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The Mediating Role of Metacognitive Awareness in the Relationship between Middle School Preservice Mathematics Teachers' Beliefs About the Nature of Mathematics and Mathematical Resilience

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Abstract

This research seeks to investigate how metacognitive awareness (MA) mediates the connection between beliefs about the nature of mathematics (BNM) and mathematical resilience (MR). In this sense, in accordance with the purpose of the study, the use of the cross-sectional survey model, one of the quantitative research methods, was preferred. The study involves a sample of 162 preservice mathematics teachers, and data were gathered using online forms incorporating the BNM Scale, MA Scale, and MR Scale. The mediation model analyses were performed to investigate the mediating role of MA in the correlation between BNM and MR. The findings reveal significant associations among these variables, suggesting that MA partially serves as a mediator in the relationship between BNM and MR. The study underscores the significance of taking into account individuals' beliefs about mathematics and their MA in fostering MR. In this sense, the teachers can carry out classroom practices that aim to increase students' mathematical endurance by improving their beliefs and metacognitive awareness about mathematics through various strategies.

Keywords: Metacognitive Awareness, Beliefs About the Nature of Mathematics, Mathematical Resilience, Preservice Teachers

Ortaokul Matematik Öğretmen Adaylarının Matematiğin Doğasına Yönelik İnanç ve Matematiksel Dayanıklılıkları Arasındaki İlişkide Üstbilişsel Farkındalığın Aracı Rolü

Öz

Bu araştırma, üstbilişsel farkındalığın (ÜF), matematiğin doğasına ilişkin inançlar (MDİİ) ile matematiksel dayanıklılık (MD) arasındaki bağlantıya nasıl aracılık ettiğini araştırmayı amaçlamaktadır. Bu anlamda çalışmanın amacına uygun olarak nicel araştırma yöntemlerinden kesitsel anket modelinin kullanımı tercih edilmiştir. Çalışma 162 matematik öğretmeni adayından oluşan bir örneklemi içermektedir ve veri MDİİ Ölçeği, ÜF Ölçeği ve MD Ölçeği'ni içeren çevrimiçi formlar kullanılarak toplanmıştır. MDİİ ile MD arasındaki korelasyonda ÜF'nin aracılık rolünü araştırmak için aracılık analizleri yapılmıştır. Bulgular bu değişkenler arasında önemli ilişkiler olduğunu ortaya koymuştur. Bu durum ÜF'ın MDİİ ile MD arasındaki ilişkide kısmen aracılık ettiğini göstermiştir. Çalışma, MD'ı geliştirmede bireylerin matematik hakkındaki inançları ve üstbilişsel farkındalıklarını dikkate almanın önemini ortaya koymuştur. Bu anlamda öğretmenler, çeşitli stratejiler aracılığıyla öğrencilerin matematik hakkındaki inançlarını ve üstbilişsel farkındalıklarını geliştirerek matematiksel dayanıklılıklarını artırmayı amaçlayan sınıf içi uygulamaları gerçekleştirebilir.

Anahtar kelimeler: Üstbilişsel Farkındalık, Matematiğin Doğasına Yönelik İnanç, Matematiksel Dayanıklılık, Öğretmen Adayları

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INTRODUCTION

Mathematics is commonly perceived as a challenging field in society, often associated with difficulty in achieving success. Internationally, a substantial population demonstrates a pessimistic outlook on mathematics (OECD, 2013). This negative perspective towards mathematics is observed to manifest as anxiety and avoidance in individuals (Lee & Johnston-Wilder, 2017; Peker & Mirasyedioğlu, 2003). Having a positive attitude toward mathematics is a fundamental requirement for success in mathematics classes (Johnston-Wilder & Lee, 2010). Exploring how to develop a positive attitude toward mathematics is a researched topic (Lee & Johnston-Wilder, 2017). However, possessing a positive attitude alone is insufficient; it needs to be translated into behavior. The behavioral equivalent of a positive attitude towards mathematics corresponds to an individual's MR (Lee & Johnston-Wilder, 2017). MR involves approaching mathematics with perseverance and enthusiasm to discuss, investigate, and achieve (Johnston-Wilder & Lee, 2010). This resilience, encompassing personal characteristics, plays a significant role in succeeding in mathematics (Marangoz, 2024). Another factor influencing success is how well the concepts learned are understood and applied (Kroll & Miller, 1993), in other words, how much the subject is learned. Metacognition is also associated with variables such as problem-solving skills, academic achievement, thinking ability, and intelligence, playing a significant role in achieving success in mathematics (Çelik ve Arslan, 2022; Karakelle, 2012). However, MA is considered an important variable in learning (Desoete, 2009; Desoete & Veenman, 2006). MA enables individuals to recognize their cognitive processes and employ this awareness to regulate them effectively (Flavell, 1979). In essence, MA allows individuals to be conscious of their thinking processes, learning strategies, and learning approaches. Therefore, it can be said that it is a cognitive ability that enables individuals to understand and guide their mental activities. With MA, an individual becomes aware of how they think, what learning strategies they use, and how they manage these processes when faced with mathematics. Hence, this ability may make an individual more resilient to challenges in mathematics. Therefore, this study seeks to uncover the relationship between an individual's cognitive process awareness and their capacity to tackle mathematical challenges, thereby impacting their MR.

On the other hand, the BNM held by teachers and preservice teachers also have an impact on students' development of positive or negative attitudes toward the subject. Therefore, evaluating the beliefs of preservice teachers is essential for improving mathematics education (Underhill, 1988). Considering that the beliefs held by preservice teachers guide their classroom behaviors, practices, and decisions, attention should be paid to the positive or negative effects of these beliefs on education (Brantlinger, 1996; Pajares, 1992; Steinbring, 1998). Given that the beliefs of preservice teachers in their initial years of teaching can have an impact on education (Lester & Garofalo, 1987), it is crucial to identify the variables influenced by these beliefs. This study investigates how MA mediates the connection between BNM and MR.

MA can increase MR by improving individuals' ability to evaluate their own thinking and learning processes and can help individuals develop positive BNM. This study aims to investigate the mediating role of MA in MR and BNM. Thus, it was attempted to understand the psychological and cognitive dynamics related to the mathematics teaching and learning processes of preservice teachers. In addition, determining this relationship can enable educators and researchers to develop strategies that will strengthen individuals' ability to cope with difficulties and develop a more positive attitude towards mathematics. Thus, the results of the study can constitute an important resource for increasing the effectiveness of mathematics education. In pursuit of this goal, the study addresses the following research questions:

- Does BNM positively and significantly predict MR?
- Does BNM predict MA positively and significantly?
- Does MA positively and significantly predict MR?

Does MA have a significant mediating role in the relationship between BNM and MR

In this sense, the concepts of MA, BNM and MR, which form the study's conceptual framework, are discussed in more detail below.

Conceptual Framework

Metacognitive Awareness (MA)

Flavell (1987) defined the concept of metacognition as being aware of one's cognitive processes, and monitoring and regulating these cognitive processes. If an individual perceives that they are struggling more in situation A compared to situation B, believes that they need to recheck and reconsider option C without immediately accepting it, or thinks that they should carefully examine each option when faced with choices, these are examples of metacognitive processes (Flavell, 1976).

Metacognitive knowledge is described as an individual's general understanding and knowledge about their cognition. Being aware of one's capabilities, knowing the competencies required in the process, and all beliefs and knowledge in one's mind are associated with metacognitive knowledge (Özsoy, 2007). An individual with metacognitive abilities is expected to be aware of their mental activities, know what tasks to perform in the learning process, understand which learning methods work best for them, plan strategies for success in tasks, effectively use strategies, monitor the learning situation, and comprehend how much knowledge they have acquired. The feature of metacognition enhancing an individual's success (Pintrich, 2004) can also serve teachers' goal to increase student achievement. The clear MA of teachers and learners increases the likelihood of students developing these skills (Schunk, 1989). Additionally, metacognitive awareness plays a crucial role in the problem-solving process. Research indicates that metacognition facilitates students’ success in solving mathematical problems (Schoenfeld, 2016). Individuals with metacognitive awareness are better equipped to manage their own learning processes and develop more systematic approaches to complex problems (Pintrich, 2002). Furthermore, metacognition serves as an intrinsic motivator that enhances the smooth progression of mathematical cognitive activities (Tay et al., 2024). Therefore, examining the levels of MA and the development of preservice teachers in this process is important.

Beliefs about the Nature of Mathematics (BNM)

According to Thompson (1992), beliefs hold considerable sway over the learning and teaching dynamics within mathematics. Belief is defined as an understanding, proposition, and assumption that an individual accepts as true (Philipp, 2007). Additionally, Thompson (1992) has considered belief equal to meaning, concept, rule, proposition, and mental images. The common understanding about belief is that it is influenced by experiences (Ponte, 1994), varies from person to person (Philipp, 2007), is based on cognitive and affective foundations (Pehkonen, 2004). Mathematical belief, on the other hand, is defined as the reflection of experiences related to mathematical concepts (Silver, 1985; Thompson, 1982) and the assessments made as a result of these experiences (Raymond, 1997).

Preservice teachers’ BNM is divided into two categories: seeing mathematics as a set of rules and procedures or viewing it as a process of exploration and discovery (Tatto et al., 2008). Preservice teachers who perceive mathematics as a set of rules and procedures are thought to adopt a traditional approach of merely transmitting knowledge in their classes (Ernest, 1989). Moreover, those who see mathematics as a process of exploration and discovery are expected to create various environments for their students to engage in mathematical activities (Baş et al., 2015), with the belief that such environments enhance success (Ernest, 1989; Thompson, 1992). Since teachers' beliefs influence classroom practices and these practices impact student achievement (Dede & Karakuş, 2014), the forms of BNM are considered significant for teacher competence (Gess-Newsome, 2015).

Mathematical Resilience (MR)

In a general sense, resilience refers to personal qualities that lead to success despite encountering challenging situations (Masten, 2001). MR, on the other hand, is defined as the attitude individuals display towards learning mathematics despite facing difficulties in the learning process (Johnston-Wilder and Lee, 2010). Kooken et al. (2016) have identified three components of MR: value, struggle, and growth. The value component is expressed by how individuals perceive the importance of mathematics in achieving their goals, while the struggle component refers to the effort individuals exert in the face of challenges encountered in learning mathematics. The growth component represents individuals' self-belief contributing to their knowledge accumulation in mathematics. The characteristics of resilient individuals have been highlighted by various researchers. According to Benard (1996), resilient individuals possess social skills, independence, a sense of belonging to their community, and a positive outlook on the future. Resilient individuals are open to innovations (Rak & Patterson, 1996), believe in overcoming potential difficulties in the future (Bland et al., 1994), explain success or failure with internal reasons, increase their efforts, and have resilient personality traits (Masten, 2001). Furthermore, resilient individuals are characterized by their ability to develop solutions to problems (Jew et al., 1999), awareness of emotions, thoughts, and behaviors, initiative (Vazquez, 2000), problem-solving skills (Masten, 2001), a sense of humor (Vance & Sanchez, 1998; Vazquez, 2000), and optimism (Rak & Patterson, 1996).

Resilience, although rooted in individual personality traits, can be negatively affected by certain factors in MR. Intensive schedules and exams, the abstract and independent processing of mathematics from daily life, and teachers' authoritarian and unsympathetic attitudes increase students' anxiety levels. As the anxiety level rises, students' psychological resistance and MR decrease (Johnston-Wilder & Lee, 2010). Lee and Johnston-Wilder (2014) have stated that MR can be developed. In mathematics education, behaviors that nourish MR include motivating students to succeed in the face of challenges, exposing them to problem situations that require critical thinking, and assigning responsibilities that students can overcome (Van de Walle, 2013). Individuals who are resilient in the face of challenges in mathematics class have the motivation to continue working, and exhibit MR have a higher likelihood of success. Having this characteristic is a significant factor in enhancing mathematical achievement (Kooken et al., 2013). The abstract nature of mathematical concepts, requiring more effort in learning, emphasizes the importance of MR (Pekdemir, 2019).

It is a long established fact that a reader will be distracted by the readable content of a page when looking at its layout. The point of using Lorem Ipsum is that it has a more-or-less normal distribution of letters, as opposed to using 'Content here, content here', making it look like readable English. Many desktop publishing packages and web page editors now use Lorem Ipsum as their default model text, and a search for 'lorem ipsum' will uncover many web sites still in their infancy. Various versions have evolved over the years, sometimes by accident, sometimes on purpose.

METHOD

Research Design

In the data collection phase of this research, a quantitative research approach was adopted, utilizing the survey model. Specifically, the chosen survey model is the cross-sectional survey, wherein data is gathered from the sample at a single point in time, as outlined by Fraenkel, Wallen, and Hyun (2012).

Study Group

This study's sample comprised 162 preservice mathematics teachers who were registered in the Mathematics Teaching Department of the Faculty of Education at a university situated in the western region of Turkey during the spring semester of the academic year 2021-2022. The sample distribution consisted of 124 female and 38 male participants, encompassing 1st grade students (f=48), 2nd grade students (f=44), 3rd grade students (f=41), and 4th grade students (f=29) specializing in middle school mathematics education. Purposeful sampling was employed, specifically the convenient situation sampling method, to ensure meaningful results in similar situations. The researcher chose participants who were easily accessible for the sake of expediency and practicality in the sampling process, which is characteristic of the convenient situation sampling method (Merriam, 2009).

Data Collection Tools and Process

The study utilized three separate scales, namely the "MR Scale," "MA Scale," and "BNM Scale," as tools for gathering data. For these scales, necessary permissions were obtained via email from the researchers who developed or adapted the scale.

MA Inventory: The MA Inventory, adapted into Turkish by Akın et al. (2007), was originally formulated by Schraw and Dennison (1994). Comprising a total of 52 items, the inventory is structured into eight subscales. Respondents utilized a 5-point Likert scale to rate statements, ranging from "Always (5)" to "Never (1)." Scored on a 5-point Likert scale, the inventory generates scores spanning from 52 to 260, with higher scores indicative of a heightened level of MA. Devoid of negative items, the scale's elevated scores signify a greater degree of MA. To assess the internal consistency of the MA Inventory items in the present study, Cronbach's alpha was computed, yielding a highly reliable test score with $\alpha = 0.953$. Following this criterion, the measurements derived from items related to the MA Inventory concerning attitudes toward mathematics are deemed highly reliable.

BNM Scale: The Scale for BNM, crafted by Akyıldız and Dede (2019), adopts a format based on a 5-point Likert scale. This instrument assigns the following ratings to statements: "Strongly Agree (5)," "Agree (4)," "Undecided (3)," "Disagree (2)," and "Strongly Disagree (1)." Elevated scores on this scale signify reinforced convictions regarding the nature of mathematics. Comprising 41 items, the scale unfolds into two distinct dimensions: "Related Belief" and "Separated Belief." In the realm of Related Belief, perspectives focus on progress-oriented mathematics and utility-oriented mathematics. Meanwhile, Separated Belief encapsulates viewpoints centered on instrument-focused mathematics and goal-focused mathematics. Progress-focused belief involves the generation of knowledge, the exploration of mathematics, and its interrelation with other scientific disciplines. Utility-focused belief perceives mathematics as a tool employed to address daily life needs.

Instrument-focused belief regards mathematics as a disconnected set of concepts, formulas, operations, and rules. Goal-focused belief assesses mathematics as an abstract, deductive science containing precise information (Akyıldız & Dede, 2019). For the present study, the reliability of the scale was computed as .877 to scrutinize the internal consistency of items within the BNM Scale, affirming the reliability of the test scores.

MR Scale: Gürefe and Akçakın (2018) adapted the MR Scale, originally developed by Kooken et al. (2016), for use in Turkish. This scale comprises twenty-four items, which are divided into three factors: "Value," "Struggle," and "Development." The "Value" factor consists of eight items, the "Struggle" factor includes six items, and the "Development" factor encompasses five items. Respondents rate their agreement or disagreement with these items on a 7-point Likert scale, ranging from "Strongly Agree" to "Strongly Disagree." Notably, six items are phrased negatively, and their scores are reversed during scoring. The total score on the scale ranges from 19 to 133, with higher scores indicating greater levels of MR. To assess the internal consistency of the scale items, Cronbach's alpha coefficient was calculated resulting in a reliable test score with $\alpha = 0.746$. According to established reliability standards, a Cronbach's alpha coefficient surpassing 0.70 signifies sufficient reliability, while a coefficient surpassing 0.90 indicates exceptionally high reliability (Şeker & Gençdoğan, 2006). Consequently, the metrics derived from items related to the MR Scale, specifically those about attitudes toward mathematics, can be deemed trustworthy.

The data for the study were collected during the spring semester of the academic year 2021-2022. The scales utilized in the research were developed using the Google Forms platform and were administered to preservice teachers within a classroom environment. The online forms automatically collect data and can often provide instant analysis. This can help researchers save time and evaluate results more quickly. In addition, since printing and distributing physical forms is costly, online forms are more economical by reducing these costs. For these reasons, the use of online forms has been preferred. The researcher sent the forms to the email addresses of the preservice teachers and requested them to fill them out. During this process, the researcher informed the preservice teachers about the voluntary basis of participation and the confidentiality of their responses. The preservice teachers were given one class hour to respond to the items in the scale. It was assumed that the preservice teachers answered the items in the scale accurately and sincerely.

Data Analysis

Before analyzing the data set, the completion of missing data and the normality and linearity values of the variables were examined (Tabachnick & Fidell, 2013). It was determined that there was no incorrect or missing data in the data set. For normality check, the kurtosis and skewness coefficients were initially calculated. These coefficients being between -1.5 and +1.5 indicate that the data have a normal distribution (Tabachnick & Fidell, 2013). When the linearity assumption was examined through the scatter plots of the variables constituting the model separately, it was determined that the linearity assumption was met. Subsequently, the Pearson Product-Moment Correlation Analysis was performed to detect the relationships between variables. After identifying a significant relationship between variables, a mediation model was constructed, and for the mediation model analysis, Model 4 of the Process v4.2 extension in IBM SPSS 26.0 developed by Hayes (2017) was utilized. To ensure the reliability of the study, Cronbach's alpha reliability coefficients for each scale were recalculated, and it was determined that the scales were reliable.

FINDINGS

Descriptive Statistics and Relationships Between Variables

To determine the relationships between middle school mathematics preservice teachers' BNM, MA, and MR, the Pearson Correlation Coefficient was calculated (see Table 1).

Table 1. Descriptive statistics and Pearson's correlations between the study variables 1

Variables	M	SD	1	2	3	Kurtosis	Skewness
(1) BNM	160.64	14.184	-	.532**	.231**	1.152	.392
(2) MA	184.34	26.019	.532**	-	.279**	.415	.119
(3) MR	112.30	8.671	.231**	.279**	-	1.098	-.668

Before delving into relationships between variables, it is necessary to test the assumption of normality for the collected data. In this context, the skewness and kurtosis coefficients of the scales have been examined. The results showed that the skewness values of the variables ranged from -0.668 to 0.392, and their kurtosis values ranged from 0.415 to 1.152. When the ratio of skewness and kurtosis values to their own standard errors is examined, being close to 0 within the ±1.5 limits is interpreted as another indicator of normal distribution (Tabachnick & Fidell, 2013). In this context, it has been determined that the dataset meets the assumption of normal distribution.

According to Evans (1996), correlation coefficients are grouped into categories. These classifications help interpret the strength of the relationship between variables. Specifically, a coefficient ranging from .00 to .19 is considered very weak, .20 to .39 signifies a weak relationship, and .40 to .59 indicates a moderate relationship. The study employed the Pearson correlation test, and the outcomes are presented in Table 2. This statistical analysis reveals the relationships among pre-service teachers' MR, MA, and BNM. MR and MA: the Pearson correlation coefficient ($r = .279$, $p < .05$) suggests a positive but weak and statistically significant relationship between pre-service teachers' MR and MA. This implies that as MR increases, there is a weak yet meaningful increase in MA among pre-service teachers. MR and BNM: The analysis also shows a positive and weak significant relationship ($r = .231$, $p < .05$) between MR and BNM. This indicates that individuals with higher levels of MR tend to hold positive BNM. MA and BNM: Lastly, there is a positive and moderately significant relationship ($r = .532$, $p < .05$) between pre-service teachers' MA and their BNM. This suggests a stronger correlation between MA and positive BNM. In summary, these correlation results provide valuable insights into the interconnections among MR, MA, and BNM among pre-service teachers.

The results of mediator analysis

The potential mediation of MA in the influence of BNM on MR was examined utilizing the Process macro, as outlined by Hayes (2017). To ascertain this mediating effect, the bootstrapping technique advocated by Preacher and Hayes (2004) was applied. In this context, a regression analysis was conducted. This method has been suggested by various authors to yield more reliable results than the traditional causal steps method by Baron and Kenny (1986) and the Sobel test (Hayes, 2017). The analyses utilized the bootstrap resampling option, and 5,000 resamples were taken for the analysis. In analyses conducted using the bootstrap technique, support for mediating effect hypotheses is only possible if the confidence interval's lower and upper bounds do not include zero (0) within the 95% confidence interval (MacKinnon et al., 2004).

In the model developed within the framework of the research, we aimed to discern both the direct and indirect impacts of MA on the relationship between BNM and MR. BNM positively and significantly predicted both MA ($ß=0.97$, $t=7.93$) and MR ($ß=0.14$, $t=2.99$). Moreover, MA also positively and significantly predicted MR ($ß=0.07$, $t=2.44$). Interpreting the ß values, it is observed that in the absence of a mediator variable, MR increases by 0.14 standard deviations when BNM increases by one standard deviation. This effect was deemed statistically significant ($p < .01$). Additionally, it was identified that MR increases by 0.07 when MA increases by one standard deviation. After incorporating the mediator variable (MA) into the model, the regression coefficient between BNM and MR was calculated as 0.07, which is lower than in the base model. This reduction was found to be significant ($p < 0.001$), indicating that MA serves as a partial mediator in the relationship between BNM and MR. Confidence intervals for the indirect effect were calculated based on a sample size of 5000 individuals (see Table 2).

Table 2. Bootstrapping results for the mediation model

Pattern paths	Bootstrap values Coefficient	SE	t	95% CI Lower	Upper	K^2
Model without mediator						
Total effect BNM → MR	.1410*	.0470	2.998	.0481	.2339	.1160
Model with MA as the mediator						
Direct effect BNM → MR	.0701	.0547	1.281	-.0379	.1781	
Indirect effect of CT BAMN → MA → MR	.0709*	.0356		.0068	.1448	

*p<.05, p>.05 K^2=Fully standardized size of indirect effect (mediation effect)

Upon examining the table, it is evident that the indirect effect, signifying the mediating role of MA in the relationship between BNM and MR, is 0.709. Statistically, when analyzing the indirect effect within the 95% bootstrap confidence interval, it was found to be different from zero and above zero (between BootLLCI=.0068 and BootULCI=.1448), indicating statistical significance. This outcome affirms that MA is a mediating variable in the relationship between BNM and MR. The significance holds true in both the research and bootstrapping samples (5000) (MacKinnon et al., 2004).

In mediation models, another pivotal parameter is the fully standardized size of the indirect effect, denoted as $K^2 = 0.1160$. In interpreting effect sizes, evaluations are categorized as low effect if K^2 is close to .01, medium effect if K^2 is close to .09, and high effect if K^2 is close to .25 (Preacher & Kelley, 2011). This effect size of $K^2 = 0.11$ is considered medium according to the criteria set by Preacher and Kelley (2011). Therefore, the analysis leads to the conclusion that MA serves as a substantial and statistically significant mediator in the correlation between BNM and MR.

DISCUSSION & CONCLUSION

The research indicates that preservice teachers' BNM significantly predicts MR. So, how an individual thinks about mathematics, and their beliefs in this regard, can help predict how resilient they might be when confronted with mathematics. This statement emphasizes that beliefs about mathematics are a determinant that influences an individual's ability to cope with mathematical challenges. Belief, defined as the understanding, proposition, and assumption accepted as true by an individual (Philipp, 2007), and mathematical belief, the norms created through one's experiences with mathematics (Raymond, 1997), play a crucial role in mathematics (Thompson, 1992). MR, on the other hand, is the attitude individuals exhibit towards learning despite facing difficulties (Johnston-Wilder and Lee, 2010). Therefore, it can be said that students' value of mathematics and their ability to cope with challenges in mathematics are positively influenced by their belief in mathematics. Similarly, students who hold negative beliefs about mathematics are generally prone to avoiding mathematical subjects, and thus, it can be said that their MR is low. Individuals resilient in mathematics possess characteristics that enable them to explain success or failure through internal reasons, putting in more effort to increase their achievements (Masten, 2001). Marangoz (2024) supported the finding in her study, in which she determined that students with high endurance in mathematics were more successful. MR also allows individuals to overcome existing negative attitudes towards mathematics by changing their mindsets (Lee & Johnston-Wilder, 2010). Thus, nurturing resilient individuals in mathematics can contribute to their overall success. Additionally, belief not only supports learning but also contributes to the learning process (McLeod & McLeod, 2002; Palsdottir, 2007). It significantly influences students' self-assessment of their abilities, willingness to engage in mathematical activities, and attitudes toward mathematics (NCTM, 1989). Therefore, it can be said that increasing belief in mathematics would be beneficial in enhancing students' resilience and contributing to them in multiple aspects.

Another finding obtained through the research is that MA significantly influences MR. As MA involves being aware of one's knowledge, actions, and abilities (Özsoy, 2007), it can be said that individuals who are aware of themselves are more successful in coping with challenges in mathematics. MA enables individuals to know and internalize their strengths and weaknesses. An individual aware of their weaknesses can make the necessary efforts to strengthen those aspects, which can positively impact their struggle against challenges. Indeed, individuals with MA may find it easier to cope with challenges in mathematics. In this case, the positive contribution of high levels of MA to mathematical, resilience becomes inevitable. Individuals with metacognitive awareness may possess deeper understanding and problem-solving skills in mathematics. By monitoring their thought processes, they can develop various strategies for addressing problems and identify appropriate solutions. Consequently, it can be said that individuals with metacognitive awareness can employ more effective strategies, focus their attention better, and enhance their problem-solving abilities while tackling mathematical problems. Indeed, these individuals exhibit greater resilience against difficulties in mathematics. This situation can undoubtedly lead to an improvement in students' mathematical achievements. Furthermore, MA allows individuals to be aware of the thinking processes they use in planned learning and problem-solving environments and to regulate this process (Brown, 1978). Individuals with metacognitive awareness can manage their mathematical learning processes more effectively, being more conscious when selecting, applying, and evaluating learning strategies.

The research reveals an additional insight, indicating that MA plays a significant mediating role in the correlation between the beliefs held by preservice teachers regarding the nature of mathematics and their level of MR. To put it differently, individuals who maintain positive beliefs about mathematics demonstrate a slightly significant impact on their MA, contributing to their ability to be resilient when confronted with mathematical challenges. The framework of MR encompasses key elements such as a developing mindset, valuing the subject, dealing with difficulty, nurturing curiosity, demonstrating perseverance, and seeking support (Lee & Johnston-Wilder, 2017). From this perspective, it can be inferred that individuals with MR tend to exhibit a greater curiosity about and value for mathematics. Undoubtedly, a belief in the intrinsic value of mathematics becomes a necessary foundation for truly valuing the subject. In light of this, the notion that BNM could indeed influence one's resilience becomes not only conceivable but also plausible. As a matter of fact, Morkoyunlu and Saltık Ayhanöz (2023) showed in their study that there is a significant relationship between belief in mathematics and mathematical endurance.

Individuals characterized by high levels of MR demonstrate a noteworthy ability to persist in learning despite encountering challenges, as highlighted by Johnston-Wilder (2013). This persistence not only fortifies their daily lives but also contributes positively to their professional careers, as emphasized in the findings by Ariyanto et al. (2017). Additionally, research indicates that resilient individuals, in general, tend to possess elevated self-esteem, effectively navigate through problems (Jew et al., 1999), and exhibit proficient problem-solving skills (Masten, 2001). Efficient educational practices require individuals who are socially and psychologically well-rounded. Recognizing and cultivating variables that can impact individuals' resilience becomes pivotal for achieving this goal. Emotional characteristics, such as resilience and belief, are regarded as significant predictors of the development of cognitive activities and overall success (Ma & Kishor, 1997). As demonstrated in this study, the cultivation of positive beliefs about mathematics and the enhancement of MA emerge as crucial factors in augmenting MR. This underscores the importance of fostering a supportive educational environment that nurtures not only mathematical skills but also the emotional and cognitive attributes contributing to resilience. Group work and interactive activities, using various teaching methods that can appeal to different learning styles, can help students understand mathematical concepts better, which can contribute to the development of their positive feelings and attitudes towards mathematics.

Suggestions

This study emphasizes the mediating role of MA in the relationship between BNM and mathematical endurance to understand the connection between them. That is, MA is thought to be an intermediate factor in understanding how mathematics-related beliefs affect mathematical endurance. In this context, a person's beliefs about mathematics may reflect how that person behaves when encountering mathematics and the effects of this on mathematical endurance. MA refers to the individual's ability to be aware of his or her thinking processes, strategies, and learning approaches. Therefore, the mediating role of MA in the relationship between mathematical endurance and mathematical beliefs emerges as an important factor in understanding and explaining this relationship. Teachers can implement classroom practices aimed at developing students' beliefs about mathematics through various strategies. These include recognizing and celebrating even small successes, relating mathematical concepts to real-life scenarios, and enhancing engagement by incorporating mathematical games and interactive activities. Additionally, teachers can encourage students to reflect on their thought processes in a manner that supports mathematical achievement by asking questions such as, “How did you solve this problem?” or “What strategies did you use?” Allowing students time to evaluate their learning processes is also essential. Furthermore, fostering collaboration through group work enables students to share their thought processes and learn from one another. In this context; teachers should assess students' beliefs about mathematics and develop strategies to reinforce positive beliefs while understanding and addressing negative ones. This can positively impact students' attitudes towards mathematics. In addition to this, teachers should understand students' differences and acknowledge that each student's level of MA is different. It is essential to support students in setting personal goals and developing their learning strategies. Besides, MA is crucial for problem-solving skills. For example, metacognitive awareness assists individuals in determining which problem-solving strategies to employ. By recognizing their own thinking processes, individuals can select the most appropriate strategies. Furthermore, difficulties encountered during problem-solving can induce anxiety and stress. Metacognitive awareness enables individuals to identify and manage these emotions, allowing them to focus on the problem more effectively. Teachers can enhance MA by providing opportunities for students to explore and use different problem-solving strategies.

Limitations

The study was conducted with preservice teachers studying in the mathematics education department of a state university. To further investigate the effects of the scales addressed in the study, it would be beneficial to increase the number of participants by also applying them to preservice teachers studying at different universities.

Statements of Publication Ethics

The research obtained approval by the Uşak University Social and Humanities Ethics Committee the university ethics committee for research purposes. Meeting date: 12.05.2022.

Researchers’ Contribution Rate

Authors	Literature review	Method	Data Collection	Data Analysis	Results	Conclusion
Author 1	☒	☒	☒	☒	☒	☒
Author 2	☒	☒	☒	☐	☒	☒

Conflict of Interest

We confirm that there are no conflicts of interest associated with this research.

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