# MODIFIED EXPONENTIAL TYPE ESTIMATOR FOR POPULATION MEAN USING AUXILIARY VARIABLES IN STRATIFIED RANDOM SAMPLING

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**Abstract**

Technology's perpetual vicissitude and product models' distinction in industrial market have a crucial effect on forecasting demand for spare components. In order to set forth the future demand rates for products, inventory managers repetitively update their prognostications. Bayesian model is utilizing a prior probability distribution for the injunctive authorization rate which was habituated in order to get optimum levels of account over a number of periods. However, under sundry demand rates like intermittent demand, Bayesian Model's performance has not been analyzed. With the help of a research question, the study investigates that circumstance.

*Keywords: Stratified random sampling, exponential type estimates, Auxiliary information, Mean squared error, Efficiency*
*Jel Code: C10*

# TABAKALI RASGELE ÖRNEKLEMEDE YARDIMCI DEĞİŞKENLER KULLANARAK KİTLE ORTALAMASI İÇİN DEĞİŞTİRİLMİŞ ÜSTEL TİP TAHMİN EDİCİ

**Özet**

Bu çalışmada, kitle ortalaması için yardımcı değişken bilgisi kullanarak yeni bir üstel tip tahmin edici tabakalı örneklemede geliştirilmiştir. Elde edilen tahmin edicinin etkinliğini değerlendirebilmek için, ilk olarak literatürdeki bazı tahmin ediciler incelenmiş ve önerilen stratejinin optimum özelliği incelenmiştir. Önerilen tahmin edicinin özelliğini değerlendirebilmek için optimallik koşulu altında benzetim çalışması ve gerçek veri uygulamaları yapılmıştır. Sonuçlar elde edilen tahmin edicinin var olan oran ve çarpım tahmin edicilerinden ve tabakalı örnekleme düzeninde yansız tahmin ediciden daha etkin olduğunu göstermiştir.

*Anahtar Kelimeler : Tabakalı rasgele örnekleme, üstel tip tahmin ediciler, Yardımcı değişken, Hata Kareler Ortalaması, Etkinlik*
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## 1. INTRODUCTION

Sample surveys play important role in social science research and also in interdisciplinary research. One of the most popular sample designs used by survey researchers is simple random sampling design which is often applied without consideration of the population random variable one wants to study. When the population distribution is highly skewed, to the right or to the left, simple random sampling might not be the most appropriate sampling design to study the nature of the population. To obtain more precise estimates of some parameters the researcher should consider the importance of the sample units or the data from a sample drawn from a population differently. One of the sampling designs that apply different weights

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to the sampling units drawn from different subsets of the population is stratified random sampling (SRS) design. In stratified random sampling, the population is partitioned into a number of strata and a simple random sample is drawn from each stratum independently from the others. For the estimation of the population parameters in such a design, one uses different weights to the units drawn from different strata to obtain unbiased estimates. By this way, the estimates of the population characteristics from SRS design are usually more precise than those from other designs [1].

In a sampling survey situation, the investigators often collect observations from more than one variable, including the variable of interest y and some auxiliary variables x's. For example, to estimate the average household living expense, the variable of interest is the living expense of a household, and the auxiliary variable can be the total income, the number of household members, the social status or the residential area of the household. For obtaining a better inference, one would like to utilize the information provided by the auxiliary variable to make the best use of the survey data. It is well known that the use of auxiliary information at the estimation stage improves the precision of estimates of the population mean or total. Ratio, product and regression methods of estimation are good examples in this context. If the correlation between study variable y and the auxiliary variable x is positive (high), the ratio method of estimation envisaged by Cochran [2] is used. On the other hand if the correlation between y and x is negative (high), the product method of estimation envisaged by Robson [3] and revisited by Murthy [4] can be employed quite effectively. Diana [5] suggested a class of estimators of the population mean using one auxiliary variable in the stratified random sampling and examined the MSE of the estimators up to the k-th order of approximation. Kadilar and Cingi [6], Singh and Vishwakarma [7, 8], Singh et al. [9] proposed estimators in stratified random sampling. There are also some recent studies proposing estimators depending on the exponential function. Bahl and Tuteja [10], Singh et al. [9, 11] suggested some exponential ratio type estimators for the SRS.

In this study, under stratified random sampling without replacement scheme (SRSWOR), we suggest an exponential type estimator to estimate the population mean of the study variable which is more efficient than the traditional estimators. The outline of the paper is as follows: in Section 2, we consider several estimators of the finite population mean that are available in literature. The proposed estimators are given in Section 3 along with the corresponding MSE expressions. In Section 4, we provide theoretical comparisons to evaluate the performances of the proposed and existing estimators. Real data applications are provided in Section 5 and an empirical

study is conducted in Section 6, and some concluding remarks are given in Section 7.

## 2. EXISTING ESTIMATORS

Let us assume that a finite population $U = (u_1, u_2, ..., u_N)$ of size N, and let y and x, respectively, be the study and auxiliary variables associated with each unit $u_j$ ($j = 1, 2, ..., N$) of the population. Let the population size, N, is stratified into L strata with h-th stratum containing $N_h$ units, where $h = 1, 2, ..., L$, such that $\sum_{h=1}^{L} N_h = N$. A simple random sample of size $n_h$ is drawn without replacement from the h-th stratum such that $\sum_{h=1}^{L} n_h = n$. Let $(y_{hi}, x_{hi})$ denote the observed values of the variables y and x on i-th of the h-th stratum, where $i = 1, 2, ..., N_h$ and $h = 1, 2, ..., L$.

It is well known that the variance of the sample mean estimator $(\bar{y}_1)$ under SRS is given by

$$Var(\bar{y}_1) = \sum_{h=1}^{L} w_h^2 \gamma_h S_{yh}^2 \qquad (1)$$

When information is available on x that is positively correlated with y, the ratio estimator is suitable for estimating the population mean. For example, the area of tillage can be considered as a useful auxiliary variable when the harvest is the population quantity of interest. Also, the amount of food resource can be used as an auxiliary variable when the number of certain species of animal is of primary interest. Hansen et al. [12] suggested a combined ratio estimator for estimating the population mean of the study variable $\bar{Y}$

$$\bar{y}_2 = \frac{\bar{y}_{st}}{\bar{x}_{st}} \bar{X} = \hat{R}\bar{X},$$

where $\bar{y}_{st} = \sum_{h}^{L} w_h \bar{y}_h$, $\bar{x}_{st} = \sum_{h}^{L} w_h \bar{x}_h$, and $\bar{X} = \sum_{h}^{L} w_h \bar{X}_h$. Here $\bar{y}_h = \sum_{i=1}^{n_h} \frac{y_{hi}}{n_h}$, $\bar{x}_h = \sum_{i=1}^{n_h} \frac{x_{hi}}{n_h}$ and $w_h = N_h / N$ is the stratum weight. Similar expressions for x can also be defined.

The mean squared error (MSE) of $\bar{y}_2$, to a first degree of approximation, is given by

$$MSE(\bar{y}_2) \cong \sum_{h=1}^{L} w_h^2 \gamma_h (S_{yh}^2 + R^2 S_{xh}^2 - 2R S_{yxh}) \quad (2)$$

where $\gamma_h = \dfrac{1 - f_h}{n_h}$, $f_h = \dfrac{n_h}{N_h}$. Here, $R = \dfrac{\overline{Y}}{\overline{X}} = \dfrac{\overline{Y}_{st}}{\overline{X}_{st}}$ is the population ratio, $S_{yh}^2$ is the population variance of a study variable, $S_{xh}^2$ is the population variance of the auxiliary variable and $S_{yxh}$ is the population covariance between study and auxiliary variables in the stratum h.

When there is a negative high correlation between y and x in the SRS, the product estimator for $\overline{Y}$ is defined by

$$\overline{y}_3 = \dfrac{\overline{y}_{st}}{\overline{X}} \overline{x}_{st}$$

and the MSE of the product estimator is given by

$$MSE(\overline{y}_3) \cong \sum_{h=1}^{L} w_h^2 \gamma_h (S_{yh}^2 + R^2 S_{xh}^2 + 2 R S_{yxh}) \qquad (3)$$

Auxiliary variables are commonly used in survey sampling to improve the precision of estimates. Whenever there is auxiliary variable information available, the researchers want to utilize it in the method of estimation to obtain the most efficient estimator. In some cases, in addition to mean of auxiliary, various parameters related to auxiliary variable, such as standard deviation, coefficient of variation, skewness, kurtosis, etc. may also be known. A number of papers on ratio type estimators appeared based on different type of transformation. Kadilar and Cingi [6] introduced an estimator for the population mean using known value of some population parameters in the SRS given by

$$\overline{y}_4 = \dfrac{\overline{y}_{st}}{\overline{x}_{st,a,b}} \overline{X}_{st,a,b},$$

where $\qquad \overline{x}_{st,a,b} = \displaystyle\sum_{h=1}^{L} w_h (a_h \overline{x}_h + b_h) \qquad$,

$\overline{X}_{st,a,b} = \displaystyle\sum_{h=1}^{L} w_h (a_h \overline{X}_h + b_h)$ and $a_h$, $b_h$ are the functions of known parameters of the auxiliary variable such as coefficient of variation $C_{xh}$, coefficient of kurtosis $\beta_{2h}$ etc.

The MSE of the estimator $\overline{y}_4$ is given by

$$MSE(\overline{y}_4) \cong \sum_{h=1}^{L} w_h^2 \gamma_h (S_{yh}^2 + R_{a,b}^2 a_h^2 S_{xh}^2 - 2 R_{a,b} a_h S_{yxh}) \quad (4)$$

where $R_{a,b} = \dfrac{\overline{Y}_{st}}{\overline{X}_{st,a,b}} = \dfrac{\displaystyle\sum_{h=1}^{L} w_h \overline{Y}_h}{\displaystyle\sum_{h=1}^{L} w_h (a_h \overline{X}_h + b_h)}$ .

Bahl and Tuteja [10] suggested an exponential ratio type estimator for the population mean in simple random sampling. Motivated by Bahl and Tuteja [10], Singh et al. [13] adapted this estimator to the SRS as

$$\overline{y}_5 = \overline{y}_{st} \exp\left( \dfrac{\overline{X}_{st} - \overline{x}_{st}}{\overline{X}_{st} + \overline{x}_{st}} \right).$$

The MSE of the estimator $\overline{y}_5$ is obtained as

$$MSE(\overline{y}_5) \cong \sum_{h=1}^{L} w_h^2 \gamma_h \left( S_{yh}^2 - R S_{yxh} + \dfrac{R^2}{4} S_{xh}^2 \right) \qquad (5)$$

Kadilar and Cingi [14] proposed the following estimator

$$\overline{y}_6 = k \overline{y}_{st} \exp\left( \dfrac{\overline{X}_{st} - \overline{x}_{st}}{\overline{X}_{st} + \overline{x}_{st}} \right).$$

The MSE of the estimator $\overline{y}_6$ is given by

$$MSE(\overline{y}_6) = k^2 \sum_{h=1}^{L} w_h^2 \gamma_h \left( S_{yh}^2 - 2 R S_{yxh} + R^2 S_{xh}^2 \right) + (k-1)^2 \overline{Y}^2$$

$$(6)$$

where $k = \dfrac{\overline{Y}^2}{\displaystyle\sum_{h=1}^{L} w_h^2 \gamma_h \left( S_{yh}^2 - 2 R S_{yxh} + R^2 S_{xh}^2 \right) + \overline{Y}^2}$ .

## 3. PROPOSED EXPONENTIAL ESTIMATOR

Motivated by Singh et al. [13], we define a modified exponential type estimator for estimating $\overline{Y}$ in the stratified random sampling as

$$\overline{y}_{PR} = \overline{y}_{st} \left( \dfrac{\overline{x}_{st}}{\overline{X}_{st}} \right)^{\alpha} \exp\left[ \dfrac{\overline{X}_{st,a,b} - \overline{x}_{st,a,b}}{\overline{X}_{st,a,b} + \overline{x}_{st,a,b}} \right] \qquad (7)$$

where a and b are suitably chosen scalars and $\alpha$ is a constant. Here, $\qquad \overline{x}_{st,a,b} = \displaystyle\sum_{h=1}^{L} w_h (a_h \overline{x}_h + b_h) \qquad$,

$\overline{X}_{st,a,b} = \displaystyle\sum_{h=1}^{L} w_h (a_h \overline{X}_h + b_h)$ and $a_h$, $b_h$ are the functions of known parameters of the auxiliary variable such as coefficient of variation $C_{xh}$, coefficient of kurtosis $\beta_{2h}$ etc.

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In order to obtain the MSE of $\overline{y}_{PR}$, let us define $\overline{y}_{st} = \overline{Y}(1 + e_0)$ and $\overline{x}_{st} = \overline{X}(1 + e_1)$ such that

$$E(e_0) = E(e_1) = 0 \quad , \quad E(e_0^2) = \frac{1}{\overline{Y}^2} \sum_{h=1}^{L} w_h^2 \gamma_h S_{yh}^2 \quad ,$$

$$E(e_1^2) = \frac{1}{\overline{Y}^2} \sum_{h=1}^{L} w_h^2 \gamma_h S_{yh}^2 \quad , \quad \text{and}$$

$$E(e_0 e_1) = \frac{1}{\overline{X}\overline{Y}} \sum_{h=1}^{L} w_h^2 \gamma_h S_{yxh} .$$

Expressing Equation (7), in terms of e's, we have

$$\overline{y}_{PR} = \overline{Y}(1 + e_0)(1 + e_1)^{\alpha} \exp\left[ \frac{a\overline{X} - a\overline{X}(1 + e_1)}{a\overline{X} + 2b + a\overline{X}(1 + e_1)} \right]$$

where $\theta = \dfrac{a\overline{X} + b}{2(a\overline{X} + b)}$. We obtain

$$\overline{y}_{PR} = \overline{Y}(1 + e_0)(1 + e_1)^{\alpha} \exp\left[ -\theta e_1 (1 + \theta e_1)^{-1} \right] \quad (8)$$

Expanding the right hand side of Equation (8) and retaining terms up to the second power of e's, we have

$$\overline{y}_{PR} = \overline{Y}(1 + e_0)(1 + e_1)^{\alpha} \left[ -\theta e_1 \left( 1 - \theta e_1 + (\theta e_1)^2 - ... \right) \right]$$

$$= \overline{Y}(1 + e_0) \left[ 1 + \alpha e_1 + \frac{\alpha(\alpha - 1)}{2} e_1^2 + ... \right] \quad (9)$$

$$* \left( 1 - \theta e_1 + \frac{(\theta e_1)^2}{2!} \right) \left( 1 - (\theta e_1)^2 \right)$$

Using Equation (9), we get

$$= \overline{y}_{PR} - \overline{Y}$$

$$\cong \overline{Y} \left[ \begin{array}{l} e_0 + \alpha e_1 + \alpha e_1 e_0 - \theta e_1 - \dfrac{(\theta e_1)^2}{2} - \\[2mm] \theta e_0 e_1 - \alpha \theta e_1^2 + \dfrac{\alpha(\alpha - 1)}{2} e_1^2 \end{array} \right] \quad (10)$$

Squaring Equation (10) and then taking expectation of both sides, we get the MSE of the estimator $\overline{y}_{PR}$ as

$$= \text{MSE}(\overline{y}_{PR})$$

$$= \sum_{h=1}^{L} w_h^2 \gamma_h \left( S_{yh}^2 + \alpha^2 S_{xh}^2 + \theta^2 S_{xh}^2 + 2\alpha R S_{yxh} - 2\theta R S_{yxh} - 2\alpha\theta S_{xh}^2 \right)$$

$$(11)$$

We obtain the optimum $\alpha$ to minimize $\text{MSE}(\overline{y}_{PR})$. Differentiating $\text{MSE}(\overline{y}_{PR})$ with respect to $\alpha$ and equating the derivative to zero, optimum value of $\alpha$ is given by

$$\alpha_{opt} = \frac{\displaystyle\sum_{h=1}^{L} w_h^2 \gamma_h (\theta S_{xh} - R S_{yh})}{\displaystyle\sum_{h=1}^{L} S_{xh}} .$$

Substituting the value of $\alpha_{opt}$ in Equation (11), we get the minimum value of $\text{MSE}(\overline{y}_{PR})$ as

$$\text{MSE}_{\min}(\overline{y}_{PR}) = V(y_{st})(1 - \rho_{st}^2) = \sum_{h=1}^{L} w_h^2 \gamma_h S_{yh}^2 (1 - \rho_c^2) \quad (12)$$

where $\rho_c$ is combined correlation coefficient in the stratified sampling across all strata. It is calculated as

$$\rho_c^2 = \frac{\left( \displaystyle\sum_{h=1}^{L} w_h^2 \gamma_h \rho_h S_{yh} S_{xh} \right)^2}{\displaystyle\sum_{h=1}^{L} w_h^2 \gamma_h S_{yh}^2 \sum_{h=1}^{L} w_h^2 \gamma_h S_{xh}^2} .$$

## 4. EFFICIENCY COMPARISONS

In this section, we compare the MSE of traditional estimators $\overline{y}_i$, $i = 1, 2, ..., 6$, with the MSE of the optimal proposed estimator $\overline{y}_{PR}$. From Equations (1)-(6) and Equation (12), we have

$$[MSE(\overline{y}_1) - MSE_{\min}(\overline{y}_{pr})] = \sum_{h=1}^{L} w_h^2 \gamma_h (S_{xh} - R S_{yh})^2 > 0$$

$$[MSE(\overline{y}_2) - MSE_{\min}(\overline{y}_{pr})] = \sum_{h=1}^{L} w_h^2 \gamma_h (S_{xh} - R S_{yh})^2 > 0$$

$$[MSE(\overline{y}_3) - MSE_{\min}(\overline{y}_{pr})] = \sum_{h=1}^{L} w_h^2 \gamma_h (S_{xh} + R S_{yh})^2 > 0$$

$$[MSE(\overline{y}_4) - MSE_{\min}(\overline{y}_{pr})] = \sum_{h=1}^{L} w_h^2 \gamma_h (S_{xh}/2 - R S_{yh})^2 > 0$$

$$[MSE(\overline{y}_5) - MSE_{\min}(\overline{y}_{pr})] = \sum_{h=1}^{L} w_h^2 \gamma_h (S_{xh}\theta - R S_{yh})^2 > 0$$

These are always true. Hence, the estimator $\overline{y}_{PR}$ under optimum condition will be more efficient than the traditional estimators $\overline{y}_i$, $i = 1, 2, ..., 6$, in all conditions.

## 5. APPLICATION TO REAL DATA SETS

In this section, the performance of the proposed estimator are assessed with that of the existing estimators for certain natural populations. Therefore, we have considered three natural populations for the assessment of the performance of the proposed estimators with that of the existing estimators. The description of the populations and the required values of the parameters are shown in the Table 1.

The Population I is taken from [6]. It is concerning the number of teachers as study variable and the number of students as auxiliary variable in both primary and secondary schools for 923 districts at six regions (as 1: Marmara 2: Aegean 3: Mediterranean 4: Central Anatolia 5: Black Sea 6: East and Southeast Anatolia) in Turkey in 2007. The Population II is taken from Kadilar and Cingi [6]. In this data set, Y is the apple production amount in 854 villages of Turkey in 1999, and x is the numbers of apple trees in 854 villages of Turkey in 1999. Tha data are stratified by the region of Turkey from each stratum. Population III is taken from the Japan Meteorological Society [14]. The number of rainy days is the study variable and the total sunshine hours is the auxiliary variable.

Note that the Neyman allocation is used to allocate sample to strata based on the strata variances and similar sampling costs in the strata. It provides the most precision for estimating a population mean given a fixed total sample size. Neyman allocation assigns sample units within each stratum proportional to the product of the population stratum size and the within-stratum standard deviation so that minimum variance for a population mean estimator can be achieved. The equation for the Neyman allocation is given by

$$n_h = n \frac{N_h S_h}{\sum_{h=1}^{L} N_h S_h}.$$

In Table 1, we observe that the correlations between auxiliary and study variables are positive for the Populations I-II. Therefore, the ratio estimator is used for the estimation of the population mean. Similarly, the product estimator is used for the Population III since the value of coefficient of correlation is negative. Then, the MSE and PRE values of the traditional and proposed estimators are obtained based on Populations I, II, and III using Equations (1) to (7) and Equation (11), respectively. These values are given in Table 2. From the values of Table 2, it is observed that $\overline{y}_{PR}$ estimators have the smallest MSE values among all existing estimators. The estimator with the highest PRE is also considered to be the most efficient that the other estimator. From this result, we can conclude that the proposed estimator is more efficient than others for all data sets. Note that $\overline{y}_4$ requires the auxiliary variable information, on the other hand, one can reach the minimum MSE value using the proposed estimator without auxiliary variable information.

## 6. SIMULATION STUDY

In this section, a simulation study is conducted to compare the performance of the proposed estimator with existing estimators in the SRS under different conditions such as different $\rho$, h and/or n. As seen from Table 2, $MSE(\overline{y}_5)$ of Singh et al. [9] is close to $MSE(\overline{y}_{PR})$ for the Population I, so we decide to compare the performance of proposed estimator with $\overline{y}_5$. Therefore, we have simulated sample (with SRSWOR) using R software (version 2.14.0). Bivariate random observations were generated from a bivariate normal distribution. For each condition, a pseudo population with size N = 1000 was generated. For each sample selected, we calculated $\overline{y}_5$ and $\overline{y}_{PR}$. The empirical MSE is defined as the average of the squared errors of the 1000 random sample and calculated as $MSE = \frac{1}{1000} \sum_{i=1}^{} (\overline{y}_i - \overline{Y})^2$. Furthermore, the empirical Relative Efficiency (RE) is defined as the ratio of the empirical MSE's of $\overline{y}_{PR}$ to $\overline{y}_5$ .It is calculated as $RE = \frac{MSE(\overline{y}_5)}{MSE(\overline{y}_{pr})}$. While performing a simulation study, we use the following steps in sequence:

1. Population size N=1000
2. Generate $\mathbf{x} = (x_1, ..., x_N)$ as the fixed values of auxiliary variable.
3. Generate the fixed values of population variable $\mathbf{y} = (y_1, ..., y_N)$ based on a bivariate normal population model with a given correlation coefficient $\rho$ between x and y.
4. Select a random sample of $(x, y)$ with sample size n.
5. Stratified the sample into h strata based on a given condition of x, and calculate $\overline{y}_{PR}$ and $\overline{y}_5$ .
6. For each case of correlation coefficient or h, 1000 different sample were selected to calculate the empirical MSE for $\overline{y}_{PR}$ and $\overline{y}_5$, denoted as $MSE(\overline{y}_{PR})$ and $MSE(\overline{y}_5)$, respectively.
7. Calculate RE of $MSE(\overline{y}_{PR})$ to $MSE(\overline{y}_5)$.

In this simulation, we would like to study the impacts of the population correlation coefficient $\rho$, number of strata h, and sample size on RE. First we fixed the sample size as n=100, and then simulate RE under different h and $\rho$. The results are summarized in Figure 1. From Figure 1, it can be seen that the performance of $\overline{y}_{PR}$ is superior to $\overline{y}_5$ most of the time. RE of $\overline{y}_{PR}$ to $\overline{y}_5$ increases as h increases, that is, the performance of $\overline{y}_{PR}$ is more preferable as the number of strata increases. On the other hand, RE increases as $\rho$ increases. The RE can be as high as more than 15 when the population correlation coefficient is high and the number of strata is 8.

Another simulation study was also conducted to

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examine the impact of sample size n as well as $\rho$ on RE. As seen from Table 2, MSE of the product estimator $\bar{y}_3$ is close to $\bar{y}_{PR}$. So, we compare the proposed estimator with the product estimator. The number of strata is chosen to be 3, which is not a large number of strata when N=1000, so that the simulation would be fair to both methods. The results are summarized in Figure 2. It is

clear that $\bar{y}_{PR}$ is superior to $\bar{y}_3$ as what appeared in Figure 1. Additionally, sample size seems to be as decisive as $\rho$ and h. With a smaller size n=20, the RE is less than the other cases when n=50, 70 and 100.

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**Table 1.** Statistics of the populations

| Stratum | Population I | | | | | | Population II | | | | | | Population III | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 1 | 2 | 3 | 4 | 5 | 6 | 1 | 2 |
| $N_h$ | 127 | 117 | 103 | 170 | 205 | 201 | 106 | 106 | 94 | 171 | 204 | 173 | 10 | 10 |
| $n_h$ | 31 | 21 | 29 | 38 | 22 | 39 | 9 | 17 | 38 | 67 | 7 | 2 | 4 | 4 |
| $\bar{Y}_h$ | 703.74 | 413 | 573.17 | 424.66 | 267.03 | 393.84 | 1536 | 2212 | 9384 | 5588 | 967 | 404 | 149.7 | 102.6 |
| $\bar{X}_h$ | 20804.59 | 9211.79 | 14309.30 | 9478.85 | 5569.95 | 12997.59 | 127 | 117 | 103 | 170 | 205 | 201 | 1630 | 2036 |
| $C_{xh}$ | 1.465 | 1.648 | 1.925 | 1.922 | 1.526 | 1.777 | 2.02 | 2.10 | 2.22 | 3.84 | 1.75 | 1.91 | 0.063 | 0.050 |
| $C_{yh}$ | 1.256 | 1.562 | 1.803 | 1.909 | 1.512 | 1.807 | 4.18 | 5.22 | 3.19 | 5.13 | 2.47 | 2.34 | 0.09 | 0.122 |
| $S_{xh}$ | 30486.751 | 15180.769 | 27549.697 | 18218.931 | 8497.776 | 23094.141 | 49189 | 57461 | 160757 | 285603 | 45403 | 18794 | 102.17 | 103.26 |
| $S_{yh}$ | 883.835 | 644.922 | 1033.467 | 810.585 | 403.654 | 711.723 | 6425 | 11552 | 29907 | 28643 | 2390 | 946 | 13.470 | 12.610 |
| $\rho_h$ | 0.936 | 0.996 | 0.994 | 0.983 | 0.989 | 0.965 | 0.82 | 0.86 | 0.90 | 0.99 | 0.71 | 0.89 | -0.779 | -0.503 |
| $\gamma_h$ | 0.024 | 0.039 | 0.025 | 0.020 | 0.041 | 0.021 | 0.102 | 0.049 | 0.016 | 0.009 | 0.138 | 0.006 | 0.150 | 0.150 |
| $w_h^2$ | 0.019 | 0.016 | 0.013 | 0.034 | 0.049 | 0.048 | 0.015 | 0.015 | 0.012 | 0.04 | 0.057 | 0.041 | 0.500 | 0.500 |

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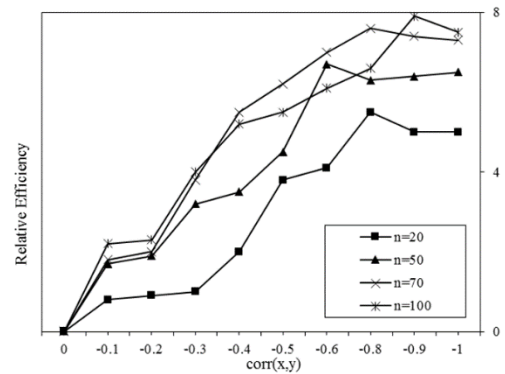htttp://www.alphanumericjournal.com/

**Table 2.** PREs and MSEs of different estimators of population mean with respect to sample mean $\bar{y}_1$ for the populations.

| Estimator | Population I | | Population II | | Population III | |
|---|---|---|---|---|---|---|
| | PRE | MSE | PRE | MSE | PRE | MSE |
| $\bar{y}_1$ | 100.000 | 2247.600 | 100.000 | 673477.704 | 100.000 | 25.534 |
| $\bar{y}_2$ | 65.303 | 3441.807 | 317.650 | 212018.592 | * | * |
| $\bar{y}_3$ | * | * | * | * | 168.097 | 15.190 |
| $\bar{y}_{41}$ | 246.322 | 912.465 | 158.378 | 425234.956 | 51.951 | 49.150 |
| $\bar{y}_{42}$ | 379.600 | 592.097 | 325.780 | 206728.053 | 50.195 | 50.870 |
| $\bar{y}_5$ | 979.393 | 229.489 | 188.869 | 356584.710 | 70.284 | 36.330 |
| $\bar{y}_6$ | 11.700 | 19210.676 | 79.096 | 851473.482 | 80.689 | 31.645 |
| $\bar{y}_{PR}$ | 1150.221 | 195.406 | 333.204 | 202122.000 | 171.174 | 14.917 |

*: Not applicable
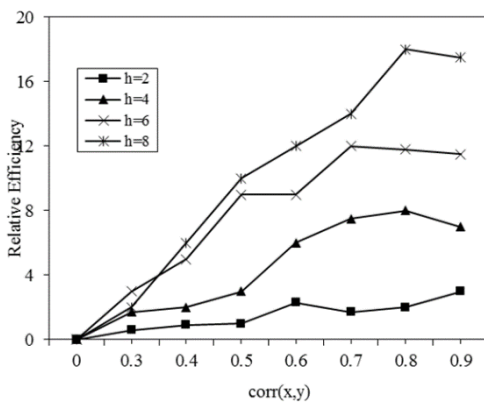


**Figure 1.** Relative efficiency of $\bar{y}_{PR}$ to $\bar{y}_5$



**Figure 2.** Relative efficiency of $\bar{y}_{PR}$ to $\bar{y}_3$

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