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# Digital Systems Model Order Reduction with Substructure Preservation and Fuzzy Logic Control

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**Abstract**: A digital system model order reduction (MOR) technique with substructure preservation and fuzzy logic control is presented in this paper. The reduction procedure is performed utilizing the following two steps; model transformation, and system approximation. The transformation process is achieved by utilizing the Lyapunov Sylvester equation, which internally allows for the substructure preservation. The reduction process is achieved using the singular perturbation approximation technique that deals with Multi-Time-Scale systems (MTS). In some of the MTS systems, some of the dynamics may be eliminated since they usually have negligible effect on the overall system response. The reduced order model is then controlled using a fuzzy logic control. The proposed method of model reduction and control is investigated by observing the results of a simulated example. Results of investigation show the achievement of new models with the following advantages; reduced order models, stable models, and controlled model. This is all achieved while having the original dominant same dynamics in the new models, which emphasizes the potential of the proposed technique.

Keywords: Model order reduction, Multi-time-scale, Fuzzy logic control, Digital systems

## Introduction

MOR has been a powerful technique in the field of engineering and computational science, which aims to simplify complex mathematical models while retaining their essential characteristics (Lu, et al., 2021). This process involves reducing the dimensionality of a system, which can be critical in situations where high-fidelity simulations or detailed models are computationally expensive or time-consuming (Patalano, et al., 2021). By creating simplified, lower-dimensional representations of these systems, MOR enables faster simulations, design optimization, and real-time control, making it a valuable tool in various industries, including aerospace, mechanical engineering, and electronics (Mendonca et al., 2019). Research in MOR encompasses various disciplines and has numerous applications. Some of these applications, for example electronic systems with integrated circuits and electronic systems, can be faster and more efficient in performance.

Many researchers have focused on the process of MOR while proposing and investigating different techniques from traditional to artificial intelligence (AI) approaches. This may be seen as in using intrinsic differential equations as in (Desai et al., 2013), genetic algorithms as in (Alsmadi et al., 2011a), invasive weed optimization as in (Abu-Al-Nadi et al., 2013), and artificial neural networks as in (Alsmadi et al., 2012; Alsmadi et al., 2011b). Some of the reduced order modelling approaches such as matching Markov parameters as proposed by Krajewski et al. (1995), were introduced to ensure stability of the reduced order model. A popular technique for

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obtaining reduced order models is the Krylov subspace as presented by Salimbahrami et al. 2005), however stability of the reduced order model is not guaranteed. Another important group of modelling algorithms is the eigenvalue preservation technique where important eigenvalues of the system are retained to find suitable lower order models as investigated by Alsmadi et al. (2011a) and Abu-Al-Nadi et al. (2013).

In spite of the high focus on reduced order modelling of continuous systems, a little has been devoted to digital systems. Recently, Maulik et al. (2022) have considered MOR for Digital models, which form the basis of autonomous off-road vehicles. They presented derivation and simulation outcomes with custom-built virtual modules of powertrain, electrical, and control systems in a problem-solving environment. (Hartmann, Herz, & Wever, 2018) addressed the advantages of model order reduction for digital model-based system engineering and real-time thermal control of electric motors. Some optimization techniques like genetic algorithm, as in (Tse et al., 2001), particle swarm optimization, as in (Deepa et al., 2011), and artificial neural networks, as in (Alsmadi et al., 2011b), have also been introduced for reduced order modelling of digital systems. However, it is found that no method can provide acceptable results for all kinds of systems with fuzzy logic control utilizing the Lyapunov-Sylvester Equation, which transforms the two-time-scale system into a decoupled model. This decoupling allows for the use of MOR utilizing the singular perturbation approximation.

#### **Problem Formulation**

Consider a digital system given by the following difference equation described by

$$y(k) + a_1 y(k-1) + a_2 y(k-2) + \dots + a_n y(k-n) = b_0 u(k) + b_1 u(k-1) + \dots + b_{\overline{n}} u(k-\overline{n})$$
(1)

where u(k) is the input and y(k) is the output of the system at the  $k^{th}$  sampling instant. Transforming this equation into the Z-domain produces the following pulse transfer function

$$G(z) = \frac{Y(z)}{U(z)} = \frac{b_0 z^{\bar{n}} + b_1 z^{\bar{n}-1} + \dots + b_{\bar{n}}}{z^n + a_1 z^{n-1} + \dots + a_n}$$
(2)

with  $\overline{n} \le n$  for a stricktly proper system. The characteristic polynomial contains the system dominant and none dominant poles with distinct, repeated, or complex and the system is referred to as an  $n^{th}$  order model. The corresponding desired reduced  $r^{th}$  order model is given by

$$G_r(z) = \frac{Y(z)}{U(z)} = \frac{b_0 z^{\bar{r}} + b_1 z^{\bar{r}-1} + \dots + b_{\bar{r}}}{z^r + a_1 z^{r-1} + \dots + a_r}$$
(3)

where some of the coefficients  $a_i$  ( $i = 1, 2, \dots, r$ ) and  $b_i$  ( $i = 0, 1, 2, \dots, \bar{r}$ ) may be zeros as long as  $\bar{r} \le r$ , since the  $\bar{r} > r$  term represents an improper system. For MOR and use of the singular perturbation approximation, the system in Equation (1) maybe transformed into the state space form given as following

$$x(k+1) = Ax(k) + Bu(k) \tag{4}$$

$$y(k) = Cx(k) + Du(k)$$
<sup>5</sup>

where  $x \in \Re^n$  is the state vector,  $u \in \Re^p$  and  $y \in \Re^m$  are the input and output vectors respectively,  $A \in \Re^{n \times n}, B \in \Re^{n \times p}, C \in \Re^{q \times n}, D \in \Re^{q \times p}$  are matrices of appropriate dimensions with *n*, *p*, and *q* are the system order, number of inputs, and number of outputs respectively. The corresponding desired reduced  $r^{\text{th}}$  order model is obtained as:

$$x_r(k+1) = A_r x_r(k) + B_r u(k)$$
(6)

$$y_r(k) = C_r x_r(k) + D_r u(k) \tag{7}$$

which is obtained as will be illustrated in the following subsections. The fuzzy logic approach will be used to design the PID control produced by modifying the system input signal by the following factor

$$K_{P} + \frac{K_{i}T_{s}z}{z-1} + \frac{K_{d}(z-1)}{T_{s}z}$$
(8)

obtained using the Forward Euler form where  $K_p$ ,  $K_d$ ,  $K_i$  are the PID parameters and  $T_s$  is the sampling time.

#### **Transformation and MOR**

It is very well known that the system behavior is generally controlled by the characteristic polynomial seen in Equation (2), which provides the system dynamics named system poles. The poles of continuous and discrete time systems are illustrated as seen in Figure 1 (Fadali & Visioli, 2019).



Figure 1. Discrete and continuous time system poles

Motivated by two-time-scale observations, the higher  $|\sigma_1|$  (seen in the s-domain of Figure 1) the more confidently one may eliminate that dynamic. This observation may also be illustrated as seen in Figure 2. As seen here, relatively the farthest roots (poles) from the origin become insignificant. On the other hand, for discrete systems, the closest characteristic roots from the origin become insignificant.



Figure 2. Significant and insignificant dynamics of discrete and continuous time system

Motivated by the singular perturbation approximation, investigated by BaniHani et al. (2009) model transformation of digital systems is performed with the dominant dynamics set properly in the system state matrix. The system shown in Equation (2) can simply be transformed into a state space representation shown by the form given in Equations (4) and (5). Hence, for the  $n^{th}$  order digital system, the dynamics can be represented by:

$$\lambda = [\lambda_1, \lambda_2, \dots, \lambda_n] \tag{9}$$

Deriving the system as in Equations (4) and (5), the state space model may be represented by different formats. As it is well known, some forms are referred to as controllable, observable, Jordan (modal), and some are general. For a singularly perturbed digital model with order reduction objectives, the derived model may not be suitable for the simplification, as in the case where fast and slow system dynamics are coupled together. Hence, system dynamics decoupling becomes one of the potential choices. Adapting this choice, the state x is transformed into  $\hat{x}$  as following:

$$x(k) = T \hat{x}(k) \tag{10}$$

provided that T is the transformation matrix. Following the standard procedure of system transformation, as illustrated by(Trentelman et al. (2001), substituting Equation (10) into Equations (4) and (5) yields

$$T \hat{x}(k+1) = AT \hat{x}(k) + Bu(k)$$
 (11)

$$\hat{y}(k) = CT \,\hat{x}(k) + D \,u(k)$$
 (12)

Multiplying Equation (10) by the inverse of *T* yields the transformed model:

$$\hat{x}(k+1) = T^{-1}AT\,\hat{x}(k) + T^{-1}Bu(k) \tag{13}$$

Hence, the overall transformed model may then be given by

$$\hat{x}(k+1) = \hat{A}\hat{x}(k) + \hat{B}u(k)$$
 (14)

$$\hat{y}(k) = \hat{C}\hat{x}(k) + \hat{D}u(k)$$
 (15)

where  $\hat{A} = T^{-1}AT$  which has the decoupled system dynamics,  $\hat{B} = T^{-1}B$ ,  $\hat{C} = CT$ , and  $\hat{D} = D$ . It is important to notice that the models in Equations (4) and (5) and Equations (14) and (15) both have the same system characteristics i.e., same eigenvalues as presented by Trentelman et al. (2001). For this transformed model, a reduced order model is to be obtained by truncating the unnecessary dynamics.

For the system in Equations (4) and (5), the proposed dominant dynamics reduced order modelling procedure is achieved by maintaining the full order dominant poles as a subset in the reduced order model. Thus, the transformed state matrix  $\hat{A}$ , in the continuous form of Equation (14), is designed to have the following decoupling format:

$$\hat{A} = \begin{bmatrix} \lambda_{1} & a_{12} & a_{13} & a_{14} & \cdots & a_{1r} \\ 0 & \lambda_{2} & a_{23} & a_{24} & \cdots & & \\ & \ddots & \ddots & \vdots & & & \\ \cdot & 0 & \lambda_{\overline{b}} & & & \vdots \\ \cdot & 0 & \lambda_{\overline{b}} & & & \vdots \\ \cdot & 0 & -\alpha_{1} & \alpha_{1} & & & \\ 0 & 0 & 0 & \ddots & a_{(r-2)r} \\ & & 0 & \sigma_{\overline{p}} & \alpha_{\overline{p}} \\ 0 & \cdot & \cdot & 0 & -\alpha_{\overline{p}} & \sigma_{\overline{p}} \end{bmatrix}$$
(16)

where the original system dominant poles (real and/or complex) are preserved in the diagonal, seen as  $\lambda_i$ , i = 1, 2, ...  $\overline{b}$  (real) and  $\sigma_i \pm \alpha_i$ ,  $i = 1, 2, ... \overline{p}$  (complex). Notice that for this reduced order model,  $r = (\overline{b} + 2\overline{p}) < n$ . To ensure that the dominant poles are preserved in the reduced order model, the following condition is be satisfied:

$$\lambda_{\text{dominant}} \coloneqq |\lambda_1| < |\lambda_2| < \dots < |\lambda_{\overline{b}}|, \ |\lambda| < |\sigma|, \text{ and } |\sigma_1 \pm \alpha_1| < |\sigma_2 \pm \alpha_2| < \dots > |\sigma_{\overline{p}} \pm \alpha_p|$$
(17)

Taking into account that if  $|\lambda_i| > |\sigma_i \pm \alpha_i|$ , then Equation (17) is to be redefined accordingly if necessary. For simplicity, the modal form is chosen, which implies that all upper triangular elements seen in Equation (16) as  $a_{ij}$  are set to zero. Now, for the system state matrix in Equation (13), which now has the form in Equation (16), the matrix *T* is found using the following Lyapunov-Sylvester Equation as proposed by (Wachspress, 1988):

$$XT + TY = Q \tag{18}$$

where  $X \in \Re^{n \times n}$ ,  $Y \in \Re^{m \times m}$ , and  $Q \in \Re^{n \times m}$  are given matrices. Equation (18) has a unique solution *T* if and only if the following condition is satisfied

$$\varphi(X) \cap \varphi(Y) = 0 \tag{19}$$

where  $\phi(X)$  denotes the spectrum of the matrix X.

To find the transformation matrix, T in Equation (18), the definition of  $\hat{A}$  shown in Equation (14) is written as:

$$T^{-1}AT - \hat{A} = 0 \tag{20}$$

To satisfy the condition of Equation (18), the following modification is introduced

$$T^{-1}AT - \hat{A} = \varepsilon \tag{21}$$

where  $\varepsilon$  is a very small number. Rearranging Equation (21) and multiplying it by the matrix T, yields:

$$AT - T(\hat{A} + \varepsilon) = 0 \tag{22}$$

To determine the transformation matrix T, the X and Y matrices are transformed into a complex Schur form, which results in obtaining a model state matrix  $\hat{A}$  that has the system poles decoupled according to their potential contributions in the system behavior. Now, based on the proposed dynamics decoupling, the well knownn method of singular perturbation technique is used in this paper. This method performs the reduction by eliminating the decoupled fast dynamics and focuses on the slow dominant dynamics as illustrated in (Zoran et al, 2001), which yields

$$x_r(k+1) = A_{11} x_r(k) + A_{12} x_o(k) + B_1 u(k)$$
(23)

$$\gamma x_o(k+1) = A_{21} x_r(k) + A_{22} x_o(k) + B_2 u(k)$$
(24)

$$y(k) = C_1 x_r(k) + C_2 x_o(k) + Du(k)$$
(25)

where  $x_r$  represent the dominant dynamics and  $x_o$  represents the non dominant dynamics. Assuming that  $x_o(k+1)$  has reached its quasi steady state ( $\gamma = 0$ ), as proposed by Kokotovic et al. (1986), Equation (24) can be rewritten as following:

$$0 = A_{21}x_r(k) + A_{22}x_o(k) + B_2u(k) \implies x_o(k) = -A_{22}^{-1}(A_{21}x_r(k) + B_2u(k))$$
(26)

Providing that  $A_{22}$  is nonsingular, substituting  $x_o$  given in Equation (26) into Equations (24) and (25) yields the following reduced order model:

$$x_r(k+1) = (A_{11} - A_{12}A_{22}^{-1}A_{21})x_r(k) + (B_1 - A_{12}A_{22}^{-1}B_2)u(k)$$
(27)

$$y(k) = (C_1 - C_2 A_{22}^{-1} A_{21}) x_r(k) + (D - C_2 A_{22}^{-1} B_2) u(k)$$
(28)

### **Fuzzy Logic Control**

Fuzzy logic operation is based on searching for a suitable answer from a range of views in a similar way of how human thinking is processed. There are four main components in a fuzzy logic controller which are presented as shown in Figure 3.



Figure 3. Fuzzy logic control.

In the fuzzification step, the values of input variables are measured and the input data are converted into suitable linguistic values. The crisp values are then transformed into fuzzy sets by means of fuzzifier. The inference engine uses IF-THEN rules and simulates human reasoning process. The rule base stores the knowledge. Defuzzification is the step which converts the fuzzy values into crisp values to be implemented and control the system outputs. The overall control scheme is implemented as shown in Figure 4 (Tiwari et al., 2019). The sate space form here is only as a result of the reduced modelling. The PID controller parameters are provided by the fuzzy logic designer.



Figure 4. Reduced order model with fuzzy control.

### **Illustration and Discussion**

In this section, we will consider a digital model described by the following 7<sup>th</sup> order for a supersonic jet engine inlet investigated by (Telescu et al, 2013)

$$G(z) = \frac{2.0434z^6 - 4.9825z^5 + 6.57z^4 - 5.8189z^3 + 3.636z^2 - 1.4105z + 0.2997}{z^7 - 2.46z^6 + 3.433z^5 - 3.333z^4 + 2.546z^3 - 1.584z^2 + 0.7478z - 0.252}$$

with poles given as  $[0.6843 \pm 0.5820i, 0.8913, 0.2988 \pm 0.7574i, -0.1987 \pm 0.6993i]$ . Telescu et al. obtained the following 5<sup>th</sup> order model using their proposed method

$$G_r(z) = \frac{2.043z^5 - 3.057z^4 + 2.195z^3 - 1.545z^2 + 0.8617z}{z^5 - 1.518z^4 + 1.270z^3 - 1.032z^2 + 0.7539z - 0.3156}$$

with poles given as  $[0.8320, -0.2318 \pm 0.7612i, 0.5748 \pm 0.5183i]$ , which are unrelated to the original system dynamics. On the other hand, the substructure preservation technique produced a lower dimension (3<sup>rd</sup> order) model with a transfer function given by

$$G_r(z) = \frac{1.5152z^3 - 3.1238z^2 + 2.4042z - 0.63125}{z^3 - 2.2598z^2 + 2.0266z - 0.71915}$$

To investigate the performance of the reduced order models, the full and both reduced order models were simulated to step inputs with results presented as shown in Figure 5 (with (b) for better viewing).



Figure 5. Step response of the full and reduced order models.

In addition to that, the differences between the two methods, and based on the previous simulation results presented in Figures 5, comparisons are presented clearly in Table 1. In this table, please note the use of SSP is to denote Substructure preservation, and Tstp is to denote Time of convergence for step response.

.

Full Order         Proposed educed order         Telescu et al reduced order	
<b>Model</b> Given $1.5152z^3 - 3.1238z^2 + 2.4042z - 0.63125 = 2.043z^5 - 3.057z^4 + 2.195z^3 - 1.545z^4$	$z^2 + 0.8617z$
$z^{3}-2.2598z^{2}+2.0266z-0.71915$ $z^{5}-1.518z^{4}+1.270z^{3}-1.032z^{2}+0.725z^{2}-1.032z^{2}-$	7539z-0.3156
Model $7^{\text{th}}$ $3^{\text{rd}}$ $5^{\text{th}}$	
Order	
<b>SSP</b> $\{0.6843 \pm 0.5820i, 0.8913\}$ $\{0.8320, -0.2318 \pm 0.7612i, 0.57318 \pm 0.7612i, 0.57312i, 0.57318 \pm 0.7612i, 0.57318i, 0.57312i, 0.57$	$748 \pm$
Subset of the full order model 0.5183i}	
(Substructure preservation) Not related to the full order syst	em
(No Substructure preservation)	
Tstp 0.5 second No convergence	

Implementing a PID controller design while focusing on minimizing the overshoot, settling time, and steady state error, the following system was obtained

$$\frac{2.729z^4 - 7.2022z^3 + 5.7432z^2 - 0.7341z - 0.5280}{z^4 - 2.9169z^3 + 2.8909z^2 - 1.0201z + 0.0467}$$

Results of simulation are shown as presented in Figure 6. As observed, all three factored have been obtained with requirements as specified.



Figure 6. Step responses for the reduced order model, with and without controller.

#### Conclusion

In this paper, a technique for digital MOR with substructure preservation and fuzzy logic control is presented. The reduction process is performed using system transformation and singular perturbation approximation. The transformation is the Sylvester equation for a state space model while forcing the system state matrix to preserve the dominant original dynamics in the transformed model. The singular perturbation technique is then used to perform the reduction while preserving a substructure of the original system in the reduced model. A fuzzy logic control is then implemented to produce the suitable PID controller parameters. The parameters are selected to achieve the desired response while maintaining relatively low overshoot, low settling time, and low steady state error. Results of investigations show the achievement of all the considered specifications as seen in the section of illustration and discussion. Results have also been compared with some other methods where outperformance is clearly seen in the presented method.

# **Scientific Ethics Declaration**

The authors declare that the scientific ethical and legal responsibility of this article published in EPSTEM journal belongs to the authors.

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