banach extension theorem for interval valued functions.

Molodtsov [11] initiated a new theory of linear functional analysis by starting the theory of soft sets. Then, Maji et all. [12], [13] introduced several operations on soft sets. After that, many research works have been done in soft set theory such as [14], [15], [16] . Also, Das and Samanta introduced the idea of soft linear spaces in [17]. Next, Samanta et all. [18], [19] presented some new concepts about the soft set theory such as soft convex set, soft semi norm, soft Minkowski's functionals on a soft linear space and soft pseudo metric.

Based on our studies with related to quasilinear spaces and studies of Samanta and Das, in [21], Bozkurt defined soft quasilinear spaces and soft normed quasilinear spaces which are generalization of the soft linear spaces and soft normed linear spaces, respectively. In the same study, Bozkurt obtained new results about soft quasilinear spaces.

In this paper, we have introduced a concept of soft quasilinear operator over soft quasilinear spaces which extends the notion of quasilinear operator. Also, we studied some properties of soft quasilinear operators with illustrating examples. Further, we have defined inverse of a soft quasilinear operator and its some different properties from inverse of soft linear operators are obtained.

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2. Preliminaries

Firstly, we give the definition of quasilinear space, normed quaslinear space and some its basic properties given by Aseev [1]. After, we give the concepts of soft quasilinear space and soft normed quaslinear space given by [21]. Now, let's continue with the definition of Aseev:

Definition 2.1. [1] A quasilinear space over a field $\mathbb{R}$ is a set Q with a partial order relation "$\preceq$", with the operations of addition $Q \times Q \to Q$ and scalar multiplication $\mathbb{R} \times Q \to Q$ satisfying the following conditions:
$(Q1)$ $q \preceq q$,
$(Q2)$ $q \preceq z$ if $q \preceq w$ and $w \preceq z$,
$(Q3)$ $q = w$ if $q \preceq w$ and $w \preceq q$,
$(Q4)$ $q + w = w + q$,
$(Q5)$ $q + (w + z) = (q + w) + z$,
$(Q6)$ there exists an element $\theta \in Q$ such that $q + \theta = q$,
$(Q7)$ $\alpha \cdot (\beta \cdot q) = (\alpha \cdot \beta) \cdot q$,
$(Q8)$ $\alpha \cdot (q + w) = \alpha \cdot q + \alpha \cdot w$,
$(Q9)$ $1 \cdot q = q$,
$(Q10)$ $0 \cdot q = \theta$,
$(Q11)$ $(\alpha + \beta) \cdot q \preceq \alpha \cdot q + \beta \cdot q$,
$(Q12)$ $q + z \preceq w + v$ if $q \preceq w$ and $z \preceq v$,
$(Q13)$ $\alpha \cdot q \preceq \alpha \cdot w$ if $q \preceq w$,
for every $q, w, z, v \in Q$ and every $\alpha, \beta \in \mathbb{R}$.

If an element q has an inverse, then it is called *regular*. If an element q has no inverse, then it is called *singular*. Also, Q_r express for the set of all regular elements in Q and Q_s imply the sets of all singular elements in Q. Besides, Q_r, Q_d and $Q_s \cup \{0\}$ are subspaces of Q, where Q_r regular subspace of Q, Q_d symmetric subspace of Q and $Q_s \cup \{0\}$ singular subspace of Q [2].

Definition 2.2. [1] Let Q be a quasilinear space. A function $\|.\|_Q : Q \longrightarrow \mathbb{R}$ is named a norm if the following circumstances hold:
$(NQ1)$ $\|q\|_Q > 0$ if $q \neq 0$,
$(NQ2)$ $\|q + w\|_Q \leq \|q\|_Q + \|w\|_Q$,
$(NQ3)$ $\|\alpha \cdot q\|_Q = |\alpha| \cdot \|q\|_Q$,
$(NQ4)$ if $q \preceq w$, then $\|q\|_Q \leq \|w\|_Q$,
$(NQ5)$ if for any $\varepsilon > 0$ there exists an element $q_\varepsilon \in Q$ such that, $q \preceq w + q_\varepsilon$ and $\|q_\varepsilon\|_Q \leq \varepsilon$ then $q \preceq w$ for any elements $q, w \in Q$ and any real number $\alpha \in \mathbb{R}$.

A quasilinear space Q is called normed quasilinear space with a norm defined on it. Let Q be a normed quasilinear space. Then, Hausdorff or norm metric on Q is defined by

$$h_Q(q, w) = \inf \left\{ r \geq 0 : q \preceq w + a_1^r, w \preceq q + a_2^r, \|a_i^r\| \leq r \right\}.$$

Definition 2.3. [1] Let Q and W be quasilinear spaces. Then a quasilinear operator $\lambda : Q \to W$ is a function satisfying
$(QO1)$ $\lambda\left(\alpha \cdot q\right) = \alpha \cdot \lambda\left(q\right)$,
$(QO2)$ $\lambda\left(q + w\right) \preceq \lambda\left(q\right) + \lambda\left(w\right)$,
$(QO3)$ $\lambda\left(q\right) \preceq \lambda\left(w\right)$ if $q \preceq w$ for any $q, w \in Q$ and $\alpha \in \mathbb{R}$.

Definition 2.4. [11] Let U be an universe and E be a set of parameters. Let $P(U)$ denote the power set of U and A be a non-empty subset of E. A pair (F, A) is called a soft set over U, where F is mapping given by $F : A \to P(U)$. A soft set (F, E) over U is said to be absolute soft set denoted by $\widetilde{U}$ if for all $\varepsilon \in E$, $F(\varepsilon) = U$.

Definition 2.5. [20] Let X be a non-empty set and E be a non-empty parameter set. Then a function $\varepsilon : E \to X$ is said to be a soft element of X. A soft element ε of X is said to belongs to a soft set A of X, which is denoted by $\varepsilon \widetilde{\in} A$, if $\varepsilon\left(e\right) \in A\left(e\right), \forall e \in E$.

Now, we will give the notion of soft quasilinear space, soft normed quasilinear space, soft quasi vector and some results related this notions.

Definition 2.6. [21] Let (G, P) be a non-null soft set over a quasilinear space Q. Then (G, P) is called a soft quasilinear space over Q if $G(p)$ is a subquasilinear space of Q for every $p \in Supp(G, P)$.

Definition 2.7. [21] Let (G, P) is a soft quasilinear space of Q. A soft element of Q is said to be a soft quasi vector of (G, P). A soft element of the soft set $(\mathbb{R}, P)$ is said to be a soft scalar.

Definition 2.8. [21] Let (G, P) is a soft quasilinear space of Q over $\mathbb{R}$ and $(F, P) \subseteq (G, P)$ is a soft set over (G, P). Then (F, P) is called a soft subquasilinear space of (G, P) whenever (F, P) is quasilinear space with identical partial ordering and identical operations on Q.

Proposition 2.1. *[21] Let (G, P) be a soft quasilinear space over Q. Then*
a) $0 \cdot q = \Theta$, for all $q \in (G, P)$,
b) $k \cdot \Theta = \Theta$, for all soft scalar k,
c) $(-1) \cdot q = -q$, for all $q \in (G, P)$.

Let Q be a quasilinear space, Q is also our initial uiverse set and P be the non-empty set of parameters. Let $\widetilde{Q}$ be the absolute soft quasilinear space i.e., $G(p) = Q$, $\forall p \in P$, where $(G, P) = \widetilde{Q}$. Let $SQV\left(\widetilde{Q}\right)$ be the collection all soft quasi vectors over $\widetilde{Q}$. We use the notation $\widetilde{q}, \widetilde{w}$ to denote soft quasi vectors of a soft quasilinear space and $\widetilde{\alpha}$ to denote soft real numbers whereas $\overline{\alpha}$ will denote a particular type of soft real numbers such that $\overline{\alpha}(\lambda) = \alpha$, for all $\lambda \in P$.

Theorem 2.1. *[21] The set $SQV\left(\widetilde{Q}\right)$ is a quasilinear space with the relation "$\widetilde{\preceq}$"*

$$\widetilde{q}\widetilde{\preceq}\widetilde{w} \Leftrightarrow \widetilde{q}(\lambda) \preceq \widetilde{w}(\lambda)$$

the sum operation

$$(\widetilde{q} + \widetilde{w})(\lambda) = \widetilde{q}(\lambda) + \widetilde{w}(\lambda)$$

and the soft real-scalar multiplication

$$(\widetilde{\alpha} \cdot \widetilde{q})(\lambda) = \widetilde{\alpha}(\lambda) \cdot q(\lambda)$$

for every $\widetilde{q}, \widetilde{w}$ soft vectors of $SQV\left(\widetilde{Q}\right)$, $\forall \lambda \in P$ and for every soft real numbers $\widetilde{\alpha}$.

Definition 2.9. [21] Let $SQV\left(\widetilde{Q}\right)$ be a soft quasilinear space and $\widetilde{N} \subset SQV\left(\widetilde{Q}\right)$ be a subset. If $\widetilde{N}$ is a soft quasilinear space, then $\widetilde{N}$ is said to be a soft quasilinear subspace of $SQV\left(\widetilde{Q}\right)$ and stated by $SQV\left(\widetilde{N}\right) \subset SQV\left(\widetilde{Q}\right)$.

Definition 2.10. [21] Let $SQV\left(\widetilde{Q}\right)$ be a soft quasilinear space. Then a mapping $\|.\| : SQV\left(\widetilde{Q}\right) \to \mathbb{R}^+(\mathbb{R})$ is said to be a soft norm on the soft quasilinear space $SQV\left(\widetilde{Q}\right)$, if $\|.\|$ satisfies the following conditions:

(SNQ1) $\|\widetilde{q}\| \widetilde{>} \widetilde{0}$ if $\widetilde{q} \neq \widetilde{\theta}$ for all $\widetilde{q} \in SQV\left(\widetilde{Q}\right)$,

(SNQ2) $\|\widetilde{q} + \widetilde{w}\| \widetilde{\leq} \|\widetilde{q}\| + \|\widetilde{w}\|$ for all $\widetilde{q}, \widetilde{w} \in SQV\left(\widetilde{Q}\right)$,

(SNQ3) $\|\widetilde{\alpha} \cdot \widetilde{q}\| = |\widetilde{\alpha}| \|\widetilde{q}\|$ for every $\widetilde{q} \in SQV\left(\widetilde{Q}\right)$ and for every soft scalar $\widetilde{\alpha}$,

(SNQ4) if $\widetilde{q}\widetilde{\preceq}\widetilde{w}$, then $\|\widetilde{q}\| \widetilde{\leq} \|\widetilde{w}\|$ for all $\widetilde{q}, \widetilde{w} \in SQV\left(\widetilde{Q}\right)$,

(SNQ5) if for any $\widetilde{\epsilon} \widetilde{>} \widetilde{0}$ there exists an element $\widetilde{z}$ $SQV\left(\widetilde{Q}\right)$ such that $\widetilde{q}\widetilde{\preceq}\widetilde{w} + \widetilde{z}$ and $\|\widetilde{z}\| \widetilde{\leq} \widetilde{\epsilon}$ then $\widetilde{q}\widetilde{\preceq}\widetilde{w}$.

Definition 2.11. [21] Let $\left(\widetilde{Q}, \|.\|\right)$ be a soft normed quasilinear space. Soft Hausdorff metric or soft norm metric on $\widetilde{Q}$ is defined by equality

$$h_{\widetilde{Q}}(\widetilde{q}, \widetilde{w}) = \inf\left\{\widetilde{r} \geq \widetilde{0} : \widetilde{q}\widetilde{\preceq}\widetilde{w} + \widetilde{a_1^r}, \widetilde{w}\widetilde{\preceq}\widetilde{q} + \widetilde{a_2^r}, \left\|\widetilde{a_i^r}\right\| \widetilde{\leq} \widetilde{r}\right\}.$$

Same as the definition of Hausdorff metric on normed quasilinear space, we obtain $\widetilde{q}\widetilde{\preceq}\widetilde{w} + (\widetilde{q} - \widetilde{w})$ and $\widetilde{w}\widetilde{\preceq}\widetilde{q} + (\widetilde{w} - \widetilde{q})$ for every $\widetilde{q}, \widetilde{w} \in SQV\left(\widetilde{Q}\right)$.

$$h_{\widetilde{Q}}(\widetilde{q}, \widetilde{w}) \widetilde{\leq} \|\widetilde{q} - \widetilde{w}\|.$$

Here, we should note that $h_{\widetilde{Q}}(\widetilde{q},\widetilde{w})$ may not equal to $\|\widetilde{q}-\widetilde{w}\|$ since $\widetilde{Q}$ is a soft quasilinear space.

Definition 2.12. [21] A sequence of soft elements $\{\widetilde{q}_n\}$ in a soft normed quasilinear space $\left(\widetilde{Q},\|.\|\right)$ is said to be converges to a soft element $\widetilde{q}_0$ if $h_{\widetilde{Q}}(\widetilde{q}_n,\widetilde{q}_0)\to\widetilde{0}$ as $n\to\infty$.

Definition 2.13. [21] A sequence of soft elements $\{\widetilde{q}_n\}$ in a soft normed quasilinear space $\left(\widetilde{Q},\|.\|\right)$ is said to be a Cauchy sequence if corresponding to every $\widetilde{\epsilon}\widetilde{>}\widetilde{0}$, $\exists m\in\mathbb{N}$ such that $h_{\widetilde{Q}}(\widetilde{q}_i,\widetilde{q}_j)\widetilde{\leq}\widetilde{\epsilon}$ for all $i,j>m$ i.e. $h_{\widetilde{Q}}(\widetilde{q}_i,\widetilde{q}_j)\to\widetilde{0}$ as $i,j\to\infty$.

3. Main Results

Let Q and W be two soft quasilinear spaces over field $\mathbb{R}$, P be a nonempty set of parameters, $\widetilde{Q}$ and $\widetilde{W}$ be the corresponding absolute soft quasilinear spaces i.e. $\widetilde{Q}(\lambda)=Q$ and $\widetilde{W}(\lambda)=W$ for every $\lambda\in P$. We use the notations $\widetilde{q},\widetilde{w}$ and $\widetilde{z}$ to denote soft quasi vectors of a soft quasilinear space.

Definition 3.1. Let $\chi:SQV(\widetilde{Q})\to SQV(\widetilde{W})$ be an operator. Then χ is said to be soft quasilinear if
(SQO1) $\chi(\widetilde{q}+\widetilde{w})\widetilde{\preceq}\chi(\widetilde{q})+\chi(\widetilde{w})$,
(SQO2) $\chi(\widetilde{c}\cdot\widetilde{q})=\widetilde{c}\cdot\chi(\widetilde{q})$ for every soft scalar $\widetilde{c}$,
(SQO3) $\widetilde{q}\widetilde{\preceq}\widetilde{w}\Rightarrow\chi(\widetilde{q})\widetilde{\preceq}\chi(\widetilde{w})$,
for every $\widetilde{q},\widetilde{w}\in SQV(\widetilde{Q})$.

Example 3.1. If $\widetilde{Q}$ be a soft normed quasilinear space. Then the identity operator $\chi:SQV(\widetilde{Q})\to SQV(\widetilde{Q})$ such that $\chi(\widetilde{q})=\widetilde{q}$, for every soft quasi element $\widetilde{q}\in\widetilde{Q}$, is a soft quasilinear operator.

Example 3.2. Let $\mathbb{R}(P)$ be the set of all soft real numbers defined over the parameter set P and consider the absolute soft quasi set generated by $\Omega_C(\mathbb{R})$ i.e. $\widetilde{\Omega_C(\mathbb{R})}(\lambda)=\Omega_C(\mathbb{R})$. Let an operator

$$\begin{array}{rcl}\chi&:&\mathbb{R}(P)\to SQV(\widetilde{\Omega_C(\mathbb{R})})\\ \widetilde{r}&\to&\chi(\widetilde{r})=\widetilde{r}\cdot\widetilde{[1,2]}\end{array}$$

for a soft quasi vector $\widetilde{[1,2]}\in\widetilde{\Omega_C(\mathbb{R})}$. For every $\widetilde{r},\widetilde{m}\in\mathbb{R}(P)$, we have

$$\begin{array}{rcl}\chi(\widetilde{r}+\widetilde{m})&=&(\widetilde{r}+\widetilde{m})\cdot\widetilde{[1,2]}\\ &&\widetilde{\preceq}\widetilde{r}\cdot\widetilde{[1,2]}+\widetilde{m}\cdot\widetilde{[1,2]}\\ &=&\chi(\widetilde{r})+\chi(\widetilde{m}).\end{array}$$

For every soft scalar $\widetilde{c}$, we get

$$\begin{array}{rcl}\chi(\widetilde{c}\widetilde{r})&=&\widetilde{c}\widetilde{r}\cdot\widetilde{[1,2]}\\ &=&\widetilde{c}\left(\widetilde{r}\cdot\widetilde{[1,2]}\right)\\ &=&\widetilde{c}\cdot\chi(\widetilde{r}).\end{array}$$

For every $\widetilde{r},\widetilde{m}\in\mathbb{R}(P)$, if $\widetilde{r}=\widetilde{m}$ then $\widetilde{r}\cdot\widetilde{[1,2]}=\widetilde{m}\cdot\widetilde{[1,2]}$ since $\mathbb{R}(P)$ is a soft quasilinear space with relation "=". So, we obtain $\chi(\widetilde{r})\widetilde{\preceq}\chi(\widetilde{m})$.

Definition 3.2. The operator $\chi:SQV(\widetilde{Q})\to SQV(\widetilde{W})$ is said to be continuous at $\widetilde{q}\in\widetilde{Q}$ if for every sequence $\{\widetilde{q}_n\}$ of soft element of $\widetilde{Q}$ with $\widetilde{q}_n\to\widetilde{q}$ as $n\to\infty$, we have $\chi(\widetilde{q}_n)\to\chi(\widetilde{q})$ as $n\to\infty$ i.e., $h(\widetilde{q}_n,\widetilde{q})\to\overline{0}$ as $n\to\infty$ implies $h(\chi(\widetilde{q}_n),\chi(\widetilde{q}))\to\overline{0}$ as $n\to\infty$. If χ is continuous at every soft quasi element of $\widetilde{Q}$, then χ is said to be a continuous quasilinear operator.

Example 3.3. The identity operator given in Example 3.1 is continuous since $h(\chi(\widetilde{q}_n),\chi(\widetilde{q}))=h(\widetilde{q}_n,\widetilde{q})\to\overline{0}$ as $n\to\infty$.

Theorem 3.1. *Let $\widetilde{Q}$ and $\widetilde{W}$ be two soft normed quasilinear spaces. If $\chi : SQV(\widetilde{Q}) \to SQV(\widetilde{W})$ be a soft quasilinear operator, then $\chi\left(\sum\limits_{k=1}^{n} \widetilde{c_k} \cdot \widetilde{q_k}\right) \widetilde{\leq} \sum\limits_{k=1}^{n} \widetilde{c_k}\chi\left(\widetilde{q_k}\right)$, $\widetilde{c_k}$ are soft scalars.*

Proof. For $n = 1$ the inequality is satisfied. We consider that the conclusion is true for $(n-1)$ i.e.,

$$\chi\left(\sum_{k=1}^{n-1} \widetilde{c_k} \cdot \widetilde{q_k}\right) \widetilde{\leq} \sum_{k=1}^{n-1} \widetilde{c_k}\chi\left(\widetilde{q_k}\right), \widetilde{c_k}.$$

From here,

$$\begin{aligned}
\chi\left(\sum_{k=1}^{n} \widetilde{c_k} \cdot \widetilde{q_k}\right) &= \chi\left(\sum_{k=1}^{n-1} \widetilde{c_k} \cdot \widetilde{q_k} + \widetilde{c_n} \cdot \widetilde{q_n}\right) \\
&\widetilde{\leq} \chi\left(\sum_{k=1}^{n-1} \widetilde{c_k} \cdot \widetilde{q_k}\right) + \chi\left(\widetilde{c_n} \cdot \widetilde{q_n}\right) \\
&= \sum_{k=1}^{n-1} \widetilde{c_k} \cdot \chi\left(\widetilde{q_k}\right) + \widetilde{c_n} \cdot \chi\left(\widetilde{q_n}\right) \\
&= \sum_{k=1}^{n} \widetilde{c_k} \cdot \chi\left(\widetilde{q_k}\right).
\end{aligned}$$

□

Definition 3.3. Let $\chi : SQV(\widetilde{Q}) \to SQV(\widetilde{W})$ be a soft quasilinear operator, where $\widetilde{Q}$ and $\widetilde{W}$ are soft normed quasilinear spaces. The operator χ is called bounded if there exists some positive soft real number $\widetilde{N}$ such that for all $\widetilde{q} \in \widetilde{Q}$, $\|\chi\left(\widetilde{q}\right)\| \widetilde{\leq} \widetilde{N} \|\widetilde{q}\|$.

Theorem 3.2. *Let $\chi : SQV(\widetilde{Q}) \to SQV(\widetilde{W})$ be a soft quasilinear operator, where $\widetilde{Q}$ and $\widetilde{W}$ are soft normed quasilinear spaces. If χ is bounded then χ is continuous.*

Proof. Assume that χ is bounded. Then there exists a positive soft real number $\widetilde{N}$ such that for all $\widetilde{q} \in \widetilde{Q}$, $\|\chi\left(\widetilde{q}\right)\| \widetilde{\leq} \widetilde{N}$ $\|\widetilde{q}\|$. Let $\widetilde{q_n} \to \widetilde{q}$ as $n \to \infty$ i.e., for every $\epsilon > 0$ there exists a $n_0 \in \mathbb{N}$ such that

$$\widetilde{q_n} \widetilde{\leq} \widetilde{q} + \widetilde{q^{\epsilon}_{1n}}, \ \widetilde{q} \widetilde{\leq} \widetilde{q_n} + \widetilde{q^{\epsilon}_{2n}}, \ \left\|\widetilde{q^{\epsilon}_{in}}\right\| \widetilde{\leq} \frac{\widetilde{\epsilon}}{\widetilde{N}}$$

for all $n \geq n_0$. Then

$$\chi\left(\widetilde{q_n}\right) \widetilde{\leq} \chi\left(\widetilde{q}\right) + \chi\left(\widetilde{q^{\epsilon}_{1n}}\right), \ \chi\left(\widetilde{q}\right) \widetilde{\leq} \chi\left(\widetilde{q_n}\right) + \chi\left(\widetilde{q^{\epsilon}_{2n}}\right)$$

and

$$\left\|\chi\left(\widetilde{q^{\epsilon}_{in}}\right)\right\| \widetilde{\leq} \widetilde{N} \left\|\widetilde{q^{\epsilon}_{in}}\right\| \leq \widetilde{\epsilon}.$$

Therefore, $\chi\left(\widetilde{q_n}\right) \to \chi\left(\widetilde{q}\right)$ as $n \to \infty$. So χ is continuous at $\widetilde{q} \in \widetilde{Q}$. Since $\widetilde{q} \in \widetilde{Q}$ is arbitrary, χ is continuous. □

Theorem 3.3. *Suppose a soft quasilinear operator $\chi : SQV(\widetilde{Q}) \to SQV(\widetilde{W})$, where $\widetilde{Q}$ and $\widetilde{W}$ are soft normed quasilinear spaces, satisfies the condition: for $\mu \in Q$ and $\lambda \in P$,*

$$\left\{\chi\left(\widetilde{q}\right)(\lambda) : \widetilde{q} \widetilde{\in} \widetilde{Q} \text{ such that } \widetilde{q}(\lambda) = \mu\right\}$$

is a singleton set. Then for each $\lambda \in P$, $\chi_\lambda : Q \to W$ defined by $\chi_\lambda(\mu) = \chi\left(\widetilde{q}\right)(\lambda)$, for all $\mu \in Q$, $\widetilde{q} \widetilde{\in} \widetilde{Q}$ such that $\widetilde{q}(\lambda) = \mu$, is a quasilinear operator.

Proof. From the above condition, χ_λ is well defined for every $\lambda \in P$. Since χ is a soft quasilinear operator, χ_λ satisfies soft quasilinear operator conditions for $\forall \lambda \in P$:

For every $\mu, \nu \in Q$ and soft scalar $\widetilde{c}$, we get

$$\chi_\lambda(\mu+\nu) = \chi\left(\widetilde{q}+\widetilde{q^\imath}\right)(\lambda) \widetilde{\leq} \chi\left(\widetilde{q}\right)(\lambda) + \chi\left(\widetilde{q^\imath}\right)(\lambda) = \chi_\lambda(\mu) + \chi_\lambda(\nu),$$

$$\chi_\lambda(\widetilde{c}\cdot\mu) = \chi\left(\widetilde{c}\cdot\widetilde{q}\right)(\lambda) = \widetilde{c}\cdot\chi\left(\widetilde{q}\right)(\lambda) = \widetilde{c}\cdot\chi_\lambda(\mu),$$

$$\mu \leq \nu \Rightarrow \mu = \widetilde{q}(\lambda) \widetilde{\leq} \widetilde{q^\imath}(\lambda) = \nu \Rightarrow \chi\left(\widetilde{q}\right)(\lambda) \widetilde{\leq} \chi\left(\widetilde{q^\imath}\right)(\lambda) \Rightarrow \chi_\lambda(\mu) \leq \chi_\lambda(\nu).$$

Therefore, the soft quasilinear operator χ satisfying above condition gives a parametrized family of crisp quasilinear operators. □

Theorem 3.4. *Let $\{\chi_\lambda : Q \to W, \lambda \in P\}$ be a family of crisp quasilinear operators from quasilinear space Q to the quasilinear space W. Then there exists a soft quasilinear operator $\chi : SQV(\widetilde{Q}) \to SQV(\widetilde{W})$, defined by $\chi\left(\widetilde{q}\right)(\lambda) = \chi_\lambda(\mu)$ if $\widetilde{q}(\lambda) = \mu$ and $\lambda \in P$; which satisfies Theorem 3.3 and $\chi(\lambda) = \chi_\lambda$ for every $\lambda \in P$.*

Proof. Let $\widetilde{q}\widetilde{\in}\widetilde{Q}$ be an arbitrary soft quasi element and $\chi : SQV(\widetilde{Q}) \to SQV(\widetilde{W})$, by $\chi\left(\widetilde{q}\right)(\lambda) = \chi_\lambda(\mu)$ if $\widetilde{q}(\lambda) = \mu$ for every $\lambda \in P$. Also, $\widetilde{q^\imath}\widetilde{\in}\widetilde{Q}$ be any soft quasi element, $\lambda \in P$ and $\widetilde{q^\imath}(\lambda) = \nu$. Then, we get

$$\begin{aligned} \chi\left(\widetilde{q}+\widetilde{q^\imath}\right)(\lambda) &= \chi\left(\widetilde{q}(\lambda)+\widetilde{q^\imath}(\lambda)\right) \\ &= \chi_\lambda(\mu+\nu) \\ &\widetilde{\leq} \chi_\lambda(\mu)+\chi_\lambda(\nu) \\ &= \chi\left(\widetilde{q}\right)(\lambda)+\chi\left(\widetilde{q^\imath}\right)(\lambda). \end{aligned}$$

For every soft scalar $\widetilde{c}$, we obtain

$$\begin{aligned} \chi\left(\widetilde{c}\cdot\widetilde{q}\right)(\lambda) &= \chi\left(\widetilde{c}(\lambda)\cdot\widetilde{q}(\lambda)\right) \\ &= \chi_\lambda(\widetilde{c}(\lambda)\cdot\mu) \\ &= \widetilde{c}(\lambda)\cdot\chi_\lambda(\mu) \\ &= \widetilde{c}(\lambda)\cdot\chi\left(\widetilde{q}\right)(\lambda). \end{aligned}$$

Let us consider $\widetilde{q}\widetilde{\leq}\widetilde{q^\imath}$ such that $\widetilde{q}(\lambda) = \mu$ and $\widetilde{q^\imath}(\lambda) = \nu$ for arbitrary $\widetilde{q}, \widetilde{q^\imath}\widetilde{\in}\widetilde{Q}$ and arbitrary $\lambda \in P$. Then, we have $\widetilde{q}(\lambda) \widetilde{\leq} \widetilde{q^\imath}(\lambda)$ for $\lambda \in P$. From here, we get $\chi_\lambda(\mu)\widetilde{\leq}\chi_\lambda(\nu)$ since χ_λ is a soft quasilinear operator. So, we have $\chi\left(\widetilde{q}\right)(\lambda) \widetilde{\leq} \chi\left(\widetilde{q^\imath}\right)(\lambda)$. Therefore, $\chi : SQV(\widetilde{Q}) \to SQV(\widetilde{W})$ is a soft quasilinear operator. □

Lemma 3.1. *Let $\left(\widetilde{Q}, \|.\|, P\right)$ be a soft normed quasilinear space and a soft quasi norm $\|.\|$ satisfies the condition:*

$$\text{For } \mu \in Q \text{ and } \lambda \in P,\ \left\{\|\widetilde{q}\|(\lambda) : \widetilde{q}(\lambda) = \mu\right\} \text{ is a singleton set.}$$

Then for every $\lambda \in P$, $\|.\|_\lambda : Q \to \mathbb{R}^+$ defined by $\|\mu\|_\lambda = \|\widetilde{q}\|(\lambda)$, for every $\mu \in Q$ and $\widetilde{q}\widetilde{\in}\widetilde{Q}$ such that $\widetilde{q}(\lambda) = \mu$, is a quasi norm on Q.

Proof. Let $\|.\|_\lambda : Q \to \mathbb{R}^+$ defined by $\|\mu\|_\lambda = \|\widetilde{q}\|(\lambda)$, for every $\mu \in Q$, $\lambda \in P$ and $\widetilde{q}\widetilde{\in}\widetilde{Q}$ such that $\widetilde{q}(\lambda) = \mu$. For every $\mu \in Q$, $\|\mu\|_\lambda = \|\widetilde{q}\|(\lambda) \widetilde{\geq} \overline{0}$. If $\|\mu\|_\lambda = 0$, then $\|\widetilde{q}\|(\lambda) = \|\widetilde{q}(\lambda)\| = \theta = \mu$. For every soft scalar $\widetilde{c}$, we obtain $\|\widetilde{c}\cdot\mu\|_\lambda = \|\widetilde{c}\cdot\widetilde{q}\|(\lambda) = \widetilde{c}\cdot\|\widetilde{q}\|(\lambda) = \widetilde{c}\cdot\|\mu\|_\lambda$. Also, $\widetilde{q^\imath}\widetilde{\in}\widetilde{Q}$ be any soft quasi element, $\lambda \in P$ and $\widetilde{q^\imath}(\lambda) = \nu$. Then, for every $\mu, \nu \in Q$, we get $\|\mu+\nu\|_\lambda = \left\|\widetilde{q}+\widetilde{q^\imath}\right\|(\lambda) = \left\|\widetilde{q}(\lambda)+\widetilde{q^\imath}(\lambda)\right\| \widetilde{\leq} \|\widetilde{q}(\lambda)\| + \left\|\widetilde{q^\imath}(\lambda)\right\| = \|\widetilde{q}\|(\lambda) + \left\|\widetilde{q^\imath}\right\|(\lambda) = \|\mu\|_\lambda + \|\nu\|_\lambda$. If $\mu \leq \nu$, then $\widetilde{q}(\lambda) \widetilde{\leq} \widetilde{q^\imath}(\lambda)$. Since $\|.\|$ is a soft quasi norm, we obtain $\|\widetilde{q}\|(\lambda) \widetilde{\leq} \left\|\widetilde{q^\imath}\right\|(\lambda)$. So, we have $\|\mu\|_\lambda \leq \|\nu\|_\lambda$. Lastly, for every $\epsilon \geq 0$ there exist an element $\xi_\epsilon \in Q$ such that $\mu \leq \nu + \xi_\epsilon$ and $\|\xi_\epsilon\|_\lambda \leq \epsilon$. Here, there exist an element $\widetilde{q_\epsilon} \in \widetilde{Q}$ such that $\widetilde{q_\epsilon}(\lambda) = \xi_\epsilon$. Thus, we get $\widetilde{q}(\lambda) \widetilde{\leq} \widetilde{q^\imath}(\lambda) + \widetilde{q_\epsilon}(\lambda)$ for $\widetilde{q}(\lambda) = \mu$, $\widetilde{q^\imath}(\lambda) = \nu$ and $\widetilde{q_\epsilon}(\lambda) = \xi_\epsilon$. On the other hand, we obtain $\|\xi_\epsilon\|_\lambda = \|\widetilde{q_\epsilon}\|(\lambda) = \|\widetilde{q_\epsilon}(\lambda)\| \widetilde{\leq} \widetilde{\epsilon}$ since $\|\xi_\epsilon\|_\lambda \leq \epsilon$. From $\left(\widetilde{Q}, \|.\|, P\right)$ is a soft normed quasilinear space, we have $\widetilde{q}(\lambda) \widetilde{\leq} \widetilde{q^\imath}(\lambda)$. This gives $\mu \leq \nu$. □

Theorem 3.5. *Let $\widetilde{Q}$ and $\widetilde{W}$ be soft normed quasilinear space which for $\mu \in Q$ and $\lambda \in P$, $\left\{\|\widetilde{q}\|(\lambda) : \widetilde{q}(\lambda) = \mu\right\}$ is a singleton set. Let $\chi : SQV(\widetilde{Q}) \to SQV(\widetilde{W})$ be a soft quasilinear operator satisfying for $\mu \in Q$ and $\lambda \in P$,*

$$\left\{\chi\left(\widetilde{q}\right)(\lambda) : \widetilde{q}\widetilde{\in}\widetilde{Q} \text{ such that } \widetilde{q}(\lambda) = \mu\right\}$$

is a singleton set. If χ is continuous then χ is bounded.

Proof. The proof is similar to the soft linear counterpart. □

Let's examine the inverse of a soft quasilinear operator χ. Let $\chi : SQV(\widetilde{Q}) \to SQV(\widetilde{W})$ be a soft quasilinear operator where $\widetilde{Q}$ and $\widetilde{W}$ are soft normed quasilinear spaces. $\left\{\chi(\widetilde{q}) : \widetilde{q}\widetilde{\in}\widetilde{Q}\right\}$ is the range set of χ.

Theorem 3.6. *Let $\chi : SQV(\widetilde{Q}) \to SQV(\widetilde{W})$ be q soft quasilinear operator. If χ^{-1} exists then*

$$\chi(\widetilde{q}) = \theta \text{ implies } \widetilde{q} = \theta.$$

Proof. Assume χ^{-1} exists i.e. $\chi(\widetilde{q}) = \chi(\widetilde{q^\imath})$ implies $\widetilde{q} = \widetilde{q^\imath}$. Let $\widetilde{q^\imath} = \theta$, then

$$\chi(\widetilde{q}) = \chi(\theta) = \theta$$

implying thereby

$$\widetilde{q} = \theta.$$

But, the converse of above theorem is not true. That is, if $\chi(\widetilde{q}) = \theta$ implies $\widetilde{q} = \theta$, then χ^{-1} may not be exists. Clearly, we know that this requirement exists in linear soft quasilinear spaces, that is, in soft linear spaces. Let's give an example related to soft quasilinear operators and it's inverse. □

Example 3.4. $\mathbb{R}(P)$ be the set of all soft real numbers defined over the parameter set P. Let an operator

$$\begin{array}{ccl} \chi & : & \mathbb{R}(P) \to SQV(\widetilde{\Omega_C(\mathbb{R})}) \\ \widetilde{r} & \to & \chi(\widetilde{r}) = \begin{cases} [-\widetilde{r}, \widetilde{r}] : \ \widetilde{r} \geq \widetilde{0} \\ [\widetilde{r}, -\widetilde{r}] : \ \widetilde{r} < \widetilde{0}. \end{cases} \end{array}$$

Clearly, $\chi(\widetilde{r}) \in SQV(\widetilde{\Omega_C(\mathbb{R})})$ for every $\widetilde{r} \in \mathbb{R}(P)$. Now, for every $\widetilde{r}, \widetilde{m} \in \mathbb{R}(P)$:

1) If $\widetilde{r}, \widetilde{m} \geq \widetilde{0}$, then $\widetilde{r} + \widetilde{m} \geq \widetilde{0}$. So, we get

$$\begin{aligned} \chi(\widetilde{r} + \widetilde{m}) &= [-(\widetilde{r} + \widetilde{m}), (\widetilde{r} + \widetilde{m})] \\ &= [-\widetilde{r} - \widetilde{m}, \widetilde{r} + \widetilde{m}] \\ &= [-\widetilde{r}, \widetilde{r}] + [-\widetilde{m}, \widetilde{m}] \\ &= \chi(\widetilde{r}) + \chi(\widetilde{m}). \end{aligned}$$

2) If $\widetilde{r}, \widetilde{m} < \widetilde{0}$, then $\widetilde{r} + \widetilde{m} < \widetilde{0}$. So, we have

$$\begin{aligned} \chi(\widetilde{r} + \widetilde{m}) &= [(\widetilde{r} + \widetilde{m}), -(\widetilde{r} + \widetilde{m})] \\ &= [\widetilde{r} + \widetilde{m}, -\widetilde{r} - \widetilde{m}] \\ &= [\widetilde{r}, -\widetilde{r}] + [\widetilde{m}, -\widetilde{m}] \\ &= \chi(\widetilde{r}) + \chi(\widetilde{m}). \end{aligned}$$

3) Let $\widetilde{r} \geq \widetilde{0}$ and $\widetilde{m} < \widetilde{0}$. If $\widetilde{r} + \widetilde{m} < \widetilde{0}$, then, we get

$$\begin{aligned} \chi(\widetilde{r} + \widetilde{m}) &= [(\widetilde{r} + \widetilde{m}), -(\widetilde{r} + \widetilde{m})] \\ &= [\widetilde{r} + \widetilde{m}, -\widetilde{r} - \widetilde{m}] \\ &\ \widetilde{\preceq} [\widetilde{r}, -\widetilde{r}] + [\widetilde{m}, -\widetilde{m}] \\ &= \chi(\widetilde{r}) + \chi(\widetilde{m}). \end{aligned}$$

If $\widetilde{r} + \widetilde{m} \geq \widetilde{0}$, then, we get

$$\begin{aligned} \chi(\widetilde{r} + \widetilde{m}) &= [(\widetilde{r} + \widetilde{m}), -(\widetilde{r} + \widetilde{m})] \\ &= [\widetilde{r} + \widetilde{m}, -\widetilde{r} - \widetilde{m}] \\ &\ \widetilde{\preceq} [\widetilde{r}, -\widetilde{r}] + [\widetilde{m}, -\widetilde{m}] \\ &= \chi(\widetilde{r}) + \chi(\widetilde{m}). \end{aligned}$$

4) Let $\widetilde{r} < \widetilde{0}$ and $\widetilde{m} \geq \widetilde{0}$. If $\widetilde{r} + \widetilde{m} < \widetilde{0}$, then, we get

$$\begin{aligned} \chi(\widetilde{r}+\widetilde{m}) &= [(\widetilde{r}+\widetilde{m}), -(\widetilde{r}+\widetilde{m})] \\ &= [\widetilde{r}+\widetilde{m}, -\widetilde{r}-\widetilde{m}] \\ &\widetilde{\preceq} [\widetilde{r}, -\widetilde{r}] + [\widetilde{m}, -\widetilde{m}] \\ &= \chi(\widetilde{r}) + \chi(\widetilde{m}). \end{aligned}$$

If $\widetilde{r} + \widetilde{m} \geq \widetilde{0}$, then, we get

$$\begin{aligned} \chi(\widetilde{r}+\widetilde{m}) &= [(\widetilde{r}+\widetilde{m}), -(\widetilde{r}+\widetilde{m})] \\ &= [\widetilde{r}+\widetilde{m}, -\widetilde{r}-\widetilde{m}] \\ &\widetilde{\preceq} [\widetilde{r}, -\widetilde{r}] + [\widetilde{m}, -\widetilde{m}] \\ &= \chi(\widetilde{r}) + \chi(\widetilde{m}). \end{aligned}$$

If $\widetilde{r} \geq \widetilde{0}$, then $\widetilde{c}.\widetilde{r} \geq \widetilde{0}$ for every soft positive scalar $\widetilde{c}$. Thus, we get

$$\chi(\widetilde{c}.\widetilde{r}) = [-\widetilde{c}.\widetilde{r}, \widetilde{c}.\widetilde{r}] = \widetilde{c} \cdot [-\widetilde{r}, \widetilde{r}] = \widetilde{c} \cdot \chi(\widetilde{r}).$$

If $\widetilde{r} < \widetilde{0}$, then $\widetilde{c}.\widetilde{r} < \widetilde{0}$ for every soft positive scalar $\widetilde{c}$. Thus, we get

$$\chi(\widetilde{c}.\widetilde{r}) = [\widetilde{c}.\widetilde{r}, -\widetilde{c}.\widetilde{r}] = \widetilde{c} \cdot [\widetilde{r}, -\widetilde{r}] = \widetilde{c} \cdot \chi(\widetilde{r}).$$

If $\widetilde{r} \geq \widetilde{0}$, then $\widetilde{c}.\widetilde{r} < \widetilde{0}$ for every soft negative scalar $\widetilde{c}$. Thus, we get

$$\chi(\widetilde{c}.\widetilde{r}) = [\widetilde{c}.\widetilde{r}, -\widetilde{c}.\widetilde{r}] = \widetilde{c} \cdot [\widetilde{r}, -\widetilde{r}] = \widetilde{c} \cdot \chi(\widetilde{r}).$$

If $\widetilde{r} < \widetilde{0}$, then $\widetilde{c}.\widetilde{r} \geq \widetilde{0}$ for every soft negative scalar $\widetilde{c}$. Thus, we get

$$\chi(\widetilde{c}.\widetilde{r}) = [-\widetilde{c}.\widetilde{r}, \widetilde{c}.\widetilde{r}] = \widetilde{c} \cdot [-\widetilde{r}, \widetilde{r}] = \widetilde{c} \cdot \chi(\widetilde{r}).$$

If $\widetilde{c} = \widetilde{0}$, then

$$\chi(\widetilde{c}.\widetilde{r}) = \chi\left(\widetilde{0}.\widetilde{r}\right) = \chi\left(\widetilde{\theta}\right) = \left\{\widetilde{0}\right\}.$$

For every $\widetilde{r}, \widetilde{m} \in \mathbb{R}(P)$, if $\widetilde{r} = \widetilde{m}$ then

$$\chi(\widetilde{r}) = \begin{cases} [-\widetilde{r}, \widetilde{r}] : \ \widetilde{r} \geq \widetilde{0} \\ [\widetilde{r}, -\widetilde{r}] : \ \widetilde{r} < \widetilde{0}. \end{cases}$$

equal to

$$\chi(\widetilde{m}) = \begin{cases} [-\widetilde{m}, \widetilde{m}] : \ \widetilde{m} \geq \widetilde{0} \\ [\widetilde{m}, -\widetilde{m}] : \ \widetilde{m} < \widetilde{0}. \end{cases}$$

Therefore, we obtain $\chi(\widetilde{r}) \widetilde{\preceq} \chi(\widetilde{m})$ for every $\widetilde{r}, \widetilde{m} \in \mathbb{R}(P)$. So, the operator χ is a soft quasilinear operator. Further, ıf $\chi(\widetilde{r}) = \widetilde{\theta}$ then $\widetilde{r} = \widetilde{0}$. But, χ is not an one to one mapping. Because, $-\widetilde{2} \neq \widetilde{2}$ for $-\widetilde{2}, \widetilde{2} \in \mathbb{R}(P)$, but $\chi\left(-\widetilde{2}\right) = \left[-\widetilde{2}, \widetilde{2}\right] = \chi\left(\widetilde{2}\right)$.

Remark 3.1. We know from the soft linear operators, if χ^{-1} exists for a soft linear operator χ, then χ^{-1} is a soft linear. But, this situation may not be true for a soft quasilinear operators. Now, let 's give an example to illlustrate this situation.

Example 3.5. Let $\mathbb{R}(P)$ be the set of all soft real numbers defined over the parameter set P and consider the absolute soft quasi set generated by $\Omega_C(\mathbb{R})$ i.e. $\widetilde{\Omega_C(\mathbb{R})}(\lambda) = \Omega_C(\mathbb{R})$. Let an operator

$$\begin{aligned} \chi &: \ \mathbb{R}(P) \to SQV(\widetilde{\Omega_C(\mathbb{R})}) \\ \widetilde{r} &\to \ \chi(\widetilde{r}) = \widetilde{r} \cdot \widetilde{[-1,0]} \end{aligned}$$

for a soft quasi vector $\widetilde{[-1,0]} \in \widetilde{\Omega_C(\mathbb{R})}$. χ is a soft quasilinear operator. So, χ^{-1} is exists since χ is an one to one soft quasilinear operator. Also, for $\widetilde{1} \cdot \widetilde{[-1,0]} \in SQV(\widetilde{\Omega_C(\mathbb{R})})$ and $\widetilde{\frac{1}{2}} \cdot \widetilde{[-1,0]} \in SQV(\widetilde{\Omega_C(\mathbb{R})})$, we have

$$\widetilde{\frac{1}{2}} \cdot \widetilde{[-1,0]} \widetilde{\subseteq} \widetilde{1} \cdot \widetilde{[-1,0]}$$

but

$$\widetilde{\frac{1}{2}} \neq \widetilde{1}.$$

Thus, χ^{-1} is not a soft quasilinear operator since χ^{-1} does not satisfy condition $\widetilde{q} \widetilde{\preceq} \widetilde{w} \Rightarrow \chi(\widetilde{q}) \widetilde{\preceq} \chi(\widetilde{w})$.

Theorem 3.7. *Let $\widetilde{Q}$ and $\widetilde{W}$ be a soft normed quasilinear spaces which satisfy the condition: $\{\|\widetilde{q}\|(\lambda) : \widetilde{q}(\lambda) = \mu\}$ is a singleton set for $\mu \in Q$ and $\lambda \in P$. Let $\chi : SQV(\widetilde{Q}) \to SQV(\widetilde{W})$ be a soft quasilinear operator, χ^{-1} be a continuous soft quasilinear operator and χ^{-1} which satisfy the condition:*

$$\left\{\chi^{-1}(\widetilde{w})(\lambda) : \widetilde{w} \widetilde{\in} \widetilde{W} \text{ such that } \widetilde{w}(\lambda) = \kappa\right\}$$

is a singleton set for $\kappa \in W$ and $\lambda \in P$. Then there exists a soft real number $\widetilde{k} \widetilde{\geq} \widetilde{0}$ such that $\widetilde{k}\,\|\widetilde{q}\| \widetilde{\leq} \|\chi(\widetilde{q})\|$ for every $\widetilde{q} \widetilde{\in} \widetilde{Q}$.

Proof. Assume that χ^{-1} is exists and continuous. We obtain χ^{-1} bounded by Theorem 3.5 since $\{\|\widetilde{q}\|(\lambda) : \widetilde{q}(\lambda) = \mu\}$ is a singleton set for $\mu \in Q$ and $\lambda \in P$ and $\left\{\chi^{-1}(\widetilde{w})(\lambda) : \widetilde{w} \widetilde{\in} \widetilde{W} \text{ such that } \widetilde{w}(\lambda) = \kappa\right\}$ is a singleton set for $\kappa \in W$ and $\lambda \in P$. Thus, there exists a positive soft real number $\widetilde{N} \widetilde{\geq} \widetilde{0}$ such that $\left\|\chi^{-1}(\widetilde{w})\right\| \widetilde{\leq} \widetilde{N}\,\|\widetilde{w}\|$ for every $\widetilde{w} \in \widetilde{W}$. There exists $\widetilde{q} \widetilde{\in} \widetilde{Q}$ such that $\chi^{-1}(\widetilde{w}) = \widetilde{q}$. Thus, we obtain $\|\widetilde{q}\| \widetilde{\leq} \widetilde{N}\,\|\chi(\widetilde{q})\|$. If we take $\widetilde{k} = \frac{1}{\widetilde{N}}$, then we get $\widetilde{k}\,\|\widetilde{q}\| \widetilde{\leq} \|\chi(\widetilde{q})\|$. □

Theorem 3.8. *Let $\widetilde{Q}$ be a soft Banach quasilinear space and $\widetilde{W}$ be a soft normed quasilinear spaces which satisfy the condition: $\{\|\widetilde{q}\|(\lambda) : \widetilde{q}(\lambda) = \mu\}$ is a singleton set for $\mu \in Q$ and $\lambda \in P$. Also, $\chi : SQV(\widetilde{Q}) \to SQV(\widetilde{W})$ be a bijective continuous soft quasilinear operator satisfying the condition: $\left\{\chi(\widetilde{q})(\lambda) : \widetilde{q} \widetilde{\in} \widetilde{Q} \text{ such that } \widetilde{q}(\lambda) = \mu\right\}$ is a singleton set for $\mu \in Q$ and $\lambda \in P$. If χ^{-1} is continuous then $\widetilde{W}$ is a soft Banach quasilinear space.*

Proof. Let $\{\widetilde{w_n}\}$ be a Cauchy sequence in $\widetilde{W}$. Then there exists a $n_0 \in \mathbb{N}$ such that

$$\widetilde{w_n} \widetilde{\leq} \widetilde{w_m} + \widetilde{w_{1n}^{\epsilon}},\ \widetilde{w_m} \widetilde{\leq} \widetilde{w_n} + \widetilde{w_{2n}^{\epsilon}},\ \left\|\widetilde{w_{in}^{\epsilon}}\right\| \widetilde{\leq} \frac{\widetilde{\epsilon}}{\widetilde{N}}$$

for all $n, m \geq n_0$. Since χ^{-1} is continuous

$$\chi^{-1}(\widetilde{w_n}) \widetilde{\leq} \chi^{-1}(\widetilde{w_m}) + \chi^{-1}\left(\widetilde{w_{1n}^{\epsilon}}\right),\ \chi^{-1}(\widetilde{w_m}) \widetilde{\leq} \chi^{-1}(\widetilde{w_n}) + \chi^{-1}\left(\widetilde{w_{2n}^{\epsilon}}\right)$$

i.e.

$$\widetilde{q_n} \widetilde{\leq} \widetilde{q_m} + \widetilde{q_{1n}^{\epsilon}},\ \widetilde{q_m} \widetilde{\leq} \widetilde{q_n} + \widetilde{q_{2n}^{\epsilon}}$$

for every $\chi(\widetilde{q_n}) = \widetilde{w_n}$ for $\widetilde{q_n} \widetilde{\in} \widetilde{Q}$. There exists $\widetilde{q_{in}^{\epsilon}} \widetilde{\in} \widetilde{Q}$ such that $\chi^{-1}\left(\widetilde{w_{in}^{\epsilon}}\right) = \widetilde{q_{in}^{\epsilon}}$. Thus, from the above theorem, we obtain

$$\left\|\widetilde{q_{in}^{\epsilon}}\right\| \widetilde{\leq} \widetilde{N}\left\|\chi\left(\widetilde{q_{1n}^{\epsilon}}\right)\right\| = \widetilde{N}\left\|\widetilde{w_{in}^{\epsilon}}\right\| \widetilde{\leq} \widetilde{\epsilon}.$$

Thus, $\{\widetilde{q_n}\}$ is a Cauchy sequence in $\widetilde{Q}$. Since $\widetilde{Q}$ is complete, $\widetilde{q_n} \to \widetilde{q_0}$ for some $\widetilde{q_0} \in \widetilde{Q}$. So, there exists a $n_0 \in \mathbb{N}$ such that

$$\widetilde{q_n} \widetilde{\leq} \widetilde{q_0} + \widetilde{q_{1n}^{\epsilon}},\ \widetilde{q_0} \widetilde{\leq} \widetilde{q_n} + \widetilde{q_{2n}^{\epsilon}},\ \left\|\widetilde{q_{in}^{\epsilon}}\right\| \widetilde{\leq} \widetilde{\epsilon}$$

for all $n \geq n_0$. Thus, we get

$$\widetilde{w_n} \widetilde{\leq} \widetilde{w_0} + \widetilde{w_{1n}^{\epsilon}},\ \widetilde{w_0} \widetilde{\leq} \widetilde{w_n} + \widetilde{w_{2n}^{\epsilon}},\ \left\|\widetilde{w_{in}^{\epsilon}}\right\| \widetilde{\leq} \widetilde{\epsilon}$$

for $\chi(\widetilde{q_n}) = \widetilde{w_n}$ and $\chi(\widetilde{q_0}) = \widetilde{w_0}$ since χ is continuous. Therefore, $\widetilde{W}$ is a soft Banach quasilinear space. □

4. Conclusion

In this work, the notion of soft quasilinear operator is defined. Also, some consistent theorems and conclusions related with soft quasilinear operators are obtained. Lastly, the inverse of a soft quasilinear operator is described and its some basic properties are worked.

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