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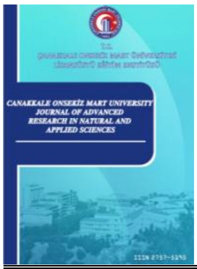

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Mathematical Analyses of the Upper and Lower Possibilistic Mean – Variance Models and Their Extensions to Multiple Scenarios

Furkan Göktaş[1,*]

[1]Department of Business Administration, Faculty of Management, Karabuk University, Karabuk, Türkiye

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Research Article

Abstract − Possibility theory is the one of the most important and widely used uncertainty theories because it is closely related to the imprecise probability and expert knowledge. The possibilistic mean - variance (MV) model is the counterpart of the Markowitz's MV model in the possibility theory. There are variants of the possibilistic MV model, which are called as the upper and lower possibilistic MV models. However, to the best of our knowledge, analytical solutions and exact efficient frontiers of these variants are not presented in the literature when the possibility distributions are given with trapezoidal fuzzy numbers. In this study, under this assumption, we make mathematical analyses of the upper and lower possibilistic MV models and derive their analytical solutions and exact efficient frontiers. Based on the max-min optimization framework, we also propose their extensions where there are multiple upper (lower) possibilistic mean scenarios. We show that the proposed extensions have the ease of use as the upper and lower possibilistic MV models. We also illustrate and compare the upper and lower possibilistic mean - variance models and their proposed extensions with an explanatory example. As we expect, we see that these extensions can be effectively used in portfolio selection by conservative investors.

***Keywords** – Max-min optimization, portfolio selection, possibility theory, scenario analysis, trapezoidal fuzzy numbers.*

1. Introduction

The studies in which the possibility theory are used for the first time in portfolio selection are Tanaka & Guo (1999) and Tanaka, Guo & Türksen (2000). In these studies, the exponential possibility distributions are used for asset returns. The possibilistic counterpart of the Markowitz's MV model is proposed in Carlsson, Fuller & Majlender (2002) where the possibility distributions are given with trapezoidal fuzzy numbers. The Sequential Minimal Optimization (SMO) algorithm is given in Zhang, Zhang & Xiao (2009) for its solution. When the possibility distributions are given with triangular fuzzy numbers, it is shown in Taş, Kahraman & Güran (2016) that the possibilistic MV model reduces to a linear optimization problem. When the possibility distributions are given with triangular fuzzy numbers, mathematical analysis of the possibilistic MV model is studied in Göktaş & Duran (2020).

The variants of the possibilistic MV model, which are called as the upper and lower possibilistic MV models are proposed in Zhang, Wang, Chen & Nie (2007). When the possibility distributions are given with trapezoidal fuzzy numbers and asset weights are bounded, these variants are studied in Zhang (2007). There are also lots of studies about the possibilistic portfolio selection as reviewed in Zhang, Li & Guo (2018) and Fuller & Harnati (2018). On the other hand, the analytical solutions and exact efficient frontiers of these variants are not presented in the literature. Thus, in this study, we make mathematical analyses of these variants. In this regard, we derive their analytical solutions and exact efficient frontiers when the possibility distributions are given with trapezoidal fuzzy numbers. We also propose their extensions to multiple upper (lower) possibilistic mean scenarios by using the max-min optimization framework. Furthermore, we show that the max-min

[1] furkangoktas@karabuk.edu.tr
* Corresponding Author

optimization problems in the proposed extensions reduce to the linear optimization problems as in the upper and lower possibilistic MV models. That is, the proposed extensions can be solved with the known algorithms in the literature.

The rest of paper is organized as follows. In Section 2.1, we make mathematical analyses of the upper and lower possibilistic MV models when the possibility distributions are given with trapezoidal fuzzy numbers. In Section 2.2, under the same assumption, we propose their extensions where there are multiple upper (lower) possibilistic mean scenarios. In Section 3, we illustrate and compare the upper and lower possibilistic MV models and their proposed extensions with an explanatory example. Then, we conclude the paper with Section 4.

2. Methods

2.1. The Upper and Lower Possibilistic Mean – Variance Models

The upper (lower) possibilistic MV model does not capture the negative dependence (Corazzo & Nardelli, 2019). Thus, in this study, we assume that short positioning is not allowed in the portfolio selection problem as the risk-free asset. Then, feasible set (S) in the portfolio selection problem is given as below where w is the weight vector of assets and the weight of the i^{th} asset is equal to w_i and n is the number of the assets.

$$S=\left\{w:\sum_{i=1}^{n}w_i=1\,and\;w_i\geq 0,\forall i\right\} \tag{2.1}$$

The membership function of trapezoidal fuzzy number (a,b,α,β) is given as below. See Klir & Yuan (1995), Zimmermann (2001) and Kosinski (2006) for detailed information about the fuzzy numbers and the possibility theory.

$$A(t)=\begin{cases}1-\dfrac{a-t}{\alpha}, a-\alpha\leq t\leq a\\ 1, a\leq t\leq b\\ 1-\dfrac{t-b}{\beta}, b\leq t\leq b+\beta\\ 0, otherwise\end{cases} \tag{2.2}$$

Graphical representation of (2.2) is given in Figure 1.

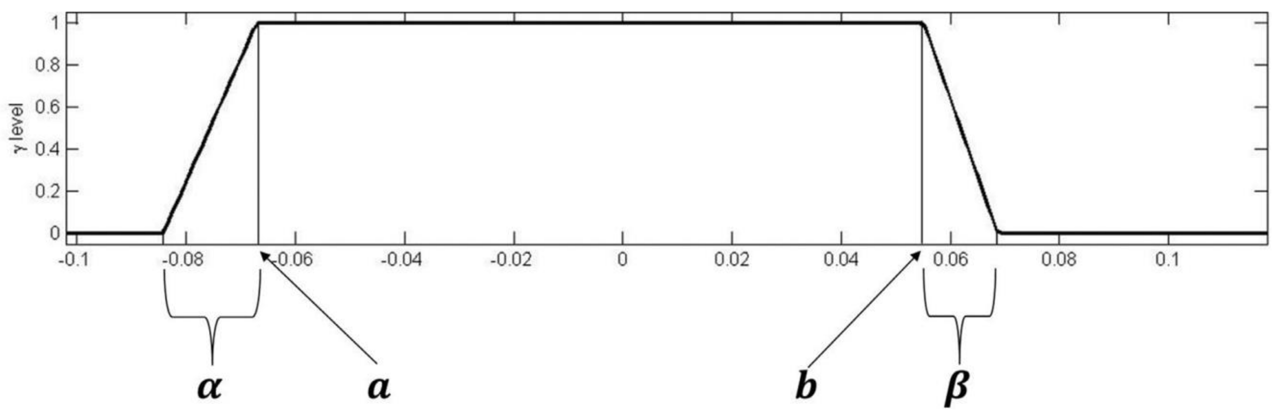

Figure 1. The membership function of trapezoidal fuzzy number (Corazzo & Nardelli, 2019).

Upper and lower possibilistic means are calculated as below for (a,b,α,β) respectively (Zhang, 2007). Here, $b+(1-\gamma)\beta$ is the upper bound of γ cut whereas $a-(1-\gamma)\alpha$ is the lower bound of the γ cut.

$$M^*(A)=2\int_0^1\big(b+(1-\gamma)\beta\big)\gamma d\gamma=b+\frac{\beta}{3}$$
$$M_*(A)=2\int_0^1\big(a-(1-\gamma)\alpha\big)\gamma d\gamma=a-\frac{\alpha}{3} \tag{2.3}$$

Upper and lower possibilistic variances are calculated as below for (a,b,α,β) respectively (Zhang, 2007).

$$Var^*(A)=2\int_0^1\Big[M^*(A)-\big(b+(1-\gamma)\beta\big)\Big]^2\gamma d\gamma=\frac{\beta^2}{18}$$
$$Var_*(A)=2\int_0^1\Big[M_*(A)-\big(a-(1-\gamma)\alpha\big)\Big]^2\gamma d\gamma=\frac{\alpha^2}{18} \tag{2.4}$$

Upper (lower) possibilistic mean of portfolio is the weighted average of upper (lower) possibilistic means of the assets. This information is also valid for upper (lower) possibilistic standard deviation. That is, upper (lower) possibilistic correlation between any two asset is equal to 1 (Zhang, 2007; Corazzo & Nardelli, 2019). Based on this information, the upper possibilistic MV model can be given with the following bi-objective linear maximization problem where $(a_i,b_i,\alpha_i,\beta_i)$ is the possibility distribution of i^{th} asset's return (r_i) for all i. Here, the first (second) objective is to maximize (minimize) upper possibilistic mean (standard deviation) of portfolio.

$$\max_{w\in S}\left\{\begin{array}{l}\sum_{i=1}^{n}w_i\left(b_i+\frac{\beta_i}{3}\right),\\ -\sum_{i=1}^{n}w_i\left(\frac{\beta_i}{3\sqrt{2}}\right)\end{array}\right\} \tag{2.5a}$$

The lower possibilistic MV model can be given with the following bi-objective linear maximization problem where the first (second) objective is to maximize (minimize) the lower possibilistic mean (standard deviation) of portfolio.

$$\max_{w\in S}\left\{\begin{array}{l}\sum_{i=1}^{n}w_i\left(a_i-\frac{\alpha_i}{3}\right),\\ -\sum_{i=1}^{n}w_i\left(\frac{\alpha_i}{3\sqrt{2}}\right)\end{array}\right\} \tag{2.5b}$$

Based on the weighted sum method, the efficient frontier of (2.5a) is equal to the efficient frontier of the following linear maximization problem where the weight (c) of the first objective varies on [0,1]. The optimal solution of (2.6a) is defined as the upper possibilistic efficient portfolio (UEP) for given c in (0,1). If the optimal solution of (2.6a) for c in{0,1}is also a Pareto optimal solution of (2.5a), then it is also called as the upper possibilistic efficient portfolio.

$$\max c\sum_{i=1}^{n} w_i\left(b_i+\frac{\beta_i}{3}\right)+(1-c)\left(-\sum_{i=1}^{n} w_i\left(\frac{\beta_i}{3\sqrt{2}}\right)\right)$$

$$s.t. \sum_{i=1}^{n} w_i = 1 \qquad (2.6a)$$

$$w_i \geq 0, \forall i$$

Based on the weighted sum method, the efficient frontier of (2.5b) is equal to the efficient frontier of the following linear maximization problem where the weight (c) of the first objective varies on [0,1]. The optimal solution of (2.6b) is defined as the lower possibilistic efficient portfolio (LEP) for given c in (0,1). If the optimal solution of (2.6b) for c in {0,1} is also a Pareto optimal solution of (2.5b), then it is also called as the lower possibilistic efficient portfolio.

$$\max c\sum_{i=1}^{n} w_i\left(a_i-\frac{\alpha_i}{3}\right)+(1-c)\left(-\sum_{i=1}^{n} w_i\left(\frac{\alpha_i}{3\sqrt{2}}\right)\right)$$

$$s.t. \sum_{i=1}^{n} w_i = 1 \qquad (2.6b)$$

$$w_i \geq 0, \forall i$$

Remark: When c=1 in (2.6a), the portfolio (MaxUPM) that maximizes upper possibilistic mean is found. When c=0 in (2.6a), the portfolio (MinUPS) that minimizes upper possibilistic standard deviation is found. When c=1 in (2.6b), the portfolio (MaxLPM) that maximizes lower possibilistic mean is found. When c=0 in (2.6b), the portfolio (MinLPS) that minimizes lower possibilistic standard deviation is found.

Based on the Fundamental Theorem of Linear Programming, we have the following cases. Similar results are also valid for the lower possibilistic efficient portfolio.

i. The upper possibilistic efficient portfolio consists of only one asset, which is a Pareto optimal solution of (2.5a) and the unique solution of (2.6a). We call this asset as a non-dominated asset.

ii. The upper possibilistic efficient portfolio consists of any convex combination of two or more assets, which are Pareto optimal solutions of (2.5a) and alternative optimal solutions of (2.6a).

For given upper possibilistic standard deviation e, the upper possibilistic efficient portfolio is found with the following linear maximization problem.

$$\max \sum_{i=1}^{n} w_i\left(b_i+\frac{\beta_i}{3}\right)$$

$$s.t. \sum_{i=1}^{n} w_i\left(\frac{\beta_i}{3\sqrt{2}}\right)=e$$

$$\sum_{i=1}^{n} w_i = 1 \qquad (2.7)$$

$$w_i \geq 0, \forall i$$

Based on the Simplex algorithm, we have the following results. Similar results are also valid for the lower possibilistic efficient portfolio.

i. If the optimal solution of (2.7) is a degenerate solution, the upper possibilistic efficient portfolio consists of only one asset, which is a Pareto optimal solution of (2.5a).

ii. If the optimal solution of (2.7) is a nondegenerate solution and there are not alternative optimal solutions, the upper possibilistic efficient portfolio is equal to any convex combination of two assets, which are Pareto optimal solutions of (2.5a).

iii. If there are alternative optimal solutions of (2.7), the upper possibilistic efficient portfolio is equal to any convex combination of three or more assets, which are Pareto optimal solutions of (2.5a).

iv. The upper possibilistic efficient frontier is exactly constructed by connecting the non-dominated assets with line segments since upper possibilistic mean and standard deviation are the linear functions of *w*.

Upper possibilistic performance can be defined as upper possibilistic mean over upper possibilistic standard deviation. Then, the portfolio (MaxUP) that maximizes it found as below under the assumption that at least one asset's upper possibilistic performance is positive. By definition, MaxUP is an upper possibilistic efficient portfolio.

$$\begin{aligned}
&\max \left(UP(w) := \frac{\sum_{i=1}^{n} w_i \left(b_i + \frac{\beta_i}{3} \right)}{\sum_{i=1}^{n} w_i \left(\frac{\beta_i}{3\sqrt{2}} \right)} \right) \\
&\sum_{i=1}^{n} w_i = 1 \\
&w_i \geq 0, \forall i
\end{aligned} \tag{2.8}$$

(2.8) is a linear fractional problem and its optimal solution(s) are either a corner point or an edge of the feasible set (Biswas, Verma & Ojha, 2017). That is, we have the following cases. Similar results are also valid for the portfolio (MaxLP) that maximizes the lower possibilistic performance.

i. MaxUP consists of only one asset, which uniquely maximizes upper possibilistic performance.

ii. MaxUP consists of any convex combination of two or more assets, which maximize upper possibilistic performance.

2.2. The Proposed Extensions of Upper and Lower Possibilistic Mean – Variance Models

In this study, we assume that the possibility distributions are determined based on the past data in the upper and lower possibilistic MV models similar to Taş et al. (2016). In our proposed extensions, we call this case as a base scenario. In each additional scenario, we assume that these distributions are shifted by $\Theta_{i,j}$, which are determined by the *k* experts based on their future perspectives. Clearly, $\Theta_{i,j}$ is positive when the j^{th} expert is more optimistic about the future return of the i^{th} asset than its past return.

$$r_i = \left(a_i + \theta_{i,j}, b_i + \theta_{i,j}, \alpha_i, \beta_i \right), \forall i \tag{2.9}$$

In each additional scenario, upper (lower) possibilistic variances remain the same whereas upper (lower) possibilistic means change by $\Theta_{i,j}$. Then, based on (2.5a), we propose the extension of upper possibilistic MV model to multiple upper possibilistic mean scenarios as below.

$$\max_{w\in S}\min_{j}\left\{\begin{array}{l}\sum_{i=1}^{n} w_i\left(b_i+\frac{\beta_i}{3}+\theta_{i,j}\right),\\ -\sum_{i=1}^{n} w_i\left(\frac{\beta_i}{3\sqrt{2}}\right)\end{array}\right\} \tag{2.10a}$$

Based on (2.5b), we propose the extension of lower possibilistic MV model to multiple lower possibilistic mean scenarios as below. The max-min strategy guarantees the robustness (Rüstem, Becker & Marty, 2000). Thus, we call (2.10b) as the robust lower possibilistic MV model whereas we call (2.10a) as the robust upper possibilistic MV model.

$$\max_{w\in S}\min_{j}\left\{\begin{array}{l}\sum_{i=1}^{n} w_i\left(a_i-\frac{\alpha_i}{3}+\theta_{i,j}\right),\\ -\sum_{i=1}^{n} w_i\left(\frac{\alpha_i}{3\sqrt{2}}\right)\end{array}\right\} \tag{2.10b}$$

Based on the weighted sum method and definition of minimum function, the following linear maximization problem gives the same efficient frontier with (2.10a). We call the optimal solution of (2.11a) as the robust upper possibilistic efficient portfolio (RUEP) for given c in (0,1). If the optimal solution of (2.11a) for c in {0,1} is also a Pareto optimal solution of (2.10a), then we also call it as the robust upper possibilistic efficient portfolio.

$$\begin{aligned}&\max c\eta+(1-c)\left(-\sum_{i=1}^{n} w_i\left(\frac{\beta_i}{3\sqrt{2}}\right)\right)\\ &s.t.\ \eta\le\sum_{i=1}^{n} w_i\left(b_i+\frac{\beta_i}{3}+\theta_{i,j}\right)\ \textit{for all } j\\ &\quad\sum_{i=1}^{n} w_i=1\\ &\quad w_i\ge 0,\forall i\end{aligned} \tag{2.11a}$$

Based on the weighted sum method and definition of minimum function, the following linear maximization problem gives the same efficient frontier with (2.10b). We call the optimal solution of (2.11b) as the robust lower possibilistic efficient portfolio (RLEP) for given c in (0,1). If the optimal solution of (2.11b) for c in {0,1} is also a Pareto optimal solution of (2.10b), then we also call it as the robust lower possibilistic efficient portfolio.

$$\max c\eta + (1-c)\left(-\sum_{i=1}^{n} w_i \left(\frac{\alpha_i}{3\sqrt{2}}\right)\right)$$

$$s.t.\ \eta \le \sum_{i=1}^{n} w_i \left(a_i - \frac{\alpha_i}{3} + \theta_{i,j}\right) \text{ for all } j \tag{2.11b}$$

$$\sum_{i=1}^{n} w_i = 1$$

$$w_i \ge 0, \forall i$$

Remark: When c=1 in (2.11a), the portfolio (MaxWUPM) that maximizes the worst-case upper possibilistic mean is found. When c=1 in (2.11b), the portfolio (MaxWLPM) that maximizes the worst-case lower possibilistic mean is found. When c=0 in (2.11a), MinUPS is found. When c=0 in (2.11b), MinLPS is found. MinUPS and MinLPS are defined in Section 2.1.

We define the portfolio (MaxWUP) that maximizes the worst-case upper possibilistic performance with the following max-min problem based on (2.8) where we assume that at least one portfolio's worst-case upper possibilistic performance (mean) is positive.

$$\max_{w\in S} \min_{j} \left(UP(w,\theta_j) := \frac{\sum_{i=1}^{n} w_i \left(b_i + \frac{\beta_i}{3} + \theta_{i,j}\right)}{\sum_{i=1}^{n} w_i \left(\frac{\beta_i}{3\sqrt{2}}\right)} \right) \tag{2.12a}$$

Due to the definition of minimum function, (2.12a) is equivalent to the following maximization problem.

$$\max \frac{\eta}{\sum_{i=1}^{n} w_i \left(\frac{\beta_i}{3\sqrt{2}}\right)}$$

$$s.t.\ \eta \le \sum_{i=1}^{n} w_i \left(b_i + \frac{\beta_i}{3} + \theta_{i,j}\right) \text{ for all } j \tag{2.12b}$$

$$\sum_{i=1}^{n} w_i = 1$$

$$w_i \ge 0, \forall i$$

(2.12b) is equivalent to the following maximization problem where π is a positive variable.

$$
\begin{aligned}
&\max \pi\eta \\
&s.t.\ \pi\eta \le \pi\sum_{i=1}^{n} w_i\left(b_i + \frac{\beta_i}{3} + \theta_{i,j}\right) \text{ for all } j \\
&\pi\sum_{i=1}^{n} w_i\left(\frac{\beta_i}{3\sqrt{2}}\right) = 1 \qquad (2.12c) \\
&\pi\sum_{i=1}^{n} w_i = \pi \\
&\pi \ge 0 \text{ and } w_i \ge 0, \forall i
\end{aligned}
$$

After some arrangements, we see that (2.12c) is equivalent to the following linear maximization problem. Thus, we derive MaxWUP by standardizing the optimal solution of (2.12d).

$$
\begin{aligned}
&\max z \\
&s.t.\ z \le \sum_{i=1}^{n} \tilde{w}_i\left(b_i + \frac{\beta_i}{3} + \theta_{i,j}\right) \text{ for all } j \\
&\sum_{i=1}^{n} \tilde{w}_i\left(\frac{\beta_i}{3\sqrt{2}}\right) = 1 \qquad (2.12d) \\
&\tilde{w}_i \ge 0, \forall i
\end{aligned}
$$

Similarly, we derive the portfolio (MaxWLP) that maximizes the worst-case lower possibilistic performance by standardizing the optimal solution of (2.13).

$$
\begin{aligned}
&\max z \\
&s.t.\ z \le \sum_{i=1}^{n} \tilde{w}_i\left(a_i - \frac{\alpha_i}{3} + \theta_{i,j}\right) \text{ for all } j \\
&\sum_{i=1}^{n} \tilde{w}_i\left(\frac{\alpha_i}{3\sqrt{2}}\right) = 1 \qquad (2.13) \\
&\tilde{w}_i \ge 0, \forall i
\end{aligned}
$$

3. Results and Discussion

In this section, we illustrate and compare the upper and lower possibilistic MV models and their proposed extensions where the possibility distributions of five risky assets (R1, R2, R3, R4 and R5) in the base scenario are as in Zhang (2007) and (3.1). Since $a_i = b_i$ for all i, these are specifically triangular fuzzy numbers. In this section, we take c=0.5 in finding UEP, LEP, RUEP and RLEP. That is, the two objectives in the portfolio selection problem are equally-weighted.

$$r_1 = (0.073, 0.073, 0.054, 0.087)$$
$$r_2 = (0.105, 0.105, 0.075, 0.102)$$
$$r_3 = (0.138, 0.138, 0.096, 0.123) \quad (3.1)$$
$$r_4 = (0.168, 0.168, 0.126, 0.162)$$
$$r_5 = (0.208, 0.208, 0.168, 0.213)$$

Based on (3.1), we calculate the important metrics for the upper possibilistic MV model as in Table 1. We show the best values in bold. We see that the all risky assets are non-dominated assets in this model.

Table 1
The important metrics for the upper possibilistic MV model.

Risky Assets	Upper P. Mean	Upper P. Std.	Upper Perf.	(2.6a) for c=0.5
R1	0.102	**0.0205**	4.9741	0.0407
R2	0.139	0.0240	5.7816	0.0575
R3	0.179	0.0290	**6.1742**	0.0750
R4	0.222	0.0382	5.8140	0.0919
R5	**0.279**	0.0502	5.5573	**0.1144**

Based on (3.1), we calculate the important metrics for the lower possibilistic MV model as in Table 2. We show the best values in bold. We see that the all risky assets are non-dominated assets in this model.

Table 2
The important metrics for the lower possibilistic MV model.

Risky Assets	Lower P. Mean	Lower P. Std.	Lower Perf.	(2.6b) for c=0.5
R1	0.055	**0.0127**	4.3212	0.0211
R2	0.08	0.0177	4.5255	0.0312
R3	0.106	0.0226	**4.6846**	0.0417
R4	0.126	0.0297	4.2426	0.0482
R5	**0.152**	0.0396	3.8386	**0.0562**

By using the best values in Table 1 (Table 2) with the order of its columns, we derive the following results. We see that the upper and lower possibilistic MV models give the same optimal portfolios, which are not diversified.

i. MaxUPM (MaxLPM) consists of only R5.

ii. MinUPS (MinLPS) consists of only R1.

iii. MaxUP (MaxLP) consists of only R3.

iv. UEP (LEP) consists of only R5.

We assume that the shifting parameters ($\Theta_{i,j}$), which are determined based on the expert knowledge are as in Table 3. Clearly, the first expert predicts that the all possibility distributions are as in (3.1) whereas the first, second and third experts predict that the possibility distribution of R3 is as in (3.1).

Table 3
The shifting parameters.

Risky Assets	Scenario 1	Scenario 2	Scenario 3	Scenario 4
R1	0	0.1	0.05	-0.05
R2	0	0.05	-0.05	0.1
R3	0	0	0	-0.05
R4	0	-0.05	0.05	-0.1
R5	0	-0.1	-0.05	0.05

We uniquely find the some optimal portfolios in the robust upper possibilistic MV model as in Table 4. We see that RUEP is equal to MaxWUPM in our example. We also see that its proposed extension gives more diversified portfolios than the upper possibilistic MV model.

Table 4
Some optimal portfolios in the robust upper possibilistic MV model.

Risky Assets	MaxWUP	MaxWUPM	RUEP
R1	0	0.5	0.5
R2	0.5	0	0
R3	0	0	0
R4	0.5	0	0
R5	0	0.5	0.5

We uniquely find the some optimal portfolios in the robust lower possibilistic MV model as in Table 5. We see that its proposed extension gives more diversified portfolios than the lower possibilistic MV model. We also see that MaxWLP (RLEP) is different from MaxWUP (RUEP).

Table 5
Some optimal portfolios in the robust lower possibilistic MV model.

Risky Assets	MaxWLP	MaxWLPM	RLEP
R1	0.3333	0.5	0
R2	0.3333	0	0.5
R3	0.3333	0	0
R4	0	0	0.5
R5	0	0.5	0

In Figure 2, we give the exact efficient frontier of the upper possibilistic MV model and the approximated efficient frontier of its proposed extension respectively where UP is equal to MaxUP. Since the all risky assets are non-dominated in the upper possibilistic MV model, its efficient frontier is constructed by connecting them with line segments. We see that the robust upper possibilistic efficient portfolios are close to the efficient frontier of the upper possibilistic MV model whereas the upper possibilistic efficient portfolios are not close to the its proposed extension's efficient frontier. That is, the robust upper possibilistic efficient portfolios give satisfactory results in the base scenario and are robust to worst-case scenario unlike the upper possibilistic efficient portfolios.

In Figure 3, we give the exact efficient frontier of the lower possibilistic MV model and the approximated efficient frontier of its proposed extension respectively where WLPM is equal to MaxWLPM. Since the all risky assets are non-dominated in the lower possibilistic MV model, its efficient frontier is constructed by connecting them with line segments. We see that the robust lower possibilistic efficient portfolios are close to the efficient frontier of the lower possibilistic MV model whereas the lower possibilistic efficient portfolios are not close to the its proposed extension's efficient frontier. Furthermore, MaxWLP is the efficient portfolio in the lower possibilistic MV model. That is, the robust lower possibilistic efficient portfolios give satisfactory results in the base scenario and are robust to worst-case scenario unlike the lower possibilistic efficient portfolios.

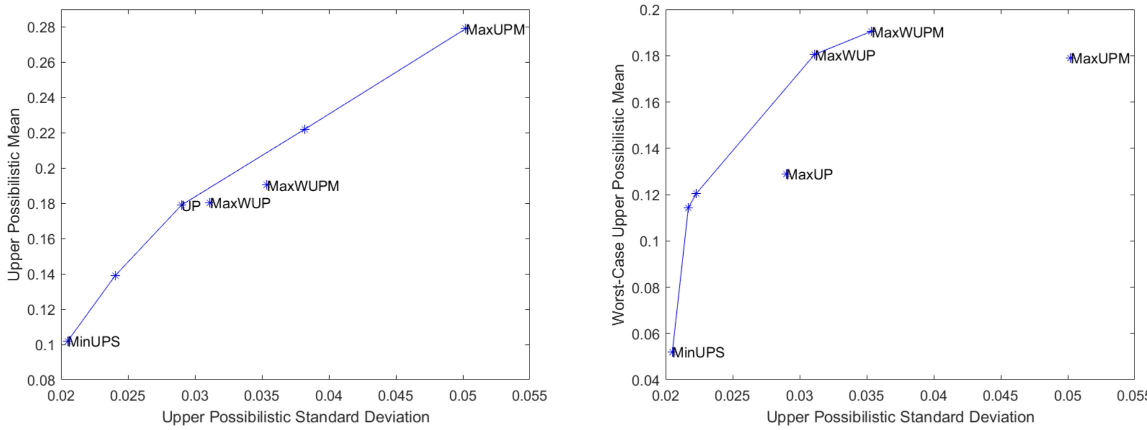

Figure 2. The efficient frontiers of the upper possibilistic MV model and its proposed extension.

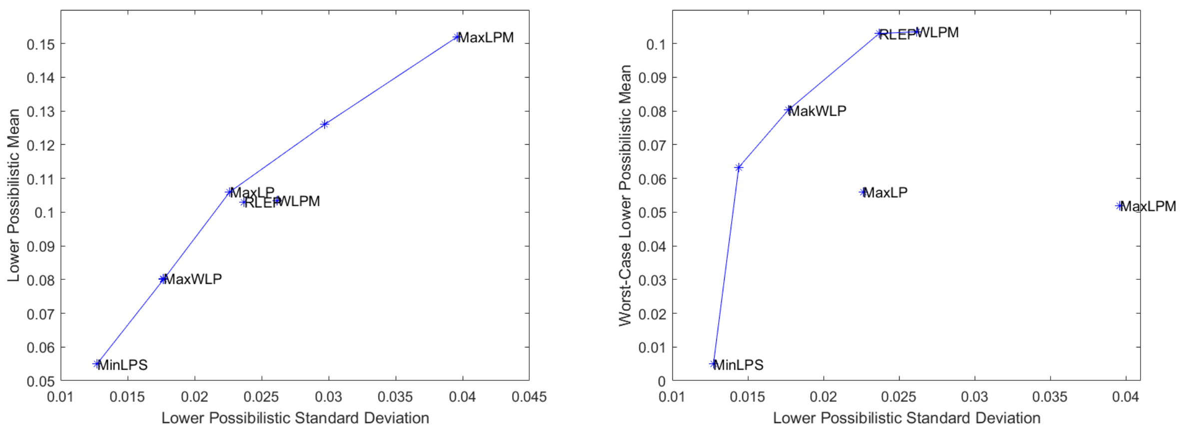

Figure 3. The efficient frontiers of the lower possibilistic MV model and its proposed extension.

4. Conclusion

In this study, we derive the analytical solutions and exact efficient frontiers of the upper (lower) possibilistic MV model under certain assumptions. We also propose its extension to multiple upper (lower) possibilistic mean scenarios based on the max-min optimization framework. Since the max-min optimization strategy guarantees the robustness, we call its proposed extension as the robust upper (lower) possibilistic MV model. We show that its proposed extension can be solved by using the linear optimization algorithms. Furthermore, we derive more diversified and conservative optimal portfolios with its proposed extension than the upper (lower) possibilistic MV model in our illustrative example. Thus, its proposed extension is more suitable for conservative investors than the upper (lower) possibilistic MV model in portfolio selection. On the other hand, it may not be preferable for non-conservative investors due to the worst-case orientation.

Conflicts of Interest

The author declares no conflict of interest.

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