Muş Alparslan Üniversitesi Fen Bilimleri Dergisi
Muş Alparslan University Journal of Science
ISSN:2147-7930

Cilt/*Volume*:**1** **Sayı**/ *Issue*:**2** **Aralık**/*December*: **2013**

THE COLLECTIVE EXCITATIONS AND RESONANCES IN ATOMIC NUCLEI

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ABSTRACT

In the present paper using the random phase approximation (QRPA) method we carried out more complete and accurate calculations for the collective excitations in nuclei. The self-consistent determination of the effective interactions for all collective modes is performed more precisely on the basis translational, rotational and Galilean invariance requirements. The calculations of Ml transitions to excitations with $I^{\pi}K = 1^{+}1;1^{+}0$ and their contribution to the sum rules indicate the existence of a spin-flip Ml resonance in the region of energies 8-10 MeV in rare-earth and 6-9 MeV in actinide nuclei. In well deformed nuclei the calculations predict electric dipole and quadrupole giant resonances between 11 and 16 MeV energy. The region of localization of the isoscalar E2 (K=1) resonance corresponds to the energy ~70 $A^{-1/3}$ MeV, which agrees with experimental data (E_x~65 $A^{-1/3}$ MeV). The theory predicts 1^{-} giant dipole resonance splitting into two components with K=0 and K=1 at energy around 12 MeV and 16 MeV, respectively. Rather collective states arise in PDR region below nucleon threshold energy. The charge-exchange Fermi and GT resonances in odd-odd nuclei are analyzed and their calculated β-decay quantities compared with other predictions.

Key Words *Microscopic model, QRPA, Collective excitations, Deformed nuclei*

ÖZET

Bu çalışmada çekirdekteki kolektif uyarılmaların tam ve daha doğru hesaplamaları Kuaziparçacık Serbest Faz Yaklaşımı kullanılarak yapılmıştır. Öteleme, dönme ve Galilean değişmezlik gereksinimleri temel alınarak efektif etkileşmeler tüm kolektif modlar için öz uyumlu olarak belirlenmiştir. $I^{\pi}K = 1^{+}1;1^{+}0$ uyarılmaları için hesaplanan M1 geçişleri ve bunların toplam kurallarına katkıları nadir toprak bölgesinde 8-10MeV ve aktinid bölgesinde 6-9MeV enerjilerinde spin-flip M1 rezonansın varlığını ortaya koymaktadır. Hesaplamalar iyi deforme çekirdekler için 11 ve 16 MeV enerji aralığında dev elektrik dipol ve kuadrupol rezonanasları öngörmektedir. İzoskaler E2 (K=1) rezonansının yerleştiği bölge deneysel veri (E_x~65 $A^{-1/3}$ MeV) ile uyumlu olarak ~70 $A^{-1/3}$ MeV enerjisine karşılık gelmektedir. Teori sırası ile 12 MeV ve 16 MeV enerjilerinde K=0 VE K=1 olarak iki yarılmış 1^{-} dev rezonansını öngörmektedir. Daha kolektif seviyeler nükleon eşik enerjisinin altındaki PDR bölgesinde ortaya çıkmaktadır. Tek-tek çekirdeklerde yük değişimli Fermi ve GT analiz edilerek hesaplanan β-bozunum nicelikleri diğer çalışmaların öngörüleri ile kıyaslanmıştır.

Anahtar Kelimeler: *Mikroskobik model, QRPA, Kolektif uyarılmalar, Deforme çekirdekler*

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1. INTRODUCTION

In the study of the structure of the nucleus, the collective excitations in which the forces between nucleons are responsible holds an important place. In these excitations the electric and the magnetic dipole vibrations have a special place because of model independence of the equations of the electro-magnetic theory. These vibrations provide key information to test the theoretical models used to investigate the characteristic of the strong interactions between nucleons and to determine their strength parameters. Considering the parity there are two different types of dipole excitations. While the levels with spin and parity $I^{\pi}=1^{+}$ has the magnetic dipole character, the levels with spin and parity $I^{\pi}=1^{-}$ has the electric dipole character. 1^{-}-states corresponding to the electric dipole vibrations in which the effective isovector interactions are responsible constitute the isovector electric dipole giant resonance (GDR) in medium and heavy nuclei in the energy range of 13 to 16 MeV. In nuclear physics, this excitation mode is well-known and it is the first collective mode investigated extensively. GDR was first observed in a photo-nuclear reaction by Baldwin and Klaiber [1]. Then, by Goldhaber and Teller it was theoretically interpreted [2] as the isovector vibrations of the mass centers of the neutron and the proton systems against each other when the center-of mass (c.m.) of the nucleus being at rest.

In spherical nuclei, the appearance of collective 1^{+}-states is connected with transitions between levels of spin-orbit doublet. The presence of the 1^{+} levels corresponding to the collective vibrations in the spherical nuclei was suggested by B. Mottelsson for the first time [3]. The existence of this mode was proved after the observation of 1^{+} level at the energy of 15 MeV in the inelastic electron scattering reactions in ^{12}C [4]. When the mean field is deformed, the j-shells split with respect to the magnetic quantum number K. As a results the dipole resonance is split into two components with K=0 and 1. This in turn leads to the mixing of the states of the multiplets.

In the even-even deformed nuclei the collective spin vibrations, in which spin-spin interactions are responsible, was predicted by [5] within the framework of the nuclear microscopic model in the early 70s. In this model, 1^{+} excitations occurs as a result of the particle-hole transition between the levels conforming the transition rules ($\Delta K= 0, \pm 1$). In well deformed rare-earth elements, in the energy range of 6-12 MeV, a large spin-flip M1 resonance corresponding to K = 0 and K = 1 branches with a maximum lying energy 9 MeV between 8 MeV and 10 MeV is predicted by the theory [6]. $K^{\pi}=1^{+}$ branch of the magnetic dipole vibrations has two different low energy and high energy modes. One of them has spin-orbital character and the other has spin-vibration character. The appearance of orbital scissors mode 1^{+}-states is connected with $\Delta K=\pm 1$ transitions between components of (2j+1)/2 doublet in which the quasiparticles filling the levels of the same j-shell near the Fermi surface. The low-energy 1^{+} states corresponding to the orbital vibrations form scissors mode with a maximum around 3 MeV [7,8]. The high-energy collective excitations form the spin-flip magnetic dipole resonances in the energy range of 7-9 MeV. In recent years the experiments have showed that there are wide M1 resonance with two humps in the energy range of 7 MeV and 11 MeV, and having a maximum of $44xA^{-1/3}$ MeV [9]. The main feature of the M1 resonance which exhibits a strong fragmentation is the deformation independence of the high-energy part of the M1 strength function and the collective character of it. A feature of these resonances which have not fully disclosed up till now (quenching event) is that the experimental values of the total M1 transition strength constitute on1y 60% of the prediction of the theory [9].

In odd-odd nuclei microscopic theory predicts charge-exchange type 0^{+}-states which form the isobaric analog of the ground state of the nucleus with M_T equal to the $T_{gr.}=(N-Z)/2$

and Gamow-Teller 1^+ states with a moment proportional to the spin operator **s**. Both 0^+-states and 1^+-states forms Fermi and Gamow-Teller (GT) resonances with high collectivity at the energy around $\Delta E_{Coulomb}$.

2. COLLECTIVE EXCITATIONS IN DEFORMED NUCLEI

The collective excitations with low multi-polarity are one of the most important subjects in nuclear structure physics. The investigation of these excitations, particularly electro-magnetic dipole (I=1) and quadrupole (I=2) excitations in the even-even and charge-exchange type of excitations as Fermi resonance (FR) with spin and parity $I^{\pi}K=0^+0$ and GT resonances (GTR) with 1^+0 and 1^+1 in odd-odd deformed nuclei provides valuable information about nuclear structure and nucleon-nucleon forces at low energy. A good examples are low-lying β- and γ- vibrations with $K^{\pi}=0^+$ and $K^{\pi}=2^+$, respectively and magnetic dipole excitations, known as scissors mode [2] and high-lying giant dipole resonance (GDR), spin-flip (SF), quadrupole, isobar analog states (IAS) and GT resonances in deformed nuclei.

In the spectra of many deformed nuclei, one has observed low-lying isoscalar excitations with $K^{\pi}=0^+$ and 2^+ that may be approximately described as β- and γ-vibrations. Great success has been achieved in the qualitative description of these vibrations in even-even deformed nuclei in terms of unified nuclear model (UNM) supposed by Bohr [10].

In the harmonic approximation, according to Bohr and Mottelson the UNM Hamiltonian has the form [11]

$$H=\frac{1}{2}B_{\lambda}\,\dot{\alpha}_{\lambda\mu}^{2}+\frac{1}{2}C_{\lambda}\,\alpha_{\lambda\mu}^{2} \qquad (1)$$

As seen in the harmonic approximation, the Hamiltonian is a quadratic function of the vibrational amplitudes $\alpha_{\lambda\mu}$ and the canonically conjugate momenta$_{\lambda\mu}$. Because of axial symmetry of the nucleus there are only two different vibration modes with the variables

$$\alpha_{20}=\beta_2\cos\gamma;\ \alpha_{22}=\alpha_{2-2}=\frac{1}{\sqrt{2}}\beta_2\sin\gamma \qquad (2)$$

The variables $\alpha_{1\mu}=0$, $\alpha_{21}=\alpha_{2-1}=0$ and $\alpha_0=0$ represent a center-of-mass displacement, a rotation motion and zero point energy of a nucleus, respectively, do not contribute to intrinsic excitations.

In UNM model the low-lying excitations involve two types of terms corresponding to $\gamma=0$ and $\gamma\neq0$, for the change of the total oscillator quantum number. In the case $\gamma=0$, $\alpha_{20}=\beta_2$ and these vibrations preserve axial symmetry and are referred to as β-vibrations with $K^{\pi}=0^+$. These vibrations occur without change in shape (see left-hand side of the top of the Figure 1). The oscillations with $\gamma\neq0$ break the axial symmetry and lead to nuclear shapes of ellipsoidal type are referred as γ-vibrations with $K^{\pi}=2^+$ (see the lower part of the Figure 1). This feature suggests an interpretation of this excitation mode in terms of a collective quadrupole vibration away from axial symmetry (γ0.)

The states with variables $\alpha_{21}=\alpha_{2-1}$ is equivalent to a rotation about an axis perpendicular to the symmetry axis, without change in shape (see right hand side of the top of the Figure 1). This mode with $I^{\pi}K=2^+K$ with $K=\pm1$, therefore, is "spurious" in the sense that it does not occur as an intrinsic (non-rotational) excitation. The absence of the $I^{\pi}K=2^+1$ mode is analogous to the absence of shape oscillations with I=1, the degrees of freedom of which correspond to center-of-mass motion [11]. The removal of the spurious 2^+K mode with $K=\pm1$ in the deformed nuclei is considered on the next part of the lecture.

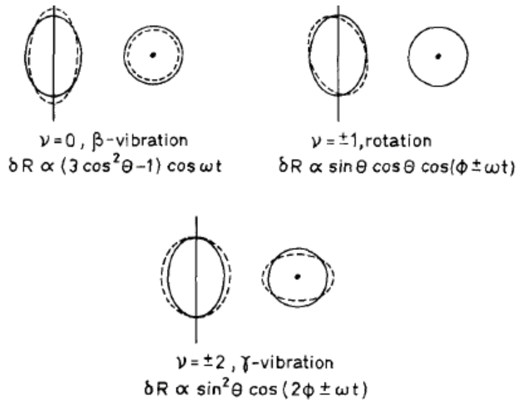

Figure 1. Quadrupole shape oscillations in a spheroidal nucleus [11]. The Figure shows projections of the nuclear shape in directions perpendicular and parallel to the symmetry axis. The upper part of the Figure shows

β-vibration ($\alpha_{20}=\beta_2\cos\gamma$), while the lower part shows γ-vibration ($\alpha_{22}=\beta_2\sin\gamma$). It has been assumed that the vibrational motion is harmonic with no interactions between the β and γ vibrational quanta α_{20} and $\alpha_{22,}$ respectively.

The occurrence of a rather low-lying states with $K^\pi=0^+$ and $K^\pi=2^+$, is a systematic feature in the spectra of the even-even deformed nuclei. The relative magnitude of ω_β and ω_γ depends on the internal structure. The evidence on the excitation energy of mass number A this mode is collected in Figure 2.

As stressed in [11] a shape oscillation hν is characterized by a large E2-transition probability for low-lying exciting states. The observed E2-transition probabilities for the excitation of β- and γ- vibrations are typically five to ten times the appropriate single-particle units. Though enhanced, these transition probabilities are considerably smaller than for the quadrupole vibrations in spherical nuclei. For the low-frequency modes, which are sensitive to the coupling scheme of the particles in partially filled shells, the effect of the deformation cannot be viewed as a small perturbation. In fact, the nuclear deformations, though numerically rather small (β_0<0.3), profoundly affect the motion of the nucleons in the partially filled shells. For the low-frequency modes in deformed nuclei, it may be possible to assign a multipole quantum number λ describing the main component in the oscillating density. However, modes with the same λ and different ν will, in general, have rather different properties and may not be related in a simple manner to the modes observed in spherical nuclei [11].

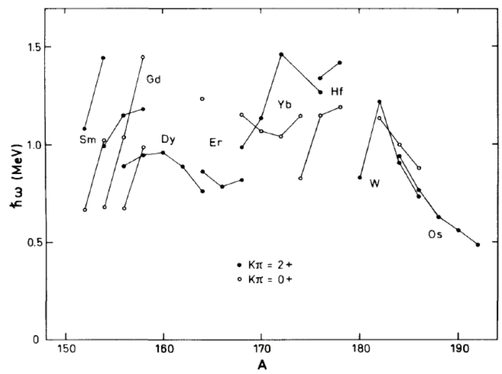

Figure 2. Systematics of β- and γ- vibrational frequencies for 150 < A < 192**.** The Figure shows the energies of the lowest 0^+, I= 0, and 2^+, I=2, intrinsic excitations. For the β- and γ-vibrations with two quanta, it has been assumed that the vibrational motion is harmonic with no interactions between the β and γ-vibrational quanta, the data are taken from the [11].

Numerical calculations have shown that UNM describes the β- and γ- vibrations as the oscillation of the surface of the nucleus and defines the main component of the multipole quantum number of the surface oscillating density. This model predicts well the rotational energy and the shape oscillation energy of the deformed nuclei. But the decay properties of the β- and γ-vibrations, predicted by the theory, are in poor agreement with experimental data [12]. The model failed to explain the appearance of the second 0^+ excited state below the energy gap in deformed nuclei. Such states were found in a number of deformed nuclei.

3. MICROSCOPIC MODEL FOR β-AND γ-VIBRATIONS

The microscopic model considers the β- and γ- vibrational states as low-energy part of the 0^+- and 2^+- quadrupole type states in the deformed nuclei. Great success has been achieved in the qualitative description of the 0^+ and 2^+ excited states in even deformed nuclei in terms of Random Phase Approximation (RPA) of the microscopic model using a simple interaction composed of a pairing force and a quadrupole force. The QRPA has been found successful in explaining low-lying multipole vibrations and giant resonances as well as the scissors mode excitations observed in deformed nuclei [13] and references therein). The QRPA is able to calculate the beta decay and double beta decay probabilities. Numerical calculations have shown that this model predicts well the energy and the B(E2) values for γ-vibrational 2^+-states. But the properties of the β-vibrational 0^+-states, predicted by the theory, are in poor agreement with experimental data[13,14] The model failed to explain the appearance of the second 0^+ excited state below the energy gap. Such states were found in a number of deformed nuclei [15-17].

An important tool in the study of the nature of the 0^+ states is the allowed β-decay. An appreciable slowing down of the decay to the first 0^+ excitation as compared with that to the ground state was observed in both cases, while the theory predicts approximately the same rate of the decay to both the ground and β-vibrational states [13].

In the microscopic theory the 0^+ states are described as superposition of two-quasi-particle states. As a rule the quasi-particle pairs occupying the levels near the Fermi surface have the largest amplitudes in the 0^+-state wave function. One or two of such pairs give the main contribution to an allowed β-decay matrix elements. The experimental data point out that the amplitudes of the quasi-particle pairs participating in the decay of the 0^+ states are smaller than those predicted by theory with the quadrupole-quadrupole interactions.

It was pointed out that a significant decrease of the diagonal amplitude may be due to the residual spin-quadrupole interactions. The spin-quadrupole force may generate a new 0^+ excitation below the energy gap. Some results of our studies were given earlier in paper [7], in which 0^+- and 2^+-states have been generated by quadrupole and spin-quadrupole interactions.

It is found from our calculations that within the framework of the model with pairing-plus-quadrupole interactions it is impossible to achieve a good fitting of calculations to both the experimental energy and the log *ft* value for the first 0^+- state. This may be obtained provided the spin-quadrupole force affects significant the energy of 0^+ states, the B(E2) and the log *ft* values. The second 0^+ excitation appears below the energy gap due to the spin-quadrupole force. The spin-quadrupole force affects slightly the energy and the B(E2) value for gamma-vibrational state, while the properties of the high-lying 2^+ states are affected much more strongly. The same hold for the effect of the spin-quadrupole force on the rate of β -decay to the gamma-vibrational and high-lying 2^+ states.

4. THE PROPERTIES OF COLLECTIVE $I^{\pi}K=1^{+}1$ STATES

At present the idea of residual spin-spin correlations in atomic nuclei is well recognized in nuclear physics. In deformed nuclei the effective g_K factors obtained from experimental magnetic moments were successfully described theoretically taking into account the spin-spin correlations [5,6,18]. Spin-dependent np correlations are responsible for the strong hindrance of Gamow-Teller β-transitions among low-energy states of nuclei [19]. So far we have only mentioned the effects which are manifested in low-energy nuclear states. The explanation of these effects requires the repulsive nature of the spin-spin correlations. Hence, the collective modes of excitations associated with these residual interactions (e.g., 1^+ states in even-mass nuclei) are expected to appear at high energy.

The Hartree-Fock potential violates many symmetries of the nuclear single-particle Hamiltonian. For instance, in the quasiparticle model, the deformed mean field potential is not rotational, translation and Galilean invariant. Therefore, 1^+-states have admixture from rotational motion of deformed nuclei, while 1^--dipole vibrations according to Goldstoun theorem contain zero energy spurious admixtures from the rotational and the center-of-mass motion [20]. For example a quadrupole deformation with $I^\pi=2^+$ with $\nu=\pm1$ is equivalent to a rotation about an axis perpendicular to the symmetry axis, without change in the shape. This mode, therefore, is "spurious" in the sense that it does not occur as an intrinsic (non-rotational) excitation. Thus, the low-lying branch of dipole excitations which does not have a vibrational nature should be separated from the internal excitation spectrum of deformed nuclei as a state with an energy $\omega_0=0$ [21]. Separating the spurious state with energy of $\omega_0=0$ from the vibrational ones is one of the fundamental requirements for microscopic models. Various methods were elaborated for the separation of the spurious state from the vibrational ones. The detailed description of the use of the QRPA for the separation of the spurious states was given in [8] and has been applied to calculate the 1^+-states in heavy deformed nuclei, but with restriction to the isoscalar part of the restoring forces. The generalization of the method for the separation of the spurious rotational state with zero energy to a realistic case in which two different isoscalar and isovector restoring interactions in the Hamiltonian has been presented in QRPA [22] and in high version of the QRPA [23]. An extension to the calculation of E1 transitions after restoration of translational invariance is discussed in [24]. Thus the microscopic calculations using rotational invariant QRPA, have shown that these models reproduce the excitation energy as well as the experimental quadratic dependence of the summed M1 strength of the mode in heavy even-even deformed nuclei. A detailed description of the use of the QRPA for the separation of the spurious states was given in [22]. Omitting the details of the solution we only present the most relevant equations. In particular, the secular equation for the excitation energy of 1^+-state, which can be written as:

$$\omega_i^2 J_{eff}(\omega_i) = \omega_i^2\left[J - 8\chi_\sigma \frac{X^2}{D_\sigma} + \frac{\omega_i^2}{\gamma_1 - F_1}\left(J_1^2 - 8\chi_\sigma \frac{J\cdot X_1^2 - 2J_1 X\cdot X_1}{D_\sigma}\right)\right] = 0 \quad (3)$$

As can be seen from Eq. (3), the spurious $\omega = 0$ solution (Goldstone branch) is separated automatically and the zero-energy solution belongs to the rotational excitation state. The remaining solutions of Eq. (3) with $\omega_i > 0$ describe the harmonic vibrations of the system, lying above the first two-quasiparticle energy. The low-lying orbital mode of the vibrations belongs to scissors mode.

The most characteristic quantity of M1 dipole 1^+-excitations is the ground-state transition width, which can be calculated with formulas:

$$\Gamma_0(M1)[meV] = 3.86\cdot\omega_i^3 B(M1)\uparrow \quad (4)$$

where the excitation energy ω_i is in MeV and the M1 transition probability B(M1) in units μ_N^2.

4.1 Deformed Rare-Earth Nuclei

The existence of the low-lying orbital magnetic dipole scissors mode states is now well established as fundamental excitations in deformed nuclei [1]. The presence of 1^+-states with magnetic character has been firstly predicted theoretically within the microscopic model [5,6,8]. Then, the 1^+-states with orbital character generally known as scissors mode has been predicted theoretically within the semi-classical two-rotor model [25]. Scissors mode is low-energy part of the 1^+-states represents the coherent isovector rotation motion of proton and neutron systems as rigid deformed bodies making scissors-like oscillations against each other, while the common deformed axis of the nucleus being at rest. After theoretical predictions, this so-called "scissors mode" has been experimentally found in the deformed ^{156}Gd nuclei by the Darmstadt group [26]. The existence of the low-lying orbital magnetic dipole scissors mode states is now well established as fundamental excitations in deformed nuclei beginning from the light nuclei (such as ^{46}Ti) up to the actinides (see refs. [27-29] and references therein). In this collective mode, the ground-state transition strength in deformed nuclei is generally fragmented and concentrated in the energy region below 4 MeV with considerable dependence on deformation.

The analysis shows that, in the rotational invariant model, the spectroscopic1^+-states were more strongly collective than in the rotational non invariant model. In the following, we will discuss magnetic and electric dipole excitations separately and compare the calculated results of $^{176-180}$Hf [30] with the experimental data of [31,32], which are shown in Figure 3.

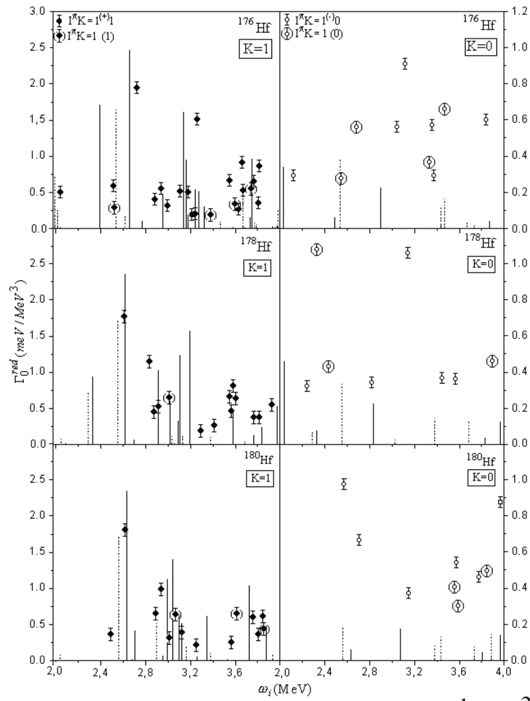

Figure 3. Energy diagram of $\Gamma_0^{\text{red.}}=\omega_i^{-3}\Gamma_0$ values calculated by QRPA method [30] and experimentally observed strengths [31,32] for $^{176-180}$Hf. In the QRPA results, M1 transitions with ΔK=1 and ΔK=0 are shown as solid lines and E1 transitions with ΔK=1 and ΔK=0 are shown as dashed lines, respectively. Full and open circles with error bars denote the experimental data for ΔK=1 and ΔK=0 excitations with tentative parity assignment, respectively. Full and open circles with error bars in parentheses denote the experimental data for tentative ΔK=1 and ΔK=0 excitations with unknown parity assignment, respectively. Because the absolute values of the ΔK=1 transitions are several times stronger than those of the ΔK=0 ones, we use different scales of the reduced transition widths for ΔK=0 and ΔK=1, respectively, shown on the right and left side of Figure.

As seen from Figure 3. the experimental distribution patterns look quite similar, with one isolated dipole excitation at an energy of $\omega_i \approx 2.722$ MeV and two bumps around $\omega_i \approx 3.2$ MeV and $\omega_i \approx 3.8$ MeV for all three nuclei under investigation. The calculations also predict one more scissors mode excitation at an energy of around $\omega_i \approx 2.38$ MeV with a high value of $\Gamma_{red.}$=1.71 meV/MeV^3 in ^{176}Hf, which decreases in ^{178}Hf to $\Gamma_{red.}$=0.93 meV/MeV^3 and disappears in ^{180}Hf. The M1 strength calculated in ^{176}Hf, which is observed in the experiment, is somewhat more fragmented than that in ^{178,180}Hf. Overall, the agreement of the calculated magnetic dipole excitations with the experimental findings is generally relatively high. Several similarities are observed on predicted and observed excitations.

The present calculation predicts strongly fragmented scissors mode $K^{\pi}=1^+$-states in the excitation energy range 2–4MeV for ^{176,178,180}Hf. The calculations show that, for the states, the orbit-to-spin ratio are mainly changed $|M_l/M_s|>1–9$ and the main contribution to B(Ml) comes mainly from the single-particle orbital matrix elements.

4.2 Deformed Actinide Nuclei

In this study, $I^{\pi}=1^+$ dipole mode excitations are systematically investigated within the rotational and translational + Galilean invariant quasiparticle random-phase approximation for ^{232}Th, ^{236}U, and ^{238}U actinide nuclei [29]. It is shown that the investigated nuclei reach a B(M1) strength structure, which corresponds to the scissors mode. The calculated mean excitation energies as well as the summed B(M1) value of the scissors mode excitations are consistent with the available experimental data.

Table 1 shows the results for the mean energy ϖ of the scissors mode excitations and the summed B(M1) strengths calculated with the rotational invariant Hamiltonian including the isoscalar plus isovector restoring forces and with the experimentally observed M1 dipole excitations in the energy region 2.0–2.5MeV. As seen from the table our calculated total B(M1) strengths for ΔK = 1 excitations are in very good agreement with the experimental ones.

Table 1. Comparison the mean energy ϖ and summed B(M1) values of the scissors mod excitations calculated with the rotational invariant Hamiltonian with experimentally observed M1 dipole excitations in (γ, γ′) [33] and (e, e′) [34] reaction in energy range 2.0-2.5 MeV.

Nuclei	RPA		(γ,γ')		(e,e′)	
	$\sum B(M1)$	ϖ	$\sum B(M1)$	ϖ	$\sum B(M1)$	ϖ
^{232}Th	3.06	2.14	2.59(25)	2.14	2.7(1.1)	2.16
^{236}U	3.22	2.24	2.94(23)	2.34	---	---
^{238}U	3.23	2.31	3.19(24)	2.27	4.0(1.7)	2.26

4.3 Transitional Nuclei with Moderate Deformation

In most cases, particularly for strongly deformed rare-earth nuclei near mid-shell, the variations of the mean excitation energy and the total M1 excitation strength of the mode are small [35,36]. However, while the global properties of the scissors mode are reasonably understood in regions of moderate to large deformations, the nature of the scissors mode is an open question in nuclei near shell closures where the simple geometrical picture of a scissors-like motion of deformed proton and neutron bodies breaks down. It would be desirable to confirm the features of the scissors mode in other γ-soft and transitional deformed nuclei.

The theory predicts several dipole K = 1$^+$ excitations at the 2-4MeV energy interval for the all investigated Ba isotopes. The comparative characteristics of the low-lying dipole K^π= 1$^+$ excitations of $^{124-134}$Ba calculated with the invariant Hamiltonian are shown in Figure 4 [37].

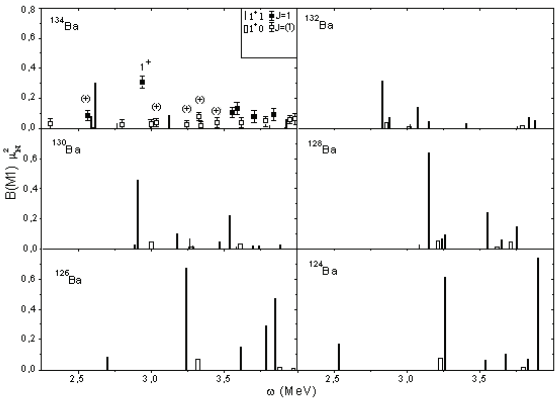

Figure 4. Energy dependence [37] of the calculated and observed B(M1) for $^{124-134}$Ba isotopes. Here only states with B(M1)>0.01 μ_N^2 is given. Predicted M1 transitions with ΔK = 1 are shown as a solid line and M1 transitions with ΔK = 0 are shown as open bars. Whereas symbol denotes the experimental data if J=1 is certain, symbol denotes the experimental data if I=1 is uncertain. (+) assign a tentative parity assignment.

The theory predicts several dipole K^π=0$^+$ and K^π =1$^+$ excitations at 2-4MeV energy region. As seen from the Figure 4, contributions of the dipole states with K = 0 (open bars) are small in the energy range 2–4MeV and are not take considered interest for the nuclei investigated. So here main discuses focused only on the K=1 transitions. Since ^{138}Ba nearly spherical nucleus [38] therefore calculated small *M*1 scissors mode strength should be negligible (B(M1)≤0.05 μ_N^2), therefore as can be seen from Figure 4, no *M*1 excitations could be observed in the semi-magic isotope ^{138}Ba below 4 MeV. Therefore a magnetic dipole excitation emerges firstly in vibrational ^{136}Ba isotopes and numbers of them increase toward deformed isotopes. As seen from the Figure 4, the calculation accounts one well pronounced ΔK=1 transitions at low energies and small groups at higher energies around 3.25MeV in nuclei considered. The calculations showed magnetic dipole excitations in Barium nuclei much smaller distributed than well deformed nuclei.

In order to study the role of the deformation on scissor mode excitation of the $^{124\text{-}136}$Ba isotopes, we investigated dependence of the summed B(M1) values for 1^+ states on δ_2^2 [37]. In the following, the calculation and experimental observations aiming systematic description of the scissors mode features are discussed with respect to the data in Figure 5. As can be seen the deformation dependence of summed strengths for the well Barium isotopes is nearly linear on δ_2^2. However, for ^{124}Ba isotope linearity of the strengths broke. The difference between two experimental summed strengths and the calculated ones is well suited with each other.

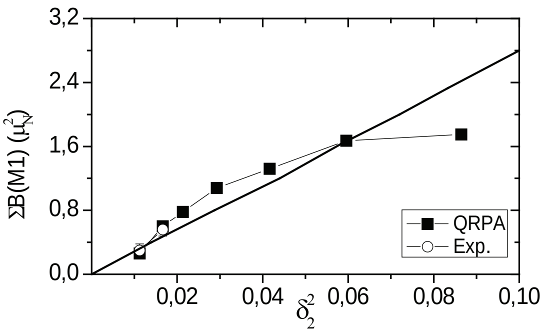

Figure 5. Summed scissors mode strength in $^{124-136}$Ba as a function of the square of the deformation parameter δ.

The extracted scissors mode strengths are shown in Figure 6 as a function of the square of the deformation parameter δ [38]. The QRPA calculations (dashed-dotted line) described above are capable to account for the B(M1) strengths in the more collective $^{122-126}$Te nuclei and slightly overestimate the experimental result for ^{130}Te. Despite the deformed single-particle basis the calculations describe the dipole strength distributions in the near closed shell nuclei ^{122,124,126,130}Te surprisingly well.

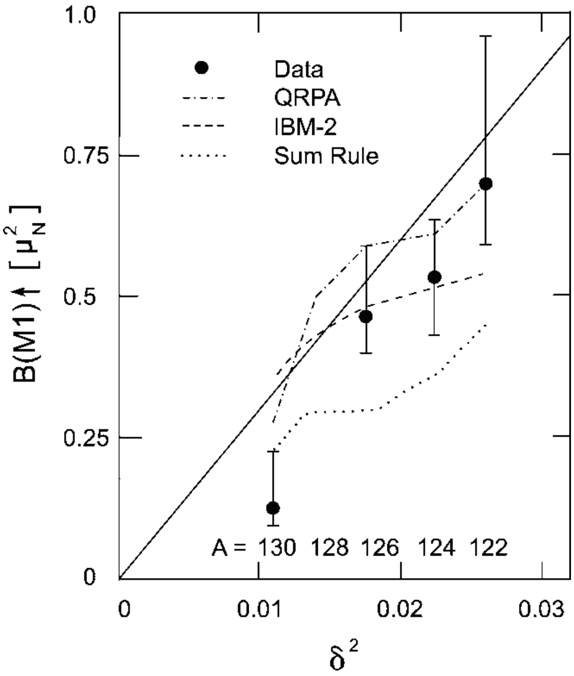

Figure 6. Summed scissors mode strength in $^{122\text{-}130}$Te as a function of the square of the deformation parameter δ.

In Figure 7, a direct comparison of the theoretical results from rotational invariant QRPA calculations (lower part) with the experimental (upper part) dipole strength distributions for the ΔK = 1 transition is given for γ-soft deformed ^{194}Pt and ^{196}Pt nuclei.

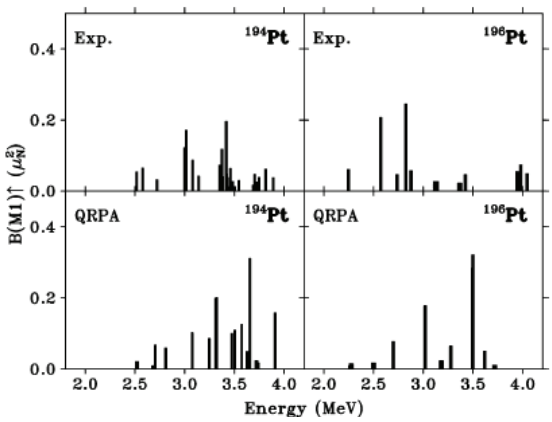

Figure 7. Comparison of the measured dipole excitation strength distribution in ^{194}Pt to the data in ^{196}Pt (upper part) and to M1strength resulting from QRPA calculations (lower part) for both nuclei as it is described in the text.

Observed 1^+ excitations in ^{194}Pt and ^{196}Pt are tentatively interpreted as the main fragments of the scissors mode based on the measured excitation strengths and our microscopic calculations [39].

In Figure 8, we compare the δ^2 dependence of the calculated B(M1) value for 1^+-states summed up to 4 MeV for the Ce, Nd and Sm isotopes.

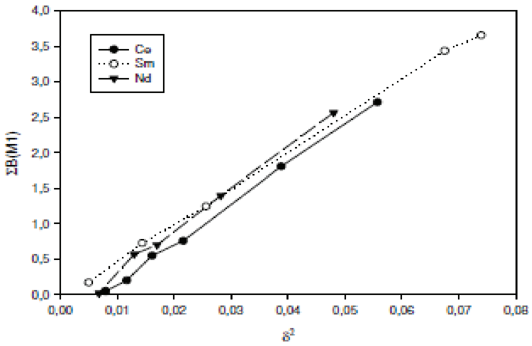

Figure 8. Dependence of the summed B(M1) values of the Ce, Nd and Sm isotopes on δ^2

As seen from Figure 8 consideration of the restoring forces reveals that summed ΣB(M1) depends on δ^2 linearly for all the isotopes under study. To obtain the nearest passed line of the theoretical points, the data is fitted and the dependence between the square of the deformation parameter and transition probability is found to be $\Sigma B(M1) = 50 \cdot \delta^2 \mu_N^2$ while for the transitional nuclei this dependence is found to be $\Sigma B(M1) = 27.8 \cdot \delta^2 \mu_N^2$. The spectra of 1^+-states for all the isotopes are similar and the states that have a high transition probability B(M1) are approximately 3 MeV.

4.4 Spin-Flip M1 Resonance

Spin-flip M1 resonance is high-energy part the Magnetic Dipole excitations, which first predicted in deformed nuclei by [5,6] and has two branches with $I^\pi K = 1^+1$ and $I^\pi K = 1^+0$. Although predicted 1^+-states in deformed nuclei more than four decades ago it still receives a strong interest in nuclear-structure physics.

At present magnetic dipole resonances ($J^\pi = 1^+$) have been found experimentally in a wide region spanning from light nuclei up to actinides [40]. These experiments show that in heavy spherical and deformed nuclei there exists a very broad Ml resonance at energies between 7 and 11 MeV centered an energy on the order of about 44 x $A^{-1/3}$ MeV. The collective spin-flip resonance (1^+-states) was observed in spherical ^{12}C at energy 15.1 MeV. Seven 1^+ states were found in the (γ, n) reaction in the energy interval 7.4-8.25 MeV in ^{208}Pb, which exhaust presumably the main oscillatory strength of Ml transitions (≈90 %). Similar states were observed in number of deformed nuclei ^{154}Sm, ^{168}Er, ^{238}U (see. [9]). The remarkable features of this mode obtained from experimental results are its independence on deformation, the quenching and a strong fragmentation in deformed and spherical nuclei. Experimental evidence for unusual fragmentation of the Ml strength in the spherical and deformed nuclei has been obtained by inelastic electron, tagged photon and threshold (γ,n) reactions. A broad bump was found in small angle inelastic (p,p') data extending from 7 to 11 MeV which was interpreted as Ml resonance [41].

The comparison of the calculated average elastic photon scattering cross section [42] with tagged photon measurements [41] is displayed below in Figure 9. Points connected by lines denote the RPA calculation and open circles are the experimental data. The experimental cross sections exhibit strong fragmentation in a wide region energy between 7 and 11 MeV. The present calculation predicts also strongly fragmented scissors mode 1^+-states up to 12 MeV in accordance with the experimental data. The theory predicts a giant spin-flip Ml resonance at energy of about 9 MeV. The group of strongly collective 1^+-states is found in the energy interval 8-10 MeV. Here, the B(M1) value may be as large as a few single-particle units. The states of this region give the main contribution to the sum rule. The calculation predicts substantially more dipole strength than is indicated by experiment for excitation energies above 8.5 MeV. However, this is largely due to the strongly decreasing g.s. branching ratio which is not taken in to account in the calculations. If one tries to correct the data for this (see [8]) a total value B(M1)=7.5±3.2 μ_N^2 is obtained in satisfactory agreement with the calculation.

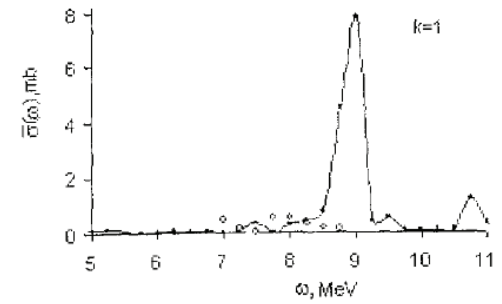

Figure 9. Tagged photon elastic scattering cross sections (in mb). Open circles are experimental data. Points connected by the lines denote the RPA calculations with ΔE=0.25 MeV.

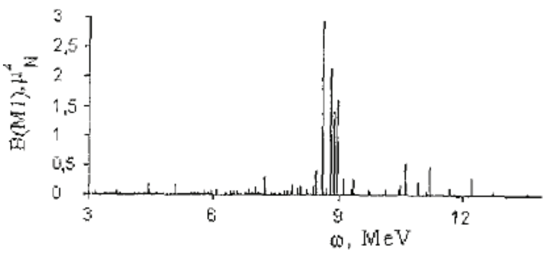

Figure 10. Fine structure of the calculated B(M1) strength distribution for $I^\pi K=1^+1$ states in ^{140}Ce

The distribution of the calculated photon elastic scattering cross-section and B(M1) strength with respect to K=1 excitations is presented in Figure 9 and Figure 10. It may be observed that the B(M1) strength is fairly strongly fragmented among the energy interval although most of the strength is concentrated around of the neutron threshold energy. A group of strongly collective 1^+ states with summed width Γ = 20.97 eV corresponding to B(Ml) = 7.97 μ_N^2 is found in the energy interval 8-9 MeV. These states give the main contribution to the resonance and they can be found in (γ,γ') and (γ,n) scattering reactions. There is a strong fragmentation of the *K* = 1 excitations at low energy and part of them at energies up to 4 MeV may be interpreted as main fragments of the scissors mode they can be found in nuclear resonance fluorescence experiments. For these 1^+ states the summed B(M1) strength is of the order of 0.6 μ_N^2 and contains about 5% of the total EWSR and NEWSR.

4.5 High Energy Part of Isoscalar Quadrupole Resonance

The properties of collective $I^\pi K=2^+1$ states in doubly even well deformed rare-earth nuclei has been investigated in the framework of the rotational invariant RPA by [8]. The calculation predicts strongly fragmented isoscalar quadrupole mode $K^\pi=1^+$-states in the excitation energy range 2-16 MeV for ^{154}Sm and ^{168}Er nuclei. The calculations show that a considerable consequence of the use of the rotational invariant model is strong fragmentation B(E2) strength in energy interval between 13 MeV and 16 MeV with summed width Γ(E2,0→2)≅300 eV. The result of the calculations show for deformed ^{154}Sm wide E2 resonance with two humps with maximums around 13 MeV and 15 MeV corresponding to K=1.

5. PROPERTIES OF ELECTRIC DIPOLE 1^- EXCITATIONS

Electric dipole vibrations in which the effective isovector interactions are responsible constitute the isovector electric dipole vibrations in medium and heavy nuclei in the energy range of 2 to 20 MeV. The electric dipole excitations have two different high energy modes. One of them Pygmy Dipole resonance (PDR) arise around below nucleon threshold energy. While second one Giant Dipole Resonance represents the coherent isovector vibration motion of proton and neutron systems as rigid bodies against each other, the common c.m. of the nucleus being at rest.

A detailed description of the use of the QRPA for the separation of the c.m. spurious1^--state in deformed nuclei was given in ref. [37]. Using effective interaction restoring translational invariant of the Hamiltonian with the dipole-dipole interaction representing the coherent isovector dipole vibrations of the proton and neutron systems against each other, while the center-of-mass (c.m.) of the nucleus being at rest.

The secular equation for the dipole1^- excitations is determined as following:

$$D(\omega_n)=\omega_i^2[M\ (1+\frac{4}{3}\chi_1(\frac{Z^2}{A^2}F_n+\frac{N^2}{A^2}F_p)-\frac{4}{3}\chi_1\omega_i^2(\frac{Z}{A}F_n-\frac{N}{A}F_p)^2]=0 \quad (5)$$

The effective charge for the E1 transitions is $e_{eff}^p = N/A$ for protons and $e_{eff}^n = -Z/A$ for neutrons [11]. Besides the transition probability, another important quantity of E1 dipole excitations is the ground-state transition width, which can be calculated with formulas:

$$\Gamma_0(E1)[meV]=0.349\cdot\omega_i^3 B(E1)\uparrow \quad (6)$$

where the excitation energy ω_i is in MeV, B(E1) in $10^{-3}\ e^2fm^2$.

5.1 Low Energy Part of the Electric Dipole Excitations

Calculation shows that, all calculated E1 excitations up to energy 4.0 MeV are single two-quasiparticle states contributing more than 99% to the wave function norm (Table 2). Such a picture is peculiar for all the three investigated actinide nuclei.

Table 2. The characteristics of several $K^\pi=1^-$-states with largest B(E1) calculated with the translational invariant Hamiltonian in ^{236}U less than 4MeV.

^{236}U			
ω_n (MeV)	$B(E1)$ ($10^{-3}\ e^2\,\mathrm{fm}^2$)	Structure $Nn_z\Lambda\Sigma$	Amplitude $\psi^i_{ss'}$
2.005	4.34	pp 521↑402↑	0.99
2.438	1.46	pp 651↓521↑	0.99
2.548	2.30	nn 752↓633↓	0.99
3.202	8.78	pp 651↓532↓	0.99
3.544	2.01	pp 633↑514↑	1.00
3.680	5.73	pp 624↓505↓	0.99

In table 2, the excitation energies, the reduced B(E1) probabilities, the single-particle asymptotic Nilsson quantum numbers (NnzΛΣ), and the quasiparticle amplitudes ψss′ are given for the ΔK = 1 branches of the E1 excitations. As seen from the table, all calculated E1 excitations are single two-quasiparticle states contributing more than 99% to the wave function norm. Such a picture is peculiar for all the three investigated actinide nuclei. Thus, the effects of the translational invariance and isolation c.m. spurious motion in the low-lying E1 dipole excitations are relatively small (admixture of the spurious state is about 1 to 2%) in nuclei and weakly independent of the mass number A. One should note that the effects of the translational invariance and isolation c.m. spurious motion in the low-lying E1 dipole excitations are relatively small (admixture of the spurious state is less than 2%) in nuclei.

Distributions of the calculated B(E1) transition strengths in the ^{238}U with respect to 1^--excitations are represented below in Figure 11.

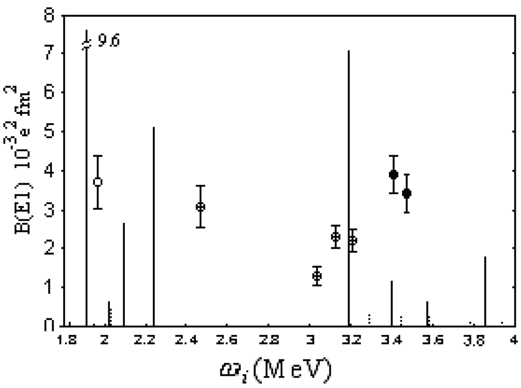

Figure 11. Comparison of the calculated *B*(E1) dipole strengths with experimentally observed ones. Calculated E1 transitions strengths with *K*=1 are shown as a solid line and _*K*=0 is a dashed line. Symbol • denotes the experimental data for E1 excitations with *K*=1. Whereas ⌽ denotes the experimental data with *K*=0 and symbol ⌽+ denotes the experimental data tentative to parity and *K* quantum number.

5.2 Pygmy Dipole Resonance

Much has been written on the deformation splitting, energy and B(E1) strength distributions of the PDR and GDR. Up to now, the important influence of spurious states on the PDR and GDR has not been examined in deformed nuclei. In connection with this, it is interesting to establish the role of the spurious RPA solutions for the PDR and GDR and investigate their effects on individual 1^- states and in E1 strength distribution. In order to determine the energy region where the admixtures of spurious state are of importance, we calculated the overlapping integrals between one-phonon states (with broken translational invariance) and the spurious state. A typical distribution of the mean squares of the overlapping integrals with respect to the energy spectrum is shown in Figure 12 for ^{238}U.

As seen from Figure 12, the effects of the translational invariance and isolation c.m. spurious motion in the low-lying E1 dipole excitations are relatively small (admixture of the spurious state is about 1 to 2% in nuclei and weakly independent of the mass number A.

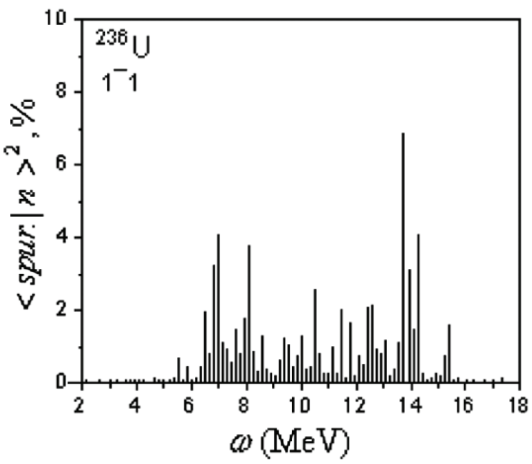

Figure 12. Distribution of the admixture of the spurious state in 1^-1 excitations calculated within the translational non invariant model in the ^{236}U nucleus.

While, the calculation shows that the effect of the spurious state rather strong for high energy part of the spectrum and does not exceed 10% in any individual 1^--state. It is found that that the main part (more than 60%) of the spurious state is spread over many levels and the larger admixtures being situated in interval between 6 MeV and 8 MeV below neutron threshold energy (PDR energy region) and GDR energy region around 14 MeV. Therefore, the effect of spurious state for the 1^--states in the energy region of the PDR and GDR should be large. It may be note that in this energy region calculation predicts spin-flip M1 resonances with K=0 and K=1. Their summed width is two order weak than the summed width of the 1^--states forming PDR at this energy region.

5.3 High Energy Part of the Electric Dipole Excitations

The effective isovector interactions are responsible constitute the GDR in medium and heavy nuclei in the energy region above nucleon threshold energy. The GDR was theoretically interpreted [2] as the isovector vibrations of the mass centers of the neutron and the proton systems against each other when the c.m. of the nucleus being at rest.

In nuclear physics, this excitation mode is well-known and it is the first collective mode investigated extensively. These states is especially interesting in the sense that it is the isovector counterpart to the spurious isoscalar c.m motion, just as the spin-flip resonance state is the isovector counterpart to the spurious rotational motion. The excitation energy of the GDR is observed to be in medium nuclei $64/A^{1/3}$ MeV and heavy nuclei $79/A^{1/3}$ MeV. Excitation energies of the GDR are well reproduced using isovector dipole-dipole forces.

In a deformed nucleus with axial symmetry, the dipole resonance is split into two components with *K=0* and 1. The experimental data confirms this result and show in the deformed ^{150}Nd nuclei wide E1 resonance with two humps with maximums in 12 MeV and 16 MeV corresponding to K=0 and K=1, respectively (see Figure 6.21 in ref [11]). The splitting between the two modes is, to leading order, proportional to the deformation:

$$E (K = l) - E(K\sim=0)\approx\bar{E}\cdot\delta \qquad (7)$$

where is the mean resonance energy.

6. CHARGE-EXCHANGE TYPE COLLECTIVE EXCITATIONS WITH 0+ AND 1+

It is well known that β-decay processes are very important to understand and the nuclear structure and nature of the weak interaction processes searching Fermi and GT transition. Beta transition process provides useful information in checking the validity of theories related to the $0\nu\beta\beta$ and $2\nu\beta\beta$, the *r*-processes, stellar collaps and supernova formation [43,44].

It has been established by now that nuclear forces are charge independent to high accuracy. The isobaric invariance is violated by the electromagnetic forces. The isospin mixing of nuclear states is basically caused by that part of the Coulomb potential which changes over the nuclear volume. Thus, isospin remains a good quantum number for the low-lying states of practically all nuclei. Quantitative estimates of the isospin admixture using single-particle and quasiparticle models are approximately an order of magnitude larger than the estimate of Bohr-Mottelson and experimental data. The residual charge-exchange interaction may cause such a difference in the mentioned estimates.

Usually, in the absence of the effective interactions the strength of the β^--transitions is concentrated in an energy region $\omega \leq 8$ MeV. When effective restoring interaction is switched on, collective 0^+ and 1^+ states at high energy are generated and they exhaust the main part(95%) of the total β^- -transition strength. The collective Fermi type 0^+ states appear at energy $\omega \approx \Delta E_{Coul.}$ and, therefore, should be coincide with the isobar analog state, as is shown in ref.[45].

The violated super-symmetry property of the pairing interaction between nucleons was restored using the Pyatov method by [43]. It is shown that the restoration of super-symmetry property of the pairing interaction is physically important in charge-exchange nuclear structure calculations. Especially the attractiveness of the restoring forces naturally affects the fixed values of the particle-hole and particle-particle constants which are very important in the two-neutrino and neutrinoless double-beta decay calculations for neutrino mass. They have shown that the isospin mixing of nuclear states is basically caused by that part of the Coulomb potential which changes over the nuclear volume. Thus, isospin remains a good quantum number for the low-lying states of all medium and heavy nuclei.

The calculated energy differences between GTR and IAR using PM have been compared with other theoretical calculations and the corresponding experimental data. It was shown that the calculated energy differences in PM were closer to the corresponding experimental data in comparison with the other calculations.

The first-forbidden transitions are a crucal test for the nuclear theory and in understanding weak interaction processes. The $0^+ \to 0^-$ first-forbidden beta decay have been investigated for some spherical nuclei by [44] and showed that the relativistic matrix elements calculated in the first approximation are 2.5 times larger than those calculated in the nonrelativistic case. The *pp* effective interaction has no significant effect on the velocity of $0^+ \to 0^-$ transitions.

7. CONCLUSIONS

To summarize, we have reported on results of the investigation of the low spin ($I \leq 2$) collective excitations strength distribution in the in heavy deformed and transitionally nuclei. It is stressed that UNM describes the β- and γ-vibrations as the oscillation of the surface of the nucleus and allows define the main component of the multipole quantum number of the surface oscillating density.

On the basis of the microscopic nuclear model it is concluded that spin-quadrupole forces may be responsible for the appearance of a new branch of collective 0^+ excitations below the energy gap. The energy of 0^+-vibrational states, the E2 transition probability and the rate of the allowed β-decay to these states significantly affected due to the spin-quadrupole force.

The properties of γ-vibrational 2^+-states are practically insensitive to the addition of this force. The spin-quadrupole force affects slightly the energy and the B(E2) value for gamma-vibrational state, while the properties of the high-lying 2^+ states are affected much more strongly. The same holds for the effects of the spin-quadrupole forces on the rate of the β-decay to the gamma-vibrational and high-lying 2^+-states. The rotational invariant QRPA causes a strong fragmentation of the $K = 1$ excitations and part of them at energies up to 4 MeV interpreted as main fragments of the scissors mode. For these 1^+ states the summed B(M1) strength is of the order of 3.0 μ_N^2 and contains about 8% of the total EWSR and NEWSR. In models with non-invariant Hamiltonian summed M1 strength exceeds the experimental values of theB(M1) more than two times. Indeed, the introduction of the restoring rotational invariance forces, essentially, reduces the B(M1) strength and increase substantially the fragmentation of the M1 strength.

The fully rotational invariant QRPA satisfactory describes of the experimental fragmentation and δ^2-dependence of the summed B(M1) value of the scissors mode in well deformed rare-earth, actinide and transitionally nuclei with moderate deformation. The agreement between the calculated excitation energies as well as the summed B(M1) values of the scissors mode and the available experimental data is quite good. For all investigated deformed nuclei the theory predicts a giant spin-flip Ml resonance at an energy interval between 8 MeV and 10 MeV.

We have investigated the dipole excitation strength distribution in less-deformed transitional nuclei, e.g., in the γ-soft nuclei $^{122\text{-}136}$Ba, $^{140\text{-}150}$Ce, $^{150\text{-}160}$Nd and ^{194,196}Pt. In all of these cases the scissors mode was observed, however, with decay properties differing considerably to the findings in well-deformed rotors because of the loss of axial symmetry and the establishment of the *d*-parity quantum number.

In translational invariant QRPA all calculated E1 excitations up to energy 4.0 MeV are single two-quasiparticle states contributing more than 99% to the wave function norm. Such a picture is peculiar for all the deformed nuclei. Rather collective states arise in PDR region below nucleon threshold energy. In deformed nuclei theory predicts 1^- giant dipole resonance splitting into two components with K=0 and K=1 at energy around 12 MeV and 16 MeV, respectively.

In odd-odd nuclei Fermi transitions and isospin mixing are affected not only by the Coulomb potential but also by the correllation caused by restoring isobaric invariance forces which gives a good agreement with the corresponding experimental data. The isotopic invariance of nuclear forces and self-consistently conditions make the theory free of any adjustable parameters. The restoration of super-symmetry property of the pairing interaction is physically important for GT-transition processes. The attractiveness of the restoring forces naturally affects the fixed values of the particle-hole and particle-particle constants. The relativistic matrix elements calculated in the first approximation for $0^+ \rightarrow 0^-$ are 2.5 times larger than those calculated in the nonrelativistic case. The *pp* effective interaction has no significant effect on the velocity of $0^+ \rightarrow 0^-$ transitions.

Acknowledgment

Discussion with Drs. E. Guliyev, F. Ertuğral, H. Yakut and Z. Zenginerler are gratefully acknowledged. I like to thank Dr. N.Çakmak for very interesting discussions and for making available their theoretical results. I am indebted to E.Tabar and H. Guliyev for their help in carrying out the numerical calculations and the preparation of the present article.

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