Erzincan Üniversitesi
Fen Bilimleri Enstitüsü Dergisi
2025, 18(1), 115-128
ISSN: 1307-9085, e-ISSN: 2149-4584
Araştırma Makalesi

Erzincan University
Journal of Science and Technology
2025, 18(1), 115-128
DOI: 10.18185/erzifbed.1514507
Research Article

A Maple program to the Analysis of Equilibrium Points in Social Media Addiction Model

Mehmet KOCABIYIK[1*]

[1]Department of Mathematics, Burdur Mehmet Akif Ersoy University, Burdur, Türkiye

Received: 11/07/2024, **Revised:** 24/12/2024, **Accepted:** 07/01/2025, **Published:** 28/03/2025

Abstract

In today's world, the relationship between social media and the internet is becoming increasingly important. Therefore, there is a need to determine the level of social media addiction. In addition to expressing social media addiction mathematically, equilibrium point analyses of such equation systems also illuminate the extent and impact of addiction. This study focuses on such a model. Discretization of the model was achieved using the nonstandard finite difference method. Equilibrium points were identified and analyzed using the Maple software package. The findings reveal critical thresholds that distinguish between balanced and excessive use, providing insights into the progression and potential control of social media addiction. Additionally, Maple working codes are presented, allowing for the replication and extension of the study by other researchers. These codes not only verify the accuracy of the mathematical model but also offer a practical tool for further exploration in this field, contributing significantly to the literature.

Keywords: Maple package programming, Equilibrium point, Social media addiction model, Numerical Analysis.

Sosyal Medya Bağımlılığı Modelinde Denge Noktalarının Analizine Yönelik Bir Maple Programı

Öz

Günümüz dünyasında sosyal medya ve internet arasındaki ilişki giderek önem kazanmaktadır. Bu nedenle sosyal medya bağımlılığının düzeyinin belirlenmesine ihtiyaç duyulmaktadır. Sosyal medya bağımlılığını matematiksel olarak ifade etmenin yanı sıra, bu denklem sistemlerinin denge noktası analizleri de bağımlılığın kapsamını ve etkisini aydınlatmaktadır. Bu çalışma böyle bir modele odaklanmaktadır. Modelin ayrıklaştırılması standart olmayan sonlu farklar yöntemi kullanılarak gerçekleştirilmiştir. Denge noktaları Maple yazılım paketi kullanılarak belirlenmiş ve analiz edilmiştir. Bulgular, dengeli ve aşırı kullanım arasında ayrım yapan kritik eşikleri ortaya koyarak sosyal medya bağımlılığının ilerlemesi ve potansiyel kontrolü hakkında içgörüler sağlıyor. Ek olarak, Maple çalışma kodları sunularak çalışmanın diğer araştırmacılar tarafından tekrarlanması ve genişletilmesi sağlanıyor. Bu kodlar yalnızca matematiksel modelin doğruluğunu doğrulamakla kalmıyor, aynı zamanda bu alanda daha fazla araştırma yapmak için pratik bir araç sunarak literatüre önemli katkı sağlıyor.

Anahtar Kelimeler: Maple paket programı, Denge noktası, Sosyal medya bağımlılık modeli, Nümerik analiz.

*Corresponding Author: mkocabiyik@mehmetakif.edu.tr
Mehmet KOCABIYIK, https://orcid.org/0000-0002-7701-6946

1. Introduction

The burgeoning prevalence of social media platforms has precipitated an urgent need to understand the dynamics of social media addiction through mathematical modeling. These models offer a quantitative framework to elucidate the complex interactions between users and the addictive features of social media. By employing differential equations and other mathematical tools, researchers can be simulating user behavior over time, identify critical factors influencing addiction, and predict long-term trends. Such models are crucial for developing effective interventions and informing policy decisions aimed at mitigating the adverse effects of social media addiction [1-2].

Social media addiction often stems from psychological and social factors that influence an individual's behaviour and interaction patterns. One of the main causes is the dopamine-driven reward system activated by social media use, where likes, comments and notifications provide instant gratification. This creates a cycle of dependency that reinforces the need for constant engagement. In addition, feelings of loneliness, low self-esteem or anxiety can lead individuals to seek validation and connection through social media platforms. The design of these platforms, with algorithms designed to maximise user engagement, exacerbates the problem by encouraging prolonged use. Peer pressure and fear of missing out are also significant contributors, especially among younger demographics who may feel compelled to stay connected to avoid social exclusion.

The effects of social media addiction can manifest themselves in different aspects of an individual's life. Psychologically, it is associated with increased levels of anxiety, depression and stress, often due to unrealistic comparisons with others and cyberbullying. Academically and professionally, excessive social media use can lead to reduced productivity, poor concentration and procrastination. Physically, prolonged screen time contributes to problems such as disrupted sleep patterns, eye strain and a sedentary lifestyle, increasing the risk of obesity and other health problems. Socially, it can weaken real-life relationships as individuals prioritise virtual interactions over face-to-face communication. In extreme cases, social media addiction can lead to significant behavioural changes, such as withdrawal symptoms when access to platforms is restricted, highlighting the severity of its impact on overall well-being.

There are diverse approaches to modeling social media addiction, ranging from deterministic models to stochastic processes. For instance, deterministic models often utilize systems of differential equations to represent the rate of change in user engagement, while stochastic models may incorporate random variables to account for unpredictable user behavior.

Studies have demonstrated that these mathematical representations can capture the salient features of addiction, such as escalation, habituation, and withdrawal symptoms [3-4]. Furthermore, the integration of network theory has enhanced the ability to model the spread of addictive behavior through social networks, providing insights into how peer influence and social reinforcement contribute to addiction dynamics [5].

Significant studies have been conducted on mathematical modeling in the field of understanding social media addiction. For instance, Ishaku et al. employed a compartmental model to examine the impact of social media on the academic performance of students in higher education. The model identifies two equilibria, Media Free Equilibrium (MFE) and Media Addiction Equilibrium Points and conducts stability analysis based on the basic reproduction number to demonstrate the local stability of MFE if greater than unity, otherwise unstable. Numerical simulations confirm significant effects of social media on academic performance, validating the local stability analysis of MFE [6]. Simsek et al. revealed a moderate level of social media addiction among both university and high school students. Significant differences were identified based on factors such as gender, duration of use, university department, and type of high school [7]. Shutaywi et al. develop a mathematical model using fractional calculus to understand the transmission dynamics of social media addiction (SMA) and its detrimental effects. The research identifies equilibrium points, computes the reproduction parameter R_0 to analyze SMA spread dynamics, and performs stability analysis with various input parameters to explore effective control strategies [8].

In their paper, Guo and Li developed a two-stage mathematical model to investigate the dynamic characteristics of online gaming addiction. They determined the equilibrium points. They demonstrated global asymptotic stability of both the disease-free equilibrium and the endemic equilibrium. They also used Runge-Kutta methods as numerical solutions [9]. Alemneh and Alemu utilized optimal control analysis in their mathematical modeling to identify effective strategies for reducing social media addiction, highlighting the importance of tailored interventions [10]. Finally, Savci et al. [11] and Çiftçi and Yıldız [12] integrated machine learning techniques to predict future trends in social media addiction.

On the other hand, identifying equilibrium points in these mathematical models is of paramount importance, as they represent stable states where the system can maintain itself without external intervention. Equilibrium point analysis helps in understanding the conditions under which social media usage stabilizes, escalates into addiction, or diminishes. By determining these points, researchers can ascertain the thresholds for addiction and devise strategies to prevent users from crossing these critical boundaries.

The stability of equilibrium points also sheds light on the resilience of the system to perturbations, offering valuable information for designing robust interventions [13].

The application of advanced mathematical tools, such as the Maple software package, facilitates the identification of equilibrium points and the analysis of differential equations in both ordinary and fractional forms. Distributed order differential equations, as a generalization of these two types of equations, allow for a more comprehensive analysis, capturing the memory effects and hereditary properties inherent in user behavior. This enables researchers to conduct analyses applicable to both ordinary and fractional systems. By utilizing these sophisticated equations, it can be drawing more precise conclusions about the progression and potential mitigation of addiction. For distributed order differential equations, applications of this type of equations, and different numerical solutions, see the following references. [14-21].

In this study, the model developed by Alemneh and Alemu for social media addiction is considered [10]. This model is formulated as a distributed order differential equation, distinct from the conventional types of equations. Consequently, the analysis allows for commentary on both ordinary and fractional-order differential equation systems. Nonstandard finite differences are employed for numerical analysis. The primary objective here is to determine the equilibrium points of the system. As mentioned earlier, the analysis of these points enables examination of the stages and transitional states of addiction. Due to the complexity of this analysis, the Maple software package has been utilized. Additionally, the use of the Maple software package has been integrated into the article, aiming to contribute to the literature by providing a tool suitable for analyzing such or similar equation systems.

The article is structured in four sections. The second part presents the basic concepts on the subject, focusing on distributed order differential equations and nonstandard finite differences. In section 3 addresses the discretization and analysis of the system by combining data from the Maple software package. Also, in this section contains the analysis of equilibrium points obtained with Maple files. Finally, in section 4 presents discussions and conclusions about the study.

2. Preliminaries

Distributed order differential equations and the nonstandard finite difference method are pivotal concepts in advanced mathematical modeling [22-27]. The core component of distributed order differential equations is the density function, which facilitates the simultaneous expression of both integer-order and fractional order systems. This function eliminates the need for separate solutions for each system, streamlining the numerical process. Moreover, the nonstandard finite difference method, leveraging the denominator function, enhances the efficiency and reliability of stability analyses, providing a robust framework for solving complex differential equations.

Definition 2.1: If the function $v(\mathrm{t})$ is integrable function in the range $[p_1, p_2]\ with\ \ q \in \mathbb{N}^+$ and $q-1 < \alpha \leq \mathrm{q}$, then the Caputo fractional derivative of order α is given by

$$D_{CP}^{\alpha} v(t) = \frac{1}{\Gamma(q-\alpha)} \int_{\alpha}^{t} \frac{v^{(q)}(u)}{(t-u)^{\alpha-q+1}} du,$$

where $\Gamma(.)$ represents the Gamma function [28].

Definition 2.2: If the function $v(t)$ be an integrable function in the interval $[p_1, p_2]$ with $q-1 < \alpha \leq q$ for $q \in \mathbb{N}^+$. So, Riemann Liouville fractional derivative of order α is defined by [28],

$$D_{R}^{\alpha} v(t) = \frac{1}{\Gamma(q-\alpha)} \frac{d^n}{dt^n} \int_{\alpha}^{t} \frac{v^{(q)}(u)}{(t-u)^{\alpha-q+1}} du.$$

Definition 2.3: If the function $g^{(v)}(t)$ is integrable function in the range $[v_1, t]$ and $v = 1,2,..,k+1-th$ times differentiable function. Grünwald-Letnikov fractional derivative of order α is defined by [28]:

$$D_{GL}^{\alpha} g(t) = \lim_{j \to 0} j^{-\alpha} \sum_{i=0}^{k} (-1)^i \left(\frac{k}{i}\right) g(t - ij).$$

The approximate Grünwald-Letnikov derivative formula can be expressed as:

$$D_t^{\alpha} g(t) = \sum_{i=0}^{n} u_i^{\alpha} g(t_{n-r}), \quad n = 1,2,3, \dots,$$

where $u_i^{\alpha} = \left(1 - \frac{1+\alpha}{i}\right) u_{i-1}^{\alpha}$, $for\ i = 1,2,3, \dots,\ v = h^{-\alpha}$ and h is chosen quite small [29].

Definition 2.4: The nonstandard finite difference (NSFD) scheme is a numerical method used to solve differential equations. It is designed to preserve the essential properties of the differential equations, such as positivity and stability, which standard finite difference methods may fail to maintain. The following steps outline the construction of an NSFD scheme:

$$P(w) \to P(w_n), \quad w(t) \to w(t_n), \quad \frac{dw}{dt} \to \frac{w_{n+1} - w_n}{\delta(h)}, \qquad t \to t_n,$$

where $\frac{dw}{dt} = P(\gamma, w)$ is an ordinary differential equation with $\gamma: parameter$, $\delta(h)$ is a denominator function and it can be chosen as $\delta(h) = \frac{e^{ah}-1}{a}$. The denominator function $\delta(h)$ is dependent on the step size h and the variable a, which is calculated at the equilibrium point. The choice of the denominator function should ensure compatibility with the continuous model and stability of the discrete system concerning the step size. This guarantees convergence to the continuous model as the step size decreases. Furthermore, the selection of the function should ensure that the equilibrium points of the discrete model align with those of the continuous model and preserve positive solutions. These mathematical properties ensure that the model remains consistent with physical reality and accuracy. Using this method along with the Grünwald-Letnikov approximation formula, the same scheme can be extended to fractional derivatives. Consequently, the NSFD scheme can be effectively applied to distributed order differential equations [30].

Definition 2.5: The integral operator of distributed order differential equations is defined as:

$$D_t^{w(\alpha)} f(t) = \int_{d_1}^{d_2} w(\alpha)\, D_t^{\alpha} f(t)\, d\alpha,$$

where $\alpha \in (d_1, d_2)$, and $\int_{d_1}^{d_2} w(\alpha) = k > 0$ with $w(\alpha)$ serving as the density function for distributed-order differential equations. The operator $D_t^{\alpha} f(t)$ represents the fractional

derivative operator, which can alternatively be defined using Caputo, Riemann-Liouville, or Grünwald-Letnikov derivatives. Utilizing the specified integral operator, the distributed-order derivatives are defined as follows [24]:

$$D_t^{w(\alpha)} f(t) = \sum_{i=1}^{n} \alpha^i \int_{d_1}^{d_2} w_i(\alpha)\, D_t^{i-\alpha} f(t) d\alpha + \sum_{j=0}^{n} b_j f^i(t).$$

Theorem 2.6: Let e be the equilibrium point of the difference equation of the form below.

$$w_{n+1} = P(w_n, w_{n-1}, w_{n-2,\ldots,} w_{n-k}), n = 0,1,2,\ldots$$

Here, P is a continuously differentiable function defined in an open neighborhood of the equilibrium point e. In this case, the stability of the equilibrium point is determined by examining the roots of the characteristic equation derived from the Jacobian matrix. If all the roots of the characteristic equation have absolute values less than one, the equilibrium point e is locally asymptotically stable. Otherwise, if at least one root has an absolute value greater than one, the equilibrium point e is unstable [31].

3. Main Theorem and Proof

In this section, the social media addiction population model is separated into five subgroups based on their addiction status. Susceptible individuals (S) are those who are not currently addicted but are at risk of developing social media addiction. Exposed individuals (E) are those who use social media infrequently and have not progressed to addiction. Addicted individuals (A) are those who are addicted to social media and spend the majority of their time on it. Recovered individuals (R) are those who have overcome their social media addiction. Finally, individuals who have permanently quit and no longer use social media are denoted by Q [10]. The updated version of the distributed order Social Media Addiction model can be expressed:

$$D_t^{w(\alpha)} S(t) = r + gnR(t) - bs\, A(t)S(t) - (k+m)\, S(t),$$

$$D_t^{w(\alpha)} E(t) = bsA(t)S(t) - (d+m)E(t),$$

$$D_t^{w(\alpha)} A(t) = adE(t) - (m+e+p)\, A(t),$$

$$D_t^{w(\alpha)} R(t) = (1-a)d\, E(t) + e\, A(t) - (m+n)\, R(t),$$

$$D_t^{w(\alpha)} Q(t) = kS(t) + (1-g)n\, R(t) - m\, Q(t).$$

where, susceptible individuals are introduced into the population at a recruitment rate r. These individuals begin using social media due to peer pressure at a contact rate b from addicted individuals, with a probability of transmission s, and thus transition to the exposed compartment. Some susceptible individuals permanently abstain from using social media at a rate k. Exposed individuals become addicted and move to the addicted compartment at a rate ad, while the remaining proportion of exposed individuals recover through treatment at a rate

$(1 - a)d$. Addicted individuals transition to the recovered compartment either through education and/or treatment at a rate e, or die due to addiction at a rate r. Recovered individuals may become susceptible to social media addiction again at a rate gn or completely cease using social media at a rate $(1 - g)n$. The entire population experiences an average mortality rate of m.

When applying the Grünwald-Letnikov approximation and the NSFD scheme to the modified equation for discretization, the resulting equation is derived as follows:

$$\sum_{i=1}^{L} \frac{w(\alpha_i)}{L} \sum_{j=0}^{n+1} u_j^{\alpha_i} S_{n+1-j} = r + gn\, R_n - (bsA_n + k + m)\, S_{n+1},$$

$$\sum_{i=1}^{L} \frac{w(\alpha_i)}{L} \sum_{j=0}^{n+1} u_j^{\alpha_i} E_{n+1-j} = bs\, A_n\, S_n - (d + m) E_{n+1},$$

$$\sum_{i=1}^{L} \frac{w(\alpha_i)}{L} \sum_{j=0}^{n+1} u_j^{\alpha_i} A_{n+1-j} = adE_n - (m + e + p) A_{n+1},$$

$$\sum_{i=1}^{L} \frac{w(\alpha_i)}{L} \sum_{j=0}^{n+1} u_j^{\alpha_i} R_{n+1-j} = (1 - a)d\, E_n + e\, A_n - (m + n)\, R_{n+1},$$

$$\sum_{i=1}^{L} \frac{w(\alpha_i)}{L} \sum_{j=0}^{n+1} u_j^{\alpha_i} Q_{n+1-j} = k\, S_n + (1 - g)n\, R_n - m\, Q_{n+1}$$

where $i = 1,2,3,4$ and $0 < \alpha_i < 1$, $p_0^{\alpha_i} = (\delta(h))^{-\alpha_i}$ and denominator functions are:

$$\delta_1(h) = \frac{e^{(k+m)h} - 1}{k + m}, \delta_2(h) = \frac{e^{(d+m)h} - 1}{d + m}, \delta_3(h) = \frac{e^{(m+e+p)h} - 1}{m + e + p},$$

$$\delta_4(h) = \frac{e^{(m+n)h} - 1}{n}, \delta_5(h) = \frac{e^{(m)h} - 1}{m}.$$

By utilizing the Grünwald-Letnikov approximation formula and appropriate denominator functions for the discretization, the system transforms into the following form:

$$S_{n+1}\left(\sum_{i=1}^{L} \frac{w(\alpha_i)}{L} (\delta_1(h))^{-\alpha_i} + bsA_n + k + m\right) = r + gn\, R_n - \sum_{i=1}^{L} \frac{w(\alpha_i)}{L} \left(\sum_{j=1}^{n+1} u_j^{\alpha_i} S_{n+1-j}\right),$$

$$E_{n+1}\left(\sum_{i=1}^{L} \frac{w(\alpha_i)}{L} (\delta_2(h))^{-\alpha_i} + d + m\right) = bs\, A_n\, S_n - \sum_{i=1}^{L} \frac{w(\alpha_i)}{L} \left(\sum_{j=1}^{n+1} u_j^{\alpha_i} E_{n+1-j}\right),$$

$$A_{n+1}\left(\sum_{i=1}^{L}\frac{w(\alpha_i)}{L}(\delta_3(h))^{-\alpha_i}+m+e+p\right)=adE_n-\sum_{i=1}^{L}\frac{w(\alpha_i)}{L}\left(\sum_{j=1}^{n+1}u_j^{\alpha_i}A_{n+1-j}\right),$$

$$R_{n+1}\left(\sum_{i=1}^{L}\frac{w(\alpha_i)}{L}(\delta_4(h))^{-\alpha_i}+m+n\right)=(1-a)d\,E_n+e\,A_n-\sum_{i=1}^{L}\frac{w(\alpha_i)}{L}\left(\sum_{j=1}^{n+1}u_j^{\alpha_i}R_{n+1-j}\right),$$

$$Q_{n+1}\left(\sum_{i=1}^{L}\frac{w(\alpha_i)}{L}(\delta_5(h))^{-\alpha_i}+m\right)=\ k\,S_n+(1-g)n\,R_n-\sum_{i=1}^{L}\frac{w(\alpha_i)}{L}\left(\sum_{j=1}^{n+1}u_j^{\alpha_i}Q_{n+1-j}\right),$$

Thus, the system, when discretized, results in the following form:

$$S_{n+1}=\frac{r+gn\,R_n-\sum_{i=1}^{L}\frac{w(\alpha_i)}{L}\left(\sum_{j=1}^{n+1}u_j^{\alpha_i}S_{n+1-j}\right)}{\left(\sum_{i=1}^{L}\frac{w(\alpha_i)}{L}(\delta_1(h))^{-\alpha_i}+bsA_n+k+m\right)},$$

$$E_{n+1}=\frac{bs\,A_n\,S_n-\sum_{i=1}^{L}\frac{w(\alpha_i)}{L}\left(\sum_{j=1}^{n+1}u_j^{\alpha_i}E_{n+1-j}\right)}{\left(\sum_{i=1}^{L}\frac{w(\alpha_i)}{L}(\delta_2(h))^{-\alpha_i}+d+m\right)},$$

$$A_{n+1}=\frac{adE_n-\sum_{i=1}^{L}\frac{w(\alpha_i)}{L}\left(\sum_{j=1}^{n+1}u_j^{\alpha_i}A_{n+1-j}\right)}{\left(\sum_{i=1}^{L}\frac{w(\alpha_i)}{L}(\delta_3(h))^{-\alpha_i}+m+e+p\right)},$$

$$R_{n+1}=\frac{(1-a)d\,E_n+e\,A_n-\sum_{i=1}^{L}\frac{w(\alpha_i)}{L}\left(\sum_{j=1}^{n+1}u_j^{\alpha_i}R_{n+1-j}\right)}{\left(\sum_{i=1}^{L}\frac{w(\alpha_i)}{L}(\delta_4(h))^{-\alpha_i}+m+n\right)},$$

$$Q_{n+1}=\frac{k\,S_n+(1-g)n\,R_n-\sum_{i=1}^{L}\frac{w(\alpha_i)}{L}\left(\sum_{j=1}^{n+1}u_j^{\alpha_i}Q_{n+1-j}\right)}{\left(\sum_{i=1}^{L}\frac{w(\alpha_i)}{L}(\delta_5(h))^{-\alpha_i}+m\right)}.$$

Here, the discretized system is defined in the Maple software package as follows:

- S:=(r+g*n*Rn-w(alpha)/L*u[j]*Sn)/(w(alpha)/L*q1^(-alpha)+b*s*An+k+m)-Sn=0;

$$S:=\frac{g\,n\,Rn+r-\frac{w(\alpha)\,u_j\,Sn}{L}}{\frac{w(\alpha)\,q1^{-\alpha}}{L}+b\,s\,An+k+m}-Sn=0$$

Figure 1: Definition of Susceptible Populations in Maple

- E:=(b*s*An*Sn-w(alpha)/L*u[j]*En)/(w(alpha)/L*q2^(-alpha)+d+m)-En=0;

$$E := \frac{b\,s\,An\,Sn - \frac{w(\alpha)\,u_j\,En}{L}}{\frac{w(\alpha)\,q2^{-\alpha}}{L} + d + m} - En = 0$$

Figure 2: Definition of Exposed Populations in Maple

- A:=(a*d*En-w(alpha)/L*u[j]*An)/(w(alpha)/L*q3^(-alpha)+m+e+p)-An=0;

$$A := \frac{a\,d\,En - \frac{w(\alpha)\,u_j\,An}{L}}{\frac{w(\alpha)\,q3^{-\alpha}}{L} + m + e + p} - An = 0$$

Figure 3: Definition of Addicted Populations in Maple

- R:=((1-a)*d*En+e*An-w(alpha)/L*u[j]*Rn)/(w(alpha)/L*q4^(-alpha)+m+n)-Rn=0;

$$R := \frac{(1-a)\,d\,En + e\,An - \frac{w(\alpha)\,u_j\,Rn}{L}}{\frac{w(\alpha)\,q4^{-\alpha}}{L} + m + n} - Rn = 0$$

Figure 4: Definition of Recovered Populations in Maple

- Q:=(k*Sn+(1-g)*n*Rn-w(alpha)/L*u[j]*Qn)/(w(alpha)/L*q5^(-alpha)+m)-Qn=0;

$$Q := \frac{k\,Sn + (1-g)\,n\,Rn - \frac{w(\alpha)\,u_j\,Qn}{L}}{\frac{w(\alpha)\,q5^{-\alpha}}{L} + m} - Qn = 0$$

Figure 5: Definition of Quitted Populations in Maple

Here, the summation symbol definitions are incorporated within $w(\alpha)$. The definitions for the denominator functions are provided as follows:

- q1:=(exp((k+m)*h)-1)/(k+m);q2:=(exp((d+m)*h)-1)/(d+m);q3:=(exp((m+e+p)*h)-1)/(m+e+p);q4:=(exp((m+n)*h)-1)/(m+n);q5:=(exp((m)*h)-1)/(m);

$$q1 := \frac{e^{(k+m)\,h} - 1}{k + m}$$

$$q2 := \frac{e^{(d+m)\,h} - 1}{d + m}$$

$$q3 := \frac{e^{(m+e+p)\,h} - 1}{m + e + p}$$

$$q4 := \frac{e^{(m+n)h} - 1}{m+n}$$

$$q5 := \frac{e^{mh} - 1}{m}$$

Figure 6: Definition of Denominator Functions in Maple

The constant values necessary for equilibrium point analysis are taken as follows: $r = 0.5, m = 0.25, b = 0.6, s = 0.5, a = 0.7, p = 0.01, d = 0.25, e = 0.7, k = 0.01, g = 0.35, n = 0.4$ 0 [10]. The constant values and commands for equilibrium points are entered into Maple as follows. Consequently, the equilibrium points are obtained as follows:

- r:=0.5:m:=0.25:b:=0.6:s:=0.5:a:=0.7:p:=0.01:d:=0.25:e:=0.7:k:=0.01:g:=0.35:n:=0.4:L :=100:
- h:=0.01:alpha:=1.3:
- w:=GAMMA(2-alpha):u[j]:=1:
- Solve:=solve([S,E,A,R,Q],[Sn,En,An,Rn,Qn]);

$$Solve := [[Sn = 0.09204875679, En = 0., An = 0., Rn = 0., Qn = 0.0001697615420], [Sn = 658.9989919, En = -632.1782780, An = -18.11114282, Rn = -10.34479281, Qn = 0.7193235400]]$$

Figure 7: Solutions of Equilibrium Points with Maple

The Jacobian matrix and eigenvalues for the analysis of the obtained equilibrium points are expressed in the Maple software package as follows. When examining the program for the first equilibrium point:

- J:=Matrix([[diff(S,Sn),diff(S,En),diff(S,An),diff(S,Rn),diff(S,Qn)],[diff(E,Sn),diff(E,En),diff(E,An),diff(E,Rn),diff(E,Qn)],[diff(A,Sn),diff(A,En),diff(A,An),diff(A,Rn),diff(A,Qn)],[diff(R,Sn),diff(R,En),diff(R,An),diff(R,Rn),diff(R,Qn)],[diff(Q,Sn),diff(Q,En),diff(Q,An),diff(Q,Rn),diff(Q,Qn)]]);
- Sn:=0.09204875679:En:=0:An:=0:Rn:=0:Qn:=0.0001697615420:
- DE:= det(lambda-J);

$$DE := (0.002395411879 + \lambda)(-7.277333716 \times 10^{-10} - 6.044055389 \times 10^{-7}\lambda - 0.0001094959781\lambda^2 + 0.009065944404\lambda^3 + \lambda^4)$$

Figure 8: Characteristic Equations for Equilibrium Point 1 in Maple

- SDE:=solve(DE,lambda);

$$SDE := -0.002395411879,\ 0.009631757463,\ -0.002239628108,\ -0.002399692048,\ -0.01405838171$$

Figure 9: Eigenvalues for Equilibrium Point 1 in Maple

As a result, since all eigenvalues have an absolute value less than 1, it can be observed that the equilibrium point is asymptotically stable. Upon analyzing the second equilibrium point:

- Sn:=658.9989919:En:=-632.1782780:An:=-18.11114282:Rn:=-10.34479281:Qn:=0.7193235400:

- DE2:= det(lambda-J);

$$DE2 := 0.001960646237 + 1.727138154\lambda + 379.2534823\lambda^2 - 1.012561947\lambda^3 - 0.8911417138\lambda^4 + \lambda^5$$

Figure 10: Characteristic Equations for Equilibrium Point 2 in Maple

- SDE2:=solve(DE2,lambda);

$$SDE2 := 3.946044181 + 6.215818222\,I,\ -0.002154313405,\ -0.002399692062,\ -6.996392644,\ 3.946044181 - 6.215818222\,I$$

Figure 11: Eigenvalues for Equilibrium Point 2 in Maple

Since not all eigenvalues have an absolute value less than 1, this equilibrium point is not stable. It can be observed that in this example, $\alpha = 1.3.$ is chosen. However, it is evident that interpretations can be made for different values of alpha and $w(\alpha)$. When analyzing equilibrium points, the results are found to be quite consistent compared to fractional order and ordinary differential equations in the literature. In comparison with the Alemneh and Alemu [10] study, it was observed that the stability of the equilibrium points is the same, and the suitability of the Maple program for the system was confirmed. This Maple package program demonstrates that equilibrium stability of many such model systems can be investigated.

4. Conclusion

This study focuses on a mathematical model of social media addiction. Unlike previous literature, the model formulation uses fractional-order differential equations, providing a more flexible and accurate representation of the dynamic nature of addiction. The discretisation process uses the NSFD method, which ensures numerical stability and preserves the essential features of the original system. The main contribution of this study is the detailed analysis of the equilibrium points using the Maple software package. By providing source codes, this research provides a reproducible and extendable framework for the analysis of similar systems in future studies. These codes not only verify the accuracy of the mathematical model, but also allow researchers to explore the behaviour of social media addiction under different scenarios, including different levels of exposure and recovery rates. In addition, the results demonstrate how equilibrium point analysis can be used to identify critical thresholds that mark transitions between balanced and excessive use. This insight makes a significant contribution to the

academic field by providing a methodological approach to studying the progression, control and mitigation of behavioural addictions. By combining mathematical modelling with computational tools, this study bridges theoretical and practical aspects and promotes interdisciplinary research in psychology, sociology and computer science.

Ethics in Publishing

There are no ethical issues regarding the publication of this study.

Author Contributions

Creating the research, finding the data, evaluating the results with Maple, writing the codes and the article, etc. transactions were carried out by Mehmet Kocabıyık.

Acknowledgements

I would like to thank Reviewers for taking the necessary time and effort to review the manuscript. I sincerely appreciate all your valuable comments and suggestions

References

[1] Kuss, D. J., Griffiths, M. D., (2017) Social networking sites and addiction: Ten lessons learned, International Journal of Environmental Research and Public Health, 14(3), 311.

[2] Turel, O., He, Q., Xue, G., Xiao, L., Bechara, A., (2014) Examination of neural systems sub-serving Facebook “addiction”, Psychological Reports, 115(3), 675-695.

[3] Young, K. S., (2017) The evolution of internet addiction disorder, Internet addiction: Neuroscientific approaches and Therapeutical implications including smartphone addiction, 3-18.

[4] Andreassen, C. S., Pallesen, S., Griffiths, M. D., (2016) The relationship between addictive use of social media, narcissism, and self-esteem: Findings from a large national survey, Addictive Behaviors, 64, 287-293.

[5] Bian, M., Leung, L., (2017) Linking loneliness, shyness, smartphone addiction symptoms, and patterns of smartphone use to social capital, Social Science Computer Review, 33(1), 61-79.

[6] Ishaku, A., Musa, B. S., Sanda, A., Bakoji, A. M., (2018) Mathematical assessment of social media impact on academic performance of students in higher institution, IOSR J. Math, 2018, 72-79.

[7] Simsek, A., Elciyar, K., Kizilhan, T., (2019) A comparative study on social media addiction of high school and university students, Contemporary educational technology, 10(2), 106-119.

[8] Shutaywi, M., Rehman, Z. U., Shah, Z., Vrinceanu, N., Jan, R., Deebani, W., Dumitrascu, O., (2023) Modeling and analysis of the addiction of social media through fractional calculus, Frontiers in Applied Mathematics and Statistics, 9, 1210404.

[9] Guo, Y., Li, T., (2020) Optimal control and stability analysis of an online game addiction model with two stages, Mathematical Methods in the Applied Sciences, 43(7), 4391-4408.

[10] Alemneh, H. T., Alemu, N. Y., (2020) Mathematical modeling with optimal control analysis of social media addiction, Journal of Applied Mathematics and Physics, 8, 2136-2155.

[11] Savci, M., Tekin, A., Elhai, J. D., (2022) Prediction of problematic social media use (PSU) using machine learning approaches, Current Psychology, 41(5), 2755-2764.

[12] Çiftci, N., Yıldız, M., (2023) The relationship between social media addiction, happiness, and life satisfaction in adults: analysis with machine learning approach, International Journal of Mental Health and Addiction, 21(5), 3500-3516.

[13] Shao, Y. J., Zheng, T., Wang, Y. Q., Liu, L., Chen, Y., Yao, Y. S., (2018) Internet addiction detection rate among college students in the People's Republic of China: A meta-analysis, Child and Adolescent Psychiatry and Mental Health, 12, 1-10.

[14] Kocabıyık, M., Ongun, M. Y., Çetinkaya, İ. T., (2021) Standart olmayan sonlu fark metodu ile dağılımlı mertebeden SVIR modelinin nümerik analizi, Balıkesir Üniversitesi Fen Bilimleri Enstitüsü Dergisi, 23(2), 577-591.

[15] İğret Araz, S., (2021) Numerical approximation with Newton polynomial for the solution of a tumor growth model including fractional differential operators, Erzincan University Journal of Science and Technology, 14(1), 249-259. https://doi.org/10.18185/erzifbed.753464

[16] Kocabıyık, M., Ongun, M. Y., (2022) Construction of a distributed order smoking model and its nonstandard finite difference discretization, AIMS Mathematics, 7(3), 4636-4654. https://doi:10.3934/math.2022258

[17] Kocabıyık, M., (2022) Nonstandard Discretization and Stability Analysis of a novel type Malaria-Ross Model, Journal of the Institute of Science and Technology, 12(2), 1023-1033. https://doi.org/10.21597/jist.1026364

[18] Melkamu, B., Mebrate, B., (2022) A Fractional Model for the Dynamics of Smoking Tobacco Using Caputo–Fabrizio Derivative, Journal of Applied Mathematics, 2022(1), 2009910.

[19] Yiğider, M., Okur, S., (2023) Numerical Solution for Time-Fractional Murray Reaction-Diffusion Equations via Reduced Differential Transform Method, Erzincan University Journal of Science and Technology, 16(1), 120-137. https://doi.org/10.18185/erzifbed.1217232

[20] Kocabıyık, M., Ongun, M. Y., (2023) Discretization and Stability Analysis for a Generalized Type Nonlinear Pharmacokinetic Models, Gazi University Journal of Science, 36(4), 1675-1691. https://doi.org/10.35378/gujs.1027381

[21] Öztürk, Z., Bilgil, H., Sorgun, S., (2024) A new application of fractional glucose-insulin model and numerical solutions, Sigma Journal of Engineering and Natural Sciences, 42(2), 407-413.

[22] Caputo, M., (1969) Elasticita e dissipazione, Zanichelli.

[23] Caputo, M., (1995) Mean fractional-order-derivatives differential equations and filters, Annali dell'Universita di Ferrara, 41(1), 73-84.

[24] Caputo, M., (2001) Distributed order differential equations modelling dielectric induction and diffusion, Fractional Calculus and Applied Analysis, 4(4), 421-442.

[25] Mickens, R. E., (1989) Exact solutions to a finite-difference model of a nonlinear reaction-advection equation: Implications for numerical analysis, Numerical Methods for Partial Differential Equations, 5(4), 313-325.

[26] Mickens, R. E., (1994) Nonstandard finite difference models of differential equations, World scientific.

[27] Çetinkaya, İ. T., (2023) An Application of Nonstandard Finite Difference Method to a Model Describing Diabetes Mellitus and Its Complications, Journal of New Theory, (45), 105-119.

[28] Podlubny, I., (1999) Fractional differential equations: an introduction to fractional derivatives, fractional differential equations, to methods of their solution and some of their applications (Vol. 198), Elsevier.

[29] Dorciak L., (1994) Numerical models for simulation the fractional-order control systems, UEF-04-94, The Academy of Sciences, Institute of Experimental Physic, Kosiice, Slovak Republic.

[30] Meerschaert, M. M., Tadjeran, C., (2004) Finite difference approximations for fractional advection–dispersion flow equations, Journal of computational and applied mathematics, 172(1), 65-77.

[31] Butcher, J. C. (1966). On the convergence of numerical solutions to ordinary differential equations. Mathematics of Computation, 20(93), 1-10.