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Application of Fractional *SPR* Psychological Disease Model in Turkey and Stability Analysis

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Abstract

Psychological diseases and their treatment are problems related to public health. According to data from the World Health Organization, about a billion people have either mental illness or substance use disorder problems in 2017. Mental, neurological diseases and substance use disorders account for 30 percent of the global non-fatal disease burden and 10 percent of the global disease burden. It is noted that in the world Dec 2005 and 2015, the incidence of mental health diseases increased by about 16 percent. In this study, we have created a fractional-order mathematical modeling of the population of individuals suffering from psychological diseases in a society. In this model, the total population was divided into three compartments: individuals who did not receive psychological treatment (S), individuals who received psychological support (P) and individuals who recovered after completing psychological treatment (R). As a fractional derivative, we used the Caputo derivative definitions. Numerical solutions were obtained with the help of the Euler method by performing stability analysis related to the fractional *SPR* model created for the mathematical model of psychological patients. Thus, it was interpreted by creating dynamics for the number of individuals with psychological problems in a population.

1. Introduction

Psychological diseases are the general name given for psychological disorders that prevent a person from living a healthy lifestyle that he should lead in a healthy way. Psychological diseases affect people mentally, as well as the negative situations that they have created manifest themselves physically. This condition affects the person's lifestyle quite a lot. Psychological disorders, obsessive-compulsive disorder, Depression, social phobia, panic disorder, post-traumatic stress disorder, bipolar disorder, eating disorders, Adult attention deficit and hyperactivity disorder, substance abuse, anxiety disorder, in the form of nicotine dependence can be sorted [1].
Genetic factors, brain injuries, traumatic experiences, problems that arise during childbirth, intensive substance abuse, intense drug use, eating enough people, recent natural disasters, poor living conditions, death, and mourning process, the high expectations of the community, the person to live alone, sexual abuse or emotional abuse, social cohesion, the difficulty of living, evolving technology, the pace of work intensive, universal problems, economic problems, family problems, such as psychological cause diseases [2].
Mathematical modeling of individuals undergoing psychological treatment in society has been studied using ordinary differential equation (ODE) systems [2]-[4].It is well known that psychological diseases are a common condition in today's world. We also know that mental illnesses extremely affect the social behavior of people in a population. The causes of psychological illness are also an important issue for us. Such situations sometimes arise independently of each other. In some cases, psychological diseases occur in connection with each other. For example, bodily reactions also occur in people with intense traumatic lives. Or traumatic life also triggers a panic attack condition. But in some cases, psychological diseases occur alone. At this point, we need to consider the genetic factors and environmental factors of people. From another point of view, the existing genetic factor reappears due to living conditions. A disease with a recessive gene can occur due

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to intense anxiety in the later processes. Traumatic experiences are very important in this regard. A person who does not have an anxiety problem experiences an anxiety disorder as a result of an earthquake he is experiencing. Such situations also arise with past lives [3].
The main purpose of mathematical modeling is to explain the functioning of processes by expressing real-life problems mathematically. Mathematical models have been developed to help explain a system, study the effects of its various components, and make predictions about its behavior. Recently, fractional derivatives have been used in several areas, such as mechanics, cancer therapy, ecology, neural networking, image processing, epidemiology [5]-[33].
Fractional derivative models give better results in the theory of various physical and biological processes and the control of dynamic systems than integer digit models. One of the most important reasons for this is that fractional order derivative and integral definitions have a memory property. Another important reason is that although the model is the same, the fractional orders of the equations change with each real application studied, giving specific and precise results to the relevant problem. In population models, the future status of a population depends on its past status. This is called the memory effect. The memory effect of the population can be studied by adding a delay term or using a fractional derivative in the model [5]-[33].
This paper consists of four parts. In the first chapter, the importance of fractional mathematical modeling and information about psychological diseases were given. In the second part, the formation of a fractional *SPR* psychological model, the generalized Euler Method and the stability analysis of the created model were performed. In the third part, numerical results were obtained and graphs were drawn by making a new application of the fractional *SPR* psychological model. In the fourth part, the results were given.

2. Fractional Derivation and Fractional *SPR* Psychological Disease Model

The most commonly used definitions of the fractional derivative are Riemann-Liouville, Caputo, Atangana-Baleanu and the Conformable derivative. In this study, because the classical initial conditions are easily applicable and provide ease of calculation, the Caputo derivative operator was preferred and modeling was created. The definition of the Caputo fractional derivative is given below.

Definition 2.1. *[4] Let $f(t)$ be a function that can be continuously differentiable n times. The value of the function $f(t)$ for the value of α that satisfies the condition $n-1<\alpha<n$. The Caputo fractional derivative of $\alpha-$th order $f(t)$ is defined by $D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)}\int_a^t (t-x)^{(n-\alpha-1)} f^n(x)dx$.*

Definition 2.2. *[4] The Riemann-Liouville (RL) fractional-order integral of a function $A(t)\in C_n$ $(n\geq -1)$ is given by*

$$J^{\gamma}A(t)=\frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{(\gamma-1)}A(s)ds, \quad J^0A(t)=A(t). \tag{2.1}$$

Definition 2.3. *[4] The series expansion of two-parametrized form of Mittag-Leffler function for $a,b>0$ is given by*

$$E_{a,b}(t)=\sum_{t=0}^{\infty}\frac{t^i}{\Gamma(ai+b)}. \tag{2.2}$$

2.1. The fractional *SPR* psychological disease model

The fractional *SPR* model of psychological illness basically divides a community into three main groups. The first are individuals who do not receive psychological treatment, individuals who receive psychological support, and individuals who recover by completing psychological treatment. The expression of the *SPR* psychological disease model as a system of fractional differential equations is as follows [1, 2].

$$\begin{aligned} \frac{d^{\alpha}S}{dt^{\alpha}} &= \mu N-\mu S-\beta S \\ \frac{d^{\alpha}P}{dt^{\alpha}} &= \beta S-\mu P-\sigma P-\theta P \\ \frac{d^{\alpha}R}{dt^{\alpha}} &= \sigma P-\mu R \end{aligned} \tag{2.3}$$

Where $\frac{d^{\alpha}}{dt^{\alpha}}$ is the Caputo fractional derivative of α-th order with respect to time t. All compartments and parameters are shown in Table 2.1 and Table 2.2. The initial values are defined as,

$$S(0)=S_0, \quad P(0)=P_0, \quad R(0)=R_0$$

$0<\alpha\leq 1$. Since society is divided into three compartments, it is $S+P+R=N$ and all terms are derived with respect to time

$$\frac{d^{\alpha}N}{dt^{\alpha}}=\frac{d^{\alpha}S}{dt^{\alpha}}+\frac{d^{\alpha}P}{dt^{\alpha}}+\frac{d^{\alpha}R}{dt^{\alpha}}$$

it is clear that.

Table 2.1: Variables used in the model and their meanings

Variables used in the systems	Meaning
$S(t)$	t the number of individuals who do not receive timely psychological treatment (annually)
$P(t)$	t the number of individuals receiving psychological support on time (annually)
$R(t)$	the number of individuals who recovered at the time of t (annually)
$N(t)$	Total population

Because fractional-order models have a memory feature in events related to a time variable, they show more realistic and accurate results than integer-order models [5]-[14]. Therefore, the established model was created as a fractional order. In the system of (2.3), the fractional-order differential equation for $\alpha = 1$ is reduces to a full order differential equation.

Table 2.2: Parameters and their meanings

Parameters	Meaning
β	The annual rate of initiation of psychological therapy
μ	Annual birth and natural mortality rate
σ	Annual rate of complete recovery
θ	The annual mortality rate due to psychological diseases

All individuals are born into the sensitive class. Natural birth and death rates were considered equal in the model. All births are considered to have entered the sensitive class. A large proportion of individuals who have completed treatment for psychological diseases may again develop a psychological disease of the same or another type. The parameters defined in the model do not change with time. The N population was dimensioned and new variables were created as follows.

$$s = \frac{S}{N}, \quad p = \frac{P}{N}, \quad r = \frac{R}{N}$$

It is clear from here that $s + p + r = 1$. Thus, the new form of the fractional *SPR* model is written as follows.

$$\begin{aligned} D^{\alpha} s(t) &= \mu - \mu s(t) - \beta s(t), \\ D^{\alpha} p(t) &= \beta s(t) - \mu p(t) - \sigma p(t) - \theta p(t), \\ D^{\alpha} r(t) &= \sigma p(t) - \mu r(t). \end{aligned} \tag{2.4}$$

2.2. Generalized Euler method

In this paper, we used the Generalized Euler method to solve the initial value problem with the Caputo fractional derivative. Many of the mathematical models consist of nonlinear systems and finding solutions to these systems can be quite difficult. In most cases, analytical solutions cannot be found and a numerical approach should be considered for this. One of these approaches is the Generalized Euler method [15]. $D^{\alpha} y(t) = f(t, y(t)), y(0) = y_0, 0 < \alpha \leq 1, 0 < t < \alpha$ for the initial value problem, $h = \frac{a}{n}$ impending $[t_j, t_{j+1}]$ is divided into n sub with $j = 0, 1, .., n-1$. Suppose that $y(t), D^{\alpha} y(t)$ and $D^{2\alpha} y(t)$ are continuous in range $[0, a]$ and using the generalized Taylor's formula, the following equation is obtained [15].

$$y(t_1) = y(t_0) + \frac{h^{\alpha}}{\Gamma(\alpha+1)} f(t_0, y(t_0))$$

This process will be repeated to create an array. Let $t_j = t_{j+1} + h$ such that

$$y(t_{j+1}) = y(t_j) + \frac{h^{\alpha}}{\Gamma(\alpha+1)} f(t_j, y(t_j))$$

$j = 0, 1, .., n-1$ the generalized formula in the form is obtained. For every $k = 0, 1, .., n-1$

$$\begin{aligned} S(k+1) &= S(k) + \tfrac{h^{\alpha}}{\Gamma(\alpha+1)} (\mu N - \mu S(k) - \beta S(k)), \\ P(k+1) &= P(k) + \tfrac{h^{\alpha}}{\Gamma(\alpha+1)} (\beta S(k) - \mu P(k) - \sigma P(k) - \theta P(k)), \\ R(k+1) &= R(k) + \tfrac{h^{\alpha}}{\Gamma(\alpha+1)} (\sigma P(k) - \mu R(k)) \end{aligned} \tag{2.5}$$

is obtained.

2.3. Analysis of the diseased equilibrium point and stability of the fractional *SPR* psychological model

In order to find the equilibrium point (2.4) in the system , $D^{\alpha} s = 0$, $D^{\alpha} p = 0$, $D^{\alpha} r = 0$ it is considered to be.

$$\begin{aligned} D^{\alpha} s(t) &= \mu - \mu s(t) - \beta s(t), \\ D^{\alpha} p(t) &= \beta s(t) - \mu p(t) - \sigma p(t) - \theta p(t), \\ D^{\alpha} r(t) &= \sigma p(t) - \mu r(t). \end{aligned}$$

To determine the psychologically diseased equilibrium point of the system, $p(t) \neq 0$ is taken. $E_0 = (s_0, p_0, r_0)$ including,

$$E_0 = \left(\frac{\mu}{(\mu+\beta)}, \frac{\beta\mu}{(\mu+\beta)(\mu+\sigma+\theta)}, \frac{\sigma\beta}{(\mu+\beta)(\mu+\sigma+\theta)}\right) \tag{2.6}$$

the psychologically diseased equilibrium point is achieved. The Jacobian matrix at the equilibrium point of the system

$$J(E_0)=\begin{bmatrix}-\mu-\beta & 0 & 0\\ \beta & -\mu-\sigma-\theta & 0\\ 0 & \sigma & -\mu\end{bmatrix} \tag{2.7}$$

it is obtained. The eigenvalues obtained from the Jacobian matrix (2.7) are given below.

$$\begin{aligned}\lambda_1 &= -\mu-\beta,\\ \lambda_2 &= -\mu-\sigma-\theta,\\ \lambda_3 &= -\mu\end{aligned}$$

where β,μ,σ,θ are the parameters of positively defined real numbers. It is clear that $\lambda_1<0$, $\lambda_2<0$ and $\lambda_3<0$. The equilibrium point of the system is locally asymptotically stable.

Theorem 2.4. *For all $t\geq 0$, $S(0)=S_0\geq 0$, $P(0)=P_0\geq 0$, $R(0)=R_0\geq 0$, the solutions of the system in (2.3) with initial conditions $(S(t),P(t),R(t))\in R^3_+$ are not negative.*

3. Numerical simulation of the fractional *SPR* psychological disease model for Turkey

In this section, numerical simulation and graphs of the fractional *SPR* psychological disease model for Turkey data for 2019 will be shown. Now let us obtain a numerical simulation of the fractional *SPR* model using the Generalized Euler method. Let us consider the following parameters according to the data in [18]. $S=67.850.000, P=15.000.000, R=150.000, \beta=0.05, \sigma=0.01, \mu=0.022, \theta=0.15$ and let's take size of step $h=0.1$. Hence we get the following results and tables. Using the Euler method, we obtain the following tables for given.

Table 3.1: The values of S, P and R at the moment t $\alpha=1$.

t	$S(t)$	$P(t)$	$R(t)$
0	68750000,00	15000000,00	150000,00
1	68437600,00	15070750,00	164670,00
2	68127449,28	15138650,35	179378,47
3	67819531,64	15203764,16	194122,49
4	67513831,01	15266153,31	208899,18
5	67210331,43	15325878,47	223705,76
6	66909017,04	15382999,14	238539,48
7	66609872,12	15437573,64	253397,70
8	66312881,04	15489659,16	268277,80
9	66018028,30	15539311,77	283177,24
10	65725298,49	15586586,44	298093,57
11	65434676,34	15631537,06	313024,35
12	65146146,67	15674216,46	327967,23
13	64859694,42	15714676,46	342919,92
14	64575304,62	15752967,82	357880,17

Table 3.2: The values of S, P and R at the moment t $\alpha=0.9$.

t	$S(t)$	$P(t)$	$R(t)$
0	68750000,00	15000000,00	150000,00
1	68341078,49	15092609,46	169202,55
2	67936010,89	15180336,34	188471,03
3	67534760,87	15263322,19	207798,86
4	67137292,47	15341704,93	227179,65
5	66743570,03	15415618,98	246607,23
6	66353558,26	15485195,30	266075,61
7	65967222,17	15550561,53	285579,01
8	65584527,14	15611842,02	305111,80
9	65205438,85	15669157,93	324668,56
10	64829923,29	15722627,33	344244,02
11	64457946,81	15772365,22	363833,10
12	64089476,05	15818483,67	383430,88
13	63724477,96	15861091,84	403032,59
14	63362919,82	15900296,11	422633,62

Table 3.3: The values of S, P and R at the moment t $\alpha = 0.8$.

t	$S(t)$	$P(t)$	$R(t)$
0	68750000,00	15000000,00	150000,00
1	68218403,82	15120391,89	174963,23
2	67693320,72	15232532,29	200037,88
3	67174670,87	15336732,14	225209,49
4	66662375,47	15433292,11	250464,17
5	66156356,66	15522502,86	275788,62
6	65656537,54	15604645,42	301170,07
7	65162842,15	15679991,41	326596,28
8	64675195,46	15748803,45	352055,52
9	64193523,38	15811335,35	377536,54
10	63717752,69	15867832,44	403028,57
11	63247811,09	15918531,83	428521,31
12	62783627,17	15963662,67	454004,89
13	62325130,38	16003446,41	479469,87
14	61872251,04	16038097,03	504907,21

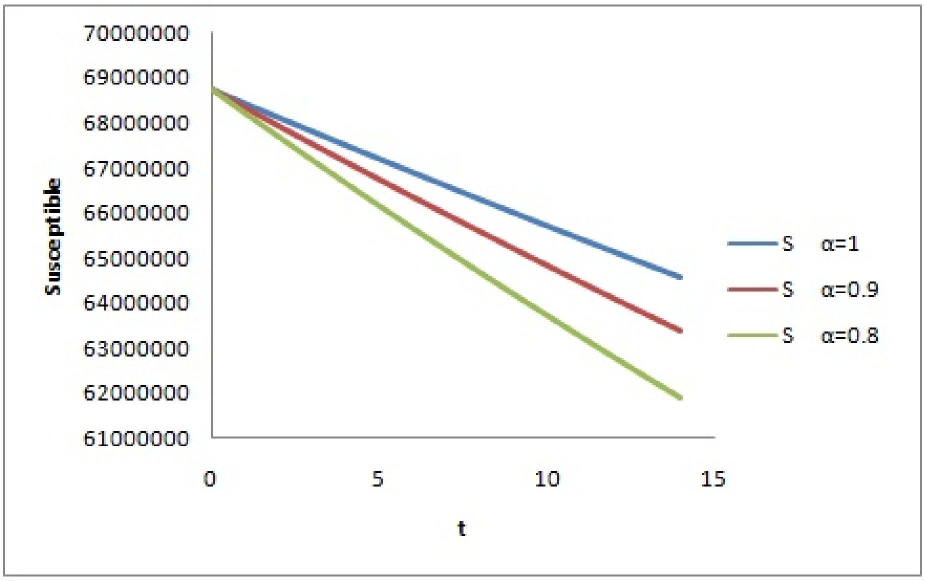

Figure 3.1: The graph of change of the S compartment model.

It is clear that, the most effective and manageable parameter of the model is β. Controlling the factors effecting human psychology, especially economic problems, will reduce the value of the β parameter. According to the observed value of the β parameter, it is seen Figure 3.1 that the number of individuals who have not yet received psychological support in Turkey is gradually decreasing. In parallel, it is clear that there will be an increase in the number of individuals in other compartments.

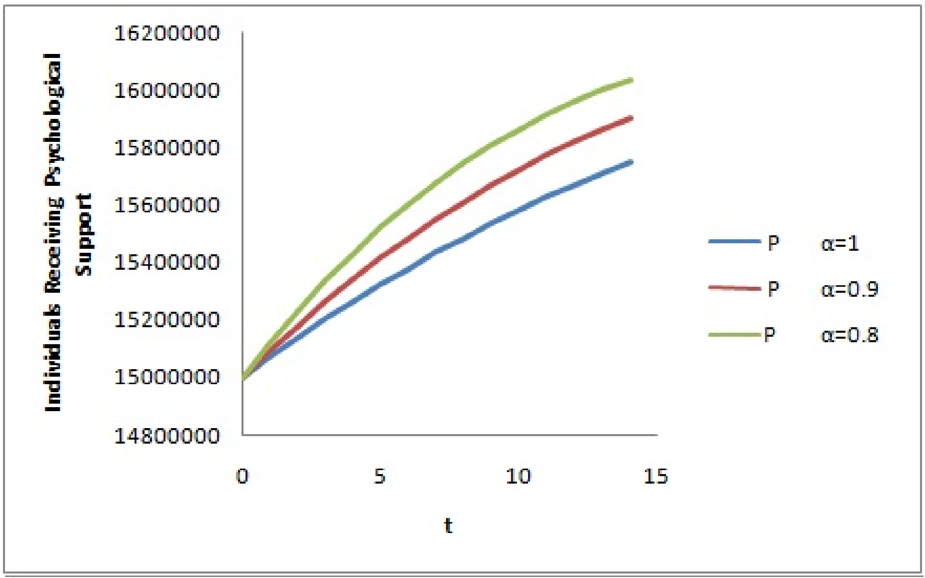

Figure 3.2: The graph of change of the P compartment model.

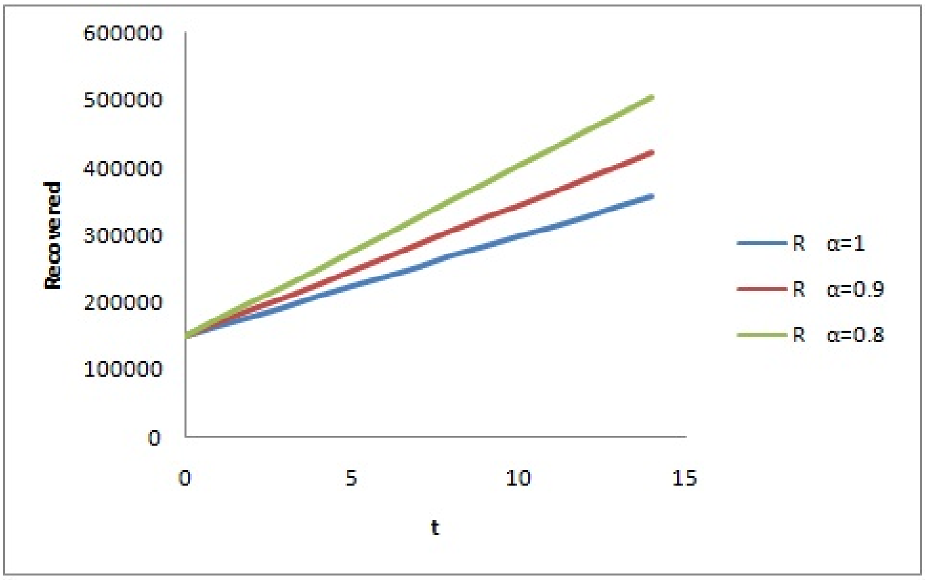

Figure 3.3: The graph of change of the R compartment model.

In Table 3.1, Table 3.2 and Table 3.3, the changes of S, P and R are observed for different states of α. By the above figures, we observe the following highlights:

* It is observed that the number of individuals who do not receive psychological treatment decreases over time (Figure 3.1).
* It is observed that the number of individuals receiving psychological support has increased over time (Figure 3.2).
* It is observed that the number of recovered individuals increases over time (Figure 3.3).
* According to the parameter values taken in this study, it is possible to reach the disease-free equilibrium point given in equation (2.6) only if the vast majority of the population is affected by the disease and receives treatment. In order to turn this negative scenario into a positive one, country managers and families should take the necessary measures and look for ways to reduce the β parameter.

4. Conclusions and Comments

In this paper, a new application of the fractional *SPR* psychological disease model was made taking into account the psychological disease data of Turkey since 2019 [18] and graphs were drawn with the help of the numerical results obtained. The psychological diseased equilibrium point of the fractional *SPR* model was obtained and stability analysis was performed. In the graphs obtained, it was observed that the number of individuals who did not receive psychological treatment decreased over time, the number of individuals who received psychological support increased over time and the number of individuals who recovered increased over time.

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References

[1] J. T. Townsend, D. E. Landon, *Mathematical models of recognition and confusion in psychology*, Math. Soc. Sci., **4**(1) (1983), 25-71.
[2] W. K. Estes, *Mathematical models in psychology*, A Handbook for Data Analysis in the Behaviorial Sciences: Volume 1: Methodological Issues, Volume 2: Statistical Issues, **3** (2014).
[3] D. Wodarz, M. A. Nowak, *Mathematical models of HIV pathogenesis and treatment*, BioEssays, **24**(12) (2002), 1178-1187.
[4] I. Podlubny, *Fractional Differential Equations*, Academy Press, San Diego CA, 1999.
[5] F. Brauer, C. Castillo-Chavez, Z. Feng, *Mathematical models in epidemiology*, Springer, New York, 2019.
[6] H. Bilgil, A. Yousef, A. Erciyes, Ü. Erdinç, Z. Öztürk, *A fractional-order mathematical model based on vaccinated and infected compartments of SARS-CoV-2 with a real case study during the last stages of the epidemiological event*, J. Comput. Appl. Math., (2022), 115015.
[7] N. T. Bailey, *The Mathematical Theory of Infectious Diseases and its Applications*, Charles Griffin & Company Ltd, 5a Crendon Street, High Wycombe, Bucks HP13 6LE, 1975.
[8] H. Hethcote, M. Zhien, L. Shengbing, *Effects of quarantine in six endemic models for infectious diseases*, Math. Biosci., **180**(1-2) (2002), 141-160.
[9] S. Wang, Y. Ding, H. Lu, S. Gong, *Stability and bifurcation analysis of SIQR for the COVID-19 epidemic model with time delay*, Math. Biosci. Eng., **18**(5) (2021), 5505-5524.
[10] B. K. Mishra, N. Jha, *SEIQRS model for the transmission of malicious objects in computer network*, Appl. Math. Model., **34**(3) (2010), 710-715.
[11] X. Liu, T. Yasuhiro, I. Shingo, *SVIR epidemic models with vaccination strategies*, J. Theor. Biol., **253**(1) (2008), 1-11.
[12] M. B. Trawicki, *Deterministic seirs epidemic model for modeling vital dynamics, vaccinations, and temporary immunity*, Mathematics, **5**(1) (2017), 7.
[13] Z. Öztürk, S. Sorgun, H. Bilgil, *SIQRV Modeli ve Nümerik Uygulaması*, Avrupa Bilim ve Teknoloji Dergisi, **28** (2021), 573-578.
[14] W. O. Kermack, A. G. McKendrick, *A contribution to the mathematical theory of epidemics*, In: Proc. R. Soc. Lond., Series A, Containing Papers of a Mathematical and Physical Character, **115**(772) (1927), 700-721.
[15] D. Yaro, S. K. Omari-Sasu, P. Harvim, A. W. Saviour, B. A. Obeng, *Generalized Euler method for modeling measles with fractional differ ential equations*, Int. J. Innov. Res. Dev., **4** (2015).
[16] Z. Öztürk, S. Sorgun, H. Bilgil, Ü. Erdinç, *New exact solutions of conformable time-fractional bad and good modified Boussinesq equations*, J. New Theory, **37** (2021), 8-25.
[17] M. Braun, M. Golubitsky, *Differential Equations and their Applications*, Springer-Verlag, New York, 1983.
[18] https://tuikweb.tuik.gov.tr/UstMenu.do.
[19] P. Kumar, V. S. Erturk, M. Vellappandi, H. Trinh, V. Govindaraj, *A study on the maize streak virus epidemic model by using optimized linearization-based predictor-corrector method in Caputo sense*, Chaos Solit. Fractals, **158** (2022), 112067.
[20] P. Kumar, V. S. Erturk, H. Abboubakar, K. S. Nisar, *Prediction studies of the epidemic peak of coronavirus disease in Brazil via new generalised Caputo type fractional derivatives*, Alex. Eng. J., **60**(3) (2021), 3189-3204.
[21] P. Kumar, V. Govindaraj, V. S. Erturk, *A novel mathematical model to describe the transmission dynamics of tooth cavity in the human population*, Chaos Solit. Fractals, **161** (2022), 112370.
[22] M. Vellappandi, P. Kumar, V. Govindaraj, *Role of fractional derivatives in the mathematical modeling of the transmission of Chlamydia in the United States from 1989 to 2019*, Nonlinear Dyn., (2022), 1-15.
[23] S. Abbas, S. Tyagi, P. Kumar, V. S. Erturk, S. Momani, *Stability and bifurcation analysis of a fractional-order model of cell-to-cell spread of HIV-1 with a discrete time delay*, Math. Meth. App. Sci., **45**(11) (2022), 7081-7095.
[24] V. S. Erturk, E. Godwe, D. Baleanu, P. Kumar, J. Asad, A. Jajarmi, *Novel fractional-order Lagrangian to describe motion of beam on nanowire*, Acta Phys. Pol. A, **140**(3) (2021), 265-272.
[25] P. Kumar, V. S. Erturk, A. Yusuf, S. Kumar, *Fractional time-delay mathematical modeling of Oncolytic Virotherapy*, Chaos Solit. Fractals, **150** (2021), 111123.
[26] A. Din, F. M. Khan, Z. U. Khan, A. Yusuf, T. Munir, *The mathematical study of climate change model under nonlocal fractional derivative*, Partial Differential Equations in Applied Mathematics, **5** (2022), 100204.
[27] E. Viera-Martin, J. F. Gomez-Aguilar, J. E. Solis-Perez, J. A. Hernandez-Perez, R. F. Escobar-Jimenez, *Artificial neural networks: a practical review of applications involving fractional calculus*, Eur. Phys. J. Spec. Top ., (2022), 1-37.
[28] V. S. Erturk, A. Ahmadkhanlu, P. Kumar, V. Govindaraj, *Some novel mathematical analysis on a corneal shape model by using Caputo fractional derivative*, Optik, **261** (2022), 169086.
[29] V. S. Erturk, A. K. Alomari, P. Kumar, M. Murillo-Arcila, *Analytic solution for the strongly nonlinear multi-order fractional version of a BVP occurring in chemical reactor theory*, Discrete Dyn. Nat. Soc., **2022** (2022), 8655340.
[30] Q. Yang, D. Chen, T. Zhao, Y. Chen, *Fractional calculus in image processing: a review*, Fract. Calc. Appl. Anal., **19**(5) (2016), 1222-1249.
[31] P. Kumar, V. Govindaraj, V. S. Erturk, M. H. Abdellattif, *A study on the dynamics of alkali-silica chemical reaction by using Caputo fractional derivative*, Pramana, **96**(3) (2022), 1-19.
[32] Z. Öztürk, H. Bilgil, Ü. Erdinç, *An optimized continuous fractional grey model for forecasting of the time dependent real world cases*, Hacettepe J. Math. Stat., **51**(1) (2022), 308-326.
[33] Ü. Erdinc, H. Bilgil, Z. Öztürk, *A novel fractional forecasting model for time dependent real world cases*, Accepted: Revstat Stat. J., (2022).