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A Clustering-based Simulated Annealing Algorithm with Taguchi Method for the Discrete Ordered Median Problem

Mustafa Serdar TOKSOY*[1]

Abstract

Researchers have studied discrete location problems for a long time because of their importance in practice. The Discrete Ordered Median Problem (DOMP) generalizes discrete facility location problems. The DOMP generalizes the main facility location problems' objective functions such as the p-median, p-center and p-centdian location problems. As these problems, also known as the problems of location-allocation, have NP-hard structure, it is inevitable to use heuristic methods for solution. In this study, a metaheuristic algorithmic suggestion will be put forward by examining the DOMP to find optimal solutions. For that purpose, we proposed a Simulated Annealing (SA) metaheuristic with K-means Clustering Algorithm in initialization for the DOMP. Novel approaches for initial solution and K-exchange algorithm-based neighborhoods for local search were analysed. In addition, best level of selected parameters were determined by Taguchi method. Forty common p-median instances derived from OR-LIB were used to test the SA performance, and the results were compared with three state-of-art algorithms in the literature. According to the computational results, 21 best solutions were obtained on instances despite gap values and CPU times increasing proportionally to the scale of the instances. In a conclusion, the proposed clustering-based SA algorithm is competitive and can be a robust alternative for the DOMP.

Keywords: Discrete ordered median problem, Simulated Annealing, Taguchi, K-means clustering.

1. INTRODUCTION

There is no doubt that people have been thinking about location decisions since cave life. The term of the facility is used intensively. It has a broad meaning, including many locations such as air and seaports, manufacturing centers, warehouses, retail outlets, schools, hospitals, childcare centers, bus stops, metro stations, electronic power stations, computer terminals, pluviometers, emergency warning sirens, and satellites. However, few of these items have been investigated in researches [1].

Location decision problems have a direct relationship with supply chain management and logistics. Therefore, location analysis and models have a great interest from many disciplines such as operations research and administrative science. The solution approaches of location models for optimality are generally crucial. The simplest models for large-scale problem examples are not easily obtained numerically. Formulation and

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solution of location models with numerical complexity were complicated until developing high-speed working computers.

Accordingly, the Discrete location theory (DLT) that processing in discrete optimization space covers a development period about the next thirty years with an expeditiously growing and developing literature [2]. Uncapacitated facility location problem, p-median problem, p-center problem, set covering problem, p-dispersion problem, maximal covering problem, hub location problem can be given fundamental facility location problems in DLT [1]. Especially, 4 location models considering the p-median problem, p-center problem, uncapacitated facility location problem, and quadratic assignment problem are considered more than other location problems in literature. These four fundamental problems are used for facility location selection and allocation of demand points to single or multiple facilities. Hence, these kinds of problems are primarily described as location-allocation problems. The demand points supplied from the candidate facilities are given in a network among these models. The general problem is the establishment of new facilities that will optimize different objectives. It is essential for such problems that distance or some measurements (trip time or cost, demand satisfaction) are less functional than others in terms of distance.

The p-median problem is one of the best-known location-allocation problems in the literature described initially by Hakimi [3]. The problem focuses on selecting the number of p facilities in a network covering a minimized weighted/unweighted distance to meet all demand. Several studies considering the total distance between demand points and facilities have been published since 1995. Following ten years, studies have increased on solution methods [4].

Another location problem called the p-center problem involves a number of p facilities in a network covering a weighted/unweighted maximum distance that must be minimized to meet all demand. As can be understood from the statement, p-median problem objectives minisum considers minimization of the total distance between demand points and facilities while p-center problem objectives minimax considers minimizing the maximum distance between demand points and facilities.

The discrete ordered median problem (DOMP) observed in this study is a general form of classic location facility problems. The solution of the DOMP contains possible facility points and a finite set of customers whose demands are met by these facilities, as in the case of other location problems.

The DOMP is firstly defined by Nickel [5] and then Boland et al. [6]. The objective function of the DOMP generalizes these well-known facility location problems such as median, center, and centdian (convex combination) problem functions. However, this generalization is not limited to these problems. It is possible to obtain solutions for different objective functions with a coefficient λ and an ordering factor embedded in the DOMP formulation. Therefore, the DOMP is a beneficial method for solving many location problems and having a simple condition in applying mentioned factors for problems [6].

As in many location problems, the DOMP is NP-hard and cannot obtain optimal solutions in an acceptable time for large-scale problems. Hence, heuristic algorithms have to be used in general. In recent years, it has been observed that metaheuristics that combine basic heuristics methods search solution space more effectively and have high-quality solutions commonly.

In this study, the solution of p-median, p-center, and p-centdian problems were observed in the DOMP simultaneously, and for that purpose, the Simulated Annealing (SA) metaheuristic was proposed for the DOMP. The SA is a local optimization method that solves complex combinatorial optimization problems. Studies about this topic point out that simulating of solids annealing process can be presented as a model [7]-[8] and can be proposed for optimization problems [9]. Since then, the model has been utilized in a large field, from scheduling problems to locational analysis, from molecular physics and chemistry to image processing.

Forty p-median instances between 100-900 nodes derived from OR-LIB [10] were used for the application. In the 2nd section of the study, a definition of the DOMP is available. The 3rd section presents the basic steps of the proposed SA approach. Computational evaluation in the 4th section contains parameter optimization by Taguchi experimental design method, application of the algorithm to test problems, and compared results with other benchmark solutions in the literature. The 5th section is composed of the conclusion and proposals.

2. DOMP DEFINITION

2.1. Literature Review

P-median, p-center, and p-centdian problems have been studied in detail because of their importance in practical applications by Daskin [11]; Drezner and Hamacher [1]; Mirchandani and Francis [2]. These studies include a finite set for potential location areas and another set for customers whose demand is supplied by these facilities. The focus point of these problems that have a significant number of alternative solution approaches proposed in the literature is the existence of fixed number locations have to be diversified areas in a specific set of candidates in which any customer can be supplied by only one facility. For each customer-facility pair, a substantial cost for customer demand must be met by a facility settled down in the determined area.

A remarkable attribute of DLP is the diversity of considered objective functions. The primary aim of the p-median problem is to minimize total service cost for all customers supplied by facilities located in all chosen areas. The p-center problem is a minimization of the maximum service cost of a customer from among chosen areas that covers all customers. As for the p-centdian problem, the primary aim is to obtain a convex combination of median and center problems. Like this, the minimization of both total cost and the highest cost is ensured. There are three standard objective functions observed in the literature, and there are specific solution methods and algorithm approaches for these problems [11].

Kalcsics et al. [12] defined necessity discrete location models for strategic supply chain management and submitted novel and flexible location models. For that purpose, Nickel [5] introduced a discrete ordered Weber problem that generalizes objective functions frequently in discrete location theory. The objective function of this problem includes a sanction implementation for the service cost of a customer related to a cost situation associated with other customers' service costs. For example, a different sanction could be applied in the presence of 5. highest customer service cost rather than 2. highest customer service cost. An "ordering" function affects the solution and makes formulation more interesting.

The generalized model that includes the "ordering" factor has been studied for large-scale planar and network type location problems [14]-[15]. Due to these various studies, the DOMP entered the literature, a specific formulation of discrete conditions [5]. Studies about the solution of the DOMP were proposed by Nickel [5] and Boland et al. [6]. Exact methods can be used in instances away from large-scale real-life problems.

The ordered median location problem with continuous, discrete, and networks was presented by Puerto and Rodriguez [15], which is based on a hierarchy of semidefinite programs that can approximate up to any degree of solution accuracy of any ordered median problem infinite-dimensional spaces. Another exact solution method was the column generation approach to solve the continuous relaxation of the model. Then, a Branch-Cut and Price algorithm was studied to moderate the size of the DOMP in competitive computational time [16].

A Lagrangean Relaxation was carried out on this formulation to produce lower and upper bounds on the optimal value of the DOMP, which contains p-median, p-center, and k-centrum problems through a parallelized algorithm [17]. Various Mixed Integer Linear Programming (MILP) formulations for the DOMP have been investigated in the literature [18] - [20]. Labbé [21] carried out a comprehensive study that presents several new formulations for the DOMP. A polyhedral study of assignment polytope of the

formulation was used that showed its proximity to the convex hull of the integer solutions.

Although there have been significant improvements in computational processes, none of the exact methods are sufficient for solving large-scale DOMP samples. Although some heuristic methods have been developed to solve medium or large scaled samples, their number is quite limited.

For that purpose, a robust tool called Evolution Program based on Genetic Algorithms (GA) within an acceptable time was submitted firstly by Holland [22], Davis [23], Goldberg et al. [24]. Variable Neighbourhood Search (VNS) is another metaheuristic method for combinatorial problems proposed by firstly Mladenovic [25] then Hansen and Mladenovic [26], which is a well-known technique used for discrete facility location problems and generally provides high-quality solutions. Domínguez-Marín et al. [27] developed two approximate heuristic solution methods, comprised of a GA, and an Evolution Program. Two GA with different coding schemes (binary coded HGA1 and integer coded HGA2) was proposed Stanimirovi'c et al. [28]. Later, Puerto et al. [29] proposed a modified VNS metaheuristic algorithm based on new neighborhood structures avoiding sorting in the evaluation of the objective function at each considered solution.

Recently, Olender and Ogryczak [30] developed a revised VNS called as REV-VNS outperforms the other methods, both in computing time and in solution quality. In the study, researchers introduced a regularization concept that intensifies the searching process for problems with a not strictly monotonic objective function.

However, all these algorithms still yield poor quality solutions for the p-center problem. There are also significant differences in the quality of solutions for other types of problems.

2.2. Mathematical Model

The mathematical model presented by Domínguez-Marín for the DOMP is as follows [31]: Let A denote a given set of M locations which $\{A = 1\ defines, \dots, M\}$ with i,...,M variables. Let c_{kj} denote a *M x M* non-negative and symmetric matrix in which total demand of customer *k* is supplied by facility *j* for $\left(c_{kj}\right)_{k,j=1,\dots M}$. $\mathrm{P} \leq M$ indicates a number of facilities must be located among candidate facilities. Let a solution for facility location problem be given by a set $X \subseteq A$ of candidates *N* under the condition that $|X| = \mathrm{P}$. It is assumed that each facility is uncapacitated in the study. So, supplying the fixed demand of customer k with the least cost by a facility *j* that is located in solution set X, can be formulated in Equation (1) as follows:

$$c_{kj} = c_k(x) = \overset{min}{i \in X} c_{kj} \tag{1}$$

The objective function ensures the difference of the problem from the classic uncapacitated p-median problem. To get this function, customer servicing cost $(c_1(X), \dots, c_M(X)$ is ordered in a non-decreasing array. σ_x is defined as a permutation on {1,...,*M*} for the following inequalities in Equation (2).

$$c_{\sigma_x(1)}(X) \leq c_{\sigma_x(2)}(X) \leq \cdots \leq c_{\sigma_x(M)}(X) \tag{2}$$

So, a valid permutation for *X* is searched for any permutation mentioned above. Cost vector associated with a given set of *X* and related ordered cost vector is presented in following as Equation (3) and (4):

$$c(X) = \left(c_1(X), \dots, c_M(X)\right) \tag{3}$$

$$c_{\leq}(X) = c_{\sigma_x(1)}(X) \leq \cdots \leq c_{\sigma_x(M)}(X) \tag{4}$$

Then, the objective function utilizes a linear cost function for *i.* least service cost of customer $c_{\sigma_x(i)}(X)$ for each *i=1,...,M* with the help of $\lambda_1 \geq 0$. Supposing that $\Lambda = (\lambda_1, \dots, \lambda_M)$ is a given vector will be required for different discrete facility location problems with $\lambda_i \geq 0$, *i=1,...,* M. Inclusive of all this information, the general formulation (Equation (5)) of the DOMP can be defined as follows.

$$\overset{min}{X \subseteq A, |X| = P} F_\Lambda(X) = (\Lambda, c_{\leq}(X)) = \sum_{i=1}^{M} \lambda_i c_{\sigma_x(i)}(X) \tag{5}$$

The structure of the DOMP taking into consideration formulation elements above is as follows in Equation (6).

$$P/D/\bullet/\bullet/\sum ord \quad (6)$$

P represents the number of candidate facilities; D represents a distance between facilities, and "ord" state represents the ordering factor. Different discrete problem solutions derived from the DOMP for (0-1) values of λ variable are noted in the classification schema of location problems proposed by Hamacher and Nickel [32] below.

Table 1 Classification of the DOMP problems

No	Formulation	Solution
1	$P/D/\Lambda = (1,1,,,,1,1)/\bullet/\sum ord$	p-median
2	$P/D/\Lambda = (0,0,,,,0,1)/\bullet/\sum ord$	p-center
3	$P/D/\Lambda = (\lambda,\lambda,,,,\lambda,1)/\bullet/\sum ord$	p-centdian

Definition of the DOMP enables to model for classic facility location problems. Classic facility location problems, which are specific cases of the DOMP, can be seen according to Λ vector ensures different varieties for objective functions below. Also, novel facility location problems can be solved. So that, by exploring this problem, varied objective function types can be obtained through a unique theoretical point of view (combining classic facility location problems) and the existence of real-life problems for different vector options.

$P\ /\ D\ /\ \Lambda = (1,1,\dots 1,1)/\bullet \sum_{\text{ord}} 1$resulted in the p-median problem. $P/D/\bullet/\bullet/\sum ord$ points out the problem, which minimizes the sum of costs for supplying the total demand of each customer.

$P\ /\ D\ /\ \Lambda = (0,0,\dots 0,1)/\bullet \sum ord$ resulted in the p-center problem. P/D/•/•/max points out the problem, which minimizes the maximal cost for supplying total demand among customers.

$P\ /\ D\ /\ \Lambda = (\mu,\mu,\dots,\mu,1)/\bullet \sum ord$ resulted in the μ-centdian problem for $0 < \mu < 1$. $P/D/\bullet/\bullet/CD_{\mu}$ points out the convex combination of median ve center objective functions.

2.3. Illustrative Example

In 2010, Daskin published a simple p-median example with a topological graph to illustrate the method of solving a simple p-median problem by utilizing spreadsheets (Figure 1).

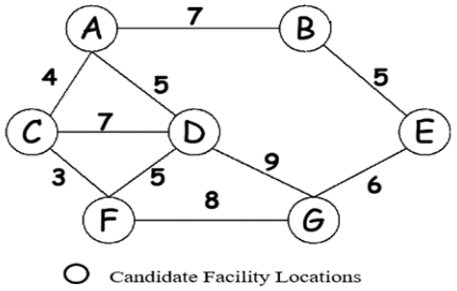

Figure 1 A graph of candidate facility locations

Let *M*= *{A,...,G}* be a set of candidate facilities in an example (Figure 1). Suppose that a decision will be made about which p=2 facilities have to be selected among them. A cost matrix is below associated with facilities and customers supplied/serviced by these facilities. Table 2 represents a distance matrix occurred by the closest distances of customers.

Table 2 Distance matrix of example for the DOMP

	A	**B**	**C**	**D**	**E**	**F**	**G**
A	0	8	5	6	13	8	15
B	8	0	12	13	6	15	12
C	5	12	0	8	17	4	12
D	6	13	8	0	16	6	10
E	13	6	17	16	0	15	7
F	8	15	4	6	15	0	8
G	15	12	12	10	7	8	0

Complete enumeration method can be used to analyse all alternative solutions by virtue of small-scale structure of the problem. For *M*=7, p=2 and C (*M*, p) =21, total, 21 number of different solutions are obtained. These solutions are ordered as $x_1 = (A,B)$, $x_2 = (A,C)$, $x_3 = (A,D)$, $x_4 = (A,E)$, $x_5 = (A,F)$, $x_6 = (A,G)$, $x_7 = (B,C)$, $x_8 = (B,D)$, $x_9 = (B,E)$, $x_{10} = (B,F)$, $x_{11} = (B,G)$, $x_{12} = (C,D)$, $x_{13} = (C,E)$, $x_{14} = (C,F)$, $x_{15} = (C,G)$, $x_{16} = (D,E)$, $x_{17} = (D,F)$, $x_{18} = (D,G)$, $x_{19} = (E,F)$, $x_{20} = (E,G)$, $x_{21} = (F,G)$.

Then, all c_{kj} variables are determined according to Equation (1) as $c_1(x_1) = 0$, $c_2(x_1) = 0$, $c_3(x_1) = 5$, $c_4(x_1) = 6$, $c_5(x_1) = 6$, $c_6(x_1) = 8$ and $c_7(x_1) = 12$ for c_{1j}. Afterwards, $c_{\leq}(A,B) = c_{\sigma_{(A,B)}(1)}(A,B) \leq .. \leq c_{\sigma_{(A,G)}(7)}(A,B)$ and $c_{\leq}(A,B) = (0,0,5,6,6,8,12)$ is obtained for $X(A,B)$ according to Equation (3) ve (4), respectively. In the p-median (2-median) solution, the vector $\Lambda = (1,1,1,1,1)$ is taken and when applied to the Equation (5), $\min_{X \subseteq A, |X| = P} F_{\Lambda}(A,B) = \Lambda, c_{\leq}(A,B)) = (1,1,1,1,1) * (0,0,5,6,6,8,12) = 37$ is calculated for $X(A,B)$.

Finally, facility C and facility E ($X(C,E)$ is determined for a 2-median optimal solution when all solutions (21) are calculated (Figure 2). The objective function value is easily obtained as 25. Facility C and facility E are supplied by themselves; facility A, facility D, and facility F are supplied by the closest selected facility C, while facility B and facility G are supplied by the facility closest selected 4.

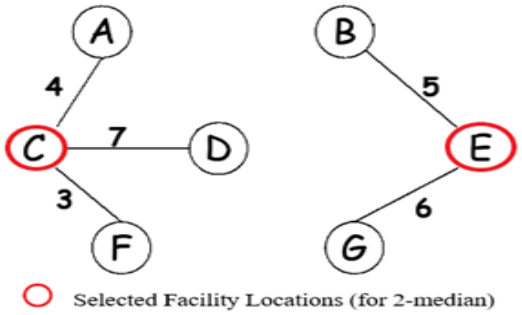

Figure 2 Solution graph of 2-median

While the same problem is analysed for the 2-center problem with $\Lambda = (0,0,0,0,1)$, an optimal solution of the problem is obtained X (1,4), and objective function value is found 12 when the solution is analysed. According to the graph, facilities 2 and 4 are supplied by themselves, facilities 3 and 5 are supplied by closest facility 1, and facility 2 is supplied by closest facility 4. If we pay attention to the solutions obtained here, it can be seen that the same solution value is obtained for p-median and p-center problems randomly, but due to the structure of $\Lambda = (\lambda_1, \ldots, \lambda_M)$ objective function values are different.

3. PROPOSED SIMULATED ANNEALING ALGORITHM

3.1. Simulated Annealing Algorithm

The Simulated Annealing (SA) is a stochastic search metaheuristic method capable of getting good solutions for combinatorial optimization problems, which is similar to the physical annealing process of the solids. SA algorithm, independent of each other, was described by Kirkpatrick et al. [7] and Cerny [8]. SA has been used to solve many combinatorial optimization problems such as traveling salesman problem, scheduling, assignment problem, network design [33] - [34].

In the SA algorithm, the control parameter is the temperature and assesses the probability of achieving a better solution for minimization problems. the SA is one of the neighborhood-based search algorithms. The descent algorithm, a simple form of neighborhood search, begins to search with an arbitrarily chosen initial solution. This solution produces a neighborhood solution through a suitable iteration mechanism, and the change in cost is calculated. If there is a cost reduction, the neighborhood solution is accepted as the new solution; otherwise, the current solution is not changed. This process continues until neighbours cannot improve the current solutions with cost improvements, and the descent algorithm ends with a local optimum.

The quality of the resolution obtained by the descent algorithm is based on the initial solution. The performance of a heuristic should not depend on only the initial solution. The initial solution should accept the bad solutions in a controlled way to reduce this dependency. To avoid the disadvantage of the descent algorithm, sometimes the neighborhood iterations leading to the raise at cost are accepted in the SA algorithm to escape the local optima traps. The acceptance or rejection of an iteration leads to an increase at cost is randomly determined in a controlled manner. The function that gives the probability of accepting the iteration leading to arise as a near as Δ in the cost function is called the accept function (Equation 7). In the following accept function, T is a control

parameter corresponding to the temperature at the physical annealing.

$$P(accept) = e^{-\left(\frac{\Delta}{T}\right)} \quad (7)$$

According to the accept function, the probability of accepting small increments in the objective function is greater than the probability of accepting large increments. Moreover, most iterations will be accepted when T is high, and if T is near zero, iterations leading to an increase in the objective function will be rejected. That is why the SA algorithm starts with a relatively high T value to prevent the solution from falling into the local optima trap. The SA algorithm continues to search by attempting a definite number of iterations at each temperature value while gradually decreases the temperature on the other hand. The parameters mentioned above are as follows:

x_{best}: The best solution for the problem.

M_{tb}: Neighbourhood solution number for each temperature level.

$T(t)$: Temperature value at t. iteration

The choice of the cooling plan has a very significant effect on the algorithm's performance. When probing specific decisions are taken, the CPU time must be used effectively, and the best solution in the SA must be closed to global optima.

The initial solution, neighborhood structure, solution space, and cost function are key factors affecting these decisions. These parameters determine the convergence speed of the SA to global optima.

Neighborhood structure must be small enough to be researched in a few of the number of iterations. Each solution in the neighborhood cluster must be reached by simple movements from each solution in the same cluster. The neighborhood solution must be produced randomly to use the run time effectively and must be chosen in which the difference between the solution and the objective function can be easily calculated.

3.2. Initialization

In this kind of classical solution technics, selecting a suitable initial population accelerates the algorithm's convergence. In the study, we used two novel initialization techniques, which are described as "sorting min to max", "the number of minimal selections" and a well-known "clustering-based solution", respectively to reach optimal/near-optimal solutions.

X_0 (I) Sorting min to max: Performs by selecting a number of *p* facilities in which the total distance of each facility to other facilities is sorted from minimum to maximum in the *M* x *M* distance matrix (see Table 3).

In Table 3, there is a list comprised of p=5 candidate facilities (②-⑦-⑨-⑬-⑳) listed according to the sum of their minimum distances (minisum) 525-642-824-953 to other all facilities (customers) (M=100).

Table 3 Sorting min to max initial solution

M	②	⑦	⑨	⑬	⑳
1	15	47	89	66	87
-	-	-	-	-	-
100	32	73	58	25	48
Total distance	525	642	724	824	953

X_0 (II) Number of minimal selections: Performs by the selection of *p* facilities that ordered minimum to maximum according to the selecting number of minimum distances of each facility to all other facilities depending on p-median values derived from *$X_0(1)$* initial solution (Table 4).

Table 4 Number of the minimal selecting initial solution

M	⑳	⑬	⑨	⑦	②
1	87	66	89	47	<u>15</u>
-	-	-	-	-	-
100	48	<u>25</u>	58	73	32
nms	5	13	22	29	31

In Table 5, the minimal distance to the first customer of the candidate facility ② is underlined as <u>15</u>, while the same measure for the candidate facility ⑬ to the hundredth customer is <u>25.</u> In this context, the total number of selection is

13 for the candidate facility ⑬ while 31 for the candidate facility ②. Ordered rows from minimal to maximum size can be seen in nms line (number of selection).

X_0(III) Clustering-based solution: Initial solution in which a number of p facility is selected by K-means clustering algorithm (KMCA).

KMCA is one of the best-known and widely used clustering algorithms in various applications, such as data mining, image processing, and machine learning. KMCA is the simplest and un tutorial clustering algorithm ever developed used to participate the given data into K number of clusters according to their characteristics or properties [35]. As a result of the clustering, although the similarity between the elements within the cluster is high, the similarity between the inter clusters is very low.

The algorithm's basic working principle is based on randomly determining K initial center points of the cluster $C_i(i = 1, \dots, K)$. For this purpose, firstly, distances of each node to cluster centers are calculated. Then, each element is assigned to the cluster at the closest distance. At the second step, the centers of the clusters are recalculated. These steps are repeated iteratively until the centers are no longer changed. Thus, conversion stability is provided. This study uses the Euclidean distance to calculate the distance between nodes and clusters. The clustering is provided by the optimization of Equation (8) given below [36].

$$minimize\ f = \sum_{i=1}^{K} \sum_{j=1, j\epsilon G_i}^{N} \left\| x_j - C_i \right\|^2 \qquad (8)$$

In Equation (8), K symbolizes the number of clusters, N symbolizes the number of nodes (vertices), x_j symbolizes the coordinate of vertex j, C_i symbolizes the coordinate of the cluster i, and G_i symbolizes the group of nodes belonging to cluster i.

The closest squared distance is determined by moving the cluster centers in space. The algorithm continuously updates the cluster center considering the assignment of all the nodes to it. Equation (9) is used for the calculation of centers as follows:

$$C_i = \frac{1}{|G_i|} \sum_{j=1, j\epsilon G_i}^{N} x_j \qquad (9)$$

The number of vertices contained in cluster i is symbolized as $|G_i|$in Equation (9).

In this study, KMCA was adopted for the DOMP. First, the K number of clusters was assumed to equal to p facilities ($K = p$). Then, all distances between the cluster centers and their nodes were calculated considering the Euclidean distance. The closed node to each cluster center was determined and accepted the facility that services other nodes (customers). Figure 3 represents the pseudo-code of KMA adopted for the DOMP.

```
Set the K (p) number of clusters center randomly;
repeat
        for each vertex do
          Calculate distance measure to each cluster;
          Assign it to the closest cluster;
        end
      Recalculate the positions of cluster centers;
until stop condition is reached;
calculate closed node for each cluster center;
accept the selected node as p-facility;
```

Figure 3 The proposed KMA pseudo code

There can be found studies in which the SA and the KMCA were used as hybrid or combined structures for different problem types in literature [37] - [43].

3.3. Local Search

After the initializing process, we developed four novel solution approaches based on K-exchange neighborhoods, one of the most widely used types of neighborhood relations, to improve the performance of the proposed SA. The algorithm improves the current solution by removing a node or nodes in the solution and replacing it with a node, not in the solution. It is implemented if an exchange of this sort can be found and improves the solution. The algorithm terminates when there is no such exchange that improves the solution.

The steps of the K-exchange algorithm is described below.

In the first step, a solution is initialized by any set of p facilities. Sites in the current solution and candidate sites not in the solution are removed in the following steps. Then, it is investigated that removing one site from the current solution and replacing it with a site that is not in the current solution will improve the objective function. If so, substitution is actualized, and the algorithm terminates. The proposed model adopted four different novel neighborhood functions for the DOMP. These functions are presented as a chromosome structure for p=5 facilities, sorted according to their total distance values.

***N(I)* Random exchange:** A non-solution random facility enters the solution in place of a random candidate in the current solution.

N(II)* Max2-exchange*: 2 non-solution random facilities enter the current solution in place of 2 facilities leading to the maximum total distance (Figure 4).

Figure 4 Max2-exchange neighborhood

In Figure 4, a solution set *X* comprised of ②, ⑨, ⑦, ⑬ and ⑳ nodes and their total distance to the other whole facilities are sorted from minimum to maximum (see Table 4). Then, nodes related to max two values (⑬ and ⑳) are exchanged with two random nodes ⑰ and ⑪ derived from other facilities in the M set.

Similar exchanges are performed according to other neighborhood conditions below.

***N(III)* Max-exchange:** A non-solution random candidate facility enters the solution in place of a facility leading to the maximum total distance.

***N(IV)* Min-exchange:** A non-solution random facility enters the solution in place of a facility leading to the minimum total distance.

***N(V)* Min2-exchange:** 2 non-solution random facilities enter the solution in place of 2 facilities leading to the minimum total distance.

The main steps of the proposed SA algorithm are as follows in Figure 5:

```
1  x0 ← Generate an initial solution by the KMCA
2  bestSolution ← x0
3  T ← maxTemperature
4  while T0 > minTemperature do
5     iter ← 0
6     while iter < maxIterations do
7           x' ←
8           Δ ← f(x') − f(xo)
9           if Δ < 0 then
10             x0 ← x
11             if f(x') < f(bestSolution) then
12                bestSolution ← x'
13          else if rand(0,1) < e^(−Δ/T) then
14             x0 ← x'
15          iter ← iter + 1
16    T ← T * u
17 return bestSolution
```

Figure 5 Pseudo-code of the proposed SA

4. COMPUTATIONAL STUDY

In the study, 40 p-median test instances [10] consisting of 100-900 nodes and 5-90 candidate solutions were used. Fundamental decisions in adopting DOMP to the SA metaheuristic were determined as; T_0= 1000-10000, M_{tb}=1-4, u = 0.999, and T=0.100 (stopping condition of the algorithm). On the condition that M defines the total number of facility M and the number of candidate facility p, two different adopted novel initial solutions x_0 and four different novel neighborhood structures (N) were proposed for the DOMP as probing specific decisions.

4.1. Data Set

The performance of the proposed SA algorithm with the Taguchi method for the DOMP was coded in MATLAB software and evaluated by the test problems [10] derived from OR-LIB literature. Hence, optimal parameter levels were observed using pmed1 data instance comprised of M=100 customers and p=5 candidate facilities. The experimental procedure was performed on

Intel(R) Core (TM) i7-6500U at 2.59 GHz with 4 gigabytes of RAM.

4.2. Parameter Tuning

Taguchi method was utilized for parameter optimization to obtain the best levels of parameters of the proposed SA in this study. Genichi Taguchi has introduced a solution called his name that will enhance realizing and evaluating experiments with his approach [44]. In this regard, it is possible to significantly reduce the number of experiments required for detailed analysis and evaluation before the experimental process. The Taguchi method is an advantageous technique for system design at high quality beyond being an experimental design technique.

The experimental design has been used to reduce variation, and Taguchi defined some criteria called signal/noise ratio as a performance criterion (Table 5) [45].

Table 5 Signal/noise ratio criterions

Target	**Signal/noise criterion**
Maximum Best	$-10log\ (\frac{\sum_{i=1}^{n}\frac{1}{y_i^2}}{n})$
Minimum Best	$-10log\ (\frac{\sum_{i}^{n} y_i^2}{n})$
Nominal Best	$-10log\ (\frac{y^{-2}}{s^2})$

The optimal parameter combination can be determined by an experimental study with the help of the various parameters and levels changing according to the structure of the problem. Several studies are available in which the Taguchi method is used for the parameter optimization for the SA algorithm [46] - [50].

In this study, four parameters with only three levels were used (neighborhood structure, initial temperature, number of solutions executed at each temperature and initial solution) as input data for Taguchi method to design an appropriate experimental pattern. Orthogonal array L9 was chosen for Taguchi method by the help of MINITAB software. Levels of parameters were determined by the help of pre-experimental studies; in other words, some of the levels were ignored which have a low significant effect on the solution quality.

To analyse the effectiveness of neighborhood structures by pre-experimental studies, five neighborhoods were investigated in preliminary studies with %100 and %20 (equal) probability, respectively. Then, the current solution was converted to the hill-climbing heuristic by dividing the P(accept) value with a big number, to reduce the probability of accepting a temporary bad solution.

According to the results of all these pre-experimental studies, obtained parameter set for the developed algorithm set is shown in Table 6 below; efficiency percentages of neighborhood structures changed as %41 N(III), %21 N(I), %19 N(II), %11 N(IV) and % 8 N(V).

Table 6 Efficiency rates of neighborhood structures

N(I)	***N(II)***	***N(III)***	***N(IV)***	***N(V)***
0 - 0.21	0.21 - 0.40	0.40 - 0.81	0.81 - 0.92	0.92 -1

In this respect, two neighborhood structures were ignored. N (4) Min-exchange and N (5) Min2-exchange with %19 and %11 efficiency percentages, respectively, were evaluated as low efficient according to other neighborhoods. All parameters and levels chosen for the proposed SA were presented in Table 7.

Table 7 Parameters and levels of the proposed SA

Symbol	**Parameter**	**Level I**	**Level II**	**Level III**
A	Neighborhood	*N(I)*	*N(II)*	*N(III)*
B	Initial temperature	*900°*	*600°*	*300°*
C	Repetition	*1*	*2*	*3*
D	Initial solution	$x_0(I)$	$x_0(II)$	$x_0(III)$

In this way, orthogonal array L9 (3^4) was utilized as the experimental pattern, and four trials were carried out for each case. Orthogonal array of parameter levels, S/N ratios and mean values obtained by MINITAB is given in Table 8.

Table 8 Orthogonal array and obtained data

Standard Array	A	B	C	D	S/N ratio	Mean
1	1	1	1	1	-75.8922	6231.25
2	1	2	2	2	-75.7373	6121.50
3	1	3	3	3	-75.5081	5961.79
4	2	1	2	3	-75.6856	6084.92
5	2	2	3	1	-75.6733	6076.50
6	2	3	1	2	-75.7708	6145.25
7	3	1	3	2	-75.7204	6109.50
8	3	2	1	3	-75.6415	6054.33
9	3	3	2	1	-75.6597	6067.00

The obtained mean S/N ratio plot for each level of the parameters was given in Figure 6. Note that the pmed-1 instance was used to determine the optimal parameter combination.

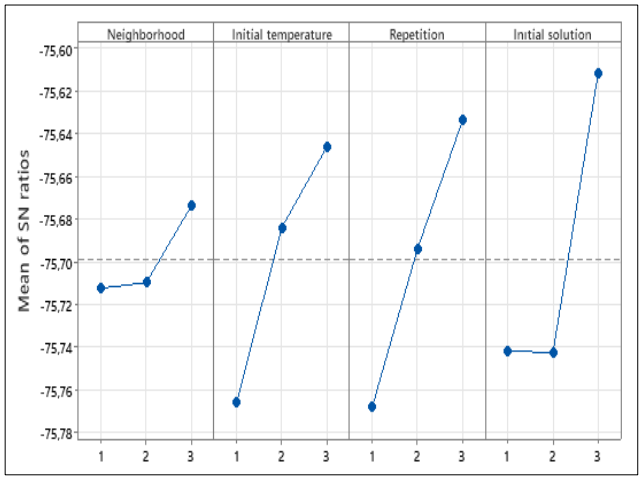

Figure 6 The mean S/N ratio plot for each level of the parameters (Alpha = 0.5)

According to Figure 6; the best level for the local search was obtained as N (3) Max-exchange, the best level for initial solutions was obtained as x_0(III) clustering-based solution, the best level for neighborhood solution number for each temperature (M_{tb}) was obtained as 3, and initial temperature (T_0) was obtained as 300^o.

It does not mean that N (I) Max2-exchange and N(II) Random-exchange does not affect the solution quality as much as N (III). Similar to the parameter set by pre-experimental studies, the max-exchange heuristic has the highest percentage value for neighborhood functions. Table 9 shows the best levels of parameters obtained with the help of the Taguchi method.

Table 9 The best levels of the parameters

N	x_0	M_{tb}	T_0
N(III)	$x_0(III)$	*3*	300^0

4.3. Computational Results

In this study, the proposed clustering-based SA algorithm was implemented in MATLAB software, and the performance of the algorithm was tested on the p-median test problems (Beasley) derived from OR-LIB [10]. For this purpose, 20 trials were run for each problem. The results of test problems were given in Table 10, with the benchmark results of the DOMP.

Table 10 includes the instance name, name of the problem, the number of facilities (M), number of facilities to supply others (p), best-known solutions (BKS), gap value (objective function value - BKS)/BKS × 100, processing times in seconds, respectively. The proposed clustering-based SA algorithm is compared with the results three state-of-the-art algorithms: Evolution Program (EP) [27] based on a GA to solve optimization problems, hybrid of a GA and a generalization of the well-known Fast Interchange heuristic (HGA1) [28] and Revised Variable Neighbourhood Search (REV-VNS) [30] in which a regularization concept that intensifies the searching process for problems with a not strictly monotonic objective function was introduced.

Table 10 The benchmarking results of algorithms for the DOMP

Instance	M	p	BKS	EP			HGA1			REV-VNS			Proposed SA		
				Best	Gap (%)	CPU	Best	Gap (%)	CPU	Best	Gap (%)	CPU	Best	Gap (%)	CPU
Pmed1	100	5	5819	**5819**	0.00	25.42	**5819**	0.00	0.93	**5819**	0.00	0.07	**5819**	0.00	11.85
Pmed2		10	4093	**4093**	0.00	37.55	**4093**	0.00	1.17	**4093**	0.00	0.19	**4093**	0.00	17.47
Pmed3		10	4250	**4250**	0.00	37.88	**4250**	0.00	1.08	**4250**	0.00	0.17	**4250**	0.00	17.60
Pmed4		20	3034	3046	0.40	61.48	**3034**	0.00	1.62	**3034**	0.00	0.54	**3034**	0.00	28.59
Pmed5		33	1355	1361	0.44	93.22	**1355**	0.00	1.91	**1355**	0.00	0.65	**1355**	0.00	43.17

Pmed6		5	7824	**7824**	0.00	36.25	**7824**	0.00	2.38	**7824**	0.00	0.23	**7824**	0.00	17.20
Pmed7		10	5631	5645	0.25	55.39	**5631**	0.00	3.33	**5631**	0.00	0.59	**5631**	0.00	26.26
Pmed8	200	20	4445	4465	0.45	91.81	**4445**	0.00	4.29	**4445**	0.00	1.79	**4445**	0.00	43.42
Pmed9		40	2734	2762	1.02	170.25	**2734**	0.00	4.64	**2734**	0.00	3.16	2745	0.40	79.65
Pmed10		67	1255	1277	1.75	290.53	1259	0.32	6.29	**1255**	0.00	3.05	1265	0.79	81.04
Pmed11		5	7696	**7696**	0.00	47.98	**7696**	0.00	4.16	**7696**	0.00	0.57	**7696**	0.00	22.18
Pmed12		10	6634	**6634**	0.00	75.63	**6634**	0.00	7.36	**6634**	0.00	1.33	**6634**	0.00	36.69
Pmed13	300	30	4374	4432	1.33	193.22	**4374**	0.00	7.55	**4374**	0.00	7.07	4386	0.26	91.84
Pmed14		60	2968	2997	0.98	359.58	2969	0.03	11.89	**2968**	0.00	11.59	2979	0.37	169.97
Pmed15		100	1729	1749	1.16	580.98	1736	0.40	15.29	1730	0.06	10.01	1739	0.56	271.46
Pmed16		5	8162	8183	0.26	56.89	**8162**	0.00	7.77	**8162**	0.00	1.10	**8162**	0.00	28.18
Pmed17		10	6999	**6999**	0.00	95.08	**6999**	0.00	14.13	**6999**	0.00	2.10	**6999**	0.00	47.42
Pmed18	400	40	4804	4880	1.48	320.38	**4809**	0.00	13.04	**4809**	0.00	15.21	4826	0.45	153.17
Pmed19		80	2845	2891	1.62	604.36	2851	0.21	26.59	2846	0.04	20.66	2862	0.59	287.42
Pmed20		133	1789	1832	2.40	963.44	1794	0.28	32.03	**1789**	0.00	29.98	1809	1.10	454.78
Pmed21		5	9138	**9138**	0.00	70.14	**9138**	0.00	8.72	**9138**	0.00	1.32	**9138**	0.00	34.49
Pmed22		10	8579	8669	1.05	116.59	**8579**	0.00	20.5	**8579**	0.00	3.67	**8579**	0.00	59.20
Pmed23	500	50	4619	4651	0.69	486.08	4624	0.11	23.55	**4619**	0.00	25.45	4631	0.26	233.86
Pmed24		100	2961	3009	1.62	924.66	2966	0.17	50.86	**2961**	0.00	43.25	2983	0.72	445.09
Pmed25		167	1828	1890	3.39	1484.13	1838	0.55	71.56	**1828**	0.00	56.14	1857	1.54	708.71
Pmed26		5	9917	9919	0.02	84.34	**9917**	0.00	14.25	**9917**	0.00	2.15	**9917**	0.00	42.56
Pmed27		10	8307	8330	0.28	136.53	**8307**	0.00	22.73	**8307**	0.00	5.42	**8307**	0.00	69.30
Pmed28	600	60	4498	4573	1.67	673.30	4500	0.04	42.87	**4498**	0.00	46.26	4530	0.71	329.32
Pmed29		120	3033	3099	2.18	1268.89	3036	0.10	85.06	3034	0.03	72.89	3050	0.56	618.06
Pmed30		200	1989	2036	2.36	2043.33	2008	0.96	110.94	1992	0.15	73.35	2015	1.31	925.05
Pmed31		5	10086	**10086**	0.00	92.67	**10086**	0.00	14.73	**10086**	0.00	2.55	**10086**	0.00	44.41
Pmed32		10	9297	9319	0.24	156.50	**9297**	0.00	34.05	**9297**	0.00	6.19	**9297**	0.00	77.77
Pmed33	700	70	4700	4781	1.72	894.19	4719	0.40	60.09	**4700**	0.00	65.46	4734	0.72	418.23
Pmed34		140	3013	3100	2.89	1762.69	3027	0.46	135.27	3016	0.10	131.11	3051	1.24	824.26
Pmed35		5	10400	**10400**	0.00	109.86	**10400**	0.00	19.44	**1040**	0.00	3.62	**10400**	0.00	53.00
Pmed36	800	10	9934	9947	0.13	182.06	9951	0.17	36.2	**9934**	0.00	9.15	9945	0.11	89.61
Pmed37		80	5057	5126	1.36	1190.25	5063	0.12	70.77	5058	0.02	110.56	5099	0.83	557.03
Pmed38		5	11060	**11060**	0.00	120.14	**11060**	0.00	27.13	**11060**	0.00	4.88	**11060**	0.00	59.65
Pmed39	900	10	9423	**9423**	0.00	207.75	**9423**	0.00	38.73	**9423**	0.00	9.40	**9423**	0.00	101.35
Pmed40		90	5128	5188	1.17	1492.59	5133	0.10	213.39	5131	0.06	158.68	5167	0.75	734.58
Average				**564.5**	**0.86**	**442.35**	**5538.50**	**0.11**	**31.71**	**5301.63**	**0.01**	**23.54**	**5545.55**	**0.33**	**208.87**
Number of best solutions				**12**			**24**			**33**			**21**		

When the best results are examined on 40 Beasley test instances in Table 10; it can be seen that the proposed clustering-based SA algorithm achieves the best result in a total of 21 instances (Pmed1, Pmed2, Pmed3, Pmed4, Pmed5, Pmed6, Pmed7, Pmed8, Pmed11, Pmed12, Pmed16, Pmed17, Pmed21, Pmed22, Pmed26, Pmed27, Pmed31, Pmed32, Pmed35, Pmed38, and Pmed39) while the other results are closed optimal solutions. Moreover, the average gap value was obtained as 0.33, and the average CPU was obtained as 208.87 seconds.

While considering average gap values of the comparison algorithms; it is observed that the proposed clustering-based SA (0.33) gives a better result than the EA (0.86) with 12 best solutions, relatively close to HGGA1 (0.11) with 24 best solutions and falls behind the REV-VNS (0.01) with 33 best solution and ranks as third. According to CPU times, the proposed clustering-based SA (208.87) ranks third again among EP (442.35), HGA1 (31.71), and REV-VNS (23.54).

5. CONCLUSION

It is seen that the location theory, which constitutes the basis of this study and especially the discrete location problems which have been studied extensively by the researchers as day-to-day and different studies will continue to increase. Therefore, the researchers have received much attention to developing a standard solution for these kinds of problems. The DOMP, which has an NP-hard structure, ensures effective and quick solutions for such problems in a single formulation. Already the studies focus on the DOMP are recent and insufficient.

This paper proposed a clustering-based SA metaheuristic, which has not been tried to solve the DOMP. The performance of the SA was compared with the p-median solutions of three state-of-the-art algorithms developed for the DOMP. Although the SA is a relatively simple and easy-to-apply metaheuristic compared to other algorithms, it is observed that it can achieve effective results for the DOMP. In order to improve the performance of the algorithm, novel initial solution methods and neighborhood structures were analysed. The KMCA was adapted to strengthen the initial solution. Moreover, The Taguchi method is utilised as a parameter tuning tool to determine the best levels of parameters used in the SA.

As a result, we developed a clustering-based SA algorithm was developed in this study. The algorithm is capable to be used in combination with other methods. According to the results, although the relatively high processing times are remarkable, the proposed algorithm is competitive and can be a robust alternative for the DOMP. Furthermore, it can be used to solve different location problems (not only p-median, p-center, and p-centdian) that are combined on a single model, simultaneously. Future work can be summarized as below:

The performance of the algorithm can be improved with other heuristic/metaheuristic algorithms in hybrid or combined structure. Neighborhood structures can be developed to take into account the processing times. Problems can be combined into a multi-objective decision-making problem. Besides, the proposed algorithm can be adapted to new models for different applications of location analysis.

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This study does not require ethics committee permission or any special permission.

The Declaration of Research and Publication Ethics

The author of the paper declare that they comply with the scientific, ethical and quotation rules of SAUJS in all processes of the paper and that they do not make any falsification on the data collected. In addition, they declare that Sakarya University Journal of Science and its editorial board have no responsibility for any ethical violations that may be encountered, and that this study has not been evaluated in any academic publication environment other than Sakarya University Journal of Science.

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