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İSTANBUL ÜNİVERSİTESİ YAYINEVİ

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Improving Forecast Accuracy Using Combined Forecasts with Regard to Structural Breaks and ARCH Innovations

Daud Ali Aser*, Esin Firuzan**

Abstract

Accurate forecasts about the future are vital in time series analyses, but accurately modeling complex structures in the data is always challenging. Two major sources of complexity are autoregressive conditional heteroskedasticity (ARCH) effects on data as well as structural breaks in the data, as these affect the quality of data and hence reduce forecast accuracy. In this regard, combining forecast types has been a helpful strategy for improving forecast accuracy for more than 50 years since Bates and Granger's (1969) original paper. Hence, this paper aims to examine if the gains from combined forecasts are sustained regarding cases with structural breaks and ARCH innovations. Moreover, the study explores which forecast combination schemes are optimal for those cases by combining the exponential smoothing (ETS), autoregressive integrated moving average (ARIMA), and artificial neural network (ANN) forecast models using simple and regression-based combination procedures. These methods are implemented in both simulated series and over empirical data from two popular Turkish stock exchanges (i.e., BIST-30 and BIST-100 Indexes). The study has found regression-based forecast combination methods to significantly improve forecast accuracy regarding cases with structural breaks and conditional heteroscedasticity. Dynamically weighted combinations show greater accuracy improvement compared to their static counterparts when the data contain a trend. Simple combination schemes, including simple averages, just perform better than single methods for ETS and ARIMA, while they barely outperform ANN. In conclusion, ANN is found to be the best-performing individual forecasting method for all cases and designs.

Keywords

Structural Break, Forecasts Combination, ARCH Effects, Artificial Neural Networks

* Daud Ali Aser (PhD Student), Dokuz Eylul University, Faculty of Science, Department of Statistics, Izmir, Turkiye
E-posta: caseyr025@gmail.com ORCID: 0000-0002-6712-5559

** **Corresponding author:** Esin Firuzan (Prof.), Dokuz Eylul University, Faculty of Science, Department of Statistics, Izmir, Turkiye
E-posta: esin.firuzan@deu.edu.tr ORCID: 0000-0002-1333-0864

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1. Introduction

Accurate forecasts about the future are vital in time series analyses, and accurately modeling complex structures in the data is always challenging for forecasters. This complexity involves determining the underlying data-generating processes, as these are not fully known in most cases. Because the observed processes are too complicated to be accurately modeled apart from in certain natural sciences (Weiss et al., 2019), models are used as approximations of real-world data-generating processes (Hansen, 2005). The sources of the complexity in data structures include unexpected changes (breaks) experienced in the model parameters regarding financial time series data that are perhaps caused by policy changes, technological advancements, financial crises, and/or natural disasters. Events like the 2008 global economic crisis and the COVID-19 pandemic are good examples of structural break causes. Another issue often experienced in time series that this study also addresses is the problem of autoregressive conditional heteroskedasticity (ARCH) effects. Many researchers have testified to the presence of structural breaks in economic and financial time series, at the forefront of which are Stock and Watson (1996), who used several standard statistical tests to examine 76 monthly U.S. economic time series relationships for model instability, as well as Koop and Potter (2001) and Siliverstovs and van Dijk (2002). Structural breaks reduce forecasting accuracy, as shown by Clements and Hendry's (1998, 1999) and Hansen's (2001) studies, and consequently mislead policy recommendations and other prediction purposes.

The traditional approach to forecasting is based on the existence of one of the available methods being the best and identifiable, but choosing the best method depends upon the features of the time series. Aggregating inputs from several forecasting techniques using forecasts is an alternative to the conventional strategy. This solves the issue of having to choose and just rely on one method and its forecasts (Winkler & Makridakis, 1983). Numerous studies have demonstrated using several different methods over the same time series and averaging the resulting forecasts to be a simple way to improve forecast accuracy. More than five decades ago, Bates and Granger (1969) wrote their seminal paper showing combined forecasts to often improve forecast accuracy. Two decades later, Clemen (1989) wrote a review paper also arguing forecast accuracy to be improved by combining multiple forecasts. Moreover, Clemen's findings showed one to be able to make impressive performance improvements in many cases by merely averaging forecasts. Although the idea of combining forecasts dates back to half a century, papers describing novel combination techniques are still published in prestigious journals, which encourages additional study (Weiss et al., 2019).

Several combining rules have been proposed when looking for an optimal combination of forecasts. The proposed techniques include a combination based on the variances and covariances of the forecast components (Bates & Granger, 1969), as well as other equal weighting methods including simple average and median forecast combinations, the ordinary least squares (OLS) regression-based combination (Granger & Ramanathan, 1984), and the eigenvector-based combination methods developed by Hsiao and Wan (2014).

Given the prominence of the forecast combination topic in both theoretical and empirical studies, if the gains of combined forecasts are sustained in specific cases such as for structural breaks and ARCH effects, then these methods should be applied more generically and have need of further study. The studies concerning this area in the literature are very limited. This current paper will examine how forecasting accuracy can be improved in the presence of structural break(s) and ARCH and generalized ARCH (GARCH) innovations, as well as which combination schemes are optimal in the considered cases. This will be done by combining the component forecasts from single models using either simple weighting techniques or regression-based weighting strategies. In the latter, we consider both time combination weights (i.e., varying combination weights [dynamic version] and time) and non-varying combination weights (i.e., static version). Before implementing the combined forecasts, component forecasts are generated from three popular single models: autoregressive integrated moving average (ARIMA), exponential smoothing (ETS), and artificial neural network (ANN). These models are implemented over simulated and empirical data while using the root mean squared forecast error (RMSE) to assess the accuracy and predictive performance of the models under study.

2. Methods

This study employs three individual methods and five forecast combination methods, which are explained in the next subsections.

2.1. Methods for Testing Structural Breaks and ARCH Effects

Before applying forecasting methods, the study tests whether the data under consideration contain structural breaks and ARCH effects. The presence of structural breaks is tested using Bai & Perron's (1998, 2003) tests, which has the nice feature of being appropriate for multiple structural breaks as well as for estimating break locations and dates. Meanwhile, ARCH effects will be checked using the Lagrange multiplier test, as proposed by Engle (1982). This test fits a linear regression model for the squared residuals and then examines the significance of the fitted model.

2.2. Forecasting Models

To forecast beyond the sample observations in both simulated and empirical data, the following individual and combined models are used:

2.2.1. Individual Models

The individual methods employed in this study involve two linear (ARIMA and exponential smoothing) models and one non-linear (artificial neural network) model. These methods are described concisely below.

2.2.1.1. Exponential Smoothing (ETS) Model

The exponential smoothing class of models was introduced in the 1950s (Winters, 1960; Holt, 2004) and are capable of producing time series forecasts by employing a weighted average of the historical values and allocating more weight to recent observations. These models consist of observation/ measurement equations that describe the observed data and state equations that describe how states (i.e., levels, trends, seasonal conditions) change over time, which is why these are called state-space models. The ETS in exponential smoothing (ETS) models stands for error, trend, and seasonal. ETS models have many variations because of number of different available trends and seasonal combinations. The trend component may be none (N), additive (A), or additive damped (A_d), while the seasonal component may be none (N), additive (A), or multiplicative (M), thus yielding nine variants for exponential smoothing methods (Hyndman & Athanasopoulos, 2018).

The component form of the *ETS*(*A,N,N*) model with additive errors is as follows:

Forecast equation $$\hat{y}_{t+1|t} = \ell_t \tag{1}$$

Smoothing equation $$\ell_t = \alpha y_t + (1-\alpha)\ell_{t-1} \tag{2}$$

By re-arranging the smoothing equation, one gets:

$$\ell_t = \ell_{t-1} + \alpha(y_t - \ell_{t-1}) = \ell_{t-1} + \alpha e_t \tag{3}$$

where ℓ_t is the estimated level and $\hat{y}_{t+1|t}$ is the one step-ahead prediction for time , t + 1 which results from the weighted average of all historical data, while $0 \leq \alpha \leq 1$ is the smoothing parameter, and $e_t = y_t - \ell_{t-1} = y_t - \hat{y}_{t|t-1}$ is the error at time *t*. Other ETS models can be written in similar fashion for each of the exponential smoothing methods.

An automated selection procedure is utilized to identify the exponential smoothing models by using the *ets* function from the *forecast* package in R environment, developed by Hyndman and Khandakar (2008). This function automatically identifies which model best suits the given time series, estimates the model parameters, and returns information about the fitted model. It can use all information criteria, but the bias-corrected Akaike criterion (AIC_c) is the default information criterion for selecting an appropriate model.

2.2.1.2. Autoregressive Integrated Moving Average (ARIMA) Model

One of the most extensively used models for time series forecasts is the ARIMA model, which was first suggested by Box and Jenkins (1970). The ARIMA model is a generalization of an ARMA model. ARIMA(p,d,q) is a non-seasonal ARIMA model with non-negative parameters p, d, and q, where p is the number of time lags (referred to as the order of the autoregressive model), d is the degree of differencing, and q is the order of the moving-average model.

ARIMA models transform a non-stationary series to a stationary series through a sequence of differencing steps. A time series y_t is integrated of order d if $\nabla^d y_t$ is stationary and

$$\nabla y_t = y_t - y_{t-1} \tag{4}$$

where y is the time series and t is the time index.

After transforming the time series into a stationary one, the estimation is done as follows:

$$\hat{y}_t = \mu + \sum_{i=1}^{p} \phi_i \, y_{t-i} + \sum_{i=1}^{q} \theta_i \, \varepsilon_{t-i} + \varepsilon_t \tag{5}$$

where ϕ_i represents the coefficients of the *AR* terms of order p, θ_i denotes the coefficients of the *MA* terms of order q, ε is a random term simulating white noise, and μ is a constant. The model parameters are estimated using maximum likelihood estimation (MLE).

For the best ARIMA model selection, this study relies on the *auto.arima* function from the *forecast* package in R, developed by Hyndman and Khandakar (2008). This function identifies the best ARIMA model based on either the Akaike information criterion (AIC), the bias-corrected AIC (AIC_c), or the Bayesian information criterion (BIC) value. This function searches among the possible models within the provided order constraints.

2.2.1.3. Artificial Neural Network (ANN) Model

Artificial neural networks (ANNs) provide a further extension of regression by establishing the sequence of layers as derived variables. The structure of ANN is characterized by three layers: 1) the input layer receives the input values of the predictors (lagged terms in this case); 2) the hidden layer(s) receives inputs from the input layer and then the sigmoid transfer function is applied to produce an output; and 3) an output layer accepts the inputs from the hidden layer and produces forecasted values. An ANN's predicted value is:

$$\hat{y}_t = \sum_{j=1}^{k} b_j h_j \tag{6}$$

$$h_j = f\left[\sum_{i=1}^{p} w_{ji} z_i(t)\right] \tag{7}$$

where $\hat{y}$ is the forecasted value, t is the time instant, k is the number of nodes in the hidden layer, p is the number of lagged inputs, b_j and w_{ji} are the respective linear and nonlinear weights of the ANN connections learned from the data, Z_i represents the i^{th} lagged term, and $f(x)$ is the sigmoid transfer function denoted by:

$$f(x) = \frac{1}{1 + e^{-x}} \tag{8}$$

In forecasting time series, the relationship is between the output (y_t) and the inputs (y_{t-1}, y_{t-2},…,y_{t-p}). Hence, the ANN model performs a nonlinear functional mapping from

the past observations (y_{t-1},y_{t-2},…,y_{t-p}) to the future y_t, such as:

$$y_t = f\left(y_{t-1}, y_{t-2}, \dots, y_{t-p}, w\right) + \varepsilon_t \tag{9}$$

Using lagged values of the time series as inputs in a neural network just as lagged values are used in a linear autoregression model is called a neural network autoregression (NNAR) model. This paper only considers feed-forward networks based on *NNAR*(*p*,*k*), where *p* indicates the lagged inputs and *k* indicates the nodes in the hidden layer (Hyndman & Athanasopoulos, 2018).

Neural network autoregression models are identified using the *nnetar* function from the forecast package in *R* as produced by Hyndman (2012). This function automatically fits a neural network model to the given time series with lagged values of the series as inputs, so it is a nonlinear autoregressive model.

2.2.2. Forecast Combinations

Forecast combinations have been a well-established strategy for improving forecasting accuracy since Bates and Granger's (1969) seminal paper. Numerous combination procedures have been proposed in theoretical and empirical studies, but so far no theoretical foundations exist regarding an ideal technique to use for combining diverse forecasts; instead, much depends on the features of the available data. Frequently used approaches for forecast combinations include simple combinations that ignore forecast error correlations and some more sophisticated combination schemes that estimate optimal combination weights such as regression-based and eigenvector weighting approaches. Even Andrawis et al. (2011) proposed combining the combined forecasts, called that strategy the hierarchical forecast combination.

To illustrate notations, this study denotes $F_{T\times P}$ as the matrix of forecasts with dimension $T\times P$, where T stands for the number of rows and P the number of columns, and f_i as the forecast obtained from the model i, where $i \in \{1 \ldots P\}$. The weight associated with that forecast in the overall combined forecast is represented as w_i , and the combined forecast as f^c.

2.2.2.1. Simple Forecasts Combination Methods

This study starts with some simple ways of combining forecasts, simple in that they ignore correlations between forecast errors and have no need to estimate the weight of each forecast to be assigned in the overall contribution.

2.2.2.1.1. Simple Average-Based Combination

This is the most intuitive approach for forecast combinations, perhaps due to its simplicity (Weiss et al., 2019). This is just the arithmetic average of single forecasts and is given as:

$$f^c = \frac{1}{P}\sum_{i=1}^{P} f_i \tag{10}$$

where f^c is the combined forecast, P is the number of single models to be combined, and f_i is the forecast obtained from model i.

This equal weighting strategy is also called the forecast combination puzzle, a term coined by Stock & Watson (2004), and may in some situations may reveal better forecasts than the sophisticated forecast combination approaches for which Clemen (1989) argued. Smith and Wallis (2009) and Claeskens et al. (2016) have provided a rigorous empirical and theoretical explanation as to why this innocent approach outperforms more sophisticated techniques .

2.2.2.1.2. Median-Based Combination

This technique is another simple location-measure combination method that is robust against outliers by using the median of the component forecasts, which can be relevant for certain applications. The simple average may not be an appropriate combination strategy when bias occurs in some of the component forecasts, as Palm and Zellner (1992) suggested, and the median combination becomes handy in such cases. The combined forecast using the median method is given as follows:

· For odd p:

$$f^c = f_{(\frac{p}{2}+0.5)} \tag{11}$$

· For even p:

$$f^c = \frac{1}{2}(f_{(\frac{p}{2})} + f_{(\frac{p}{2}+1)} \tag{12}$$

2.2.2.1.3. Bates-Granger Combination

Bates and Granger's (1969) influential paper proposed the idea of combining forecasts. Their technique uses the diagonal elements of the estimated mean squared prediction error matrix to compute combination weights. Therefore, the combined forecast is calculated as:

$$f^c = \sum_{i=1}^{P} f_i' \times \frac{\hat{\sigma}_i^{-2}}{\sum_{j=1}^{P} \hat{\sigma}_j^{-2}} \tag{13}$$

where $\hat{\sigma}_i^{-2}$ is the estimated mean squared prediction error of model i.

Despite this method being derived under the assumption of uncorrelated forecasts, it is able to work well in practice.

2.2.2.2. Regression-Based Forecasts Combination Methods

This study implements two regression-based combination methods, the ordinary least squares (OLS) regression-based combination method and the least absolute deviation-based (LAD-based) combination method. These two regression-based combinations consider both the time-invariant combination weighting (static version) and time-varying combination weighting (dynamic version) strategies. This makes the regression-based methods considered here a total of four methods.

2.2.2.2.1. Ordinary Least Squares (OLS) Regression Combination

One of the sophisticated rules regarding combining forecasts involves OLS. The idea of regression-based combinations had been developed by Crane and Crotty (1967), with Granger and Ramanathan (1984) later expanding its usage more successfully. This approach assumes the combined forecast to be a linear function of the individual forecasts, with the weights being determined by regressing the actual values of the individual forecasts.

$$y = \alpha + \sum_{i=1}^{P} w_i f_i + \varepsilon, \tag{14}$$

The combined forecast is calculated as:

$$f^c = \hat{\alpha} + \sum_{i=1}^{P} \hat{w}_i f_i, \tag{15}$$

One good feature of OLS forecast combinations is that the combined forecast is unbiased due to the intercept in the equation, even when one of the individual forecasts is biased. One drawback, however, is that it does not restrict the combination weights. Hence, they do not add up to 1 and can be negative, which complicates interpretation.

2.2.2.2.2. Least Absolute Deviation (LAD) Regression-Based Combination

Instead of minimizing the sum of squared errors when estimating the coefficient in Equation 15, one may prefer to estimate the coefficients by minimizing the absolute sum of squares. The LAD method is less sensitive to outliers and can retain its stability when the component forecasts are highly correlated in contrast to OLS. The LAD combination strategy should be preferred over OLS in the above situations (Nowotarski et al., 2014).

2.3. Implementing Combined Forecasts

Table 1 shows the combination schemes used in this paper, as implemented with the help of R functions from the package program ForecastComb developed by Weiss et al. (2019).

Table 1

R Functions for Descriptions of the Forecast Combinations Used in This Paper

Function	Description
Simple Forecast Combination Functions	
comb_SA	simple average forecast combination
comb_MED	median forecast combination
comb_BG	Bates/Granger (1969) forecast combination
Regression-Based Forecast Combination Functions	
comb_OLS	ordinary least squares (OLS) forecast combination
comb_LAD	least absolute deviation (LAD) forecast combination
rolling_combine	Computes the dynamic version of the combined forecasts (time-varying combination weights). The inside of this function needs to specify the combination method that should be used (OLS or LAD here).

2.4. Forecasting Accuracy Metrics

The RMSE is used to compare the forecast methods and assess the accuracy of the out-of-sample forecasts against the reserved observed values in the evaluation sample (i.e., the test set) and is computed as follows:

$$RMSE = \sqrt{\frac{\sum_{t=1}^{T}(\hat{y}_{t+h|t} - y_{t+h})^2}{T}} \tag{16}$$

where $\hat{y}_{t+h|t} - y_{t+h}$ states the forecast error and training data given by $\{y_1,...,y_t\}$, and the test data is given by $\{y_{t+1},y_{t+2},...\}$.

RMSE is widely used to compare forecast methods applied to an individual time series or to numerous time series with the same units (Hyndman & Athanasopoulos, 2018). Because this study's time series data simulations are replicated based on the same data generation process, RMSE is an appropriate measure for comparing the predictive performances of the considered forecast methods.

2.5. Simulation Procedures

2.5.1. Simulation Designs

The simulation considers two different designs regarding data-generating processes:

1. Design 1 is based on the AR (3) process with ARCH (2) effects as shown below,

$$y_t = \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 y_{t-3} + \varepsilon_t \tag{17}$$

$$\varepsilon_t = \sigma_t a_t \tag{18}$$

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 \tag{19}$$

where a_t is the white noise ω, and α_1 and α_2 are the parameters of the variance model. Substituting for σ_t^2 gives:

$$\varepsilon_t = a_t \sqrt{\omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2} \tag{20}$$

2. Design 2 is based on the ARIMA(3,1,0) process with ARCH (2) effects The formulation of Design 2 is same as in Design 1 except the second design process generates time series data with trend effects in addition to ARCH effects.

The simulations from both designs are conducted by specifying fixed stationary and invertible parameters. For Design 1, the parameters for the AR segment are specified as $\mu = 5, \phi_1 = 0.65, \phi_2 = -0.55$, and $\phi_3 = 0.3$, while the parameters for the ARCH segment are $\omega = 0.2, \alpha_1 = 0.45$, and $\alpha_2 = 0.3$. Similarly for Design 2, the stationary and invertible parameters for the ARIMA segment are specified as $\mu=5, \phi_1= 0.45, \phi_2 = 0.3$, and $\phi_3 = -0.75$, while the parameters for the ARCH segment are the same as those from Design 1. That these values Are chosen arbitrarily is worth noting. Figure 1 shows the graphical visualization of the simulated data.

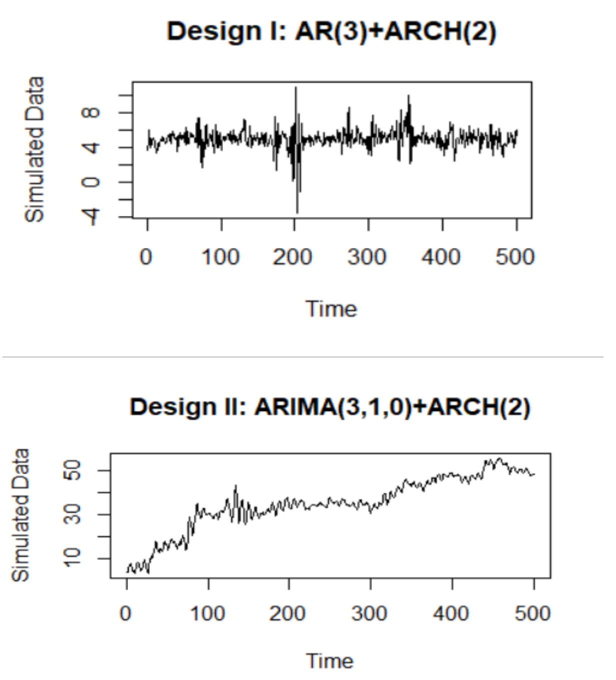
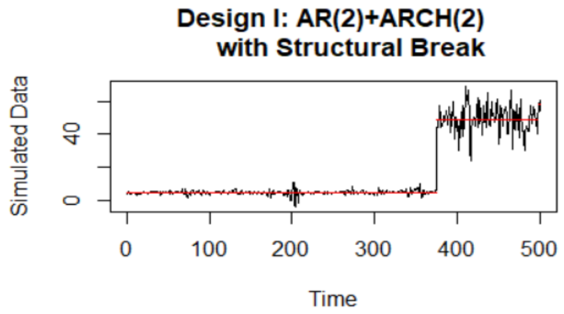


Figure 1. Graphical illustration of the simulated data

The forecasting models under study are applied to the simulated data before the structural break is placed to forecast out-of-sample observations. A single permanent structural break is allowed in the mean of the generated time series in both designs. The structural break is positioned in the 75th-percentile observation of the total sample. The break took a multiplicative form, and so the pre-structural break mean of μ becomes $\delta \mu$ after the break, with δ having a break size of 10 in this case. Figure 2 below illustrates the data after the structural break.



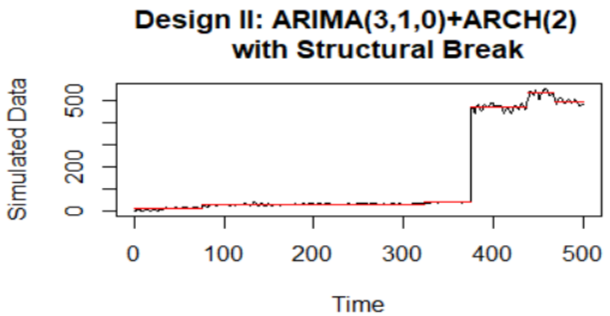


Figure 2. Simulated data after the structural break

Table 2

Summary of the Simulated Data Procedures and Train/Test Sets

Design	Case	Break Size	Break Location	Sample Size	Training Set(size)	Test set (size)
Design 1	Stable	-	-	500	480, 490, 495	20, 10, 5
	Break	10	75^{th}% Observation	500	480, 490, 495	20, 10, 5
Design 2	Stable	-	-	500	480, 490, 495	20, 10, 5
	Break	10	75^{th}% Observation	500	480, 490, 495	20, 10, 5

As shown in Table 2, the total sample size in each case ($T=n+h$) is 500,, where T is the total sample size, n is the training sample size, and h is the prediction sample size. To examine if the horizon size has an impact on forecasting accuracy, the first 480, 490, and 495 (n = 480, n = 490, and n = 495) observations are reserved for the training sample, leaving the rest as the prediction sample (h = 20, h = 10 and h = 5) in three separate experiments per case. The simulation is then repeated 3,000 times per case.

Each case generates forecasts using three single methods: ARIMA, ETS, and ANN. The study sought to improve forecasting accuracy by combining the component forecasts from single models into simple forecast combination and regression-based combination models. The simple forecast combination models considered in this study are the simple average forecast combination, median forecast combination, and Bates–Granger forecast combination. The regression-based combination models used here are the OLS forecast combination (both statistic and dynamic versions) and the least absolute deviation forecast combination (both static and dynamic versions), with RMSE being used to assess their predictive performance.

2.5.2. Simulation Results

The following tables present the RMSE values used to measure forecasting accuracy. An RMSE value being less than the others indicates that the method under consideration outperforms the other competing methods. The smaller the RMSE, the better the forecasting performance is for that given sample.

Table 3

Design I Stable Case Results

Model	***h* = 20**	***h* = 10**	***h* = 5**
	Individual Models		
ETS	0.03464	0.02664	0.02516
ARIMA	0.03150	0.02310	0.01503
ANN	0.03038	0.02159	0.01013
	Simple Forecast Combination		
Comb_SA	0.03200	0.02320	0.01586
Comb_MED	0.03125	0.02311	0.01551
Comb_BG	0.03145	0.02278	0.01437
	Regression-Based Forecast Combination		
Comb_OLS_static	0.02912	0.02106	0.01031
Comb_OLS_dynamic	0.02907	0.02109	0.01033
Comb_LAD_static	0.02909	0.02117	0.00966
Comb_LAD_dynamic	0.02906	0.02123	0.00963

[1]Table 3 presents the simulation results for AR (3) with ARCH error design before the structural break is placed. The accuracy evaluation shows ANN to outperform the other stand-alone models used here for all considered horizons. Regression-based combination models (in their static and dynamic versions) exhibit improved accuracy compared to single models and simple combinations. When compared to the static versions of the models, allowing for time-varying combination weights (dynamic version) does not appear to significantly change accuracy.

1 Comb_SA: Simple average forecast combination,
MED: Median forecast Combination,
Comb_BG: Bates-Granger forecast combination
Comb_OLS_static: Ordinary Least Squares forecast combination (statistic version)
Comb_OLS_dynamic: Ordinary Least Squares forecast combination (dynamic version)
Comb_LAD_static: Least Absolute Deviation forecast combination (statistic version)
Comb_LAD_dynamic: Least Absolute Deviation forecast combination (dynamic version)

Table 4

Design 1 with Structural Break

Model	*h* = 20	*h* = 10	*h* = 5
Individual Models			
ETS	0.6977	0.4308	0.2059
ARIMA	0.6676	0.2698	0.1788
ANN	0.4962	0.2299	0.1405
Simple Forecast Combination			
Comb_SA	0.6057	0.2696	0.1549
Comb_MED	0.6256	0.2529	0.1405
Comb_BG	0.5928	0.2646	0.1531
Regression-Based Forecast Combination			
Comb_OLS_static	0.4320	0.1676	0.1395
Comb_OLS_dynamic	0.4339	0.1683	0.1399
Comb_LAD_static	0.4428	0.1971	0.1345
Comb_LAD_dynamic	0.4451	0.1988	0.1354

Table 4 reports the simulation results for AR (3) with ARCH error design using the structural break. Again, ANN performs better than its single-model ARIMA and ETS counterparts for all considered horizons. The results also suggest that all the simple combination models perform better than the single models except for ANN. Moreover, regression-based forecast combination models perform better than any other model considered here, with the two versions of the OLS method performing the best. As in the previous case, dynamic combination weights do not change, with even the static version performing better, suggesting that the combined weight estimates do not change much over time.

Table 5

Design 2 Stable Case Results

Model	*h* = 20	*h* = 10	*h* = 5
Individual Models			
ETS	1.8587	1.2474	0.9491
ARIMA	1.7695	1.2573	0.4621
ANN	1.7107	1.1923	0.4426
Simple Forecast Combination			
Comb_SA	1.7429	1.0797	0.5097
Comb_MED	1.8063	1.1909	0.4440
Comb_BG	1.7504	1.0989	0.4904
Regression-Based Forecast Combination			
Comb_OLS_static	1.6734	1.2092	0.4141
Comb_OLS_dynamic	1.6162	1.2031	0.4150
Comb_LAD_static	1.7027	1.1716	0.4295
Comb_LAD_dynamic	1.6825	1.1682	0.4300

Table 5 reports the simulation results for the Design 2 stable case, where data are generated from AR (3) with trend and ARCH (2) errors. The results suggest ANN to exhibit the greatest accuracy compared to the non-combined models. Moreover, all simple forecast combination models perform well compared to the non-combined methods with moderate forecasting horizons ($h = 10$) and even better than ANN, but they lose their dominance for the long and short horizons. Although all regression-based models show significant improvement in forecasting accuracy, the OLS model performs the best. In contrast to Design 1, the dynamic combination weights show improvement regarding the results for both the OLS and LAD forecast combinations, suggesting that the combined weight estimates change considerably over time.

Table 6 reports the simulation results for Design 2 where the data are generated from AR (3) with trend and ARCH (2) errors after the structural break is positioned in the data. As in the previous cases, ANN is the best single model for all considered horizons. Simple combinations show some improvement with respect to ARIMA and ETS, with the simple average combination model worth mentioning as the most superior for the short horizon (h = 5).

Table 6

Design 2 with Structural Break

Model	***h* = 20**	***h* = 10**	***h* = 5**
	Individual Models		
ETS	11.1834	8.6215	5.4916
ARIMA	11.1829	8.6216	5.4857
ANN	11.0805	8.5229	5.3504
	Simple Forecast Combination		
Comb_SA	11.1034	7.2160	2.3155
Comb_MED	11.1831	8.6215	5.4857
Comb_BG	11.1058	7.2384	5.4886
	Regression-Based Forecast Combination		
Comb_OLS_static	10.9607	7.5337	4.5216
Comb_OLS_dynamic	10.7620	7.4997	4.4685
Comb_LAD_static	10.1257	7.5863	2.8762
Comb_LAD_dynamic	9.9617	7.5088	2.8344

The regression-based forecast combination methods again improve the forecasting accuracy, with dynamic combination weighting showing particularly significant improvement compared to the static counterparts.

2.6 Summary of Simulation Results

1. ANN performs better than its non-combined counterparts across all considered horizons for all examined cases and data-generating designs.

2. Simple forecast combinations (simple average, median, and Bates-Granger) perform better than ETS and ARIMA in most cases.

3. Regression-based forecast combination methods outperform all other competing models for all cases regardless of which design data is the basis or if the structural break is present or not.

4. Allowing for time-varying combination weights (dynamic version) shows a significant improvement in accuracy compared to the static counterparts with regard to trends; this suggests that the estimated combination weights fluctuate much over time. In contrast, dynamic combination weights do not change much with regard to the absence of trends.

5. Other studies, including Hsiao & Wan (2014), have shown regression-based approaches to be favorable if one of the component forecasts outperforms the rest. This also being the case in the current study, the results here agree with this conclusion.

6. Series that contain both the trend and structural break error show the simple average and Bates-Granger combinations to perform better than ANN for $h = 10$, with regression-based forecast combinations (both static and dynamic) also performing better than ANN for all horizons. This shows the simultaneous presence of trend and structural break errors to adversely affect ANN's performance.

7. The presence of trend errors in the series with an additional break in the trend similarly increases the RMSEs for all designs and across all horizons and methods. Even removing the trend error by taking the difference is unable to prevent an increase in RMSEs.

8. RMSEs are smaller for short horizons ($h = 5$) than longer horizons ($h = 10$ and $h = 20$) under all conditions. This also shows working with short-term periods to always be safer for forecasting.

3. Empirical Applications

In line with the simulation evidence, all considered models in the simulation have been applied to forecast daily closing prices of two popular Turkish financial stocks data (i.e., BIST-30 and BIST-100 Indexes). The study has taken Borsa Istanbul (BIST) data into consideration because it has the features that interest this study. As shown in Tables 7 and 8, both considered stock data indexes contain multiple structural breaks and ARCH effects.

Several studies in the literature have used various methods to forecast BIST-100 and BIST-30 Indexes. For instance, Aygören et al. (2012) studied BIST-100 Index forecasting using classical time series models such as ARIMA, numerical search models, and ANN models, arguing ANN to have outperformed the Newton numerical search models and conventional time series models. Telli and Coşkun (2016) also forecasted the BIST-100 Index using ANN models with daily data between July-November 2015 and showed the structured multilayer perceptron (MLP) model to be the best among the several tested models. Ünvan and Ergenç (2022) additionally compared the predictive ability of ANN and regression models applied to the BIST-

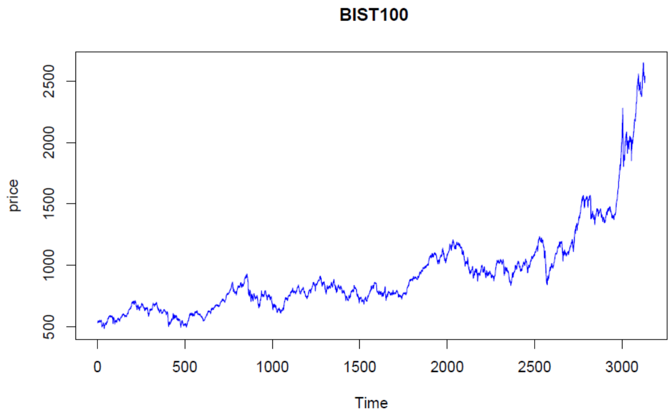
100 Index's closing prices between 2010-2020 and found ANN to perform better than the considered regression models.

Raşo and Demirci (2019) used deep learning methods to forecast the Turkish Stock Market on the BIST-30 Index from January 2016-April 2018, with their study's findings revealing the deep learning model to outperform other techniques such as support vector regression (SVR). Furthermore, Alp et al. (2020) conducted a comparative study on BIST Index price prediction, comparing the performance of ARIMA against two deep learning methods (i.e., long short-term memory [LSTM] and gated-recurrent unit [GRU]) for predicting the BIST-30, BIST-50, and BIST-100 price indexes. Their study found the ARIMA models to outperform the deep learning models with regard to predicting the considered price indexes.

Aker (2022) likewise examined price volatility of the BIST-100 Index by comparing LSTM and Facebook Prophet (Fbprophet) methods and suggested the LSTM model to have outperformed the Fbprophet model based on the RMSE, mean absolute error (MAE), and mean squared error (MSE) evaluation metrics. Pakel and Özen (2021) also investigated a volatility analysis of the BIST-100 Index using GARCH models and identified two significant shocks in the BIST-100 (i.e., currency shock and COVID-19 pandemic shock) in 2018 and 2020, respectively. Their research revealed stock market volatility to have increased significantly during the 2018-2020 period and the increase to have been more persistent during the COVID-19 pandemic. Yılmaz and Kale (2022) also analyzed the short- and long-term asymmetrical effects of companies' financial risk ratios, with their findings showing no asymmetrical relationship to be present between risk and financial ratios in the short term.

All these recent studies have indicated producing predictions based on the BIST-30 and BIST-100 Index data to still be important, as these indexes are influenced by the undesirable conditions the Turkish economy experiences in the short and long term. Some of these articles compared classical and modern forecasting methods, while others tried to model variances. Unlike the articles cited above, this study compares classical time series models and ANN models with regard to structural break and ARCH errors over BIST-30 and BIST-100 Index data, where producing high-frequency, volatile, low-error predictions is difficult.

Figure 3 shows the plot for the BIST-30 and BIST-100 Index data. For example, both considered stock datasets contain multiple structural breaks and ARCH effects, as shown in Figure 4 and Tables 7, 8, and 9. Stocks data for the Jan 4, 2010-Jun 20, 2022 period were extracted from this website (www.investing.com).



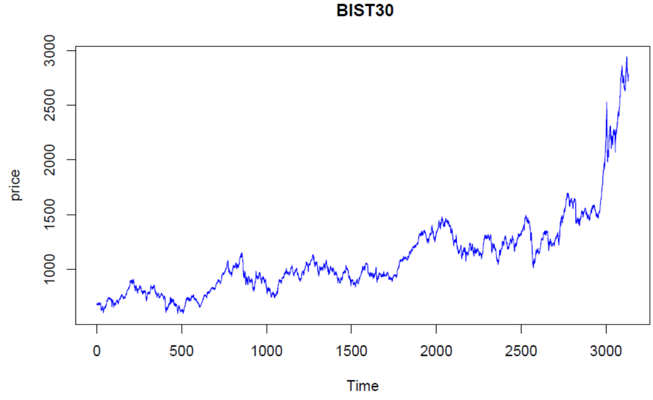


Figure 3. BIST-30 and BIST-100 Index data

3.1. Testing ARCH Effects

The Lagrange multiplier (LM) test has been used to test for the presence of ARCH effect in the series. This test is conducted using the *archTest* function in R from the *FinTS* package developed by Tsay (2005). The LM test uses the null hypothesis of no ARCH effects against the alternative hypothesis of ARCH effects presence.

Table 7

Lagrange Multiplier Test Results

Data	LM Test Statistic	*p*_value
BIST-100	771.6928	0.000
BIST-30	933.8855	0.000

Since the *p* values in Table 7 are zero or close to zero, the null hypothesis regarding residuals exhibiting no ARCH effects is rejected. Therefore, ARCH effects are concluded to be present in both the series under study here.

3.2. Structural Break(s) Testing

Before running a forecast with the models under consideration, the presence of structural breaks in the data must be checked first. This is done using Bai-Perron (2003) multiple breakpoint test with the R function *breakpoints* from the *strucchange* package.

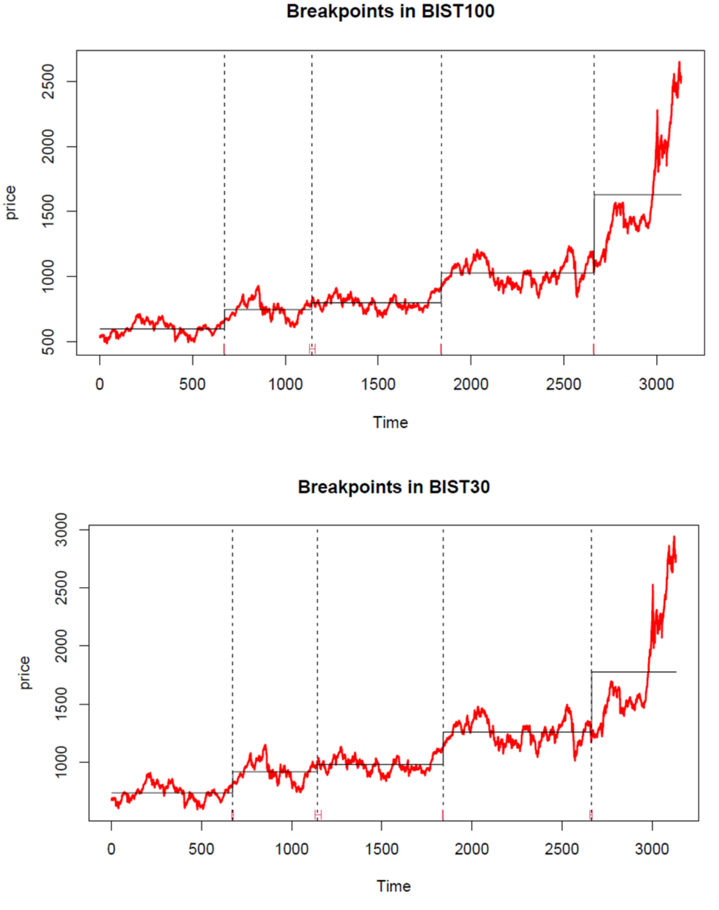


Figure 4. Break Locations and Segments.

Figure 4 suggests the data from both stock indexes to contain at least four significant structural breaks. Tables 8 and 9 show these breakpoints, the estimated break dates, and optimal segmentation accompanied by the estimated intercepts for each segment. Both the BIST-100 and BIST-30 Indexes have the same number of structural breaks and similar locations for the break dates. This study's empirical data now clearly contain structural breaks and ARCH effects, thus the time has come to proceed with the forecasting process and measure its accuracy.

Table 8

BIST100 Break Locations and Segmentation

Breakpoint	Break Date	Segment	Intercept
672	Aug. 29, 2012	Jan. 4, 2010 – Aug. 29, 2012	600.233
1,141	Jul. 14, 2014	Aug. 30, 2012 – Jul. 14, 2014	748.157
1,839	Apr. 19, 2017	Jul. 15, 2014 – Apr. 19, 2017	800.134
2,663	Aug. 7, 2020	Apr. 20, 2017 – Aug. 7, 2020	1,028.809
		Aug. 8, 2020 – Jun. 20, 2022	1,628.825

Table 9

BIST30 Breakpoints and Segmentation

Breakpoint	Break Date	Segment	Intercept
672	Aug. 29, 2012	Jan. 4, 2010 – Aug. 29, 2012	738.596
1,141	Jul. 14, 2014	Aug. 30, 2012 – Jul. 14, 2014	919.286
1,839	Apr. 19, 2017	Jul. 15, 2014 – Apr. 19, 2017	981.905
2,663	Aug. 7, 2020	Apr. 20, 2017 – Aug. 7, 2020	1,260.563
		Aug. 8, 2020 – Jun. 20, 2022	1,777.168

Table 10

Train-Test Sets Plan of the Empirical Data

Stock Index	Sample Size	Training Set(size)	Test set (size)
BIST-100	3,132	3,112, 3,122, 3,127	20, 10, 5
BIST-30	3,132	3,112, 3,122, 3,127	20, 10, 5

As Table 10 shows, the total sample size for each case ($T=n+h$) is 3,132 , where T is the total sample size, n is the training sample size, and h is the prediction sample size. Although the total sample size of the empirical data is much more than that of the simulated data, the study examined the same prediction sample sizes (horizon size) as in the simulation. In each case, the study reserves the first 3,112, 3,122 and 3,127 (i.e., $n_1 = 3112$, $n_2 = 3122$ and $n_3 = 3127$) observations as the training sample sizes and the remainder as the prediction sample size ($h = 20$, $h = 10$, and $h = 5$) in the three separate experiments for each case.

Tables 11 and 12 present the empirical results for the two popular Turkish financial stocks data indexes (i.e., BIST-30 & BIST-100). The results from the empirical data are similar to those from Design 2's simulated data. The possible explanation for this similarity is that they have similar components in the sense that both data contain ARCH effects, trends, and structural breaks.

Table 11

BIST-30 Results

Model	*h* = 20	*h* = 10	*h* = 5
Individual Models			
ETS	167.9629	160.405	23.49172
ARIMA	172.8163	156.102	23.48209
ANN	158.327	127.077	23.13754
Simple Forecast Combination			
Comb_SA	166.3248	147.7268	23.28021
Comb_MED	167.9673	156.0968	23.54386
Comb_BG	166.3106	147.2373	23.27600
Regression-Based Forecast Combination			
Comb_OLS_static	156.9727	101.259	23.05612
Comb_OLS_dynamic	135.9082	86.07532	23.15337
Comb_LAD_static	158.0176	121.7372	23.08587
Comb_LAD_dynamic	150.6015	114.8549	23.10114

Table 12

BIST-100 Results

Model	*h* = 20	*h* = 10	*h* = 5
Individual Models			
ETS	164.6264	117.8719	22.70763
ARIMA	165.9428	120.8585	22.85353
ANN	154.2198	89.08681	22.38252
Simple Forecast Combination			
Comb_SA	161.5748	109.0967	22.62900
Comb_MED	164.6226	117.7284	22.67512
Comb_BG	161.4729	108.7855	22.62865
Regression-Based Forecast Combination			
Comb_OLS_static	150.7041	69.4551	22.0219
Comb_OLS_dynamic	138.3404	55.8600	22.0339
Comb_LAD_static	153.8752	99.2405	20.8992
Comb_LAD_dynamic	150.4317	88.8441	20.8992

1. Among the non-combined models, ANN performs much better than its counterparts for all considered horizons. Unlike the simulated data, a large difference occurs between ANN and its counterparts regarding the medium (h = 10) and long (h = 20) horizons, while the difference is smaller regarding the short horizon (h = 5).

2. For simple combination models, Comb_SA and Comb_BG perform better than the non-combined models apart from ANN for all horizons.

3. The regression-based forecast combination models significantly improve forecasting accuracy. Dynamic versions of combined weights provide the best results, with the OLS combined weighting being the best performing model in both the BIST-30 and BIST-100 data sets for all horizons.

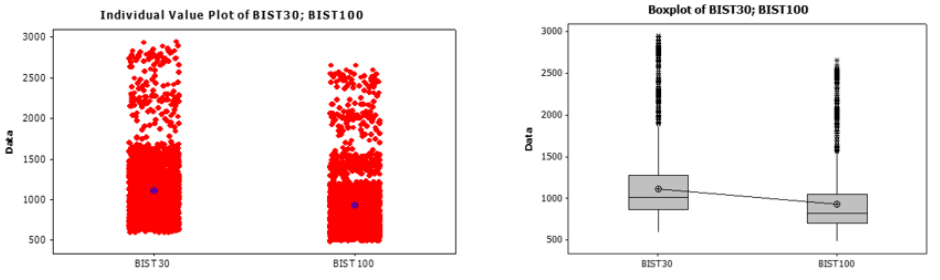


Figure 4. BIST-30 and BIST-100 means and variations

4. Both the BIST-30 and BIST-100 time series graphs show similar behaviors and structural breaks; however, the BIST-100 RMSEs are smaller for all cases. This could be explained by BIST-100 including more companies, so its average and standard deviation are lower than in the BIST-30 Index, as shown in Figure 4. The statistical significance of the difference between the variances of the two series (BIST-30 & BIST-100) was tested using the F test and Levene test, which show a statistically significant difference to exist between the two variances. The significance of the statistical difference between the means of the two series was tested using the Mann-Whitney test, which also showed a statistically significant difference to be present between the two averages. Therefore, this study's models perform better over the BIST-100 dataset because it has less variation compared to the BIST-30 Index.

5. Although the results obtained in the study regarding individual models partially parallel the results found in the reviewed studies, the current study's application forecast combination models and comparison of combined forecast performances under different conditions offer a contribution to the literature.

4. Conclusion

This study has aimed to examine if the generally accepted concepts regarding prominence of forecast combinations in forecasting accuracy is sustained with respect to specific cases of structural breaks and conditional heteroscedasticity, which are well known phenomena in financial time series. This paper has also explored which combination schemes are optimal with regard to these cases by combining the ETS, ARIMA, and ANN models, using simple and regression-based combination techniques to combine the individual components of the forecasts. These methods have been implemented using simulated and empirical-based data.

3 The study found regression-based forecast combination methods to be the best forecast combination schemes for cases with structural breaks and conditional heteroscedasticity. Allowing for time-varying combined weights (dynamic version) revealed a significant improvement in accuracy compared to the static counterparts for

the considered time series containing trends. This suggests the estimated combination of weights fluctuate greatly over time. In contrast, the dynamic combined weights did not change much for the case no trends when compared to its static counterpart. Although simple combination schemes that include the simple average model have exhibited wonderful performance in the literature on forecasting, they only perform better than ETS and ARIMA, while hardly outperforming ANN. This is an indication that structural breaks affect the performance of simple combination strategies. Furthermore, ANN was the best individual model for all cases regarding the simulated and empirical data compared to its single counterparts, even outperforming the simple forecast combination methods in most cases.

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