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Research Article

Statistical Convergence in A-Metric Spaces

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Abstract: In this paper, we will present the notion of statistical convergence in A-metric spaces, which is an important concept in summability theory. We will define statistical convergence in A-metric spaces and investigate its basic properties. Furthermore, we will explore the relationship between strongly p-Cesàro and statistical convergence in A-metric spaces.

Keywords: Statistical convergence, Cauchy sequence, A-metric spaces.

1. Introduction

Metric spaces are an important topic in mathematics, particularly in analysis and topology. Fréchet [11] was the first to introduce the notion of metric space in 1906. Since then, many researchers have been interested in the generalization of metric spaces and have published various papers on this topic [7, 14, 17–19, 25]. One of the outcomes of these studies was the concept of A-metric space, which was proposed by Abbas et al. [2] in 2015 as a generalization of S-metrics which is a generalization of metric spaces. Some fixed point theorems in this space have been studied.

The concept of statistical convergence was introduced in 1951 by Fast in [9] and Steinhaus in [26]. Afterwards, Shoenberg [24] introduced it in 1959. Since then, the properties of statistical convergence have been studied by different mathematicians and applied in several area (see, [4–6, 8, 10, 12, 13, 15, 22, 23, 27]).

Let's recall the definitions of natural density, statistical convergence and p-strongly Cesàro summability (see the references given above for details).

For a set K of positive integers, the asymptotic (or natural) density is defined as follows;

$$\delta(K) = \lim_k \frac{1}{k} |\{t \le k : t \in K\}|$$

, where $|\{t \le k : t \in K\}|$ denotes the number of elements of K not exceeding k.

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A sequence (x_t) is said to be statistically convergent to x, if for every $\varepsilon > 0$,

$$\lim_k \frac{1}{k} |\{t \leq k : |x_t - x| \geq \varepsilon\}| = 0.$$

A sequence (x_t) is said to be statistically Cauchy, if for every $\varepsilon > 0$, there exists a positive integer $S = S(\varepsilon)$ such that

$$\lim_k \frac{1}{k} |\{t \leq k : |x_t - x_S| \geq \varepsilon\}| = 0.$$

A sequence (x_t) is said to be p-strongly Cesàro summable to x, if

$$\lim_{k\to\infty} \frac{1}{k} \sum_{t=1}^{k} | x_t - x |^p = 0.$$

Recently, Bilalov and Nazarova [3] investigated statistical convergence in metric spaces. In [16], Kedian et al. defined the concept of statistical convergence in cone metric spaces. Then Nuray [21] examined statistical convergence in 2-metric spaces, and [20] investigated partial metric spaces. Recently, Abazari [1] introduced the definition of statistically convergent sequence and studied its basic properties in g-metric spaces.

In this paper, we examine the concept of a statistically convergent sequence and investigate some of its fundamental properties in A-metric spaces. Then, we discuss statistically compact spaces and characterize the statistical completeness of A-metric spaces. Furthermore, we explore the relationship between strong p-Cesàro convergence and statistical convergence in A-metric spaces.

Now let us give the basic definitions and notations that we will use in our study.

Definition 1.1 *[2] Let X be a nonempty set. A function $A : X^n \to [0, \infty)$ is called an A-metric on X if for any $x_i, a \in X, i = 1, 2, \ldots, n$ the following conditions hold;*

(A1) $A(x_1, x_2, \ldots, x_{n-1}, x_n) \geq 0$,

(A2) $A(x_1, x_2, \ldots, x_{n-1}, x_n) = 0 \Leftrightarrow x_1 = x_2 = \ldots = x_n$,

(A3) $A(x_1, x_2, \ldots, x_{n-1}, x_n) \leq \sum_{k=1}^{n} A(\underbrace{x_k, x_k, \ldots, x_k}_{n-1}, a)$.

Also the pair (X, A) is called an A-metric space.

Example 1.2 *[2] Let $X = \mathbb{R}$. A function $A : X^n \to [0, \infty)$ by*

$$A(x_1, x_2, \ldots, x_{n-1}, x_n) = \sum_{k=1}^{n} \sum_{k<j} |x_k - x_j|$$

is an A-metric space on X.

Lemma 1.3 *[2] Let (X, A) be an A-metric space. Then $A(x, x, \ldots, x, y) = A(y, y, \ldots, y, x)$ for all $x, y \in X$.*

Lemma 1.4 *[2] Let (X, A) be an A-metric space. For all $x, y \in X$, we get*

$$A(x, x, \ldots, x, z) \leq (n-1)A(x, x, \ldots, x, y) + A(y, y, \ldots, y, z)$$

and

$$A(x, x, \ldots, x, z) \leq (n-1)A(x, x, \ldots, x, y) + A(z, z, \ldots, z, y).$$

Definition 1.5 *[2] The A-metric space (X, A) is called bounded if there exists an $r > 0$ such that $A(y, y, \ldots, y, x) \leq r$ for all $x, y \in X$. Otherwise, X is unbounded.*

Definition 1.6 *[2] Let (X, A) be an A-metric space and (x_t) be a sequence in this space:*

(1) *The sequence (x_t) is said to be convergent to x, if for every $\varepsilon > 0$, there exists a positive integer t_0 such that $A(x_t, x_t, \ldots, x_t, x) < \varepsilon$ for every $t \geq t_0$.*

(2) *The sequence, (x_t) is said to be a Cauchy sequence if for every $\varepsilon > 0$, there exists a positive integer t_0 such that $A(x_t, x_t, \ldots, x_t, x_m) < \varepsilon$ for all $t, m \geq t_0$.*

2. Main Results

This section introduces the definition of statistical convergence of sequences in A-metric space and studies some basic properties. It also explores the relationship between strongly p-Cesàro and statistical convergence in A-metric spaces.

Definition 2.1 *Let (X, A) be an A-metric space. We say that a sequence (x_t) in X is statistically convergent to $x \in X$, if for every $\varepsilon > 0$,*

$$\lim_{k\to\infty} \frac{1}{k} |\{t \leq k : A(x_t, x_t, \ldots, x_t, x) \geq \varepsilon\}| = 0$$

or equivalently

$$\lim_{k\to\infty} \frac{1}{k} |\{t \leq k : A(x_t, x_t, \ldots, x_t, x) < \varepsilon\}| = 1$$

and is denoted by $x_t \xrightarrow{AS} x$. This means that $A(x_t, x_t, \ldots, x_t, x) < \varepsilon$ holds for almost all t. In this case, we write $A_{st} - \lim x_t = x$ in the sense of statistical convergence.

Definition 2.2 *Let (X, A) be an A-metric space. We say that a sequence (x_t) in X statistically Cauchy sequence, if for every $\varepsilon > 0$, there exists a positive integer $l = l(\varepsilon)$*

$$\lim_{k\to\infty} \frac{1}{k} |\{t, l \leq k : A(x_t, x_t, \ldots, x_t, x_l) \geq \varepsilon\}| = 0$$

or equivalently

$$\lim_{k\to\infty}\frac{1}{k}\left|\{t,l\leq k : A(x_t,x_t,\ldots,x_t,x_l)<\varepsilon\}\right|=1.$$

Theorem 2.3 *Any convergent sequence in an $A-$ metric space (X,A) is also statistically convergent.*

Proof Let (x_t) be a sequence in A-metric space (X,A) such that converges to $x\in X$. For every $\varepsilon>0,$ there exists a positive integer k_0 such that for all $t\geq k_0$,

$$A(x_t,x_t,\ldots,x_t,x)<\varepsilon.$$

The set

$$\mathcal{A}(k)=\{t\leq k : A(x_t,x_t,\ldots,x_t,x)<\varepsilon\},$$

then we can write

$$|\mathcal{A}(k)|=\left|\{t\leq k : A(x_t,x_t,\ldots,x_t,x)<\varepsilon\}\right|\geq k-k_0.$$

Therefore

$$\lim_{k\to\infty}\frac{|\mathcal{A}(k)|}{k}=1.$$

Hence (x_t) is statistically convergent. □

The following example shows that the converse of above theorem is not valid.

Example 2.4 *Let $X=\mathbb{R}$ and A be the metric as follows;*

$$\begin{aligned} A:\mathbb{R}^n &\to \mathbb{R}^+ \\ A\left(x_1,x_2,\ldots,x_n\right) = &\quad max\{\left|x_1-x_2\right|,\left|x_1-x_3\right|,\left|x_1-x_4\right|,\ldots,\left|x_1-x_n\right| \\ &\qquad \left|x_2-x_3\right|,\left|x_2-x_4\right|,\ldots,\left|x_2-x_n\right|, \\ &\qquad \ldots \\ &\qquad \left|x_{n-2}-x_{n-1}\right|,\left|x_{n-2}-x_n\right|, \\ &\qquad \left|x_{n-1}-x_n\right|\}. \end{aligned}$$

Consider the following sequence

$$x_t:=\begin{cases} t, & \text{if } t \text{ is square} \\ 0, & o.w. \end{cases}.$$

It is clear that (x_t) is statistically convergent while it is not convergent normally.

Theorem 2.5 *Let (X,A) be an A-metric space and (x_t) be a sequence in this space. If $x_t \xrightarrow{AS} x$ and $x_t \xrightarrow{AS} y$, then $x=y$.*

Proof For any $\varepsilon>0$, we define the following two sets

$$\mathcal{A}(\varepsilon) := \{t \in \mathbb{N} : A(x_t, x_t, \ldots, x_t, x), \geq \frac{\varepsilon}{n}\}$$

$$\mathcal{B}(\varepsilon) := \{t \in \mathbb{N} : A(x_t, x_t, \ldots, x_t, y). \geq \frac{\varepsilon}{n}\}$$

Since $x_t \xrightarrow{AS} x$ and $x_t \xrightarrow{AS} y$, then $\delta_1(\mathcal{A}(\varepsilon)) = 0$ and $\delta_1(\mathcal{B}(\varepsilon)) = 0$. Let $\mathcal{C}(\varepsilon) := \mathcal{A}(\varepsilon) \cup \mathcal{B}(\varepsilon)$, then $\delta_1(\mathcal{C}(\varepsilon)) = 0$. So $\delta_1(\mathcal{C}^c(\varepsilon)) = 1$. Suppose that $t \in \mathcal{C}^c(\varepsilon)$, then by Lemma 1.3 and Lemma 1.4, we can write

$$\begin{aligned}
A(x,x,\ldots,x,y) &\leq (n-1)A(x,x,\ldots,x,x_t) + A(x_t,x_t,\ldots,x_t,y) \\
&= (n-1)A(x_t,x_t,\ldots,x_t,x) + A(x_t,x_t,\ldots,x_t,y) \\
&< (n-1)\frac{\varepsilon}{n} + \frac{\varepsilon}{n} \\
&= \varepsilon.
\end{aligned}$$

Since $\varepsilon>0$ is arbitrarty, we have $A(x,x,\ldots,x,y)=0$. Therefore $x=y$.

□

Theorem 2.6 *Let (x_t) and (y_t) be two sequences in an A-metric space (X,A). If $y_t \xrightarrow{AS} y$ and $A(x_t,x_t,\ldots,x_t,y) \leq A(y_t,y_t,\ldots,y_t,y)$ for each $t \in \mathbb{N}$, then $x_t \xrightarrow{AS} y$.*

Proof Let $y_t \xrightarrow{AS} y$. For each $\varepsilon>0$, we can write

$$\{t \leq k : A(x_t,x_t,\ldots,x_t,y) < \varepsilon\} \supseteq \{t \leq k : A(y_t,y_t,\ldots,y_t,y) < \varepsilon\}$$

and

$$\delta(\{t \leq k : A(x_t,x_t,\ldots,x_t,y) < \varepsilon\}) \geq \delta(\{t \leq k : A(y_t,y_t,\ldots,y_t,y) < \varepsilon\}) = 1.$$

Therefore $x_t \xrightarrow{AS} y$. □

Definition 2.7 *([4]) A set $\mathcal{A} = \{t_m : m \in \mathbb{N}\}$ is said to be statistically dense in $\mathbb{N}$, if the set $\mathcal{A}(k) = \{t_j \in A : t_j \leq k\}$ has asymptotic density 1, that is,*

$$\delta(\mathcal{A}) = \lim_{k\to\infty} \frac{|\,\mathcal{A}(k)\,|}{k} = 1.$$

Definition 2.8 *Let* (X, A) *be an* A*-metric space and* (x_t) *be a sequence in* X*. A subsequence* (x_{t_m}) *of sequence* (x_t) *is said to be statistically dense in* X *if the index set* $\{t_m : m \in \mathbb{N}\}$ *is statistically dense subset of* $\mathbb{N}$*, namely,*

$$\delta(\{t_m : m \in \mathbb{N}\}) = 1.$$

The following theorem shows the equivalence between some properties of A-metric spaces. This result has been previously studied for cone metric spaces. We will give a proof for the A-metric spaces case using some techniques from [16].

Theorem 2.9 *Let* (x_t) *be a sequence in an* A*-metric space* (X, A)*. Then the followings are equivalent:*

(i) (x_t) *is statistically convergent in* (X, A)*,*

(ii) *There is a convergent sequence* (y_t) *in* X *such that* $x_t = y_t$ *for almost all* $t \in \mathbb{N}$*,*

(iii) *There is a statistically dense subsequence* (x_{t_m}) *of* (x_t) *such that* (x_{t_m}) *is convergent,*

(iv) *There is a statistically dense subsequence* (x_{t_m}) *of* (x_t) *such that* (x_{t_m}) *is statistically convergent.*

Proof $(i) \Rightarrow (ii)$ Let $\varepsilon > 0$ and $x_t \xrightarrow{AS} x \in X$. We can write

$$\lim_{k \to \infty} \frac{1}{k} \mid \{t \leq k : A(x_t, x_t, \ldots, x_t, x) < \varepsilon\} \mid = 1.$$

We can choose (t_m) as an increasing sequence in $\mathbb{N}$ such that

$$\frac{1}{k} \mid \{s \leq k : A(x_s, x_s, \ldots, x_s, x) < \frac{1}{2^m}\} \mid > 1 - \frac{1}{2^m}$$

for every $k > t_m$. We can assume that $t_m < t_{m+1}$ for each $m \in \mathbb{N}$. Now we define (y_s) as follows;

$$y_s := \begin{cases} x_s, & 1 \leq s \leq t_1 \\ x_s, & t_m < s \leq t_{m+1}, A(x_s, x_s, \ldots, x_s, x) < \dfrac{1}{2^m} \\ x, & \text{o.w.} \end{cases}.$$

Choose $m \in \mathbb{N}$ such that $\frac{1}{2^m} < \varepsilon$. Then $A(y_s, y_s, \ldots, y_s, x) < \varepsilon$ for each $s > t_m$, that is, (y_s) is convergent to x. Fix $k \in \mathbb{N}$, for $t_m < k \leq t_{m+1}$, we get

$$\{s \leq k : y_s \neq x_s\} \subseteq \{1, 2, \ldots, k\} - \{s \leq k : A(x_s, x_s, \ldots, x_s, x) < \frac{1}{2^m}\}.$$

Thus

$$\begin{aligned}\lim_{k\to\infty}\frac{1}{k}\mid\{s\le k: y_s\ne x_s\}\mid &\le \lim_{k\to\infty}\frac{1}{k}k-\lim_{k\to\infty}\frac{1}{k}\mid\{s\le k: A(x_s,x_s,\dots,x_s,x)<\frac{1}{2^m}\}\mid\\ &<1-\left(1-\frac{1}{2^m}\right)\\ &=\frac{1}{2^m}\\ &<\varepsilon.\end{aligned}$$

Hence $\lim_{k\to\infty}\frac{1}{k}\mid\{s\le k: y_s\ne x_s\}\mid=0$, that is, $\delta(\{s\in\mathbb{N}: y_s\ne x_s\})=0$. Therefore $x_s=y_s$ for almost all $s\in\mathbb{N}$.

$(ii)\ \Rightarrow\ (iii)$ Assume that (y_t) is a convergent sequence in X such that $x_t=y_t$ for almost all $t\in\mathbb{N}$. Let $\mathcal{A}=\{t\in\mathbb{N}: x_t=y_t\}$. Then $\delta(\mathcal{A})=1$. Thus $(y_t)_{t\in\mathcal{A}}$ is both a convergent sequence and a statistically dense subsequence of (x_t).

$(iii)\ \Rightarrow\ (iv)$ It is an obvious consequence of Theorem 2.3.

$(iv)\ \Rightarrow\ (i)$ Assume that there is a statistically dense subsequence (x_{t_m}) of (x_t) such that $x_{t_m}\xrightarrow{AS}x\in X$. Let $\mathcal{A}=\{t_m: m\in\mathbb{N}\}$. Then $\delta(\mathcal{A})=1$. Since, for each $\varepsilon>0$,

$$\{t\in\mathbb{N}: A(x_t,x_t,\dots,x_t,x)<\varepsilon\}\supseteq\{t_m\in\mathbb{N}: A(x_{t_m},x_{t_m},\dots,x_{t_m},x)<\varepsilon\}$$

, thus

$$\delta\left(\{t\in\mathbb{N}: A(x_t,x_t,\dots,x_t,x)<\varepsilon\}\right)\ge\delta\left(\{t_m\in\mathbb{N}: A(x_{t_m},x_{t_m},\dots,x_{t_m},x)<\varepsilon\}\right)=1,$$

we get $x_t\xrightarrow{AS}x\in X$. □

The following corollary is a direct consequence of Theorem 2.9.

Corollary 2.10 *In A-metric spaces, every statistically convergent sequence has a convergent subsequence.*

Theorem 2.11 *Let (x_t) be any statistically convergent sequence in an A-metric space (X,A), then it is statistically Cauchy.*

Proof Let (x_t) be a statistically convergent sequence in A-metric space (X,A), that is, for $\varepsilon>0$, $\lim_{k\to\infty}\frac{1}{k}\mid\{t\le k: A(x_t,x_t,\dots,x_t,x)<\frac{\varepsilon}{n}\}\mid=1$. Then we can write

$$A(x_t,x_t,\dots,x_t,x)<\frac{\varepsilon}{n}\ \text{for a.a.t},$$

and if $s := s(\varepsilon)$ is chosen so that

$$A(x_s, x_s, \ldots, x_s, x) < \frac{\varepsilon}{n}.$$

Then by Lemma 1.3 and Lemma 1.4, we get

$$\begin{aligned} A(x_t, x_t, \ldots, x_t, x_s) &\leq (n-1)A(x_t, x_t, \ldots, x_t, x) + A(x, x, \ldots, x, x_s) \\ &= (n-1)A(x_t, x_t, \ldots, x_t, x) + A(x_s, x_s, \ldots, x_s, x) \\ &< (n-1)\frac{\varepsilon}{n} + \frac{\varepsilon}{n} \\ &= \varepsilon \text{ for a.a.t.} \end{aligned}$$

So (x_t) is a statistically Cauchy sequence. □

Definition 2.12 *Let (X, A) be an A-metric space. We say that (X, A) is statistically complete if every statistically Cauchy sequence in X is also statistically convergent.*

Lemma 2.13 *Every statistically complete A-metric space is also complete.*

Proof Let (X, A) be a statistically complete A-metric space. If a sequence (x_t) be a Cauchy sequence in X, then it is a statistically Cauchy sequence in this space. Since (X, A) is statistically complete, the sequence (x_t) is statistically convergent. By Corollary 2.10, there is a subsequence (x_{t_m}) of the sequence (x_t) that converges to a point $x \in X$. Since (x_t) is a Cauchy sequence, so, for $\varepsilon > 0$, there exist a positive integer K such that

$$A(x_t, x_t, \ldots, x_t, x_s) < \frac{\varepsilon}{n}$$

for each $t, s \geq K$. Also (x_{t_m}) converges to x. So, there exists a positive integer m_0 such that $t_{m_0} \geq K$ and

$$A(x_{t_{m_0}}, x_{t_{m_0}}, \ldots, x_{t_{m_0}}, x) < \frac{\varepsilon}{n}.$$

By Lemma 1.3 and Lemma 1.4, we have

$$\begin{aligned} A(x_t, x_t, \ldots, x_t, x) &\leq (n-1)A(x_t, x_t, \ldots, x_t, x_{t_{m_0}}) + A(x_{t_{m_0}}, x_{t_{m_0}}, \ldots, x_{t_{m_0}}, x) \\ &< (n-1)\frac{\varepsilon}{n} + \frac{\varepsilon}{n} \\ &= \varepsilon \end{aligned}$$

for each $t \geq K$, that is, (x_t) converges to x. So, (X, A) is a complete A-metric space. □

In a partial metric space, Nuray [21] examined the relationship between p-strongly Cesàro summability and statistical convergence. Here following the same approach, we explain the relationship between strongly q-Cesàro convergence and statistical convergence in an A-metric space (X, A).

Definition 2.14 *Let (X, A) be an A-metric space, let (x_t) be a sequence in X, and let p be a positive real number. The sequence (x_t) is said to be p-strongly Cesàro summable to $x \in X$ if*

$$\lim_{k\to\infty} \frac{1}{k} \sum_{t=1}^{k} [A(x_t, x_t, \ldots, x_t, x)]^p = 0$$

and is denoted by $x_t \overset{A[C,p]}{\longrightarrow} x$.

Theorem 2.15 *Let (x_t) be a sequence in an A-metric space (X, A) and $p \in \mathbb{R}^{>0}$. Then*

(i) If the sequence (x_t) is strongly p-Cesàro summable to $x \in X$, then it is statistically convergent to x.

(ii) If (X, A) is bounded and the sequence (x_t) is statistically convergent to $x \in X$, then it is strongly p-Cesàro summable to x.

Proof

(i) Let (x_t) be a sequence in an A-metric space (X, A). Assume the sequence (x_t) is strongly p-Cesàro summable to $x \in X$. Then for any $\varepsilon > 0$, we can write

$$\begin{aligned} \sum_{t=1}^{k} [A(x_t, x_t, \ldots, x_t, x)]^p &= \sum_{\substack{t=1 \\ A(x_t, x_t \ldots, x_t, x) \geq \varepsilon}}^{k} [A(x_t, x_t, \ldots, x_t, x)]^p \\ &\quad + \sum_{\substack{t=1 \\ A(x_t, x_t \ldots, x_t, x) < \varepsilon}}^{k} [A(x_t, x_t, \ldots, x_t, x)]^p \\ &\geq \sum_{\substack{t=1 \\ A(x_t, x_t \ldots, x_t, x) \geq \varepsilon}}^{k} [A(x_t, x_t, \ldots, x_t, x)]^p \\ &\geq |\{t \leq k : A(x_t, x_t, \ldots, x_t, x) \geq \varepsilon\}| \, \varepsilon^p. \end{aligned}$$

This completes the proof.

(ii) Now suppose that (X, A) is bounded and (x_t) is statistically convergent to x. Since (X, A) is bounded, there exists a constant $L > 0$ such that $A(x_t, x_t, \ldots, x_t, x) < L$ for all $x_t, x \in X$.

Let $\varepsilon > 0$ be given and choose K_ε such that

$$\frac{1}{k} \mid \{t \leq k : A(x_t, x_t, \ldots, x_t, x) \geq \left(\frac{\varepsilon}{2}\right)^{\frac{1}{p}}\} \mid < \frac{\varepsilon}{2L^p}$$

for all $k > K_\varepsilon$ and set $\mathcal{A}_k = \{t \leq k : A(x_t, x_t, \ldots, x_t, x) \geq \left(\frac{\varepsilon}{2}\right)^{\frac{1}{p}}\}$. Now for all $k > K_\varepsilon$, we can write

$$\begin{aligned}\frac{1}{k}\sum_{t=1}^{k}[A(x_t, x_t, \ldots, x_t, x)]^p &= \frac{1}{k}\sum_{t\in\mathcal{A}_k}[A(x_t, x_t, \ldots, x_t, x)]^p + \frac{1}{k}\sum_{\substack{t\notin\mathcal{A}_k\\ t\leq k}}[A(x_t, x_t, \ldots, x_t, x)]^p \\ &< \frac{1}{k}\left(\frac{k\varepsilon}{2L^p}\right)L^p + \frac{1}{k}k\frac{\varepsilon}{2} = \varepsilon.\end{aligned}$$

So, (x_t) is strongly p-Cesàro summable to x. □

Declaration of Ethical Standards

The author declares that the materials and methods used in his study do not require ethical committee and/or legal special permission.

Conflicts of Interest

The author declares no conflict of interest.

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