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RESEARCH ARTICLE

INFLUENCE of SPINNING TOPOLOGICAL DEFECT on the LANDAU LEVELS ofRELATIVISTIC SPIN-0 PARTICLES

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ABSTRACT

We investigate relativistic Landau quantization of spinless particle in three dimensional space-time induced by topological defect with spin through acquiring non-perturbative solution of the corresponding Klein-Gordon equation. The obtained results allow us to analyze the alterations stemming from the background geometry on the spectrum. We observe that the background geometry can be responsible not only for shifts on the relativistic Landau levels but also for symmetry breaking of the particle-antiparticle states provided that the defect possesses non-zero spin.

Keywords: *Landau Quantization,Relativistis Quantum Mechanics,Topological Defect,Cosmic String*

1. INTRODUCTION

The history of investigations for quantum fields in curved spaces goes long way back [1-5] and analysis of the influences of curved spaces on the relativistic dynamics of physical systems has great importance in the modern physics due to the fact that these kinds of investigations have provided very interesting results [6-28]. One of the most important results of such studies is that they provide an opportunity us to see what the dependence of relativistic dynamics of a single particle or a composite system on the topological properties of the geometric background [6]. This allows us to discuss also the effect of gravity on the quantum mechanical systems [6,12]. In general, in the theoretical framework, in order to analyze the effects of curved spaces or non-trivial topologies on the quantum mechanical systems, relativistic equations such as Klein-Gordon ($\mathcal{KG}$) [10], Dirac [9], Duffin-Kemmer-Petiau [13], Vector Boson [14,15] and fully-covariant many-body equations [15,16] are used. In the literature, there exists numerous announced results for relativistic dynamics of spinning and spin-less particles in curved spaces [6-19]. The $\mathcal{KG}$ equation is used to describe the dynamics of spin-less relativistic particles [17-19] and the effects of topological defect-induced space-time backgrounds such as cosmic string space-time [19], spiral dislocation space-time [20], screw dislocation space-time [21], global monopole space-time [22] on the relativistic spin-less particles were investigated by solving the generalized $\mathcal{KG}$ equation.

On the other hand, the cosmic strings [23-25] which are stable linear topological defects were introduced first through general solutions in three dimensions [6,23-25] and background geometry

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spanned by a static or spinning cosmic string has non-trivial topology [6,23-25]. This is because of such space-time structures are not flat when viewed globally even though they are locally flat [23-25]. Due to this interesting feature, the cosmic string induced background geometry may cause for several interesting phenomena in the universe [6,23-26]. Further, the spatial part of the metric representing to cosmic string space-time (see Ref. [6]) describes also the topological defect that can appear in condensed matter mediums [15,27]. Therefore, the influence of topological defect-induced background geometries on the dynamics of physical systems has been widely studied [6,8,10-22,26,27]. In these works, the well-known quantum systems, such as quantum oscillators [8,11,13,15-22,26], single particle test fields [10,20,21,27,28], positronium or hydrogen-like low energy bound-state systems [6,12] in quantum electrodynamics are preferred to investigate the effects of space-time structures. It is also clear that analysis the effects of external magnetic field on the evolution of quantum systems is of high importance in the modern physics since such fields are exist at almost each point in the universe [9,29,30]. Hence, non-perturbative results obtained for quantum systems exposed to an external magnetic field in curved spaces are very important. In this contribution, we deal with a scalar field under the effect of external magnetic field (uniform) in the geometric background spanned by a spinning cosmic string [6,25].

This article is structured as the following. In Sec. 2, we introduce the generalized $\mathcal{KG}$ equation and then we obtain a wave equation for a scalar relativistic particle (charged) exposed to external uniform magnetic field in the background geometry induced by a spinning string source. In Sec. 3, we acquire non-perturbative results and show the dependence of spectrum on the parameters of background geometry. Then, we summarize the results and discuss to the findings in detail. Here we declare that we will prefer to use the units $\hbar = c = 1$.

2. GENERALIZED KLEIN-GORDON EQUATION

In this part, we write the generalized $\mathcal{KG}$ equation for a charged spinless particle in three-dimensional spacetime spanned by a spinning string source and obtain second order wave equation. The generalized $\mathcal{KG}$ equation is written as follows [5]

$$\frac{1}{\sqrt{|-g|}}\mathfrak{D}_\mu\left(\sqrt{|-g|}g^{\mu\nu}\mathfrak{D}_\nu\Psi\right) = m^2\Psi, \quad \mathfrak{D}_\mu = \partial_\mu + ieA_\mu, \quad (\mu,\nu = 0,1,2.), \tag{1}$$

where g stands for the determinant of the covariant metric tensor, $g^{\mu\nu}$ is the contravariant metric tensor, the letter e represents to the elementary electrical charge, A_μ is the electromagnetic 3-vector potential and Ψ is the scalar field with mass of m. It is known that the external magnetic field (uniform) is taken into account through the angular component of the 3-vector potential [29,30]. The spacetime background spanned by an idealized spinning string source is described by the following metric with signature $+,-,-$ [6,25]

$$ds^2 = dt^2 + 2\varpi dt d\vartheta - dr^2 - (\alpha^2 r^2 - \varpi^2)d\vartheta^2, \tag{2}$$

for which covariant ($g_{\mu\nu}$) and contravariant ($g^{\mu\nu}$) form of the metric tensor can be written as the following

$$g_{\mu\nu}=\begin{pmatrix}1 & 0 & \varpi\\ 0 & -1 & 0\\ \varpi & 0 & -\alpha^2r^2+\varpi^2\end{pmatrix},\qquad g^{\mu\nu}=\begin{pmatrix}1-\frac{\varpi^2}{\alpha^2r^2} & 0 & \frac{\varpi}{\alpha^2r^2}\\ 0 & -1 & 0\\ \frac{\varpi}{\alpha^2r^2} & 0 & -\frac{1}{\alpha^2r^2}\end{pmatrix}. \tag{3}$$

In Eq. (2), α relates with the angular deficit in the background and it depends on the linear mass density of the string, ϖ is the spin of the string. The Eq. (3) leads that $det(g_{\mu\nu})=\alpha^2r^2$. Now we can write explicit form of the generalized $\mathcal{KG}$ equation for a spinless relativistic particle (charged) exposed to a uniform external magnetic field in the background geometry induced by the spinning string as follows

$$\frac{1}{\sqrt{|-g|}}\partial_t\left[\sqrt{|-g|}g^{tt}\partial_t\Psi\right]+\frac{1}{\sqrt{|-g|}}\partial_t\left[\sqrt{|-g|}g^{t\vartheta}(\partial_\vartheta+ieA_\vartheta)\Psi\right]+\frac{1}{\sqrt{|-g|}}\partial_r\left[\sqrt{|-g|}g^{rr}\partial_r\Psi\right]+\frac{1}{\sqrt{|-g|}}(\partial_\vartheta+ieA_\vartheta)\left[\sqrt{|-g|}g^{\vartheta t}\partial_t\Psi\right]+\frac{1}{\sqrt{|-g|}}(\partial_\vartheta+ieA_\vartheta)\left[\sqrt{|-g|}g^{\vartheta\vartheta}(\partial_\vartheta+ieA_\vartheta)\Psi\right]-m^2\Psi=0, \tag{4}$$

where $A_\vartheta=\frac{\alpha\mathfrak{B}_0r^2}{2}$ [31]. With respect to the Eq. (2), we can factorize the wave function Ψ as follows

$$\Psi=e^{-i\omega t}e^{i\ell\vartheta}\psi(r), \tag{5}$$

in which ω and ℓ are the relativistic frequency and orbital quantum number, respectively. For the considered system, by inserting Eq. (3) and Eq. (5) into the Eq. (4), we obtain a wave equation

$$\partial_r^2\psi(r)+\frac{1}{r}\partial_r\psi(r)+\left[\omega^2\left(1-\frac{\varpi^2}{\alpha^2r^2}\right)-\frac{2\omega\varpi\ell}{\alpha^2r^2}-\frac{\omega\varpi\mathrm{B}}{\alpha}-\frac{\ell^2}{\alpha^2r^2}-\frac{\mathrm{B}\ell}{\alpha}-\frac{\mathrm{B}^2r^2}{4}-m^2\right]\psi(r), \tag{6}$$

where $\mathrm{B}=e\mathfrak{B}_0$. This second order differential equation can be reduced into a familiar form by means of a new variable, $\varrho=\frac{\mathrm{B}}{2}r^2$,

$$\partial_\varrho^2\psi(r)+\frac{1}{\varrho}\partial_\varrho\psi(\varrho)-\left[\frac{B\varrho+2m^2-2\omega^2}{4B\varrho}+\frac{(\omega\varpi+\ell)}{2\alpha\varrho}+\frac{(\omega\varpi+\ell)^2}{2\alpha\varrho^2}\right]\psi(\varrho). \tag{7}$$

3. ALTERED SPECTRUM

Here, we obtain non-perturbative spectra for the charged scalar relativistic particle exposed to an external uniform magnetic field in the space-time background induced by the spinning cosmic string. To acquire exact result, we will deal with the Eq. (7). By considering an ansatz function, $\psi(\varrho)=\varrho^{-\frac{1}{2}}\chi(\varrho)$, the wave equation in Eq. (7) can be reduced into a familiar form (see also [30])

$$\left[\partial_\varrho^2+\frac{\zeta}{\varrho}+\frac{\frac{1}{4}-\epsilon^2}{\varrho^2}-\frac{1}{4}\right]\chi(\varrho)=0,\ \ \zeta=\frac{\alpha(\omega^2-m^2)-\mathrm{B}(\omega\varpi+\ell)}{2\mathrm{B}\alpha},\quad \epsilon=\frac{\omega\varpi+\ell}{2\alpha}. \tag{8}$$

It can be verified that solution function of the Eq. (8) is given in terms of the Whittaker function, which can also be expressed in terms of Confluent Hypergeometric function [8], as $\chi(\varrho)=\mathcal{Q}\mathcal{W}_{\zeta,\epsilon}(\varrho)$. Here, $\mathcal{Q}$ is an arbitrary constant and the $\mathcal{W}_{\zeta,\epsilon}(\varrho)$ is the Whittaker function [24,25]. To be polynomial condition of the solution function is given as the following $\frac{1}{2}+\epsilon-\zeta=-n,\ (n=0,1,2..)$ [8,29,30] in which n is the radial quantum number. Through this termination, one can acquire the following spectrum of frequency

$$\omega_{n\ell}=\pm m\left\{\sqrt{1-\frac{w_c}{m}\left(2n+1+\frac{2\ell}{\alpha}\right)+\frac{{w_c}^2\varpi^2}{m^2\alpha^2}}\mp\frac{w_c\varpi}{m\alpha}\right\},\ \ \alpha=1-4\mu_s, \tag{9}$$

where $w_c=\frac{e\mathfrak{B}_0}{m}$ is the relativistic cyclotron frequency [29-33]. Here, we should underline that the ϖ(positive) is spin parameter of the cosmic string [6], α relates with angular deficit in the background geometry, $\alpha\in(0,1]$, μ_s is the linear mass density of the cosmic string [6] and orbital quantum number ℓ can take the following values: $\ell=0,\pm1\pm2..$ [10,18] in 2+1dimensions. In Eq. (9) we see that the obtained spectrum depends on the parameters of the geometric background and allows us to analyze the effects of background geometry on the relativistic Landau levels of the considered scalar particle (see Figures. 1, 2, 3, 4 and Figure 5). Here, we observe that the information about the string tension (see Ref. [6]) is carried also by the orbital quantum number ℓ. But, we lose this information if the string source is static ($\varpi=0$) for the $\ell=0$ levels.

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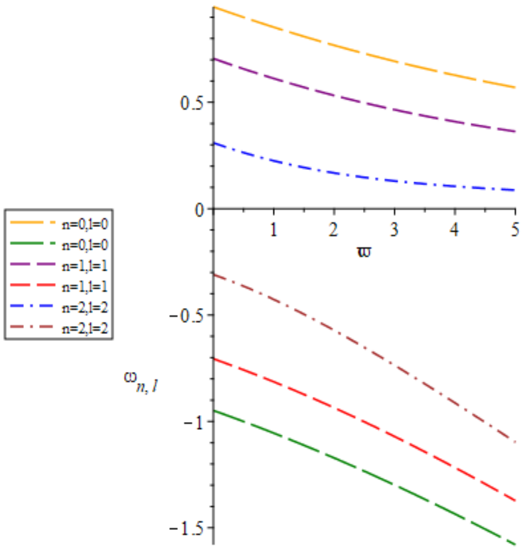

Figure 1. Effect of the spin of the string ($\propto \varpi$) on the Landau levels for $\alpha = 0.99,\ m = 1,\ e = 1, \mathfrak{B}_0 = 0.1$.

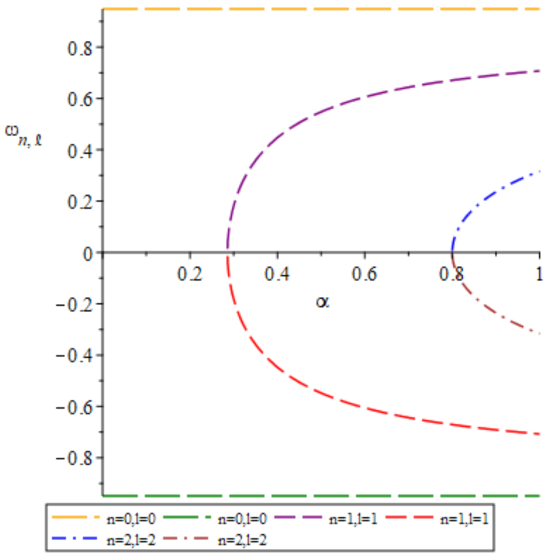

Figure 2. Influence of the α parameter on the Landau levels for $\varpi = 0,\ m = 1\ , e = 1, \mathfrak{B}_0 = 0.1$.

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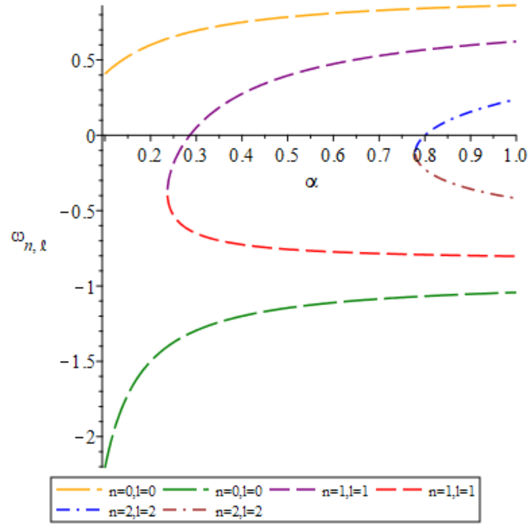

Figure 3. Effect of the parameter α on the Landau levels for $\varpi = 0.9, m = 1, \ e = 1, \mathfrak{B}_0 = 0.1$.

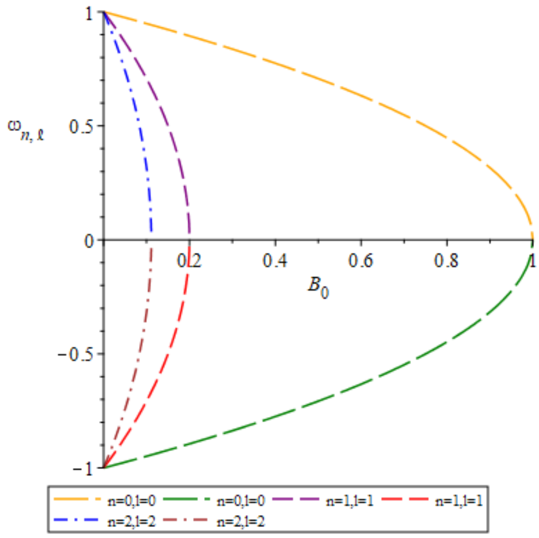

Figure 4. Effect of external magnetic field on the total frequency for $\varpi = 0, \ \alpha = 1, \ m = 1, \ e = 1$.

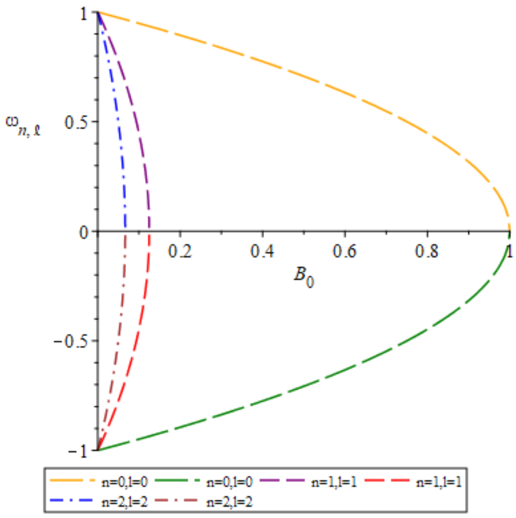

Figure 5. Effect of angular deficit parameter on the Landau levels for $\varpi = 0, \alpha = 0.4, m = 1, e = 1$.

4. CONCLUSION and DISCUSSIONS

In this contribution, we have analyzed the effects of a background geometry induced by a spinning topological defect on the relativistic Landau levels of a scalar relativistic particle exposed to an external uniform magnetic field through obtaining non-perturbative solution of the corresponding Klein-Gordon equation. The obtained spectrum of frequency (or energy) expression is given by the Eq. (9) and shows that the energy levels depend on both the spin parameter (ϖ) of the string and angular deficit ($\propto \alpha$) in the background geometry. The result in Eq. (9) can be reduced for such a flat planar system when $\varpi = 0$ and $\alpha = 1$. The Figure 4 shows that magnitude of the total energy decreases as strength of the external uniform magnetic field increases. In Figure 2 and in Figure 5, we see that the presence of angular deficit in the geometric background can affect the magnitude of the energy levels and can cause shifts in these levels. The Figure 1 and Figure 3 clearly show the string source-spanned geometric background can be responsible for symmetry breaking in the context of the energy for particle-antiparticle states provided that the cosmic string possesses non-vanishing spin.

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