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AUTONOMOUS PERPENDICULAR PARKING PATH PLANNING OF A FOUR-WHEEL STEERED VEHICLE AND RELATED PARAMETERS

Emir KUTLUAY *

Emrecan HATİPOĞLU **

Mert Kadir ASSOY **

Seda KARAOSMANOĞLU ***

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Abstract: In this paper, a path planning method is introduced to perform the autonomous perpendicular parking of a four-wheel steered vehicle. The method consists of the kinematic model of the vehicle and geometrical equations. Mentioned method is based on retrieving the vehicle from the parking spot and using the same path inversely to park the vehicle. MATLAB simulations are performed with different kinematic properties and positions. Three new equations are derived to calculate the longitudinal maneuver to locate the vehicle at ready to reverse position, the maximum approach to left side of the road during the parking maneuver, and the minimum distance between the right side of the vehicle and edge of the parking space. A methodology is introduced to define the required driving corridor by using these equations and it is shown by simulation that feasibility of the planned autonomous perpendicular parking maneuver can be evaluated by taking environmental telemetry and kinematics of the vehicle into account.

Keywords: Autonomous perpendicular parking, four-wheel steered vehicles, path planning, driver assistance systems

Dört Tekerlekten Yönlendirilebilen Araçların Otonom Dik Park Yol Planlaması ve İlgili Parametreler

Öz: Bu makalede, dört tekerlekten yönlendirilebilen bir aracın otonom dik park edebilmesi için bir yol planlama yönteminden bahsedilmektedir. Yöntem, aracın kinematik modelini ve geometrik denklemlerini temel almaktadır. Bahsedilen yöntem, aracı park yerinden çıkarmak için planlanan yolu ters yönde kullanarak aracı park etme esasına dayanmaktadır. Aracı geri parka başlamaya uygun konumlandırmak için uzunlamasına manevrayı, park manevrası sırasında yolun maksimum sol taraf kullanımını ve aracın sağ tarafı ile park yeri kenarı arasındaki minimum mesafeyi hesaplamak üzere üç yeni denklem türetilmiştir. Bu denklemler kullanılarak gerekli sürüş koridorunun tanımlanmasına yönelik bir yöntem tanıtılmış ve planlanan otonom dikey park manevrasının uygulanabilirliğinin, aracın çevresel telemetrisi ve kinematiği dikkate alınarak değerlendirilebileceği simülasyonla gösterilmiştir.

Anahtar Kelimeler: Otonom dik park, dört tekerlekten yönlendirilebilen araçlar, yol planlama, sürücü asistan sistemleri

* Hacettepe Üniversitesi Mühendislik Fakültesi, Makina Mühendisliği Bölümü 06800 Beytepe/Ankara

** Ulutek Teknoloji Geliştirme Bölgesi, Görükle Mahallesi, Üniversite-1 Caddesi Ulutek Blok No:931 İç Kapı No:6 Nilüfer/BURSA

*** Tofaş Türk Otomobil Fabrikası A.Ş., İstanbul Cd No:574, 16110 Demirtaş Dumlupınar Osb Osmangazi / Bursa

Correspondence Author: Emir Kutluay (kutluay@hacettepe.edu.tr)

1. INTRODUCTION

Vehicles with four-wheel steering to obtain advanced maneuverability are coming to the market both as concepts and in series production. Such advanced maneuverability functions are used for parking or off-road features depending on the class of the vehicle. For instance, Hyundai E-Corner is in concept phase and employs advanced maneuverability functions for autonomous intelligent parking features (Hyundai e-Corner, 2024). Hummer EV was introduced to the market in 2023 and similar functions are applied to increase off-road maneuverability and to perform parking maneuver (GMC Group, 2024).

Since a lot of vehicles can fit into limited space, perpendicular parking is stated as efficient and economical by (Petrov and Nashashibi, 2016). Vehicles with only front wheel steering are considered in this study. Perpendicular parking is especially suitable for light commercial vehicles in terms of loading and unloading from the rear door. Especially in Europe, double side parking is a common situation as mentioned by (Hatipoğlu and Kutluay, 2021) and as shown in Figure 3. Drivers have to be attentive during parking their vehicles in similar streets due to risk of crashing to left side, in addition to crashing to right side.

One of the early works in the literature, (Reeds and Shepp, 1990) describes that optimal paths for a vehicle that can go forwards and backwards can be constituted with circles of minimal radius. This method describes the calculation of the shortest path for a car to travel between two points based on the information of its starting and ending directions.

Effects and possibilities of active rear steering in various driving cases are presented by (Arvidsson and Franklin, 2012). The analyses are both performed at low speed to investigate these effects in take-off from parallel parking and are performed at high speed to examine vehicle stability. Desired rear steering angle is based on the steering angle of the front wheels and a proportional gain. Proposed parallel parking algorithm connects two circular arcs with a sinusoidal steer function. Kinematic model of a four-wheel steering car is used to model the motion of the vehicle in/on Matlab/Simulink. The values used for maximum rear wheel steering angle are 0, 2, 5 and 10 degrees. Various turning radii are calculated for different maximum rear wheel steering angles. When the vehicle has active rear steering, as the rear steering angle increases, the vehicle moves closer to the parking space boundary in take-off period. It is stated that the displacement becomes noticeable when the rear wheel steering angle is higher than 5 degrees. If the rear wheel steering angle is higher than 5 degrees, crabbing is found to be logical during take-off from the parking space. The study primarily aims to show the effect of the active rear steering on take-off maneuver from the parking space. The study primarily focuses to show the effect of the active rear steering on take-off maneuver from the parking space more than performing autonomous parking maneuvers.

Motion planning techniques that are used in autonomous driving scenarios are proposed (Gonzalez et al., 2016). These techniques are grouped as graph search-based planners, sampling based planners, interpolating curve planners and numerical optimization approaches. Interpolating curve planners section includes clothoid, polynomial, Bezier, spline curves and line and circle approaches. The main purpose of these methods is to optimize safety, comfort and energy.

A study that focused on low-speed maneuverability and highway lateral comfort of rear wheel steering vehicles is proposed by (Utbult, 2017). Curvatures of the paths taken by vehicles with different kinematics are presented. Rear end parking and "T-crossing" maneuvers are performed by vehicles with rear axle steering and with only front wheel steering. It is accented that vehicle with rear wheel steering intrudes the opposite lane less than vehicle with only front wheel steering in T-crossing case. Additionally, vehicle with rear wheel steering can park in less space. Simulations are performed with CarMaker and Simulink co-simulation environment. There are no performance metrics related to parking maneuver in this study.

Perpendicular parking strategy for vision-based autonomous parking systems is described (Nguyen and Kuhnert, 2019) in which the ego-vehicle is capable of completing its auto-park by one maneuver, or up to maximum three required maneuvers. The method provides a significant improvement in reducing the number of maneuvers as well as time to park.

An experimental electric vehicle with rear wheel steering is proposed by (Krčmář et al., 2019). Ackerman steering geometry is used. Steering angle of the rear wheel is calculated by using a sinus function. Upper limit of the rear wheel steering angle is stated as 5 degrees. Basic block diagram of the vehicle architecture is given.

A local path planning algorithm for multiple maneuver perpendicular parking of an autonomous vehicle is described by (Jeong et al., 2020). Proposed vehicle model has front wheel steering geometry. In this study, an optimal path planning algorithm based on circle and line combination in the presence of predefined obstacles is defined. The collision avoidance conditions are simulated considering the related constraints. There is no mathematical equation that includes the related parameters belong to the ego vehicle and environmental telemetry to define longitudinal distance between the right border of the parking space and rear bumper of the vehicle at the start of backward maneuvers.

Clothoid based local path planning is explained in (Kim and Chung, 2021) for automated perpendicular parking systems. Kinematic model of only front steered vehicle is presented. Simulations are performed on a planar environment model which has no obstacles. Forward and backward parking cases are presented. Steering angle is limited as 35 degrees for both directions.

An autonomous perpendicular parking study of a front wheel steered car-like vehicle is proposed by (Morales et al., 2016). Kinematic vehicle model is used. Parking path is based on straight lines and a circular arc. During the circular section of the maneuver, steering angle is constant. Proposed controller with a weighted control scheme is based on a sensor. Simulations and real-world experiment are mentioned in this study.

Sensor based method for autonomous parallel, perpendicular and angular parking is extended in (Morales et al., 2022) as a Multi Sensor Based Predictive control framework for a front wheel steered vehicle. In this study, the vehicle is capable of perceiving surrounding free parking spaces in addition to central position with lidar sensor. Parking is achieved by making the longitudinal axis of the vehicle to be collinear to the main axis of the parking spot and complete the maneuver at a certain distance from either the rear or front boundary of the parking spot.

A perpendicular parking assist for only front wheel steered vehicle is described by (Firdaus et al., 2018). Vehicle starts the maneuver in perpendicular to parking space and performs a 90-degrees maneuver. Turning radius is defined with a constant steering angle. Collision limits are given with geometric equations in the context of the proposed scenario. Fuzzy controller is described on Simulink. Open loop and close loop models are given. Various simulations are shown in this study.

Another study explains the automatic parking method for front wheel steered vehicle with fuzzy logic (Lee et al., 2017). Calculations of minimum perpendicular and parallel parking space and calculating the path are based on vehicle kinematics. RRT (Rapidly Random Tree) algorithm plans the parking path. Maneuvers are performed on daily used roads. After that, results are compared with Lidar data to compare the suitability of the real-world maneuver. The parking algorithm was implemented on “TuZhi” an autonomous vehicle developed by Wuhan University.

A planning study of autonomous perpendicular parking with fuzzy control is proposed by (Das et al., 2017). Vehicle with only front wheel steering is modeled kinematically. Flowchart of the parking assist is presented. Constant turning radius is used for turning. In order to identify free space, occupancy grid method is used. Lateral position limits of the ego vehicle at the initial instant of the maneuver are given as a figure. Membership functions of fuzzy controllers are described. The control algorithm is designed in MATLAB and validated in CarMaker.

An autonomous perpendicular reverse parking problem of front wheel steering vehicles is studied by (Petrov and Nashashibi, 2015). Constraints that required to perform a collision-free maneuver are given. A rectangular model of the vehicle is assumed. Collision-free perpendicular parking geometry in one maneuver is presented. Constant turning radius is used for turning. A tan-h type steering controller is mentioned and evaluated. In addition to simulations, a real-world test is conducted by using CyCab vehicle to verify the mentioned control logic.

An autonomous driving system implementation study for parallel and perpendicular parking is described by (Li and Tseng, 2016). Vehicle with only front wheel steering is modeled kinematically. Parking place searching, steering control, path tracking and wireless communication functions are embedded to proposed system. Steering tracking control is performed by using Fuzzy-PID. It is assumed that vehicle is perpendicular to the parking spot in the perpendicular park assist mode. Perpendicular parking geometry is given as a figure. Real-world experiment is performed with a SUV vehicle in ARTC.

A comprehensive study related to manual and autonomous driving system of a four-wheel steering vehicle is proposed by (Li et al., 2016). The autonomous mode has lane keeping, reverse and parallel parking features and based on fuzzy control and machine vision. The reverse parking maneuver is performed on a scaled narrow road. There are no equations that shows the maneuver performance of the vehicle. A test vehicle is manufactured, and in-door real time experiment is performed to evaluate the feasibility in real cases.

Maneuverability of vehicles with two steering axes is described by (Pilisiewicz and Kaczyński, 2017). Parallel, forward and backward perpendicular parking cases are analyzed to calculated optimal steering angles. There are no various ratios that define the relation between the steering angle of front inner and rear inner wheels. Values that can prove the advantages of vehicles with two steering axes in terms of maneuverability are given. Parametric equations that can be used to obtain maximum approach to left side of the road and longitudinal position at the start of reversing maneuvers are not included.

Advantages of the four-wheel steering in parallel parking is mentioned in (Hatipoglu and Kutluay, 2021). Vehicle is modeled kinematically and constant turning radius is used. Also, three new formulas are derived to prove the advantages of four-wheel steering. These formulas are related to longitudinal maneuver just before reversing maneuvers and feasibility of the maneuver within the context of the scenario. Ackerman steering is used due to low speed of the maneuver.

An autonomous parallel parking study of only front wheel steered vehicles based on Ackermann steering geometry is proposed in (Hatipoğlu et al., 2023). It is aimed to perform parallel parking maneuvers in narrow streets autonomously. Therefore, steering angle is calculated considering the environment model in addition to vehicle constraints. Algorithm has been employed for generating a path that is based on two straight lines and two identical circles. The minimum turning radius that can be used for a non-collision parking maneuver and required length of the parking spaces can be calculated with the proposed algorithm.

The literature survey indicates that the studies focusing on parking maneuvers of the four-wheel steered vehicles do not deal with performance metrics associated with different maximum rear steering angle.

Current study aims to introduce a methodology for autonomous perpendicular parking of four-wheel steered vehicles by setting the steering angles of rear wheels according to various ratios. In addition, advantages of the four-wheel steering from the perspective of autonomous perpendicular parking is demonstrated with parametric equations and numeric results. With the help of these equations, autonomous perpendicular parking maneuver performance of the vehicles with different kinematics can be evaluated quantitatively. The geometrical approach which has been used for autonomous parallel parking in (Hatipoglu and Kutluay, 2021) is utilized as an initial step and it is extended for autonomous perpendicular parking. Advantages of the four-wheel steering are described by MATLAB simulations and equations from the perspective of perpendicular parking. MATLAB Editor is used for implementing the algorithm, data acquisition and visualization. Proposed environment model is based on a frequently encountered parking scenario where vehicles are parked in parallel on one side on the street and in perpendicular on the other. Advantages of four-wheel steered vehicle are presented for different maximum steering angle limits of rear wheels and compared to only front wheel steered vehicle under different kinematic constraints. New parametric formulas are derived to show related parameters and calculate these constraints. Required driving corridor during the parking maneuver is demonstrated in this study.

2. KINEMATICS AND PATH PLANNING

Motion of the vehicle is modelled kinematically in this work. Described path planning method is based on Ackerman steering because of the low speed of the maneuver. In addition, ideal Ackerman angles are assumed without errors. Path is based on straight forward, circular backward and straight backward steps.

2.1. Vehicle Model

Vehicle is represented as a rectangular box that encapsulates the extremities of the vehicle. Similar approach has been used by (Petrov and Nashasibi, 2016). Each wheel is represented with a line and a point. The point shows the center of the tire from top view, and the line is used to control direction of each wheel. Vehicle model is given in Figure 1. Selected vehicle is 2019 model year Fiat Doblo. Dimensions of the vehicle are taken from (Fiat Doblo, 2022). Parameters are given in Table 1.

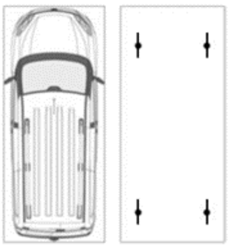

***Figure 1*:**
Vehicle Model

Table 1. Parameters & Variables

Parameters & Variables	Notation	Value
Wheelbase	l	3105 mm
Front Overhang	l_1	911 mm
Rear Overhang	l_2	740 mm
Front part of the wheelbase according to ratio	l_3	Variable
Rear part of the wheelbase according to ratio	l_4	Variable
Trackwidth	d	1530 mm
Width of the vehicle (side mirrors included)	d_0	2180 mm
Steering angle (representative, in degrees)	δ	[-35^o 35^o]

Lateral distance between the ego vehicle and the stationary vehicle	$lateral$	Variable
Instantaneous center of rotation	ICR	Variable
Turning radius	R	Variable
Center coordinates of right backward maneuver circle	(X_{RRM}, Y_{RRM})	Variable
Ratio between inner front and inner rear wheel steering angle	$ratio$	Variable
Length of the parking space	$length_{parking\ space}$	Constant
Width of the parking space	$w_{parking\ space}$	Constant
Coordinates of the vehicle rear axle center	$(X_{RAC\_ego}, Y_{RAC\_ego})$	Variable
Right front edge of the parking space	(X_{ps_3}, Y_{ps_3})	Constant
Initial position of the left front edge of the ego vehicle	$(X_{lf_{init}}, Y_{lf_{init}})$	Variable
Width of the road	$Road_{width}$	Constant
X coordinate of the left border of the road	$X_{road_{left\ border}}$	Constant

2.2. Ackerman Steering

Upper limit of the inner front wheel steering angle is assumed to be 35 degrees in suitable to mechanical limitations of the vehicle. As indicated in Equation 1, there is a ratio between steering angle of inner front wheel and steering angle of inner rear wheel. The ratio between them is selected as 3.5, 5 and 7 respectively. X-axis represents the lateral distance and Y-axis represents the longitudinal distance. Related equations are given as Equation 2-6. All equations are derived by authors. Steering geometry is given in Figure 2.

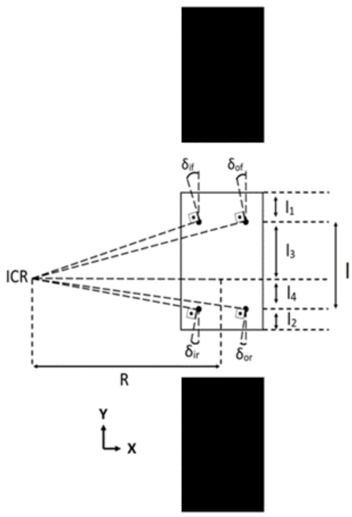

***Figure 2*:**
Ackerman Steering

$$\delta_{ir} = \frac{\delta_{if}}{ratio} \tag{1}$$

$$R = \frac{l}{tan(\delta_{if}) + tan(\delta_{ir})} + \frac{d}{2} \tag{2}$$

$$l_3 = tan(\delta_{if}) * (R - \frac{d}{2}) \tag{3}$$

$$l_4 = l\text{-} l_3 \tag{4}$$

After calculating l_3 and l_4, steering angle of outer wheels can be calculated with following formulas.

$$\delta_{of} = \left(\frac{l_3}{\left(R + \frac{d}{2}\right)}\right) \tag{5}$$

$$\delta_{or} = \left(\frac{l_4}{\left(R + \frac{d}{2}\right)}\right) \tag{6}$$

2.3. Path Planning

Environment model consists of an ego vehicle, eight parked cars, road borders and a parking space to simulate a real-world environment that is given in Figure 3. Stationary cars that have been parked in parallel are aligned among themselves and perfectly parallel to road border. Stationary cars that have been parked in perpendicular are aligned among themselves and perfectly parallel to the parking space. Parking space length is 5 meters and parking space width is 3 meters. Ego vehicle starts the parking maneuver in parallel to road borders. Road is 7 meters wide, made up of two identical 3.5 meters wide lanes. The ego vehicle cannot complete its full turn in a single maneuver in the environment model. Because of that, the road is considered as narrow. Environment model is given in Figure 4.

***Figure 3*:**
Real-world environment

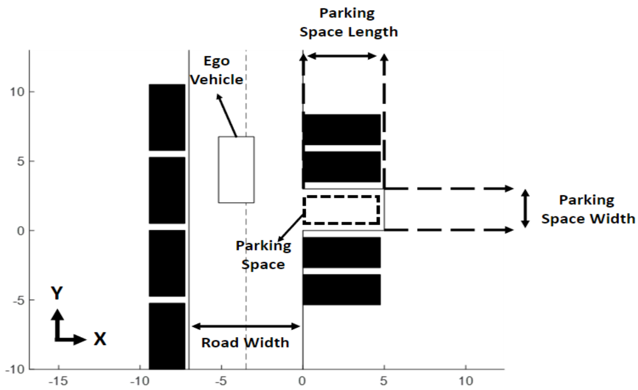

***Figure 4*:**
Environment in MATLAB Simulations

Each wheel follows a circular path with constant radius as proposed in (Das et al., 2017). It is assumed that the ego vehicle starts the backward maneuver from the perfect longitudinal position. To be able to use Ackerman equations, steering angles of the wheels can change only when the velocity equals to zero.

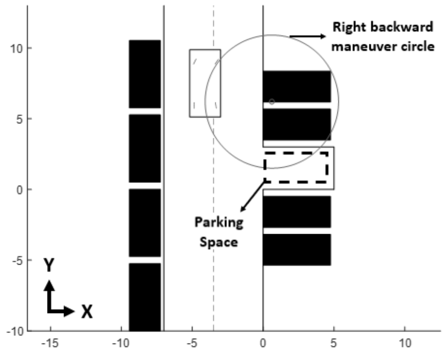

***Figure 5*:**
Maneuver circle

Center coordinates of the circle can be obtained by using environmental telemetry, vehicle dimensions, turning radius and the position of the vehicle at the start of circular backward maneuver. Maneuver circle that is used in current study is shown in Figure 5.

$$X_{RRM} = X_{RAC\_ego} + R \tag{7}$$

$$Y_{RRM} = Y_{RAC\_ego} \tag{8}$$

After the maneuver circle is constituted, the ego vehicle is able to realize the parking maneuver by using the proposed algorithm. The flowchart of the algorithm that proposed in current study is shown in Figure 6.

***Figure 6*:**
Flowchart of the proposed algorithm

3. SCENARIO AND SIMULATIONS

At the start of the scenario, the ego vehicle is in parallel to road borders. Then, it performs a straight-forward maneuver to reach the starting point of reversing maneuvers. The travelled distance is a function of environment and vehicle kinematics. After that, the ego vehicle stops and sets the steering angles. Then, it performs the circular section of the perpendicular parking maneuver. After that, the vehicle stops and sets the steering angles to zero. Finally, a straight backward maneuver is performed until the distance between rear bumper of the ego vehicle and rear border of the parking space decreases to a pre-defined value which is equal to 0.25 meters to simulate a generic limit of ultrasonic sensors. The vehicle is perfectly parallel to parking space borders at the end of the maneuver. Scenarios are given in Figure 7, Figure 8, Figure 9 and Figure 10 respectively.

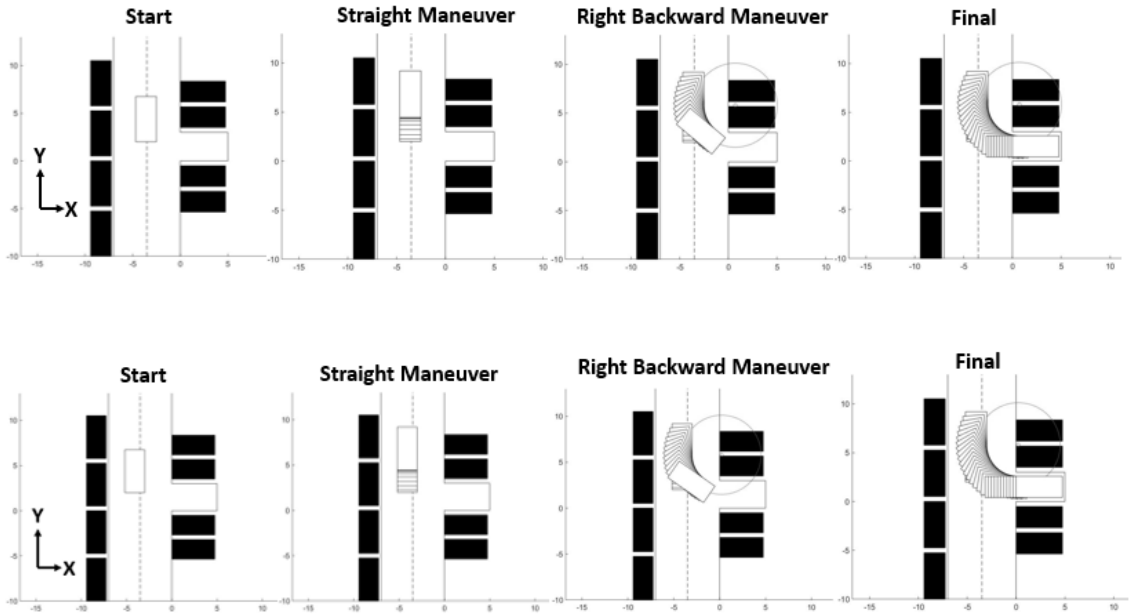

***Figure 7*:**
Parking maneuvers with lateral distances 2.5 meters & 3 meters; the ratio is 3.5 (Scenarios 1-2)

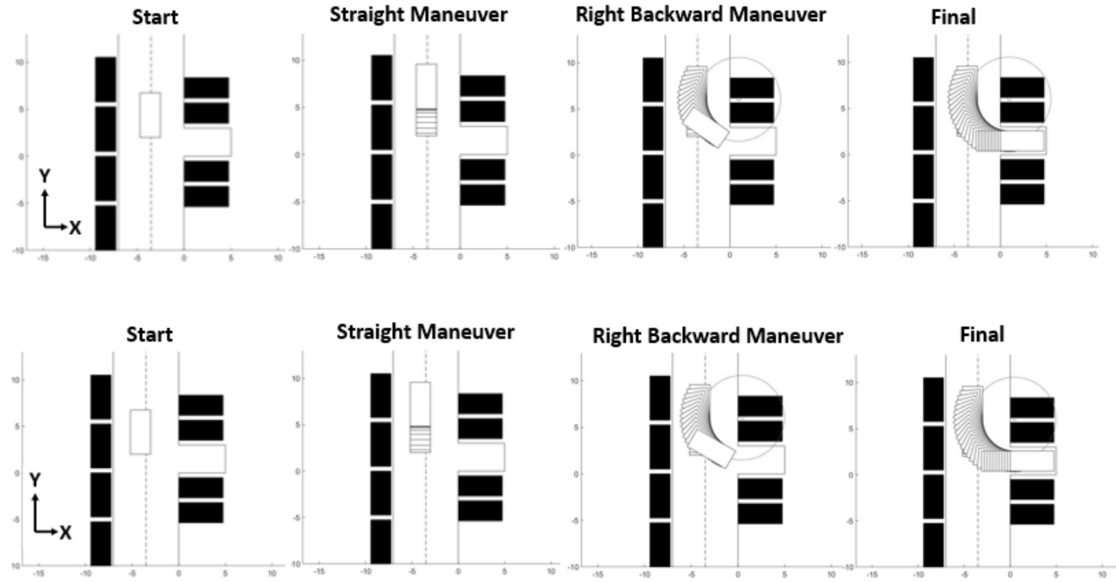

***Figure 8*:**
Parking maneuvers with lateral distances 2.5 meters & 3 meters; the ratio is 5 (Scenarios 3-4)

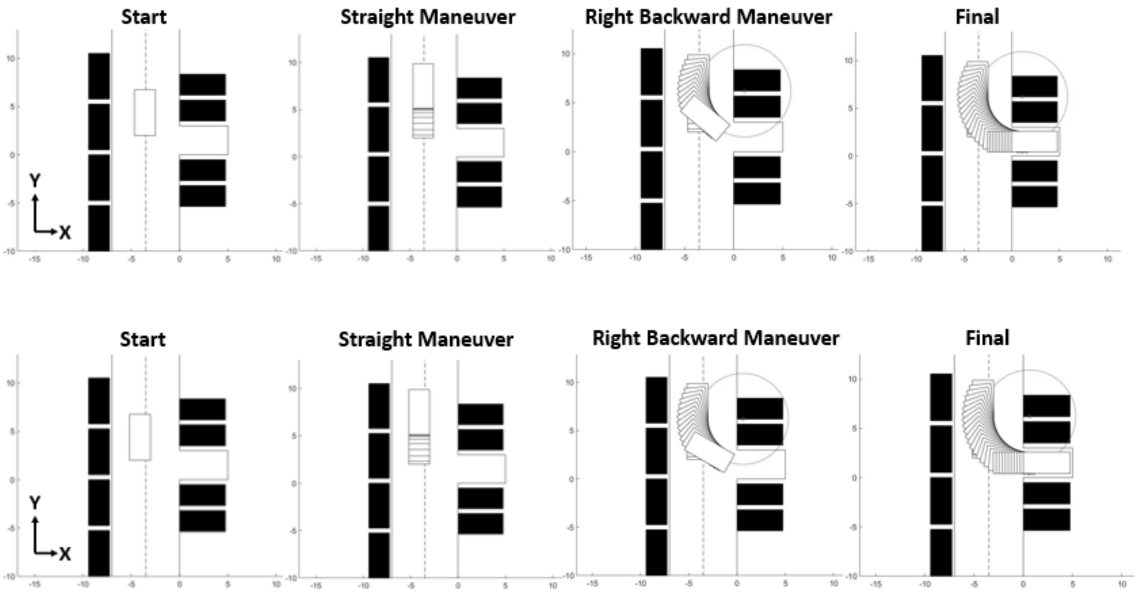

***Figure 9*:**
Parking maneuvers with lateral distances 2.5 meters & 3 meters; the ratio is 7 (Scenarios 5-6)

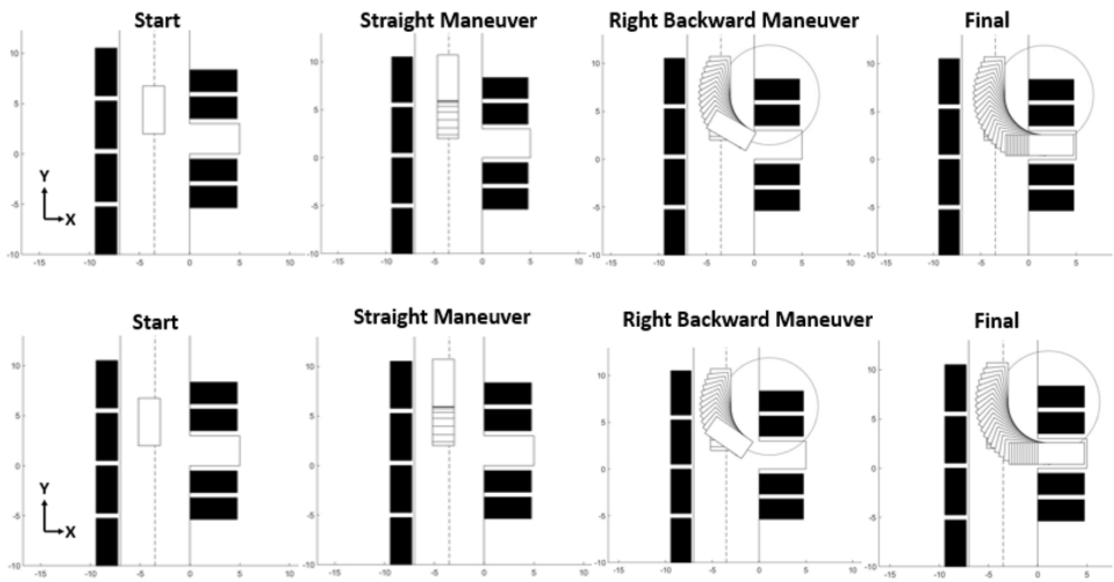

***Figure 10*:**
Parking maneuvers with lateral distances 2.5 meters & 3 meters; Ackerman for front wheel steering (Scenarios 7-8)

4. GEOMETRICAL PARKING MANEUVER EQUATIONS

The equations that are related to longitudinal forward maneuver, maximum approach to left side of the road and maximum approach to edge of the parking space are derived for the autonomous parallel parking maneuver and explained in (Hatipoğlu and Kutluay, 2021). The equations used in the current study are based on these formulas and extended for autonomous perpendicular parking.

4.1. Longitudinal Distance Difference (LDD)

A new formula is derived to show related parameters of the longitudinal distance between rear bumper of the ego vehicle and right border of the parking space at the end of straight forward maneuver. Related equation is given as Equation 9 and related figure is given in Figure 11.

$$LDD = \frac{l}{tan(\delta_{if}) + tan(\delta_{ir})} + \frac{d}{2} - (l_2 + l_4) - \frac{w_{parking\ space}}{2} \tag{9}$$

As can be seen from Equation 9, the distance is a function of steering angles of inner front wheel and inner rear wheel, width of the vehicle, dimensions of the vehicle and width of the parking space.

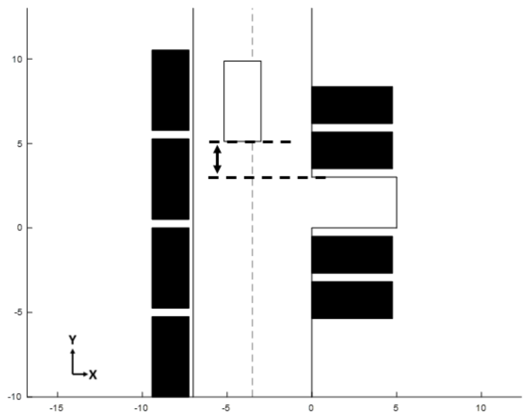

Figure 11*: *LDD

4.2. Maximum Approach to Left Side

Maximum approach to left side of the road is a significant indicator to check the feasibility of the parking maneuver in the context of proposed scenario. Because there is a crash possibility at the left side of the road during a similar maneuver, especially parking at narrow streets. The formula that is given as Equation 10 can be used to calculate the limit coordinate of the left front edge of the vehicle by benefiting the proposed circle. Related figure is given in Figure 12. This formula is constituted with the approach similar to (Hatipoglu and Kutluay, 2021). The difference between the initial position of the left front edge and calculated value by using Equation 10 gives the maximum lateral travel of the left front edge of the ego vehicle during the circular backward maneuver. Related formula is given as Equation 11. The minimum distance between the left front edge of the ego vehicle and left border of the road during the circular backward maneuver can be calculated by using Equation 12. Result of these formulas supports specifying the required driving corridor and required width of the road to perform a collision-free autonomous perpendicular parking maneuver.

$$LimitCoordinate_{Left\ Front\ Edge} = X_{RAC\_ego} + R - \sqrt{((R + \frac{d_0}{2})^2 + (l_1 + l_3)^2)} \tag{10}$$

$$Total\ travel\ to\ left = X_{lf_{init}} - LimitCoordinate_{Left\ Front\ Edge} \tag{11}$$

$$Maximum\ Approach\ to\ Left\ Side = LimitCoordinate_{Left\ Front\ Edge} - X_{road_{left\ border}} \tag{12}$$

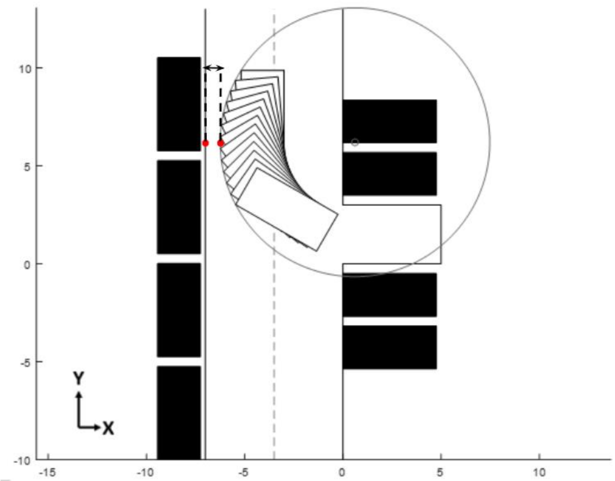

***Figure 12*:**
Maximum approach to left side

4.3. Maximum Approach to Right Side

To check the feasibility of the parking maneuver, maximum approach to right side parameter can be used as an indicator since there is a crash possibility to the right side within the context of the proposed scenario. Related formula is given as Equation 13 and related geometry is given in Figure 13. This equation is based on the proposed circle and shows the minimum distance between the edge of the parking space and projection of the right rear wheel to right border of the vehicle.

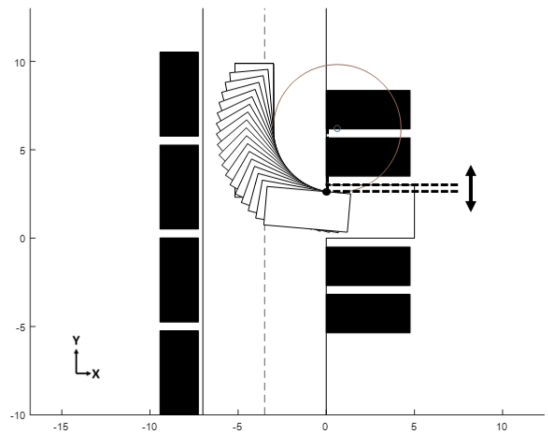

***Figure 13*:**
Maximum approach to right side

$$Maximum\ Approach_{Right\ side} = Y_{ps_3} - (Y_{RRM} - \sqrt{(((R - \frac{d_0}{2})^2 - (X_{ps_3} - X_{RRM})^2))} \quad (13)$$

Longitudinal Distance Difference (LDD), Maximum Approach to Left Side and Maximum Approach to Right Side formulas have been derived to check the feasibility of the autonomous perpendicular parking maneuver in this study. As described before, autonomous perpendicular parking maneuver consists of three main steps.

In the first step, the ego vehicle performs a straight-forward maneuver to reach the position where the circular backward maneuver starts. The longitudinal distance between the rear bumper of the ego vehicle and the right border of the parking space is called LDD and shown as “A” in Figure 14 and in Table 2.

The second step of the autonomous perpendicular parking maneuver is circular backward maneuver. Orientation of the ego vehicle is changed by 90 degrees in this step. During this maneuver, the left front edge of the ego vehicle first approaches the left side of the road and then moves away. Maximum lateral displacement of the left front edge of the ego vehicle during the circular backward maneuver is called Maximum Travel to Left and given as “B”. The minimum distance between the left front edge of the ego vehicle and left border of the road during the circular backward maneuver is called Maximum Approach to Left side and is shown as “C”. B and C are given in Figure 14 and Table 2. During the second step of the maneuver, there is a crash possibility to the right side of the ego vehicle. The minimum distance between the right front edge of the parking space and right side of the ego vehicle is called as Maximum Approach to Right Side and given as “D” in Figure 14 and Table 2.

In the last step of the autonomous perpendicular parking maneuver, the ego vehicle performs a straight backward maneuver as described in Section 3.

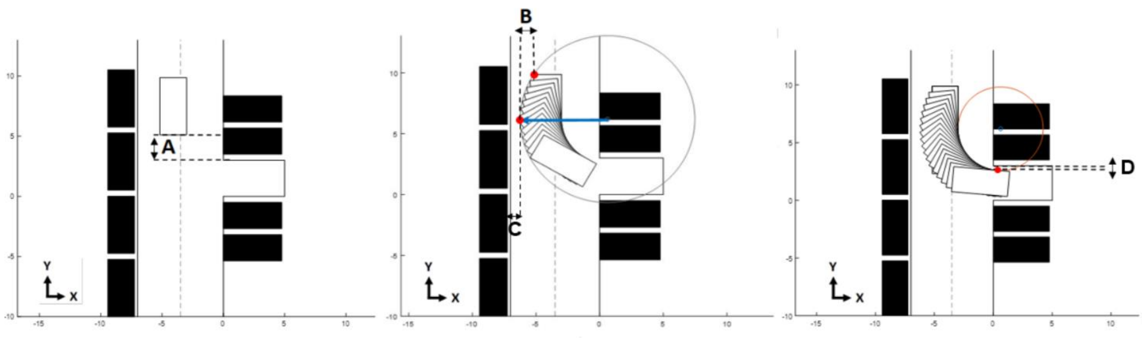

***Figure 14*:**

Calculated distances that obtained from the newly derived equations

Table 2. Scenarios

Case	Steering Angle (in degrees)	Ratio	Lateral (in meters)	A (in meters)	B (in meters)	C (in meters)	D (in meters)
1	35	3.5	2.5	1.4427	0.9770	1.343	0.3290
2	35	3.5	3	1.4427	0.9770	0.843	0.4026
3	35	5	2.5	1.8346	1.0276	1.292	0.2772
4	35	5	3	1.8346	1.0276	0.792	0.3808
5	35	7	2.5	2.1220	1.0647	1.255	0.2332
6	35	7	3	2.1220	1.0647	0.755	0.3570
7	35	-	2.5	2.9594	1.1728	1.147	0.0817
8	35	-	3	2.9594	1.1728	0.647	0.2574

Smaller ratio means that rear wheels turn more. As can be seen from the Table 2., as the ratio decreases, vehicle approaches to edge that is shown in Figure 13 less. Also, it can park to narrower street and can complete the parking with a shorter longitudinal forward maneuver.

5. CONCLUSION

An autonomous perpendicular parking algorithm that is based on geometric equations of four-wheel steered vehicles is mentioned in this paper. This algorithm depends on the parallelism between the ego vehicle and road borders and independent of the lateral position within the limits of proposed scenario and environment model. Related parameters of Longitudinal Distance Difference, Maximum Approach to Left Side and Maximum Approach to Right Side are presented. These parameters include measured environmental telemetry, vehicle kinematics and dimensions. As the ratio between inner front and inner rear wheel decreases, the vehicle can park in narrower streets, needs less longitudinal maneuver for straight forward maneuver and performs a safer parking maneuver considering the proposed edge approach. For instance, the ego vehicle can shorten the first longitudinal maneuver by up to 17% by using the proposed algorithm. Since the vehicle changes the steering angle at zero velocity, calculated distances are the smallest distances that the vehicle can move, which are required to define the constraints during the synthesis of an automated parking controller. Also, proposed study shows the necessary driving corridor during parking maneuver. As a future work, crab walking function can be investigated as a parking exit assistant.

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CONFLICT OF INTERESTS

The authors confirm that there is not any conflict of interest or common interest with any person or institution/organization.

AUTHOR CONTRIBUTION

Emir Kutluay has spent on data analysis and critical review of intellectual content. Mr Kutluay is also supervisor of Emrecan Hatipoğlu. Emrecan Hatipoğlu and Seda Karaosmanoğlu has studied on concept definition of the and preparing the draft of the article. Mert Assoy has contributed on management of conceptual process of the study and critical review of intellectual content. All authors contributed to the final approval and full responsibility sections.

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