New Theory Journal of
ISSN: 2149-1402

38 (2022) 42-51
Journal of New Theory
https://dergipark.org.tr/en/pub/jnt
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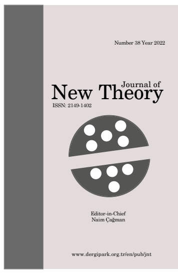

Fixed Soft Points on Parametric Soft Metric Spaces

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Article History
Received: 24 Jan 2022
Accepted: 17 Mar 2022
Published: 31 Mar 2022
10.53570/jnt.1062268
Research Article

Abstract —This manuscript is devoted to investigating the existence of fixed soft points under conditions in the parametric soft metric spaces. Since the parametric soft metric spaces are the parametric expansions of the parametric metric and soft metric spaces, the observations of the fixed-point results are meaningful to consider in such spaces.

Keywords – *Soft set, parametric soft metric, self-soft mapping, fixed-point theory.*

Mathematics Subject Classification (2020) – 54E35, 54H25

1. Introduction

Investigations on indicating the existence and uniqueness of fixed points of self-mappings have several applications in mathematics, economics, engineering, and statistics. In the mathematical aspect, fixed point theory is worth investigating by its applicability in various problems that consist of differential and integral equations, approximations, games, and so on. For these reasons, to determine the existence and uniqueness of (common) fixed points and coincident fixed points in different types of metric spaces, the researchers working in the different branches of mathematics pay attention.

The notion of a parametric metric space is defined by Hussain et al. [1]. According to this definition, the distance between the points of the space takes values according to the parameters. Since the measurement tool depends on the parameters in these spaces, it can be thought as the parameterized extension of the classical metric. This idea has taken attention by several authors and applied to the weak and strong forms of the metric spaces. Rao et al. [2] presented parametric S-metric spaces and proved common fixed-point theorems in parametric S-metric spaces. Later, Çetkin [3] introduced the concept of parametric 2-metric spaces and investigated some of their characteristics and fixed-point results. Different versions of parametric metric spaces and investigations on fixed points of the proposed spaces have been considered by several authors [4-8]. Besides, the notion of soft metric spaces introduced by Das and Samanta [9] is one of the generalizations of metric spaces based on the parameterization tool. In fact, soft metric is a crisp distance function which measures the distance between two soft points in any soft universe. That is, the parameterization tool is used just for the points of the soft universe. Nowadays, research on soft metric spaces and their fixed-point theorems is prevalent. By expanding the role of the parameterization tool in the parametric metric spaces and soft metric spaces, Tunçay and Çetkin [10] defined the concept of a parametric soft metric space and observed the basic

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features of these spaces. Hence, by using the parametric soft metric, one measures the distance between two soft points according to some parameters chosen in the soft universe.

This study aims to show the existence and uniqueness of fixed soft points and common fixed soft points of self-soft mappings described on a (complete) parametric soft metric space. Hence, by providing the parametrized distance between two soft points, where the parameters are soft real numbers, we observe extensions of some fixed-point results in the proposed spaces. This study is arranged in the following manner. In section 2, we recall some basic notions and notations necessary for the main sections. In Section 3, we discuss the existence and uniqueness of fixed soft points in parametric soft metric spaces. In section 4, we show under what conditions two self-soft mappings have common fixed soft points.

2. Preliminaries

This section mentions the concepts and the findings that we need to understand in this manuscript.

Definition 2.1. [11] Let X denote the nonempty universal set, and E represent the set of parameters. Then, the mapping $F: E \to \mathcal{P}(X)$ is called a soft set over the universe X, denoted by the pair (F, E). The collection of all soft sets over the universe X, with the set of parameters E, is represented by $SS(X, E)$.

Definition 2.2.[12] Let (F, E) and (G, E) be two soft sets over X. Then, the set operations for soft sets are defined as follows:

(1) (F, E) is a soft subset of (G, E) and write $(F, E) \sqsubseteq (G, E)$ if $F(e) \subseteq G(e)$, for each $e \in E$.

(2) the union of (F, E) and (G, E) is a soft set $(H, E) = (F, E) \sqcup (G, E)$, where $H(e) = F(e) \cup G(e)$, for all $e \in E$.

(3) the intersection of (F, E) and (G, E) is a soft set $(K, E) = (F, E) \sqcap (G, E)$, where $H(e) = F(e) \cap G(e)$, for all $e \in E$.

(4) (F, E) is called an absolute soft set denoted by $\tilde{X}$, if $F(e) = X$, for all $e \in E$.

(5) (F, E) is called a null soft set denoted by $\tilde{\emptyset}$, if $F(e) = \emptyset$, for all $e \in E$.

Definition 2.3. [13] A soft set (F, E) is called a soft real number if the mapping F is a parameterized family of nonempty bounded subsets of the real line, i.e., $F: E \to \mathcal{B}(\mathbb{R})$. For simplicity, soft real numbers are denoted by the symbols $\tilde{r}, \tilde{s}, \tilde{t}$ and the constant soft real numbers are denoted by $\bar{r}, \bar{s}, \bar{t}$. For instance, $\bar{0}$ represents the zero soft real number which means that $\bar{0}(e) = 0$, for all $e \in E$. Moreover, $\mathbb{R}(E)^*$ denotes the collection of non-negative soft real numbers.

Definition 2.4. [13] The pair $(\mathbb{R}(E)^*, \leq)$ is a partially ordered set. Here, the order " $\leq$ " is the natural order of reals over the parameters.

Definition 2.5. [9]

(1) A soft point over X is a soft set (P, E) if there is exactly one $\lambda \in E$ such that

$$P: E \to \mathcal{P}(X);\ P(e) = \begin{cases} \{x\}, & if\ e = \lambda \\ \emptyset, & if\ e \neq \lambda \end{cases}$$

Therefore, it is indicated by the symbol P_λ^x.

(2) If $P(\lambda) = \{x\} \subseteq F(\lambda)$, then $P_\lambda^x \tilde{\in} (F, E)$.

(3) $P_\lambda^x = P_\mu^y$ if and only if $\lambda = \mu$ and $x = y$. Thus, $P_\lambda^x \neq P_\mu^y$ if and only if $\lambda \neq \mu$ or $x \neq y$.

The notation $SP(\tilde{X})$ indicates the family of all soft points of the universe $\tilde{X}$.

Definition 2.6. [13] Let $\mathcal{S}$ denote a family of soft points. Then, the induced soft set by taking off all collection elements is symbolized by $SS(\mathcal{S})$. Besides, the notation $SP(F,E)$ represents the family of all soft points of the soft set (F,E).

Definition 2.7. [14] Let $(F,E_1) \in SS(X,E_1)$ and $(G,E_2) \in SS(Y,E_2)$. Then, the pair $(\varphi,\psi) \coloneqq \varphi_\psi: SS(X,E_1) \to SS(Y,E_2)$ is called a soft mapping. Here, $\varphi: X \to Y$ and $\psi: E_1 \to E_2$ are the crisp functions. In this case, the image and the preimage of the soft sets (F,E_1) and (G,E_2) under the soft mapping φ_ψ are also soft sets which are defined as follows, respectively.

$$\varphi_\psi\big((\mathrm{F},\mathrm{E}_1)\big)(\mathrm{k}) = \bigcup_{e\in\psi^{-1}(k)} \varphi(F(e)),\forall\, k \in E_2$$

and

$$\varphi_\psi^{-1}\big((\mathrm{G},\mathrm{E}_2)\big)(\mathrm{e}) = \varphi^{-1}\Big(G\big(\psi(e)\big)\Big),\forall\, e \in E_1$$

If φ and ψ are both injective (surjective), then the soft mapping φ_ψ is said to be injective (surjective).

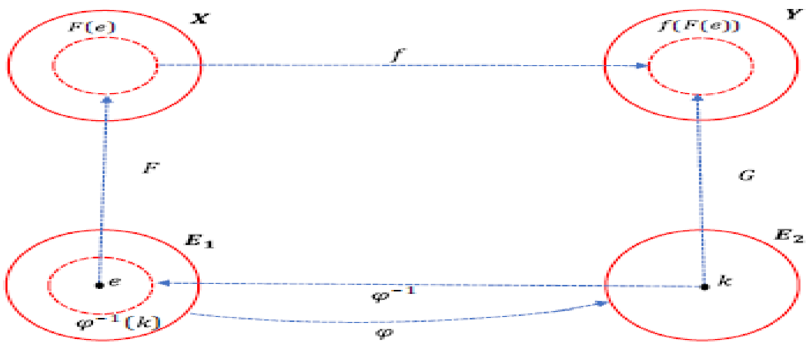

Figure 1. Graphical representation of a soft mapping

Definition 2.8. [10] A parametric soft metric on $\tilde{X}$ is a mapping $d: SP(\tilde{X}) \times SP(\tilde{X}) \times \mathbb{R}(E)^* \to \mathbb{R}(E)^*$ which satisfies the following axioms.

(P1) $d(P_\lambda^x, P_\mu^y, \bar{t}) = \bar{0}$, for all $\bar{t} \gtrsim \bar{0}$ if and only if $P_\lambda^x = P_\mu^y$.

(P2) $d(P_\lambda^x, P_\mu^y, \bar{t}) = d(P_\mu^y, P_\lambda^x, \bar{t})$, for all $\bar{t} \gtrsim \bar{0}$.

(P3) $d(P_\lambda^x, P_\mu^y, \bar{t}) \lessapprox d(P_\lambda^x, P_\gamma^z, \bar{t}) + d(P_\gamma^z, P_\mu^y, \bar{t})$, for all $P_\lambda^x, P_\mu^y, P_\gamma^z \tilde{\in} SP(\tilde{X})$ and all $\bar{t} \gtrsim \bar{0}$.

In this case, $\tilde{X}$ is said to be a parametric soft metric space and represented by the pair $(\tilde{X}, d)$.

If the parameter set is one-pointed, then this definition turns to the original parametric metric definition of Hussain et al. [1]. Besides, in a case when one considers the interval $(0,\infty)$ instead of $\mathbb{R}(E)^*$, then Definition 2.8 coincides with the definition of Bhardwaj et al. [8]. If one also considers the parameter set is one-pointed for the parameters only, then Definition 2.8 coincides with the soft metric definition of Das and Samanta [9]. Hence, our definition can be thought as the parametric extension of the parametric metric, soft metric and soft parametric metric, since both of the points and the parameters of the distance function are all have softness.

Example 2.9.[10] Define a mapping $d: SP(\tilde{X}) \times SP(\tilde{X}) \times \mathbb{R}(E)^* \to \mathbb{R}(E)^*$ by follows: for all $\bar{t} \gtrsim \bar{0}$

$$d(P_\lambda^x, P_\mu^y, \bar{t}) = \begin{cases} \bar{1}, & P_\lambda^x \neq P_\mu^y \\ \bar{0}, & P_\lambda^x = P_\mu^y \end{cases}$$

Therefore, d is a parametric soft metric over $\tilde{X}$.

Example 2.10. [10] Let $\mathbb{R}$ be the set of all reals and define a mapping $d: SP(\widetilde{\mathbb{R}}) \times SP(\widetilde{\mathbb{R}}) \times \mathbb{R}(E)^* \to \mathbb{R}(E)^*$ by follows: for each $\bar{t} > \bar{0}$,

$$d\,(P_\lambda^x, P_\mu^y, \bar{t}) = \bar{t}\big[|\bar{x} - \bar{y}| + |\bar{\lambda} - \bar{\mu}|\big]$$

Then, d is a parametric soft metric over $\widetilde{\mathbb{R}}$.

Definition 2.11.[10] Let $(\tilde{X}, d)$ be a parametric soft metric space.

(1) If $\lim_{n\to\infty} d(P_{\lambda_n}^{x_n}, P_\lambda^x, \bar{t}) = \bar{0}$, for all $\bar{t} \gtrsim \bar{0}$, then the sequence of soft points $\left\{P_{\lambda_n}^{x_n}\right\}_{n\in N}$ is convergent to a soft point $P_\lambda^x \widetilde{\in} SP(\tilde{X})$. This is denoted by $P_{\lambda_n}^{x_n} \to P_\lambda^x$ or $\lim_{n\to\infty} P_{\lambda_n}^{x_n} = P_\lambda^x$.

(2) If $\lim_{n,m\to\infty} d(P_{\lambda_n}^{x_n}, P_{\lambda_m}^{x_m}, \bar{t}) = \bar{0}$, for each $\bar{t} \gtrsim \bar{0}$, then the sequence of soft points $\left\{P_{\lambda_n}^{x_n}\right\}_{n\in N}$ satisfies the Cauchy property.

(3) If each sequence of soft points that satisfies the Cauchy property is convergent to some point in the given space, then the space is called complete.

Definition 2.12. [10] Let $\varphi_\psi: (\tilde{X}, d_1) \to (\tilde{Y}, d_2)$ be a soft mapping between parametric soft metric spaces. Then, the continuity of φ_ψ at P_λ^x in $\tilde{X}$, is described sequentially in the following manner:

if for any sequence $\left\{P_{\lambda_n}^{x_n}\right\}_{n\in N}$ such that $\lim_{n\to\infty} P_{\lambda_n}^{x_n} = P_\lambda^x$, then $\lim_{n\to\infty} \varphi_\psi\,(P_{\lambda_n}^{x_n}) = \varphi_\psi(P_\lambda^x)$.

3. Main Results

This section is devoted to investigating the existence and the uniqueness of the (common) fixed soft points of self-soft mappings in the parametric soft metric spaces.

Lemma 3.1. Let $(\tilde{X}, d)$ be a parametric soft metric space. Then, the sequence of soft points $\left\{P_{\lambda_n}^{x_n}\right\}_{n\in N}$ satisfies the Cauchy property if the following equality is satisfied for all $\bar{k} \widetilde{\in} [\bar{0}, \bar{1})$ and $n \in \mathbb{N}$.

$$d\left(P_{\lambda_n}^{x_n}, P_{\lambda_{n+1}}^{x_{n+1}}, \bar{t}\right) \lesseqqgtr \bar{k}.\, d\left(P_{\lambda_{n-1}}^{x_{n-1}}, P_{\lambda_n}^{x_n}, \bar{t}\right) \tag{1}$$

Proof. Let $m > \text{n} \geq 1$ be chosen. Then, it implies that the following for all $\bar{t} \gtrsim \bar{0}$,

$$\begin{aligned} d\left(P_{\lambda_n}^{x_n}, P_{\lambda_m}^{x_m}, \bar{t}\right) &\lesseqqgtr d\left(P_{\lambda_n}^{x_n}, P_{\lambda_{n+1}}^{x_{n+1}}, \bar{t}\right) + d\left(P_{\lambda_{n+1}}^{x_{n+1}}, P_{\lambda_{n+2}}^{x_{n+2}}, \bar{t}\right) + \cdots + d\left(P_{\lambda_{m-1}}^{x_{m-1}}, P_{\lambda_m}^{x_m}, \bar{t}\right) \\ &\lesseqqgtr \left(\bar{k}^n + \bar{k}^{n+1} + \cdots + \bar{k}^{m-1}\right) d\left(P_{\lambda_0}^{x_0}, P_{\lambda_1}^{x_1}, \bar{t}\right) \\ &\lesseqqgtr \frac{\bar{k}^n}{\bar{1} - \bar{k}}\, d\left(P_{\lambda_0}^{x_0}, P_{\lambda_1}^{x_1}, \bar{t}\right) \end{aligned} \tag{2}$$

Assume that $d(P_{\lambda_0}^{x_0}, P_{\lambda_1}^{x_1}, \bar{t}) \gtrsim \bar{0}$. Since $\bar{k} \lesssim \bar{1}$, if one taken limit as $m, n \to +\infty$ in the previous inequality, then the following is gained

$$\lim_{n,m\to\infty} d(P_{\lambda_n}^{x_n}, P_{\lambda_m}^{x_m}, \bar{t}) = \bar{0} \tag{3}$$

As a result, $\left\{P_{\lambda_n}^{x_n}\right\}_{n\in N}$ satisfies the Cauchy property. Also, in case $d(P_{\lambda_0}^{x_0}, P_{\lambda_1}^{x_1}, \bar{t}) = \bar{0}$, we have $d\left(P_{\lambda_n}^{x_n}, P_{\lambda_m}^{x_m}, \bar{t}\right) = \bar{0}$, for all $m > n$ and hence the result is clear.

Theorem 3.2. Let $(\tilde{X}, d)$ be a complete parametric soft metric space and $\varphi_\psi : (\tilde{X}, d) \to (\tilde{X}, d)$ be a surjective soft self-mapping. If there exist non-negative soft real numbers such that $\bar{\alpha} + \bar{\beta} + \bar{\gamma} \tilde{>} \bar{1}$ satisfying the following for all $P_\lambda^x, P_\mu^y \tilde{\in} SP(\tilde{X})$ and for all $\bar{t} \tilde{>} \bar{0}$.

$$d(\varphi_\psi (P_\lambda^x), \varphi_\psi (P_\mu^y), \bar{t}) \tilde{\geq} \bar{\alpha}\, d(P_\lambda^x, P_\mu^y, \bar{t}) + \bar{\beta}\, d(P_\lambda^x, \varphi_\psi (P_\lambda^x), \bar{t}) + \bar{\gamma}\, d(P_\mu^y, \varphi_\psi (P_\mu^y), \bar{t}) \tag{4}$$

then there exists a fixed soft point of φ_ψ .

Proof. By the assumptions, it is evident that φ_ψ is an injective soft mapping. Let δ_ρ denote the inverse mapping of φ_ψ , for simplicity. Choose $P_{\lambda_0}^{x_o} \tilde{\in} SP(\tilde{X})$, and set a sequence as follows:

$$P_{\lambda_1}^{x_1} = \delta_\rho \left(P_{\lambda_0}^{x_0}\right),\ P_{\lambda_2}^{x_2} = \delta_\rho \left(P_{\lambda_1}^{x_1}\right) = \delta_\rho^2 \left(P_{\lambda_0}^{x_0}\right), \cdots, P_{\lambda_{n+1}}^{x_{n+1}} = \delta_\rho \left(P_{\lambda_n}^{x_n}\right) = \delta_\rho^{\ n+1} \left(P_{\lambda_0}^{x_0}\right) \cdots$$

Let us choose $P_{\lambda_{n-1}}^{x_{n-1}} \neq P_{\lambda_n}^{x_n}$ for all positive integers (otherwise, if there exists some $P_{\lambda_{n_0}}^{x_{n_0}}$ such that $P_{\lambda_{n_0-1}}^{x_{n_0-1}} = P_{\lambda_{n_0}}^{x_{n_0}}$, then $P_{\lambda_{n_0}}^{x_{n_0}}$ is a fixed point of φ_ψ . By the condition (4), we gain the following inequalities:

$$\begin{aligned} d\left(P_{\lambda_{n-1}}^{x_{n-1}}, P_{\lambda_n}^{x_n}, \bar{t}\right) &= d\left(\varphi_\psi \left({\varphi_\psi}^{-1}\left(P_{\lambda_{n-1}}^{x_{n-1}}\right)\right), \varphi_\psi\left({\varphi_\psi}^{-1}\left(P_{\lambda_n}^{x_n}\right)\right), \bar{t}\right) \\ &\tilde{\geq} \bar{\alpha} d\left({\varphi_\psi}^{-1}\left(P_{\lambda_{n-1}}^{x_{n-1}}\right), \varphi_\psi^{-1}\left(P_{\lambda_n}^{x_n}\right), \bar{t}\right) + \bar{\beta}\, d\left({\varphi_\psi}^{-1}\left(P_{\lambda_{n-1}}^{x_{n-1}}\right), \varphi_\psi\left({\varphi_\psi}^{-1}\left(P_{\lambda_{n-1}}^{x_{n-1}}\right)\right), \bar{t}\right) \\ &+ \gamma d\left(\varphi_\psi^{-1}(P_{\lambda_n}^{x_n}), \varphi_\psi\left({\varphi_\psi}^{-1}(P_{\lambda_n}^{x_n})\right), \bar{t}\right) \\ &= \bar{\alpha} d(\delta_\rho(P_{\lambda_{n-1}}^{x_{n-1}}), \delta_\rho(P_{\lambda_n}^{x_n}), \bar{t}) + \bar{\beta} d(\delta_\rho(P_{\lambda_{n-1}}^{x_{n-1}}), P_{\lambda_{n-1}}^{x_{n-1}}, \bar{t}) + \bar{\gamma} d(\delta_\rho(P_{\lambda_n}^{x_n}), P_{\lambda_n}^{x_n}, \bar{t}) \\ &= \bar{\alpha} d\left(P_{\lambda_n}^{x_n}, P_{\lambda_{n+1}}^{x_{n+1}}, \bar{t}\right) + \bar{\beta} d\left(P_{\lambda_n}^{x_n}, P_{\lambda_{n-1}}^{x_{n-1}}, \bar{t}\right) + \bar{\gamma} d\left(P_{\lambda_{n+1}}^{x_{n+1}}, P_{\lambda_n}^{x_n}, \bar{t}\right) \end{aligned} \tag{5}$$

By arranging the right side of the previous inequality, it is obtained that

$$(\bar{1} - \bar{\beta})\, d(P_{\lambda_{n-1}}^{x_{n-1}}, P_{\lambda_n}^{x_n}, \bar{t}) \ \tilde{\geq}\ (\bar{\alpha} + \bar{\gamma})\, d(P_{\lambda_{n+1}}^{x_{n+1}}, P_{\lambda_n}^{x_n}, \bar{t}) \tag{6}$$

If $\bar{\alpha} + \bar{\gamma} = \bar{0}$, then $\bar{\beta} \tilde{>} \bar{1}$. This fact contradicts with the inequality (6). Thus, $\bar{\alpha} + \bar{\gamma}$ must be non-negative and $(\bar{1} - \bar{\beta}) \tilde{>} \bar{0}$.

This implies the following result in the case $\bar{k} = \frac{\bar{1} - \bar{\beta}}{\bar{\alpha} + \bar{\gamma}} \tilde{<} \bar{1}$ and for all $n \in \mathbb{N} \cup \{0\}$

$$d(P_{\lambda_{n+1}}^{x_{n+1}}, P_{\lambda_n}^{x_n}, \bar{t}) \ \tilde{\leq}\ \bar{k} d(P_{\lambda_{n-1}}^{x_{n-1}}, P_{\lambda_n}^{x_n}, \bar{t}) \tag{7}$$

By repeating (7) n-times, we obtain the following inequality for all $\bar{t} \tilde{>} \bar{0}$,

$$d(P_{\lambda_{n+1}}^{x_{n+1}}, P_{\lambda_n}^{x_n}, \bar{t}) \ \tilde{\leq}\ \bar{k}^n d(P_{\lambda_0}^{x_0}, P_{\lambda_1}^{x_1}, \bar{t}) \tag{8}$$

By Lemma 3.1, $\left\{P_{\lambda_n}^{x_n}\right\}_{n \in N}$ satisfies the Cauchy property.

So, by the hypothesis, $\left\{P_{\lambda_n}^{x_n}\right\}_{n \in N}$ converges to some soft point $P_\rho^\varpi \tilde{\in} SP(\tilde{X})$. Now since φ_ψ is surjective, we may write $P_\rho^\varpi = \varphi_\psi (P_\mu^y)$ for some $P_\mu^y \in SP(\tilde{X})$. Taking into consideration, we obtain that

$$\begin{aligned} d\left(P_{\lambda_n}^{x_n}, P_\rho^\varpi, \bar{t}\right) &= d\left(\varphi_\psi\left(P_{\lambda_{n+1}}^{x_{n+1}}\right), \varphi_\psi\left(P_\mu^y\right), \bar{t}\right) \\ &\widetilde{\leq} \bar{\alpha}\, d(P_{\lambda_{n+1}}^{x_{n+1}}, P_\mu^y, \bar{t}) + \bar{\beta}\, d(P_{\lambda_{n+1}}^{x_{n+1}}, \varphi_\psi(P_{\lambda_{n+1}}^{x_{n+1}}), \bar{t}) + \bar{\gamma}\, d(P_\mu^y, \varphi_\psi(P_\mu^y), \bar{t}) \\ &= \bar{\alpha} d(P_{\lambda_{n+1}}^{x_{n+1}}, P_\mu^y, \bar{t}) + \bar{\beta}\, d(P_{\lambda_{n+1}}^{x_{n+1}}, P_{\lambda_n}^{x_n}, \bar{t}) + \bar{\gamma}\, d(P_\mu^y, P_\rho^\varpi, \bar{t}) \end{aligned} \tag{9}$$

This inequality witnesses the following result in case n tends to infinity

$$\bar{0} \geq (\bar{\alpha} + \bar{\gamma})\, \mathrm{d}(P_\mu^y, P_\rho^\varpi, \bar{t}) \tag{10}$$

As a result, $P_\mu^y = P_\rho^\varpi$ is obtained as claimed.

Corollary 3.3. Let $(\tilde{X}, d)$ be a complete parametric soft metric space and $\varphi_\psi : (\tilde{X}, \mathrm{d}) \to (\tilde{X}, d)$ be a surjective self-soft mapping. If there exists a constant $\bar{k} \widetilde{\geq} \bar{1}$ satisfying for all $P_\lambda^x, P_\mu^y \widetilde{\in} SP(\tilde{X}), P_\lambda^x \neq P_\mu^y$, and for all $\bar{t} \widetilde{>} \bar{0}$,

$$d(\varphi_\psi(P_\lambda^x), \varphi_\psi(P_\mu^y), \bar{t}) \widetilde{\geq} \bar{k}\, d(P_\lambda^x, P_\mu^y, \bar{t}) \tag{11}$$

Then, there exists a unique fixed soft point of φ_ψ.

Proof. By the previous theorem, in the case $\bar{\beta} = \bar{\gamma} = \bar{0}$ and $\bar{\alpha} = \bar{k}$, the existence of the fixed soft point is clear. So, it is sufficient only to prove that the uniqueness. To do this, let us suppose the converse. That is, let $P_\rho^\varpi, P_\gamma^z$ be two fixed soft points of φ_ψ, then from condition (11), we obtain the following

$$d(P_\rho^\varpi, P_\gamma^z, \bar{t}) = d(\varphi_\psi(P_\rho^\varpi), \varphi_\psi(P_\gamma^z), \bar{t}) \widetilde{\geq} \bar{k}\, d(P_\rho^\varpi, P_\gamma^z, \bar{t}) \tag{12}$$

which implies $d(P_\rho^\varpi, P_\gamma^z, \bar{t}) = \bar{0}$, that is $P_\rho^\varpi = P_\gamma^z$ as desired.

Corollary 3.4. Let $(\tilde{X}, d)$ be a complete parametric soft metric space and φ_ψ be a surjective self soft mapping in this space. If the following is satisfied for some positive integer n and a soft constant $\bar{k} \widetilde{\geq} \bar{1}$,

$$d(\varphi_\psi^n(P_\lambda^x), \varphi_\psi^n(P_\mu^y), \bar{t}) \widetilde{\geq} \bar{k}\, \mathrm{d}(P_\lambda^x, P_\mu^y, \bar{t}) \tag{13}$$

for all $P_\lambda^x, P_\mu^y \widetilde{\in} SP(\tilde{X})$, $P_\lambda^x \neq P_\mu^y$, and for all $\bar{t} \widetilde{>} \bar{0}$, then there exists a unique fixed soft point of φ_ψ.

Proof. By the previous corollary, φ_ψ^n has a fixed soft point, such as P_ρ^ϖ. However, $\varphi_\psi^n(\varphi_\psi(P_\rho^\varpi)) = \varphi_\psi(\varphi_\psi^n(P_\rho^\varpi)) = \varphi_\psi(P_\rho^\varpi)$. So $\varphi_\psi(P_\rho^\varpi)$ is also a fixed soft point of the soft mapping φ_ψ^n. Hence $\varphi_\psi\left(P_\rho^\varpi\right) = P_\rho^\varpi$. Since the mappings φ_ψ and φ_ψ^n have the same fixed soft points. The result is obtained.

Definition 3.5. Let δ_ρ and φ_ψ be two self soft mappings of the soft universe $\tilde{X}$. Then, δ_ρ and φ_ψ are said to be weakly compatible if $\delta_\rho(P_\lambda^x) = \varphi_\psi(P_\lambda^x)$, for some $P_\lambda^x \widetilde{\in} SP(\tilde{X})$ and $\delta_\rho(\varphi_\psi(P_\lambda^x)) = \varphi_\psi(\delta_\rho(P_\lambda^x))$.

Theorem 3.6. Let $(\tilde{X}, d)$ be a complete parametric soft metric space and $\delta_\rho, \varphi_\psi : (\tilde{X}, d) \to (\tilde{X}, d)$ be the weakly compatible mappings such that $\varphi_\psi(\tilde{X}) \subseteq \delta_\rho(\tilde{X})$. If the following inequality holds for some $\bar{k} \widetilde{\geq} \bar{1}$ and for all $P_\lambda^x, P_\mu^y \widetilde{\in} \tilde{X}$,

$$d(\delta_\rho(P_\lambda^x), \delta_\rho(P_\mu^y), \bar{t}) \widetilde{\geq} \bar{k}\, d(\varphi_\psi(P_\lambda^x), \varphi_\psi(P_\mu^y), \bar{t}) \tag{14}$$

and if besides one of the images $\varphi_\psi(\tilde{X})$ or $\delta_\rho(\tilde{X})$ is complete, then these mappings have a unique common fixed soft point in $\tilde{X}$.

Proof. Let $P_{\lambda_0}^{x_0} \tilde{\in} SP(\tilde{X})$ be taken arbitrarily. Since $\varphi_\psi(\tilde{X}) \subseteq \delta_\rho(\tilde{X})$, choose $P_{\lambda_1}^{x_1}$ such that $P_{\mu_1}^{y_1} = \delta_\rho(P_{\lambda_1}^{x_1}) = \varphi_\psi(P_{\lambda_0}^{x_0})$. In general, choose $P_{\lambda_{n+1}}^{x_{n+1}}$ such that

$P_{\mu_{n+1}}^{y_{n+1}} = \delta_\rho(P_{\lambda_{n+1}}^{x_{n+1}}) = \varphi_\psi(P_{\lambda_n}^{x_n})$, then from the condition (14) we gain the following

$$d(P_{\mu_{n+1}}^{y_{n+1}}, P_{\mu_{n+2}}^{y_{n+2}}, \bar{t}) = \mathrm{d}\left(\varphi_\psi\left(P_{\lambda_n}^{x_n}\right), \varphi_\psi\left(P_{\lambda_{n+1}}^{x_{n+1}}\right), \bar{t}\right) \tilde{\leq} \frac{\bar{1}}{k} d(P_{\mu_n}^{y_n}, P_{\mu_{n+1}}^{y_{n+1}}, \bar{t}) \tag{15}$$

By repeating (15) $(n+1)$ –times, we obtain the following

$$d(P_{\mu_{n+1}}^{y_{n+1}}, P_{\mu_{n+2}}^{y_{n+2}}, \bar{t}) \tilde{\leq} \bar{\ell}^{n+1} d(P_{\mu_0}^{y_0}, P_{\mu_1}^{y_1}, \bar{t}) \tag{16}$$

where $\ell = \frac{1}{k}$. Hence for $n > m$, we have for all $\bar{t} \tilde{>} \bar{0}$,

$$\begin{aligned} d(P_{\mu_n}^{y_n}, P_{\mu_m}^{y_m}, \bar{t}) &\tilde{\leq} d(P_{\mu_n}^{y_n}, P_{\mu_{n+1}}^{y_{n+1}}, \bar{t}) + d(P_{\mu_n}^{y_n}, P_{\mu_{n+1}}^{y_{n+1}}, \bar{t}) + \dots \dots + d(P_{\mu_{m-1}}^{y_{m-1}}, P_{\mu_m}^{y_m}, \mathrm{t}) \\ &\tilde{\leq} (\bar{\ell}^n + \bar{\ell}^{n+1} + \cdots + \bar{\ell}^{m-1})\, d(P_{\mu_0}^{y_0}, P_{\mu_1}^{y_1}, \bar{t}) \end{aligned} \tag{17}$$

The previous inequality witnesses the fact that if n and m tend to infinity,

$\lim_{n,m\to\infty} d(P_{\lambda_n}^{x_n}, P_{\mu_m}^{y_m}, \bar{t}) = \bar{0}$. Therefore, $\left\{P_{\lambda_n}^{x_n}\right\}_{n\in N}$ satisfies the Cauchy property and by the hypothesis $P_{\mu_n}^{y_n} \to P_\rho^\varpi$ for some $P_\rho^\varpi \tilde{\in} SP(\tilde{X})$. Hence, we get

$$\lim_{n\to\infty} P_{\mu_n}^{y_n} = \lim_{n\to\infty} \varphi_\psi\left(P_{\lambda_n}^{x_n}\right) = \lim_{n\to\infty} \delta_\rho\left(P_{\lambda_n}^{x_n}\right) = P_\rho^\varpi \tag{18}$$

Since one of the soft images $\varphi_\psi(\tilde{X})$ or $\delta_\rho(\tilde{X})$ is complete and $\varphi_\psi(\tilde{X}) \subseteq \delta_\rho(\tilde{X})$, $\delta_\rho(P_u^v) = P_\rho^\varpi$ for some $P_u^v \tilde{\in} \tilde{X}$. Now from (14), we have for all $\bar{t} \tilde{>} \bar{0}$,

$$d\left(\varphi_\psi(P_u^v), \varphi_\psi\left(P_{\lambda_n}^{x_n}\right), \bar{t}\right) \tilde{\leq} \frac{\bar{1}}{\bar{k}} d(\delta_\rho(P_u^v), \delta_\rho(P_{\mu_n}^{y_n}), \bar{t}) \tag{19}$$

This implies the following for all $\bar{t} \tilde{>} \bar{0}$

$$d(\varphi_\psi(P_u^v), P_\rho^\varpi, \bar{t}) \tilde{\leq} \frac{\bar{1}}{\bar{k}} d(\delta_\rho(P_u^v), P_\rho^\varpi, \bar{t}) \tag{20}$$

The last inequality witnesses the fact that $\varphi_\psi(P_u^v) = P_\rho^\varpi$. Therefore $\varphi_\psi(P_u^v) = \delta_\rho(P_u^v) = P_\rho^\varpi$.

Since φ_ψ and δ_ρ are weakly compatible self-soft mappings, we have $\delta_\rho(\varphi_\psi(P_u^v)) = \varphi_\psi(\delta_\rho(P_u^v))$, that is $\delta_\rho(P_\rho^\varpi) = \varphi_\psi(P_\rho^\varpi)$. Now we show that P_ρ^ϖ is a fixed point of δ_ρ and φ_ψ. From (14), we have

$$d(\delta_\rho(P_\rho^\varpi), \delta_\rho(P_{\lambda_n}^{x_n}), \bar{t})) \tilde{\geq} \bar{k}\, d(\varphi_\psi(P_\rho^\varpi), \varphi_\psi(P_{\lambda_n}^{x_n}), \bar{t}) \tag{21}$$

If one takes limit as n → ∞ in (21), then the following inequality

$$d(\delta_\rho(P_\rho^\varpi), P_\rho^\varpi, \bar{t})) \tilde{\geq} \bar{k} d(\varphi_\psi(P_\rho^\varpi), P_\rho^\varpi, \bar{t})) \tag{22}$$

implies the fact that $\delta_\rho(P_\rho^\varpi) = P_\rho^\varpi$. Hence, we have $\delta_\rho(P_\rho^\varpi) = \varphi_\psi(P_\rho^\varpi) = P_\rho^\varpi$.

Uniqueness: Let us suppose the converse, that is, let $P_{\rho}^{\varpi} \neq P_{\gamma}^{z}$ be two common fixed points of the given self-soft mappings. Then, we have_$d(\delta_{\rho}(P_{\rho}^{\varpi}), \delta_{\rho}(P_{\gamma}^{z}), \bar{t})) \cong\geq \bar{k} d(\varphi_{\psi}(P_{\rho}^{\varpi}), \varphi_{\psi}(P_{\gamma}^{z}), \bar{t}))$, for all $\bar{t} \tilde{>} \bar{0}$,_which witnesses the fact that $P_{\rho}^{\varpi} = P_{\gamma}^{z}$. Hence, we get uniqueness.

Theorem 3.7. Let $(\tilde{X}, d)$ be a complete parametric soft metric space and $\varphi_{\psi}, \delta_{\rho}: \tilde{X} \to \tilde{X}$ be two surjective self soft mappings which satisfy the following conditions for some soft real numbers $\bar{\alpha}, \bar{\beta}$ and $\bar{k}$ such that $\bar{\alpha} > \bar{1} + 2\bar{k}$ and $\bar{\beta} > \bar{1} + 2\bar{k}$.

$$d(\varphi_{\psi}\ \delta_{\rho}(P_{\lambda}^{x}), \delta_{\rho}(P_{\lambda}^{x}), \bar{t}) + \bar{k} d(\varphi_{\psi}\, \delta_{\rho}(P_{\lambda}^{x}), P_{\lambda}^{x}, \bar{t}) \tilde{\geq} \bar{\alpha}\, d(\delta_{\rho}(P_{\lambda}^{x}), P_{\lambda}^{x}, \bar{t}) \tag{23}$$

and

$$d(\delta_{\rho}\ \varphi_{\psi}(P_{\lambda}^{x}), \varphi_{\psi}(P_{\lambda}^{x}), \bar{t}) + \bar{k} d\big(\delta_{\rho}\varphi_{\psi}(P_{\lambda}^{x}), P_{\lambda}^{x}, \tilde{t}\big) \tilde{\geq} \bar{\beta} d\big(\varphi_{\psi}(P_{\lambda}^{x}), P_{\lambda}^{x}, \bar{t}\big) \tag{24}$$

for all $P_{\lambda}^{x} \tilde{\in} SP(\tilde{X})$,all $\bar{t} \tilde{>} \bar{0}$. If one of the soft mappings is continuous, then they have a common fixed soft point.

Proof. Choose a soft point $P_{\lambda_0}^{x_0} \tilde{\in} SP(\tilde{X})$. Since φ_{ψ} is surjective, $P_{\lambda_0}^{x_0} = \varphi_{\psi}(P_{\lambda_1}^{x_1})$ for some $P_{\lambda_1}^{x_1} \tilde{\in} SP(\tilde{X})$. Since δ_{ρ} is surjective, too, $P_{\lambda_2}^{x_2} = \delta_{\rho}(P_{\lambda_1}^{x_1})$ for some $P_{\lambda_2}^{x_2} \tilde{\in} SP(\tilde{X})$. Continuing this process, we may set a sequence of soft points $\left\{P_{\lambda_n}^{x_n}\right\}_{n \in N}$ in such a way that

$$P_{\lambda_{2n}}^{x_{2n}} = \varphi_{\psi}\left(P_{\lambda_{2n+1}}^{x_{2n+1}}\right) \quad \text{and} \quad P_{\lambda_{2n+1}}^{x_{2n+1}} = \delta_{\rho}\left(P_{\lambda_{2n+2}}^{x_{2n+2}}\right), \forall\, n \in \mathbb{N} \cup \{0\} \tag{25}$$

Now for $n \in \mathbb{N} \cup \{0\}$, we have

$$d(\varphi_{\psi}\ \delta_{\rho}(P_{\lambda_{2n+2}}^{x_{2n+2}}), \delta_{\rho}(P_{\lambda_{2n+2}}^{x_{2n+2}}), \bar{t}) + \bar{k}\, d(\varphi_{\psi}\ \delta_{\rho}(P_{\lambda_{2n+2}}^{x_{2n+2}}), P_{\lambda_{2n+2}}^{x_{2n+2}}, \bar{t}) \tilde{\geq} \bar{\alpha} d\left(\delta_{\rho}\left(P_{\lambda_{2n+2}}^{x_{2n+2}}\right), P_{\lambda_{2n+2}}^{x_{2n+2}}, \bar{t}\right) \tag{26}$$

Thus, we get

$$d\left(P_{\lambda_{2n}}^{x_{2n}}, P_{\lambda_{2n+1}}^{x_{2n+1}}, \bar{t}\right) + \bar{k}\, d\left(P_{\lambda_{2n}}^{x_{2n}}, P_{\lambda_{2n+2}}^{x_{2n+2}}, \bar{t}\right) \tilde{\geq} \bar{\alpha} d\left(P_{\lambda_{2n+1}}^{x_{2n+1}}, P_{\lambda_{2n+2}}^{x_{2n+2}}, \bar{t}\right) \tag{27}$$

Since

$$d\left(P_{\lambda_{2n}}^{x_{2n}}, P_{\lambda_{2n+2}}^{x_{2n+2}}, \mathrm{t}\right) \tilde{\leq} d\left(P_{\lambda_{2n}}^{x_{2n}}, P_{\lambda_{2n+1}}^{x_{2n+1}}, \bar{t}\right) + d\left(P_{\lambda_{2n+1}}^{x_{2n+1}}, P_{\lambda_{2n+2}}^{x_{2n+2}}, \bar{t}\right)$$

Hence from (27),

$$d\left(P_{\lambda_{2n+1}}^{x_{2n+1}}, P_{\lambda_{2n+2}}^{x_{2n+2}}, \bar{t}\right) \tilde{\leq} \frac{\bar{1} + \bar{k}}{\bar{\alpha} - \bar{k}} d\left(P_{\lambda_{2n}}^{x_{2n}}, P_{\lambda_{2n+2}}^{x_{2n+2}}, \bar{t}\right) \tag{28}$$

On the other hand, we have

$$d(\delta_{\rho}\ \varphi_{\psi}(P_{\lambda_{2n+1}}^{x_{2n+1}}), \varphi_{\psi}(P_{\lambda_{2n+1}}^{x_{2n+1}}), \bar{t}) + \bar{k}\, d(\delta_{\rho}\ \varphi_{\psi}(P_{\lambda_{2n+1}}^{x_{2n+1}}), P_{\lambda_{2n+1}}^{x_{2n+1}}, \bar{t}) \tilde{\geq} \bar{\beta} d(\varphi_{\psi}(P_{\lambda_{2n+1}}^{x_{2n+1}}), P_{\lambda_{2n+1}}^{x_{2n+1}}, \bar{t}) \tag{29}$$

Thus, we have

$$d\left(P_{\lambda_{2n-1}}^{x_{2n-1}}, P_{\lambda_{2n}}^{x_{2n}}, \bar{t}\right) + \bar{k}\, d\left(P_{\lambda_{2n-1}}^{x_{2n-1}}, P_{\lambda_{2n+1}}^{x_{2n+1}}, \bar{t}\right) \tilde{\geq} \bar{\beta} d\left(P_{\lambda_{2n}}^{x_{2n}}, P_{\lambda_{2n+1}}^{x_{2n+1}}, \bar{t}\right) \tag{30}$$

Since

$$d\left(P_{\lambda_{2n-1}}^{x_{2n-1}}, P_{\lambda_{2n+1}}^{x_{2n+1}}, \bar{t}\right) \precsim d\left(P_{\lambda_{2n-1}}^{x_{2n-1}}, P_{\lambda_{2n}}^{x_{2n}}, \bar{t}\right) + d\left(P_{\lambda_{2n}}^{x_{2n}}, P_{\lambda_{2n+1}}^{x_{2n+1}}, \bar{t}\right)$$

And from (30), we have

$$d\left(P_{\lambda_{2n}}^{x_{2n}}, P_{\lambda_{2n+1}}^{x_{2n+1}}, \bar{t}\right) \precsim \frac{\bar{1} + \bar{k}}{\bar{\beta} - \bar{k}} d\left(P_{\lambda_{2n-1}}^{x_{2n-1}}, P_{\lambda_{2n}}^{x_{2n}}, \bar{t}\right) \tag{31}$$

Let $\bar{\ell} = \max\{\frac{\bar{1}+\bar{k}}{\bar{\beta}-\bar{k}}, \frac{\bar{1}+\bar{k}}{\bar{\alpha}-\bar{k}}\}$. Then, by combining (28) and (31), we have for each $n \in \mathrm{N} \cup \{0\}$ and $\bar{t} \succsim \bar{0}$

$$d\left(P_{\lambda_n}^{x_n}, P_{\lambda_{n+1}}^{x_{n+1}}, \bar{t}\right) \precsim \bar{\ell}\, d\left(P_{\lambda_{n-1}}^{x_{n-1}}, P_{\lambda_n}^{x_n}, \bar{t}\right) \tag{32}$$

By repeating (31) n-times, we get for all $n \in \mathrm{N} \cup \{0\}$ and all $\bar{t} \succsim \bar{0}$,

$$d\left(P_{\lambda_n}^{x_n}, P_{\lambda_{n+1}}^{x_{n+1}}, \bar{t}\right) \precsim \bar{\ell}^n\, d\left(P_{\lambda_0}^{x_0}, P_{\lambda_1}^{x_1}, \bar{t}\right) \tag{33}$$

By Lemma 3.1, $\left\{P_{\lambda_n}^{x_n}\right\}_{n\in N}$ satisfies the Cauchy property. Then by the hypothesis, the sequence converges to some soft point as $P_{\lambda_n}^{x_n} \to P_\rho^{\varpi}$. So, $P_{\lambda_{2n+1}}^{x_{2n+1}} \to P_\rho^{\varpi}$ and $P_{\lambda_{2n+2}}^{x_{2n+2}} \to P_\rho^{\varpi}$. If φ_ψ is continuous, then $\varphi_\psi\,(P_{\lambda_{2n+1}}^{x_{2n+1}}) \to \varphi_\psi\,(P_\rho^{\varpi})$. But $\varphi_\psi\,(P_{\lambda_{2n+1}}^{x_{2n+1}}) = P_{\lambda_{2n}}^{x_{2n}} \to P_\rho^{\varpi}$. As a result, $\varphi_\psi\,(P_\rho^{\varpi}) = P_\rho^{\varpi}$. By the surjectivity of δ_ρ, we have $\delta_\rho(P_\rho^{\varpi}) = P_\rho^{\varpi}$ for some soft point P_σ^{ϑ}. Now

$$d(\varphi_\psi\ (\delta_\rho(P_\sigma^{\vartheta}), \delta_\rho P_\sigma^{\vartheta}), \bar{t}) + \bar{k} d(\varphi_\psi\ (\delta_\rho(P_\sigma^{\vartheta}), P_\sigma^{\vartheta}, \bar{t}) \succsim \bar{\alpha} d(\delta_\rho(P_\sigma^{\vartheta}), P_\sigma^{\vartheta}, \bar{t})$$

implies that

$$\bar{k} d(P_\rho^{\varpi}, P_\sigma^{\vartheta}, \bar{t}) \succsim \bar{\alpha} d(P_\rho^{\varpi}, P_\sigma^{\vartheta}, \bar{t}).$$

Thus, we gain the following

$$d(P_\rho^{\varpi}, P_\sigma^{\vartheta}, \mathrm{t}) \precsim \frac{\bar{k}}{\bar{\alpha}} d(P_\rho^{\varpi}, P_\sigma^{\vartheta}, \bar{t})$$

Since $\bar{\alpha} \succsim \bar{k}$, we conclude that $d(P_\rho^{\varpi}, P_\sigma^{\vartheta}, \bar{t}) = \bar{0}$. So $P_\rho^{\varpi} = P_\sigma^{\vartheta}$.

Hence, $\varphi_\psi\,(P_\rho^{\varpi}) = \delta_\rho(P_\rho^{\varpi}) = P_\rho^{\varpi}$. This completes the proof as claimed.

4. Conclusion

Fixed point theory is essential in the surveys given in metric and topological spaces. Several authors have applied/embedded this theory in different metric spaces and applied sciences. Since the solutions of integral and differential equations are based on the fixed-point theory constructed in normed spaces, in this merit, we decide to investigate the existence and the uniqueness of fixed soft points of self-soft mappings in parametric soft metric spaces, which spaces the parameterization tool plays the key role. Moreover, the studies on fixed-circle results have gained attention in metric and metric-like spaces [15,16,17], nowadays. For further research, we hope to investigate some different kinds of fixed soft point theorems, some fixed-circle theorems and also, we plan to give some applications in such spaces.

Author Contributions

All authors contributed equally to this work. They all read and approved the last version of the manuscript.

Conflict of Interest

The authors declare no conflict of interest.

References

[1] N. Hussain, S. Khaleghizadeh, P. Salimi, A. A. N. Abdou, *A New Approach to Fixed Point Results in Triangular Intuitionistic Fuzzy Metric Spaces*, Abstract and Applied Analysis Article ID 690138 (2014) 16 pages.

[2] K. P. R. Rao, D. V. Babu, E. T. Ramudu, *Some Unique Common Fixed Point Theorems in Parametric S-Metric Spaces*, International Journal of Innovative Research in Science, Engineering and Technology 3 (2014) 14375–14387.

[3] V. Çetkin, *Parametric 2-Metric Spaces and Some Fixed Point Results*, New Trends in Mathematical Sciences 7(4) (2019) 503–511.

[4] Ö. Ege, İ. Karaca, *Fixed Point Theorems and an Application in Parametric Metric Spaces*, Azerbaijan Journal of Mathematics 7 (2017) 27–39.

[5] R. Jain, R. D. Daheirai, M. Ughade, *Fixed Point, Coincidence Point and Common Fixed Point Theorems under Various Expansive Conditions in Parametric Metric Spaces and Parametric b-Metric Spaces*, Gazi University Journal of Science 29(1) (2016) 95–107.

[6] N. Taş, N. Y. Özgür, *On Parametric S-Metric Spaces and Fixed-Point Type Theorems for Expansive Mappings*, Journal of Mathematics Article ID 4746732 (2016) 6 pages.

[7] H. Hosseinzadeh, Fixed Point Theorems on Soft Metric Spaces, Journal of Fixed Point Theory and Applications 19(2) (2017) 1625–1647.

[8] R. Bhardwaj, H. G. S. Kumar, B. K. Singh, Q. A. Kabir, P. Konak, *Fixed Point Theorems in Soft Parametric Metric Space*, Advances in Mathematics: Scientific Journal 9(12) (2020) 10189–10194.

[9] S. Das, S. K. Samanta, *Soft Metric*, Annals of Fuzzy Mathematics and Informatics 6 (2013) 77–94.

[10] Y. Tunçay, V. Çetkin, *Parametric Soft Metric Spaces*, in: U. Mutman, C. Özkale (Eds.), IV. The International Symposium on Engineering, Natural Sciences and Architecture, Kocaeli, Turkey, 2021, pp. 60–64.

[11] D. Molodtsov, *Soft Set Theory-First Results*, Computers and Mathematics with Applications 37 (1999) 19–31.

[12] P. K. Maji, R. Biswas, A. R. Roy, *Soft Set Theory*, Computers and Mathematics with Applications 45 (2013) 555–562.

[13] S. Das, S. K. Samanta, *Soft Real Sets, Soft Real Numbers and Their Properties*, Journal of Fuzzy Mathematics 20(3) (2012) 551–576.

[14] A. Kharal, B. Ahmad, *Mappings on Soft Classes*, New Mathematics and Natural Computation 7(3) (2011) 471–481.

[15] N. Y. Özgür, N. Taş, U. Çelik, *New Fixed-Circle Results on S-metric Spaces*, Bulletin of Mathematical Analysis and Applications 9(2) (2017) 10–23.

[16] N. Taş, N. Y. Özgür, *Some Fixed-Point Results on Parametric N_b-metric Spaces*, Communications of Korean Mathematical Society 33(3) (2018) 943–960.

[17] N. Y. Özgür, N. Taş, *Some Fixed-Circle Theorems on Metric Spaces*, Bulletin of the Malaysian Mathematical Sciences Society 42 (2019) 1433–1449.