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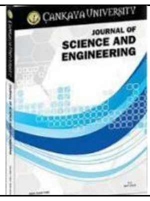

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Investigating the Viscosity and Surfactant Effect on Gravity Waves in the Ocean

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Keywords

Nonlinear Gravity Waves, Nonlinear Schrödinger Equation, Viscosity, Surfactant and Bifurcation method.

Abstract

This article deals with the dynamical behavior of weakly nonlinear gravity waves propagating including the viscosity and surfactant on the ocean free surface. Through the bifurcation method, we could predict the nature of solutions of the nonlinear Schrödinger equation (NLSE) and reduce it to the nonlinear ordinary differential equation, easily solvable. Then, bright soliton, dark soliton, and Jacobi elliptic functions solutions of this NLSE under the viscosity and surfactant effect have been derived. These obtained results are central in hydrodynamics and can predict physical phenomena such as gravity wave propagation in deep water. Moreover, they allow to enhance the decision-making process and the acquisition of radar and lidar data on the ocean surface.

1. Introduction

The dynamical behavior of water waves have been described by a Navier-Stokes and Euler Lagrange equation [1-5] and modeled by the NLSE [6-14]. This NLSE takes into account nonlinearity and dispersion in infinite water depth. Indeed, it provides special analytical solutions that depict the evolution of these water waves in time and space and thus allow the study and the understanding of this phenomenon. Several works have been devoted to the construction and the propagation interpretations of solutions of the NLSE of weakly nonlinear surface gravity waves in the ocean [15-20].

Joo et al. [21] derived a nonlinear Schrodinger equation including the viscosity and surfactant and revisited the Benjamin-Fair instability. The evolution of the water waves under the effect of soluble surfactant experimentally investigated [22]. However, in this work, the dynamical behavior of surface gravity waves under the action of viscosity and surfactant propagating on infinite depth is explained by applying the bifurcation method to the NLSE obtained by Daïka and Mbane [23].

The research findings in this paper create the scientific basis on maritime traffic management and improvement of maritime safety which could be made possible by providing the dynamical behavior of surface gravity waves propagating including the viscosity and surfactant. The investigation of this dynamical behavior improves the situational awareness of the protection of human life, ships, and oil rigs and enhances their decision-making process. Also, prior knowledge about the occurrence of this behavior on the ocean could inform maritime traffic controllers to better manage maritime traffic and navigation. It can also help improve information on the scientifically feasible reasons for acquiring radar and lidar remote sensing data from the ocean surface.

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The rest of the work is structured as follows: the dynamic equation model is presented in section two, the methods are drawn in section three, the results and the discussions are given in section four and a conclusion is provided in section five.

2. Dynamic Equation Model

This model is based on the NLSE under the impact of viscosity and surfactant for the propagation of two-dimensional nonlinear water waves on infinite-depth at the free surface. It can be presented in the form [23]:

$$ia_{\tau} + \Gamma a_{\varsigma\varsigma} + \Upsilon |a|^2 a - \mathrm{M} a = 0 \tag{1}$$

Where $\varsigma = s(x - v_g t)\cos\theta + y\sin\theta$, $\tau = s^2 t$ and $s = \dfrac{\varepsilon^{1/4}}{\lambda}$

With $\Gamma = -\dfrac{1}{8}\left(4T^2 - 8T + 1\right)\left(2 - 3\cos\theta\right)$, $\Upsilon = -\dfrac{9T^2 - 15T + 8}{4(1 - 3T)}$ and $\mathrm{M} = -\dfrac{i\lambda^2}{2\sqrt{2}} \dfrac{\kappa\left[\kappa + i\left(\sqrt{2} - \kappa\right)\right]}{\kappa^2 - \sqrt{2}\kappa + 1}$

$a(\varsigma,\tau)$ is the wave envelope, Γ, Υ and M are respectively the dispersion, nonlinearity and dissipative coefficients that depend on the Weber number κ and the surface tension coefficient T**,** of the carrier wave related to the surfactant. θ is the angle between the direction of the carrier-wave propagation and direction of propagation of the envelope, v_g is the group velocity, and λ is a proportionality constant associated with the viscosity.

Equation (1) can be reduced to the equation obtained in [21] if the angle θ is equal to zero and in [7] if the angle θ is equal to zero and without viscosity.

The free-surface elevation $\eta(x, y, t)$ is given by the relation:

$$\eta(x, y, t) = a(x, y, t)\exp\left[i\left(k_0 x - \omega\left(\vec{k}_0\right)t\right)\right] \tag{2}$$

Where $a(x, y, t)$ the wave envelope depends of x, y and t; k_0 wave number of the carrier and $\omega\left(\vec{k}_0\right)$ frequency of the carrier wave.

The equation (1) may be written into standard form using the substitutions:

$U(X,T) = a(\varsigma,\tau)\sqrt{\dfrac{\Upsilon}{2\Gamma}}\exp(iM\tau)$, $X = \varsigma$ and $T = \Gamma.\tau$ to give [14; 27]:

$$iU_T + U_{XX} + 2|U|^2 U = 0 \tag{3}$$

3. Methods

We suppose that the solutions of the equation (3) are:

$$U(X,T) = f(\xi)e^{i\mu(X,T)},\ \xi = X - 2qT,\ \mu(\varsigma,T) = qX + \delta T \tag{4}$$

Where q and δ are the positive real number to be determined.

In the framework of this transformation, the equation (3) can be reduced in the form:

$$f'' - \left(q^2 + \delta\right) f + 2 f^3 = 0 \tag{5}$$

Equation (5) is nonlinear ordinary differential equation and integrable.

3.1. Equilibrium Points and Bifurcation of Phase Portraits

Let $f' = \frac{df}{d\xi} = g$, then the equation (5) can write under the form following:

$$\begin{cases} f' = g \\ g' = \left(q^2 + \delta\right) f - 2 f^3 \end{cases} \tag{6}$$

In multiplication the equation (5) by f', the Hamiltonian writes under the form:

$$\mathrm{H} = \frac{1}{2} f'^2 - \frac{q^2 + \delta}{2} f^2 + \frac{1}{2} f^4 \tag{7}$$

The equation (7) is a conserved quantity for the equation (6).

The equilibrium points are gotten by setting in equation (6), the constraints $(f', g') = (0,0)$. Therefore, they can be written as $\left(f_0, g_0 = 0\right)$ with f_0 is the zeros of the equation (8):

$$\left(q^2 + \delta\right) f - 2 f^3 = 0 \tag{8}$$

The set of solutions of the equation (8) is:

$$f_0 = 0\,,\; f_{1,2} = \pm\sqrt{\frac{q^2 + \delta}{2}} \tag{9}$$

We have two symmetric equilibrium points f_1 and f_2 given by $f_{1,2} = \pm\sqrt{\frac{q^2 + \delta}{2}}$, which are saddle points, and fixed point $f_0 = 0$ which is a center. In this case, the separatrix connects f_1 to f_2 and is qualified as a heteroclinic orbit because two distinct unstable equilibrium points are concerned. These points are on the same potential level (the same holds for the stable ones) [24]. There are two pairs of heteroclinic orbits. The heteroclinic connection is a path in phase space that joins two different equilibrium points. This separatrix separates bounded, periodic oscillatory motions around f_0 from unbounded nonperiodic ones.

The profile of the phase portrait for different values of q for equilibrium points is depicted in Figure 1. According to Figure 1, the curve predicts the existence of kink or dark soliton as a solution.

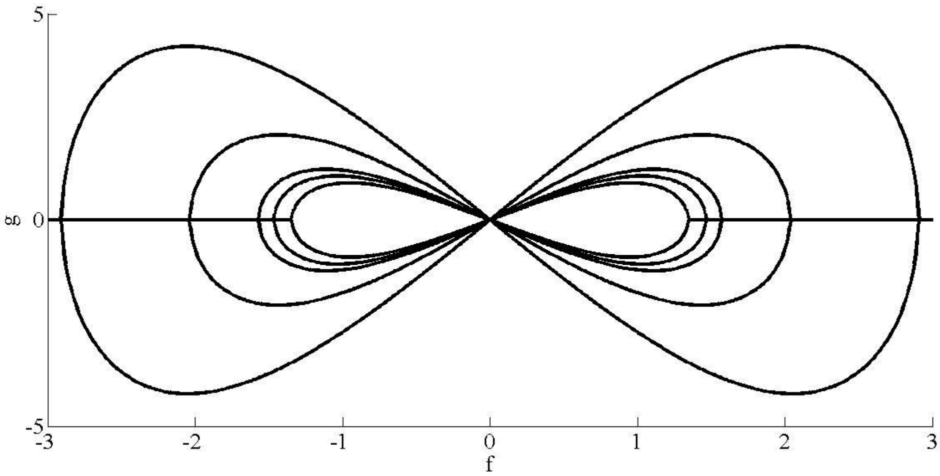

Figure 1. Phase portrait for different values of q of the equilibrium points.

3.2. Exact Soliton Solutions of the Nonlinear Schrödinger Equation

The equation (5) can be written as:

$$\frac{d^2 f(\xi)}{d\xi^2} = \frac{d}{df(\xi)}\left[\frac{1}{2}\left(\delta + q^2\right) f^2(\xi) - \frac{1}{2} f^4(\xi)\right] \tag{11}$$

The relation (11) can be transformed into:

$$\frac{df}{\sqrt{\left(\delta + q^2\right) f^2(\xi) - f^4(\xi) + B}} = d\xi \tag{12}$$

Where B is an arbitrary constant of integration.
According to the different values for the constant of integration B, the relation (12) gives the different analytic solutions as follows:

Case 1: if $B = 0$ and $\left(\delta + q^2\right) > 0$, then the soliton solution of the relation (5) is:

$$f(\xi) = \sqrt{D}\sec h\left(\sqrt{D}\xi\right) \tag{13}$$

Where $D = \left(\delta + q^2\right)$

The solution of the relation (1) is:

$$a_{1,1}(\varsigma, \tau) = \sqrt{\frac{2D\Gamma}{\Upsilon}}\sec h\left(\sqrt{D}\left(\varsigma - 2q\Gamma\tau\right)\right)\exp i\left(q\varsigma + \Omega\tau\right) \tag{14}$$

With $\Omega = \Gamma\left(\delta - M\right)$

Case 2: If $B = -4\left(\delta + q^2\right)^2$ and $\left(\delta + q^2\right) > 0$, the soliton solution of the relation (5) can be gotten:

$$f(\xi) = \sqrt{D}\tanh\left(\sqrt{D}\xi\right) \tag{15}$$

The solution of the equation (1) is:

$$a_{2,1}(\varsigma,\tau)=\sqrt{\frac{2\Gamma D}{\Upsilon}}\tan h\left(\sqrt{D}(\varsigma-2q\Gamma\tau)\right)\exp i(q\varsigma+\Omega\tau) \tag{16}$$

Case 3: Equation (11) admits three Jacobi elliptic function solutions as follows:

a) If $B=\frac{D^2m^2(m^2-1)}{(2m^2-1)^2}$ and $(\delta+q^2)>0$, the solution of the equation (11) can be found:

$$f(\xi)=\sqrt{\frac{Dm^2}{(2m^2-1)}}cn\left(\sqrt{\frac{D}{2m^2-1}}\xi\right) \tag{17}$$

The solution of the equation (1) becomes:

$$a_{3,1}(\varsigma,\tau)=\sqrt{\frac{2\Gamma}{\Upsilon}}\sqrt{\frac{Dm^2}{(2m^2-1)}}cn\left(\sqrt{\frac{D}{2m^2-1}}(\varsigma-2q\Gamma\tau)\right)\exp i(q\varsigma+\Omega\tau) \tag{18}$$

The validity of relation (17) and (18) arise on: $2m^2-1>0$

b) If $B=\frac{D^2(1-m^2)}{(2-m^2)^2}$ and $(\delta+q^2)>0$, the solution of the relation (11) is:

$$f(\xi)=\sqrt{\frac{D}{(2-m^2)}}dn\left(\sqrt{\frac{D}{2-m^2}}\xi\right) \tag{19}$$

The solution of the equation (1) becomes:

$$a_{3,2}(\varsigma,\tau)=\sqrt{\frac{2\Gamma}{\Upsilon}}\sqrt{\frac{D}{(2-m^2)}}dn\left(\sqrt{\frac{D}{2-m^2}}(\varsigma-2q\Gamma\tau)\right)\exp i(q\varsigma+\Omega\tau) \tag{20}$$

The validity of relation (19) and (20) arise on: $2-m^2>0$

c) If $B=\frac{D^2m^2}{(m^2+1)^2}$ and $(\delta+q^2)>0$, the solution of the relation (11) can be gotten:

$$f(\xi)=\sqrt{\frac{Dm^2}{(m^2+1)}}sn\left(\sqrt{\frac{D}{m^2+1}}\xi\right) \tag{21}$$

The solution of the equation (1) gets:

$$a_{3,3}(\varsigma,\tau)=\sqrt{\frac{2\Gamma}{\Upsilon}}\sqrt{\frac{Dm^2}{(m^2+1)}}sn\left(\sqrt{\frac{D}{m^2+1}}(\varsigma-2q\Gamma\tau)\right)\exp i(q\varsigma+\Omega\tau) \tag{22}$$

The constraint relation of (21) and (22) is $m^2+1\neq 0$.

The obtained results of the model, using the bifurcation method, are bright soliton, dark soliton, and Jacobi elliptic functions solutions. According to Bienvenue [25], these solutions can describe the dynamic of the solitary waves of the NLSE in various field of science and engineering. It indicates that the wave envelope propagates at the speed of the carrier wave system [26]. Besides, these results are most important in ocean engineering's such as the improvement of maritime safety and protection of human life, ships, and oil rigs. It will also help improve information on the scientifically feasible reasons for acquiring radar and lidar remote sensing data from the ocean surface. However, using an adequate parameter of the model and the constraint relations, the obtained figures in this study have been represented with the help of MATLAB software.

4. Results and Discussions

The bright soliton is explained in Figure 2, Figure 3, and Figure 4 with the different parameters. Figure 2 depicts the profile of water waves propagating under the viscosity ($\lambda=0.125$) and surfactant (T = 0.00001) effect at the value of $\theta=\pi/3$ rad. Figure 3 shows the profile of water waves propagating at the value of $\theta=\pi/4$ in the presence of viscosity ($\lambda=0.125$) and surfactant (T = 0.01). Figure 4 displays the profile of water waves propagating at the value of $\theta=\pi/3$ rad including the viscosity ($\lambda=0.125$) and surfactant (T = 0.1). In these figures, the viscosity and surfactant effect are examined with the Weber number $\kappa=\sqrt{2}$.

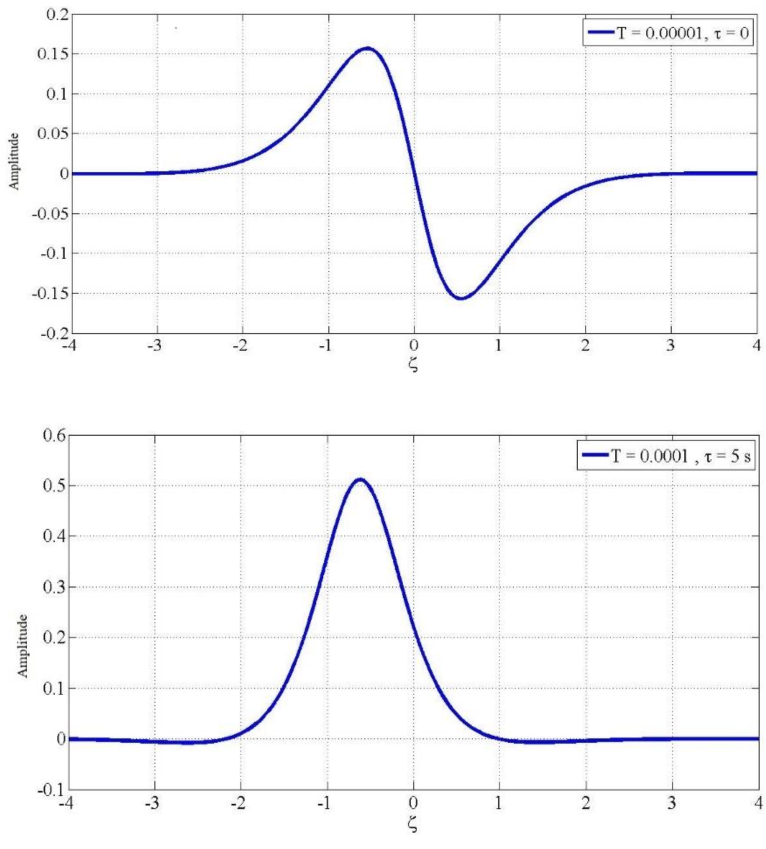

Figure 2. Profile of water waves propagating at the value of $\theta=\pi/3rad$ including the viscosity ($\lambda=0.125$) and surfactant (T = 0.00001).

However, Figure 5 stays the dark soliton and corresponds the profile of water waves propagating at the value of $\theta = \pi / 3$ rad including the viscosity ($\lambda = 0.125$) and surfactant (T = 0.01). Figure 6 displays the profile of a water waves propagating at the value of $\theta = \pi/4$ under the impact of viscosity ($\lambda = 0.5$) and surfactant (T = 0.1). It stays the Jacobi elliptic function. Furthermore, in Figure 7 the Jacobi elliptic function is stressed and can be useful in the water waves propagating at the value of $\theta = \pi/3$ rad including the viscosity ($\lambda = 0.125$) and surfactant (T = 0.01) with the Weber number $\kappa = \sqrt{2}$.

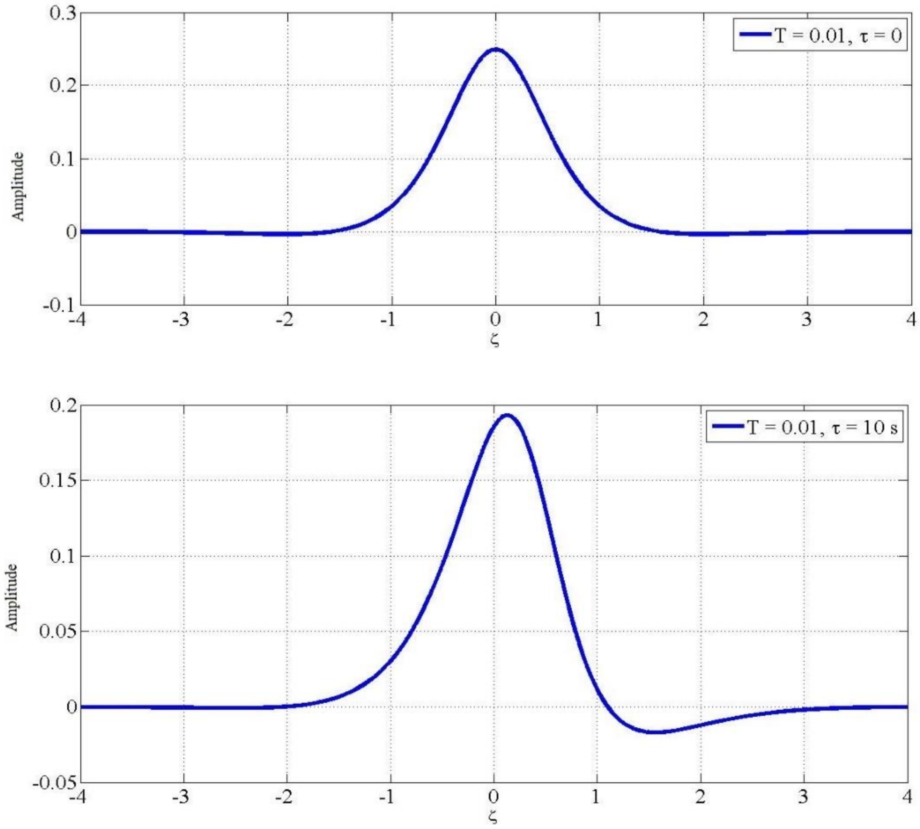

Figure 3. Profile of water waves propagating at the value of $\theta = \pi / 4 rad$ including the viscosity ($\lambda = 0.125$) and surfactant (T = 0.01).

We see the water wave propagation with surfactant and viscosity. It proves the dynamical behavior of gravity waves in the ocean including the viscosity and surfactant with the different values of the angle to the transformation and the wave direction. According to these figures, oceans' surface waves have been influenced by viscosity and surfactant.

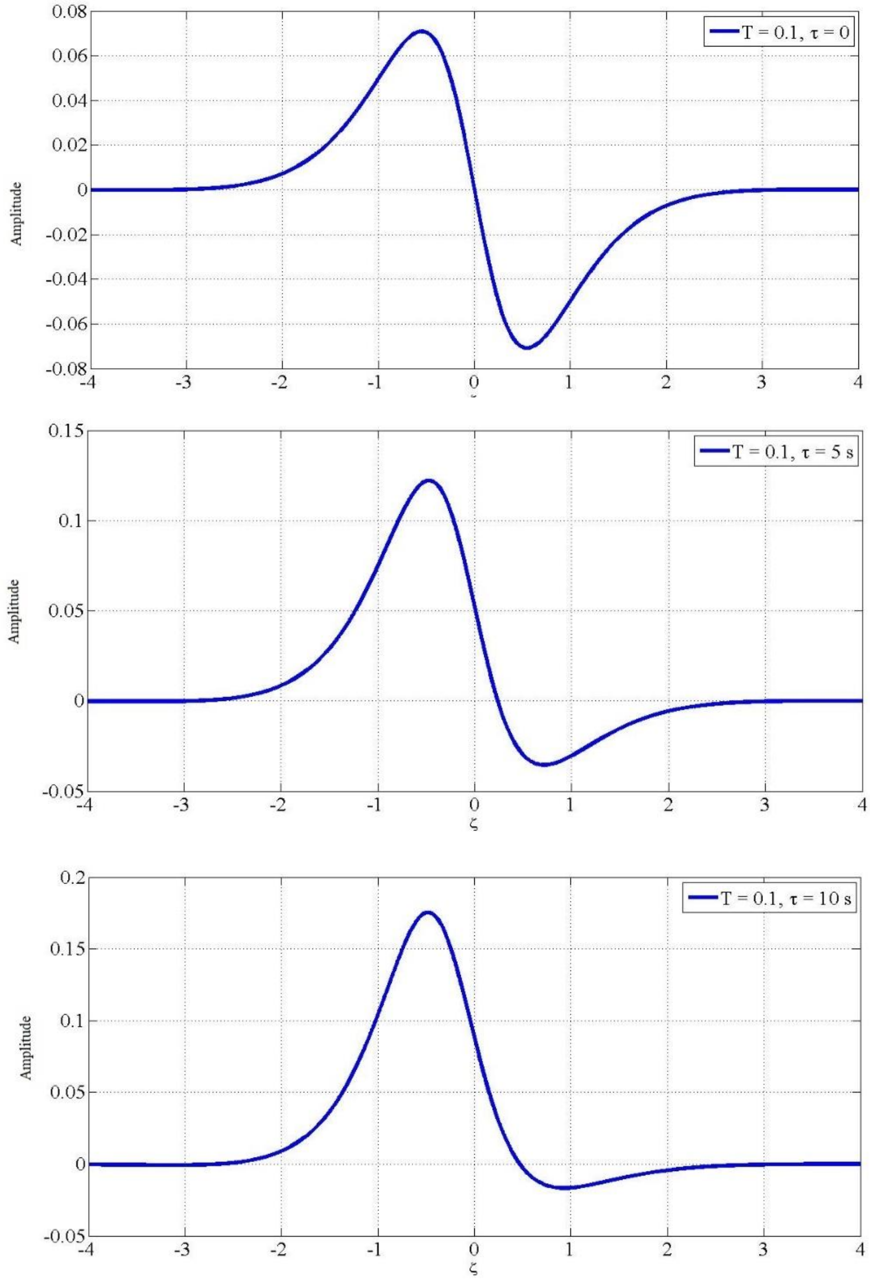

Figure 4. Profile of water waves propagating at the value of $\theta = \pi/3$ rad including the viscosity ($\lambda = 0.125$) and surfactant (T = 0.1).

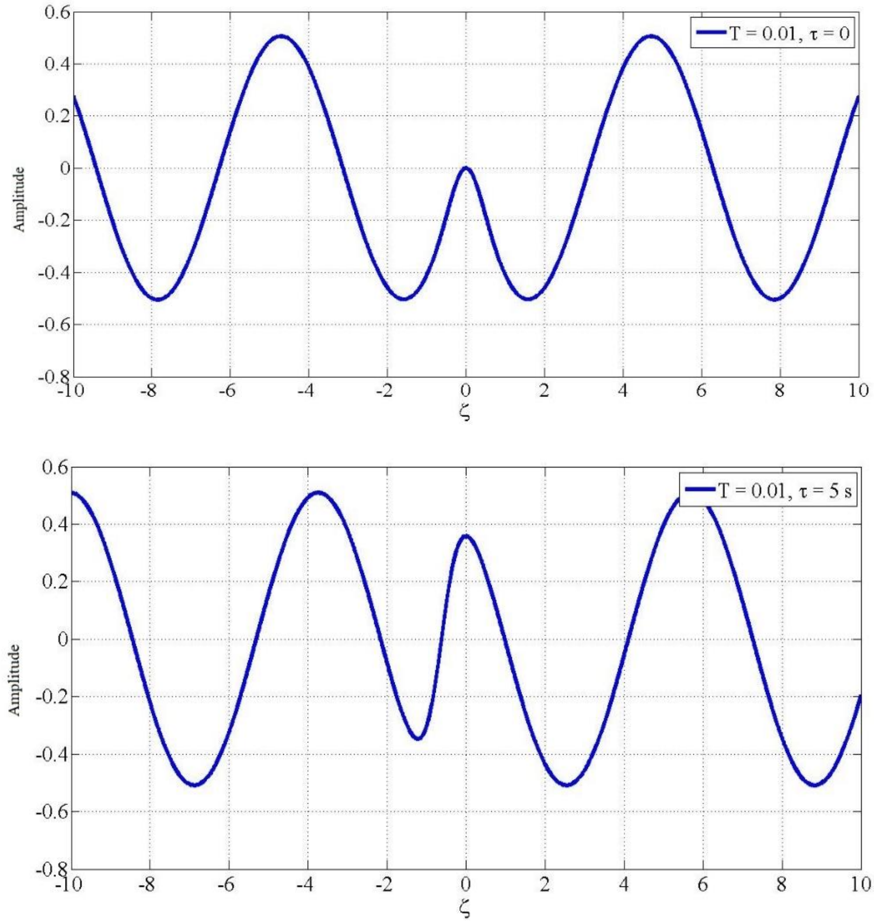

Figure 5. Profile of water waves propagating at the value of $\theta = \pi/3$ rad including the viscosity ($\lambda = 0.125$) and surfactant (T = 0.01).

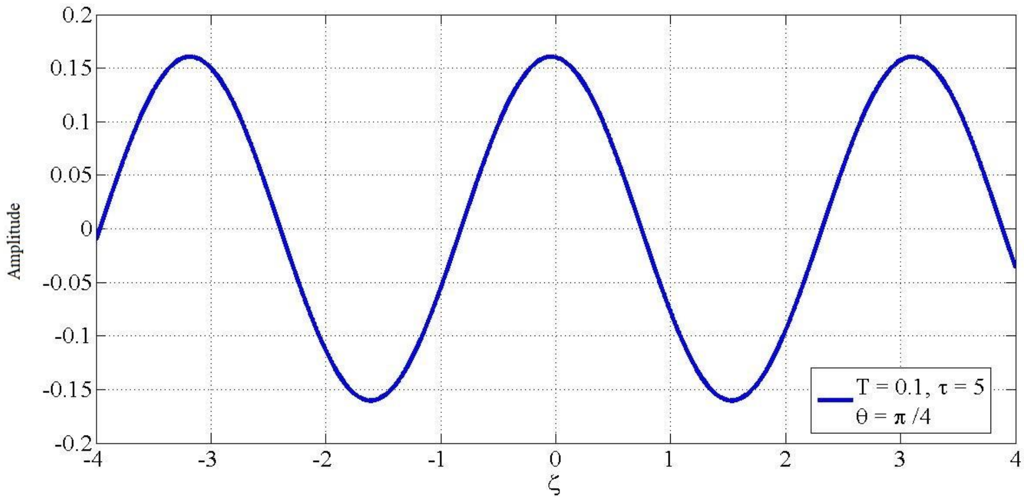

Figure 6. Profile of a water waves propagating at the value of $\theta = \pi/4$ including the viscosity ($\lambda = 0.5$) and surfactant (T = 0.1).

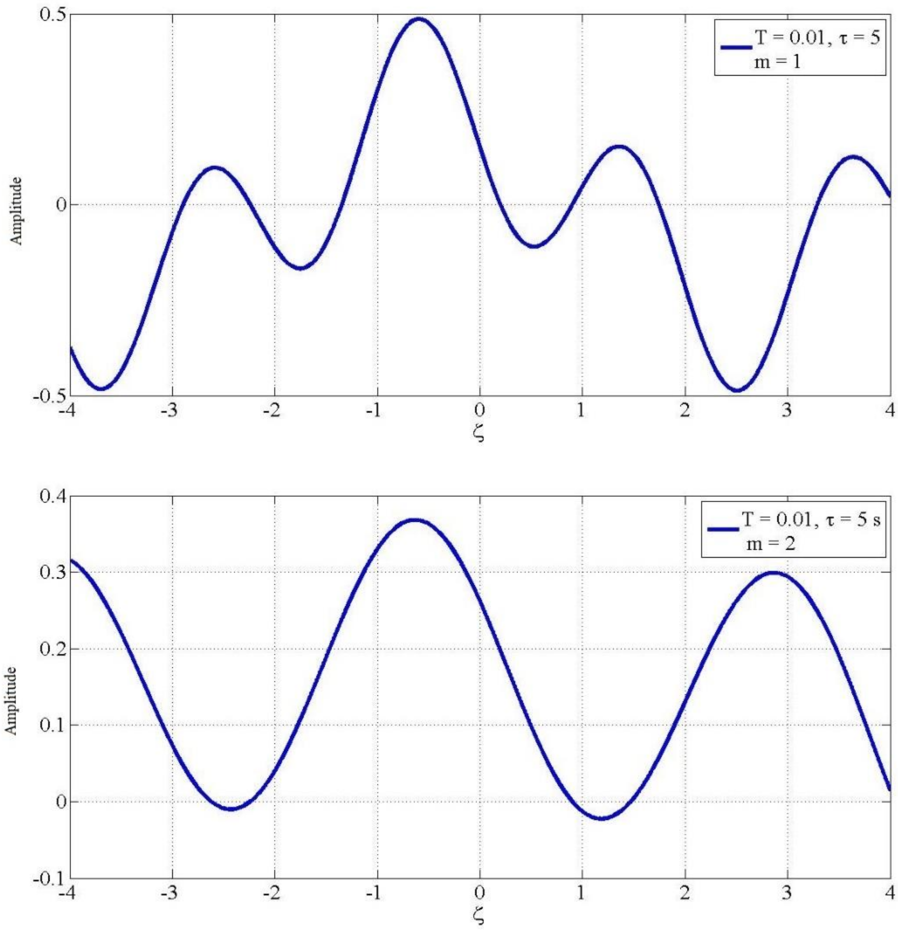

Figure 7. Profile of a water waves propagating at the value of $\theta = 0$ including the viscosity ($\lambda = 0.125$) and surfactant (T = 0.001).

5. Conclusion

In this paper, the propagation of ocean surface gravity waves including the viscosity and surfactant on infinite depth has been examined. The bifurcation theory method is effectively applied to the NLSE to construct the solutions of the propagation of two-dimensional nonlinear water waves in the deep water at the free surface in the presence of viscosity and surfactants and is used to predict the type of solution of this equation. Numerical simulations have shown a good perception of the gravity wave evolution under the action of viscosity and surfactants on the infinite depth. This study will successfully contribute to improving maritime safety and protecting human life, ships, and oil rigs. It will also help improve information on the scientifically feasible reasons for acquiring radar and lidar remote sensing data from the ocean surface. Indeed, the surfactants strongly affect the radar and lidar remote sensing images; these common surfactants, such as soaps and oils, absorb electromagnetic energy in specific regions of the spectrum; they influence the choice of wavelengths used.

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Declaration of Competing Interest

No conflict of interest was declared by the authors.

Authorship Contribution Statement

Daïka Augustin: Writing, Reviewing, Data Preparation, Editing.

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