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RESEARCH ARTICLE

Approximating Higher Order Linear Fredholm Integro-Differential Equations by an Efficient Adomian Decomposition Method

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ABSTRACT
This work presents a unique technique for the precise and efficient solution of Linear Fredholm integro-differential equations (LFDEs), the technique is based on the Modification of Adomian Decomposition Method (MADM). The MADM extends the well-known Adomian Decomposition Method (ADM) by integrating novel changes that improve convergence and computing efficiency. The LFDEs are essential for simulating a wide range of phenomena in science and engineering. Because their analytical solutions are frequently difficult to achieve, the development of efficient and trustworthy numerical approaches is required. We present an introduction of the MADM method and its important characteristics emphasizing its capacity to handle a wide range of LFDEs seen in scientific and engineering applications. We demonstrate the method's usefulness in contrast to the true approach, stressing its computational benefits and precision.

Mathematics Subject Classification (2020): 65R20, 45G15, 45B99, 45D99

Keywords: Fredholm Integro, differential Equations, Numerical Solutions, Computational Efficiency.

1. INTRODUCTION

Linear Fredholm integro-differential equations (IFDEs) are a type of mathematical model used to describe complicated events including both differential and integral elements in a variety of scientific and engineering areas. These equations are critical for understanding and forecasting real-world phenomena including heat conduction, diffusion, population dynamics, and electromagnetic fields. Regardless of their importance, analytical solutions for linear Fredholm IFDEs are frequently elusive, necessitating the development of strong numerical approaches. The Modified Adomian Decomposition Method builds upon the strengths of the original ADM while incorporating innovative adjustments to overcome limitations in convergence behavior and stability. The method involves decomposing the unknown function into a series of auxiliary functions and using a recursive scheme to obtain successive approximations. According to Abdella and Ross (2020); Acar and Dascioglu (2019); Akyuz (2006); Amin et all. (2020), integral equations are categorized into two primary categories based on the limits of integration: Fredholm and Volterra integral equations. Ayinde, James, Ishaq and Oyedepo (2022); Bogdan and Madalina (2021); Buranay, Ozarslan and Falahesar (2021), integro-differential equations are essential in both pure and practical mathematics, having numerous applications in mechanics, engineering, physics, and other fields. The behavior and evolution of many physical systems in many fields of science and engineering, including viscoelasticity, evolutionary problems, fluid dynamics, population dynamics, and many others, can be successfully modeled using Fredholm and Volterra type integrodifferential equations. Davaeeifar and Rashidainia (2017); El-Hawary and El-Sheshtawy (2010); Hosry, Nakad and Bhalekar (2020); Lofti and Alipanah (2020); Kabiru et all. (2023); Kamoh, Gyemang and Soomiyol (2019); Kurkou, Aslan, and Sezer (2017); Kabiru, Morufu and Muideen (2023); Maturi and Simbawa (2020) derived the classical operational matrices and the unknown to be approximated by First Boubaker Polynomials, with Newton-Cotes points serving as collocation points. Ming and Huanga (2017); Mishra et all. (2017) examine the existence, uniqueness, and regularity features of solutions to generic Volterra functional integral equations with non-vanishing delalys, focusing on the local representation. Ogunniran et all. (2022) developed a discrete hybrid block approach and used relevant existing concepts to test its stability, consistency, and convergence. Ogunrinde, Obayomi and Olayemi (2023); Ogunrinde et all. (2020) discussed how the Fredholm integro-differential equation has numerous applications in science, engineering, and all aspects of

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human endeavors, including kinetic theory of gases, geophysics, communication theory, mathematical economics, queuing theory, and hereditary phenomena in physics and biology. Oyedepo et all. (2023, 2022); Ramadan et all. (2016); Sabzevari (2019) proposed a collocation computing approach for solving Volterra-Fredholm integro-differential equations using fourth kind chebyshev polynomials as basis functions. According to Scathar et all. (2020); Shang and Han (2010), integral equations have applications in a variety of domains, including mathematics, physics, and engineering. The analytical solution of integral equations is quite complex, especially for application applications. Tunc (2021) investigated a linear system of integro-delay differential equations with constant time retardation. Wazwaz (2011); Yuksel et all. (2012) defined an integral equation as one in which the unknown function occurs within an integral sign.

2. DEFINITION OF TERMS

Integro-Differential Equations (IDEs): Integro-differential equations are a form of mathematical problem in which the derivatives and integrals of an unknown function are both involved. Integral terms alter the connection between a function and its rate of change in these equations, which are used to simulate a wide range of phenomena in numerous scientific and technical fields.

Fredholm Integro-Differential Equations (FIDEs): Fredholm integro-differential equations are a type of integro-differential equation that involves the use of derivatives and integrals in a mathematical formulation. These equations are named after Erik Ivar Fredholm, a Swedish mathematician who made substantial contributions to integral equations.

Linear Fredholm Integro-Differential Equations (LFIDEs): Linear Fredholm integro-differential equations are a type of integro-differential equation in which the dependent variable and its derivatives are linear. These equations, which combine differential and integral operators in a linear framework, are critical in describing a wide range of phenomena in numerous scientific and engineering fields.

Adomian Decomposition Method (ADM): The Adomian Decomposition Method is a strong analytical approach for solving nonlinear ordinary, partial differential, and integral problems. This approach, named after its originator, George Adomian, seeks approximate solutions by decomposing a given nonlinear differential equation into an endless sequence of smaller terms that may then be solved systematically.

Modified Adomian Decomposition Method (MADM): The Modified Adomian Decomposition Method improves and modifies the original Adomian Decomposition Method (ADM). It is intended to overcome some constraints and improve the ADM's convergence behavior when used to specific sorts of problems. The MADM modifies the algorithm in order to improve its efficiency and reliability for solving nonlinear ordinary and partial differential equations, as well as integral equations.

3. METHOD

To improve on the accuracies and subsequently the convergence of these approaches, we shall based our assumption on the decomposition of the source term $h(x)$ in Taylors series of the form

$$s(x) = \sum_{j=0}^{+\infty} h_i(x) \tag{1}$$

and the new recursive relation obtained as:

$$y_0(x) = k_0(x), \tag{2}$$

$$y_1(x) = k_1(x) + k_2(x) + \lambda \int_a^x h(x,t)\,(L(u_0(x)) + A_0))\,dt, \tag{3}$$

$$y_2(x) = k_3(x) + k_4(x) + \lambda \int_a^x \mathrm{h}(x,t)\,(L(u_1(x)) + A_1))\,dt, \tag{4}$$

$$\vdots$$

$$y_{j+1}(x) = k_{2(j+1)}(x) + k_{2(j+1)-1}(x) + \lambda \int_a^x \mathrm{h}(x,t)\,\left(L(u_j(x)) + \mathrm{A_j}\right)\right)dt. \tag{5}$$

And subsequently the function $u(x)$ is obtained as

$$y(x) = \sum_{j=0}^{+\infty} y_j(x). \tag{6}$$

Assuming that the nonlinear function is $P(y(x))$ therefore, below are few of the Adomian polynomials.

$$A{=}P(y_0), \tag{7}$$

$$A_1{=}y_1P'(y_0), \tag{8}$$

$$A_2{=}y_2P'(y_0)+\frac{1}{2!}y_1^2P''(y_0) \tag{9}$$

$$A_3{=}y_3P'(y_0)+y_1y_2P''(y_0)+\frac{1}{3!}y_1^3P'''(y_0), \tag{10}$$

$$A_4{=}y_4P'(y_0)+\left(\frac{1}{2!}y_2^2+y_1y_3\right)P''(y_0)+\frac{1}{2!}y_1^2y_2P'''(y_0)+\frac{1}{4}y_1^4P^{(iv)}(y_0). \tag{11}$$

Two important observations can be made here. First, A_0 depends only on y_0, A_1 depends only on y_0 and y_1, A_2 depends on y_0, y_1 and y_2, and so on. Secondly, substituting these $A_{j's}$ gives:

$$P(u)=A_0+A_1+A_2+A_3+\cdots$$

$$=P(y_0)+(y_1+y_2+y_3+\cdots)P'(y_0)+\frac{1}{2!}(y_1^2+2y_1y_2+2y_1y_3+y_2^2)P''(y_0)$$

$$+\frac{1}{3!}(y_1^3+3y_1^2y_3+6y_1y_2y_3+\cdots)P'''(y_0)+\cdots$$

$$=P(y_0)+(y-y_0)P'(y_0)+\frac{1}{2!}(y-y_0)^2P''(y_0)+\cdots$$

4. NUMERICAL EXAMPLES

Example 1: Bogdan and Madalina (2021) Consider the eighth-order linear Fredholm integro-differential equation

$$y^{(8)}(x)=y(x)-8e^x+x^2+\int_0^1 x^2y'(t)\,dt \tag{12}$$

Subject to the conditions $y(0)=1$, $y'(0)=0$, $y''(0)=-1$, $y'''(0)=-2$, $y^{(4)}(0)=-3$, $y^{(5)}(0)=-4$, $y^{(6)}(0)=-5$, and $y^{(7)}(0)=-6$.
The exact solution is $y(x)=(1-x)e^x$
Using the new Modified Adomian Decomposition Method (MADM),
We transform each term in (12) to have the following

$$\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x y^{(8)}(x)\,dxdxdxdxdxdxdxdx$$

$$=y(x)+\frac{1}{840}x^7+\frac{1}{144}x^6+\frac{1}{30}x^5+\frac{1}{8}x^4+\frac{1}{3}x^3+\frac{1}{2}x^2-1 \tag{13}$$

$$\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x y(x)\,dxdxdxdxdxdxdxdx=$$

$$\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x y(t)\,dtdxdxdxdxdxdxdx \tag{14}$$

$$\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x 8e^x\,dxdxdxdxdxdxdxdx$$

$$=-8-8x-4x^2-\frac{4}{3}x^3-\frac{1}{3}x^4-\frac{1}{15}x^5-\frac{1}{90}x^6-\frac{1}{630}x^7+8e^x \tag{15}$$

$$\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x x^2\,dxdxdxdxdxdxdxdx=\frac{1}{1814400}x^{10} \tag{16}$$

$$\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^1 x^2y'(t)\,dtdxdxdxdxdxdxdxdx =$$

$$\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^1 x^2y'(t)\,dtdxdxdxdxdxdxdxdx$$

Substitute the results of (13) – (4) into (12)
We have

$$y(x) = 9 + 8x + \tfrac{7}{2}x^2 + x^3 + \tfrac{5}{24}x^4 + \tfrac{1}{30}x^5 + \tfrac{1}{240}x^6 + \tfrac{1}{2520}x^7 + \tfrac{1}{1814400}x^{10} - 8e^x$$
$$+\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x y(t)\,dtdxdxdxdxdxdxdx$$

$$+\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^1 x^2y'(t)\,dtdxdxdxdxdxdxdxdx \tag{17}$$

Let

$$r = 9 + 8x + \frac{7}{2}x^2 + x^3 + \frac{5}{24}x^4 + \frac{1}{30}x^5 + \frac{1}{240}x^6 + \frac{1}{2520}x^7 + \frac{1}{1814400}x^{10} - 8e^x$$

Then
Expand taylor (r, x, 10)

$$= 1 - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{8}x^4 - \frac{1}{30}x^5 - \frac{1}{144}x^6 - \frac{1}{840}x^7 - \frac{1}{5040}x^8 - \frac{1}{45360}x^9$$

And

$$a_0 = 1$$

$$y_0(t) = 1$$

$$y_0'(t) = 0$$

$$g_0 = -\frac{1}{2}x^2 - \frac{1}{3}x^3$$

$$a_1 = g_0 + \int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x y_0(t)\,dtdxdxdxdxdxdxdx$$

$$+\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^1 x^2y_0'(t)\,dtdxdxdxdxdxdxdxdx \tag{18}$$

$$a_1 = -\frac{1}{2}x^2 - \frac{1}{3}x^3 + \frac{1}{40320}x^8$$

$$y_1(t) = -\frac{1}{2}t^2 - \frac{1}{3}t^3 + \frac{1}{40320}t^8$$

$$y_1'(t) = -t - t^2 + \frac{1}{5040}t^7$$

$$g_1 = -\frac{1}{8}x^4 - \frac{1}{30}x^5$$

$$a_2 = g_1 + \int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x y_1(t)\,dtdxdxdxdxdxdxdx$$

$$+\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x\int_0^1 x^2y_1'(t)\,dtdxdxdxdxdxdxdxdx \tag{19}$$

$$a_2 = -\frac{1}{8}x^4 - \frac{1}{30}x^5 - \frac{53759}{73156608000}x^{10} - \frac{1}{19958400}x^{11} + \frac{1}{20922789888000}x^{16}$$

$$y_2(t) = -\frac{1}{8}t^4 - \frac{1}{30}t^5 - \frac{53759}{73156608000}t^{10} - \frac{1}{19958400}t^{11} + \frac{1}{20922789888000}t^{16}$$

$$y_2'(t) = -\frac{1}{2}t^3 - \frac{1}{6}t^4 - \frac{53759}{7315660800}t^9 - \frac{1}{1814400}t^{10} + \frac{1}{1307674368000}t^{15}$$

$$g_2 = -\frac{1}{144}x^6 - \frac{1}{840}x^7$$

$$a_3 = g_2 + \int_0^x \int_0^x \int_0^x \int_0^x \int_0^x \int_0^x \int_0^x \int_0^x y_2(t)\, dt dx dx dx dx dx dx dx$$

$$+ \int_0^x \int_0^x \int_0^x \int_0^x \int_0^x \int_0^x \int_0^x \int_0^x \int_0^1 x^2 y_2'(t)\, dt dx dx dx dx dx dx dx dx \tag{20}$$

$$a_3 = -\tfrac{1}{144}x^6 - \tfrac{1}{840}x^7 - \tfrac{1}{159667200}x^{12} - \tfrac{1}{1556755200}x^{13} - \tfrac{53759}{129071853907476480000}x^{18} - \tfrac{1}{60822550204416000}x^{19} + \tfrac{1}{620448401733239439360000}x^{24} - \tfrac{473255926999}{542318713896960000}x^{10}$$

$$y_3(t) = -\tfrac{1}{144}t^6 - \tfrac{1}{840}t^7 - \tfrac{1}{159667200}t^{12} - \tfrac{1}{1556755200}t^{13} - \tfrac{53759}{129071853907476480000}t^{18} - \tfrac{1}{60822550204416000}t^{19} + \tfrac{1}{620448401733239439360000}t^{24} - \tfrac{473255926999}{542318713896960000}t^{10}$$

$$y_3'(t) = -\tfrac{1}{24}t^5 - \tfrac{1}{120}t^6 - \tfrac{1}{13305600}t^{11} - \tfrac{1}{119750400}t^{12} - \tfrac{53759}{7170658550415360000}t^{17} - \tfrac{1}{3201186852864000}t^{18} + \tfrac{1}{25852016738884976640000}t^{23} - \tfrac{473255926999}{54231871389696000}t^{0}$$

$$g_3 = -\frac{1}{5040}x^8 - \frac{1}{45360}x^9$$

$$a_4 = g_3 + \int_0^x \int_0^x \int_0^x \int_0^x \int_0^x \int_0^x \int_0^x \int_0^x y_3(t)\, dt dx dx dx dx dx dx dx$$

$$+ \int_0^x \int_0^x \int_0^x \int_0^x \int_0^x \int_0^x \int_0^x \int_0^x \int_0^1 x^2 y_3'(t)\, dt dx dx dx dx dx dx dx dx \tag{21}$$

$$a_4 = -\tfrac{1}{5040}x^8 - \tfrac{1}{45360}x^9 - \tfrac{1}{17435658240}x^{14} - \tfrac{1}{217945728000}x^{15} - \tfrac{1}{810967336058880000}x^{20} - \tfrac{1}{127727355429273600000}x^{21} - \tfrac{53759}{813035585631236961337344000000000}x^{26} - \tfrac{1}{5444434725209176080384000000}x^{27} + \tfrac{1}{263130836933693530167218012160000000}x^{32} - \tfrac{473255926999}{9568251416385920434176000000}x^{18} - \tfrac{25236784671012694969327}{56287079005239481938739200000000}x^{10}$$

$$y_4(t) = -\tfrac{1}{5040}t^8 - \tfrac{1}{45360}t^9 - \tfrac{1}{17435658240}t^{14} - \tfrac{1}{217945728000}t^{15} - \tfrac{1}{810967336058880000}t^{20} - \tfrac{1}{127727355429273600000}t^{21} - \tfrac{53759}{813035585631236961337344000000000}t^{26} - \tfrac{1}{5444434725209176080384000000}t^{27} + \tfrac{1}{263130836933693530167218012160000000}t^{32} - \tfrac{473255926999}{9568251416385920434176000000}t^{18} - \tfrac{25236784671012694969327}{56287079005239481938739200000000}t^{10} \tag{22}$$

$$y_n(x) = \sum_{j=0}^{4} y_j(x)$$

$$y_n(x) = 1 - \tfrac{1}{2}x^2 - \tfrac{1}{3}x^3 - \tfrac{1}{8}x^4 - \tfrac{1}{30}x^5 - \tfrac{1}{144}x^6 - \tfrac{1}{840}x^7 - \tfrac{1}{5760}x^8 - \tfrac{1}{45360}x^9 - \tfrac{46526732755175563848541270}{56287079005239481938739200000000}x^{10} - \tfrac{1}{19958400}x^{11} - \tfrac{1}{159667200}x^{12} - \tfrac{1}{1556755200}x^{13} - \tfrac{1}{17435658240}x^{14} - \tfrac{1}{217945728000}x^{15} + \tfrac{1}{20922789888000}x^{16} - \tfrac{4458475107799}{9568251416385920434176000000}x^{18} - \tfrac{1}{60822550204416000}x^{19} - \tfrac{1}{810967336058880000}x^{20} - \tfrac{1}{127727355429273600000}x^{21} + \tfrac{1}{620448401733239439360000}x^{24} - \tfrac{53759}{813035585631236961337344000000000}x^{26} - \tfrac{1}{5444434725209176080384000000}x^{27} + \tfrac{1}{263130836933693530167218012160000000}x^{32} \tag{23}$$

Example 2: Oyedepo et all. (2023) Consider the fifth-order linear Fredholm integro-differential equation

$$y^{v}(x) - x^2 y'''(x) - y'(x) - wy(x) = w^2 \cos(x) - x\sin(x) + \int_{-1}^{1} y(t)\,dt \tag{24}$$

Subject to the conditions $y(0) = 0$, $y'(0) = 1$, $y''(0) = 0$, $y'''(0) = -1$ and $y^{iv}(0) = -1$.
The exact solution is $y(x) = \sin(x)$
Using the new Modified Adomian Decomposition Method (MADM),
We obtained the following

$$y_0(x) = x \tag{25}$$

$$y_1(x) = -\frac{1}{6}x^3 - \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}wx^6 \tag{26}$$

$$\begin{aligned} y_2(x) &= \tfrac{1}{120}w^2x^5 + \left(-\tfrac{1}{5040}w^2 - \tfrac{1}{2520}\right)x^7 + \tfrac{1}{113400}wx^{10} + \tfrac{1}{17280}x^9 - \tfrac{13}{40320}x^8 - \tfrac{1}{720}x^7 \\ &+ \tfrac{1}{39916800}w^2x^{11} - \tfrac{1}{362880}wx^9 - \tfrac{1}{40320}wx^8 + \tfrac{1}{5}\left(-\tfrac{1}{1440} + \tfrac{1}{60480}w\right)x^5 \end{aligned} \tag{27}$$

Then,

$$y_n(x) = \sum_{j=0}^{2} y_j(x)$$

$$\begin{aligned} y_n(x) &= x - \tfrac{1}{6}x^3 - \tfrac{1}{24}x^4 + \tfrac{1}{120}x^5 + \tfrac{1}{720}wx^6 + \tfrac{1}{120}w^2x^5 + \left(-\tfrac{1}{5040}w^2 - \tfrac{1}{2520}\right)x^7 + \tfrac{1}{113400}wx^{10} \\ &+ \tfrac{1}{17280}x^9 - \tfrac{13}{40320}x^8 - \tfrac{1}{720}x^7 + \tfrac{1}{39916800}w^2x^{11} - \tfrac{1}{362880}wx^9 - \tfrac{1}{40320}wx^8 + \tfrac{1}{5}\left(-\tfrac{1}{1440} + \tfrac{1}{60480}w\right)x^5 \end{aligned} \tag{28}$$

When $w = 0$,
We have

$$y_n(x) = x - \frac{1}{6}x^3 - \frac{1}{24}x^4 + \frac{59}{7200}x^5 - \frac{1}{560}x^7 - \frac{13}{40320}x^8 + \frac{1}{17280}x^9 \tag{29}$$

Example 3: Ogunrinde, Obayomi and Olayemi (2023) Consider the third-order linear Fredholm integro-differential equation

$$y'''(x) = 6 + x - \int_{0}^{1} xy'(t)\,dt \tag{30}$$

Subject to the conditions $y(0) = -1$, $y'(0) = 1$, and $y''(0) = -2$.
The exact solution is $y(x) = x^3 - x^2 + x - 1$
Using the new Modified Adomian Decomposition Method (MADM),
We obtained the following

$$y_0(x) = -1 \tag{31}$$

$$y_1(x) = x - x^2 \tag{32}$$

$$y_2(x) = x^3 + \frac{1}{8}x^4 \tag{33}$$

$$y_3(x) = -\frac{7}{48}x^4 \tag{34}$$

Then,

$$y_n(x) = \sum_{j=0}^{3} y_j(x)$$

$$y_n(x) = -1 + x - x^2 + x^3 - \frac{1}{48}x^4 \tag{35}$$

5. TABLES OF RESULTS

Table 1: Numerical Results for Example 1

x	Exact	MADM	MADM_Error
0	1.000000	1.000000	0.00 E 00
0.1	0.994654	0.994654	0.00 E 00
0.2	0.977122	0.977122	1.73 E-13
0.3	0.944901	0.944901	1.01 E-11
0.4	0.895095	0.895095	1.82 E-10
0.5	0.824361	0.824361	1.72 E-09
0.6	0.728848	0.728848	1.08 E-08
0.7	0.604126	0.604126	5.09 E-08
0.8	0.445108	0.445108	1.96 E-07
0.9	0.245960	0.245961	6.45 E-07
1	0.000000	1.87 E-06	1.87 E-06

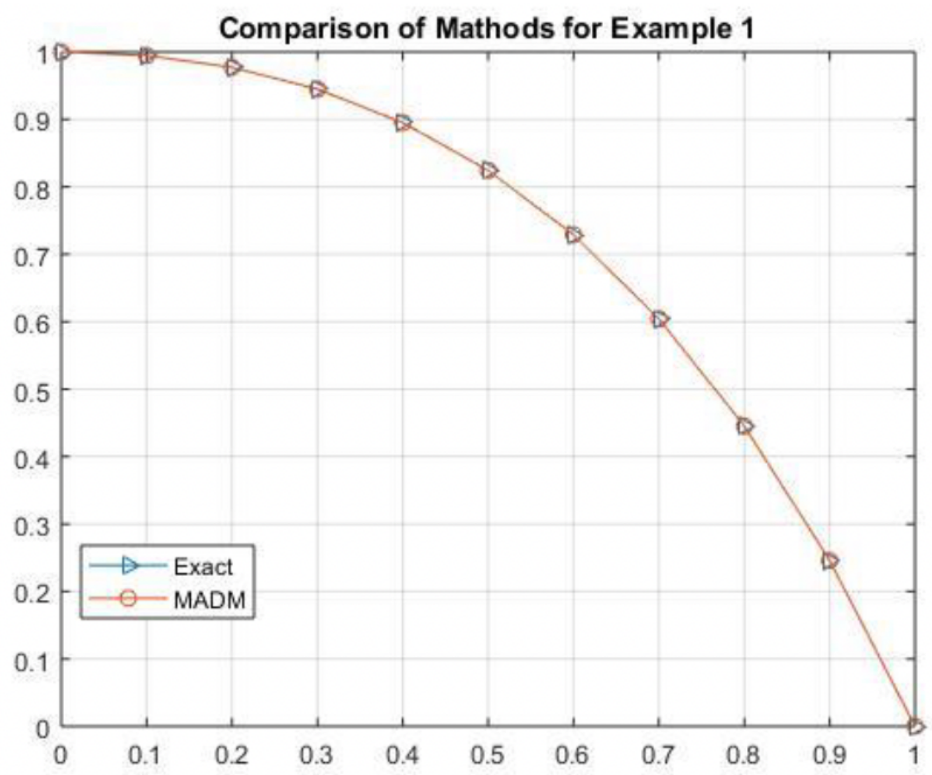

Figure 1. Graph of Comparison for Example 1

Table 2: Numerical Results for Example 2

x	Exact	MADM	MADM_Error
0	0.000000	0.000000	0.0000 E 00
0.1	0.099833	0.099829	4.1700 E-06
0.2	0.198669	0.198603	6.6700 E-05
0.3	0.295520	0.295183	3.3800 E-04
0.4	0.389418	0.388350	1.0680 E-03
0.5	0.479426	0.476812	2.6130 E-03
0.6	0.564642	0.559204	5.4380 E-03
0.7	0.644218	0.634090	1.0128 E-02
0.8	0.717356	0.699955	1.7401 E-02
0.9	0.783327	0.755195	2.8132 E-02
1	0.841471	0.798089	4.3382 E-02

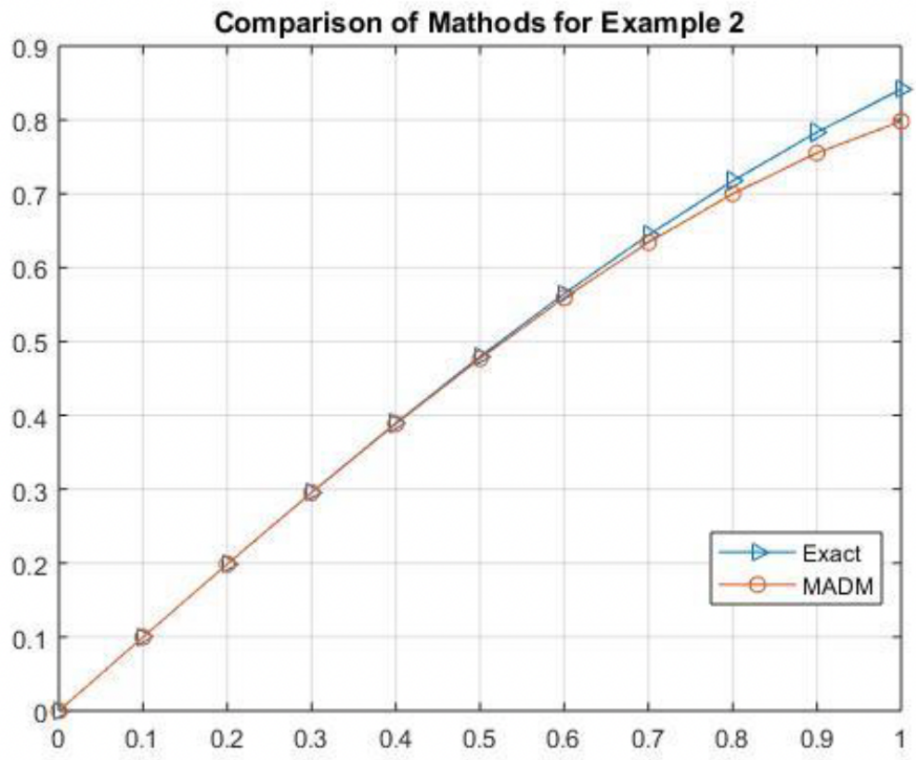

Figure 2. Graph of Comparison for Example 2

Table 3: Numerical Results for Example 3

X	Exact	MADM	MADM_Error
0	-1.000	-1.00000	0.0000 E 00
0.1	-0.909	-0.90900	2.0800 E-06
0.2	-0.832	-0.83203	3.3300 E-05
0.3	-0.763	-0.76317	1.6900 E-04
0.4	-0.696	-0.69653	5.3300 E-04
0.5	-0.625	-0.62630	1.3020 E-03
0.6	-0.544	-0.54670	2.7000 E-03
0.7	-0.447	-0.45200	5.0020 E-03
0.8	-0.328	-0.33653	8.5330 E-03
0.9	-0.181	-0.19467	1.3669 E-02
1	0.000	-0.02083	2.0833 E-02

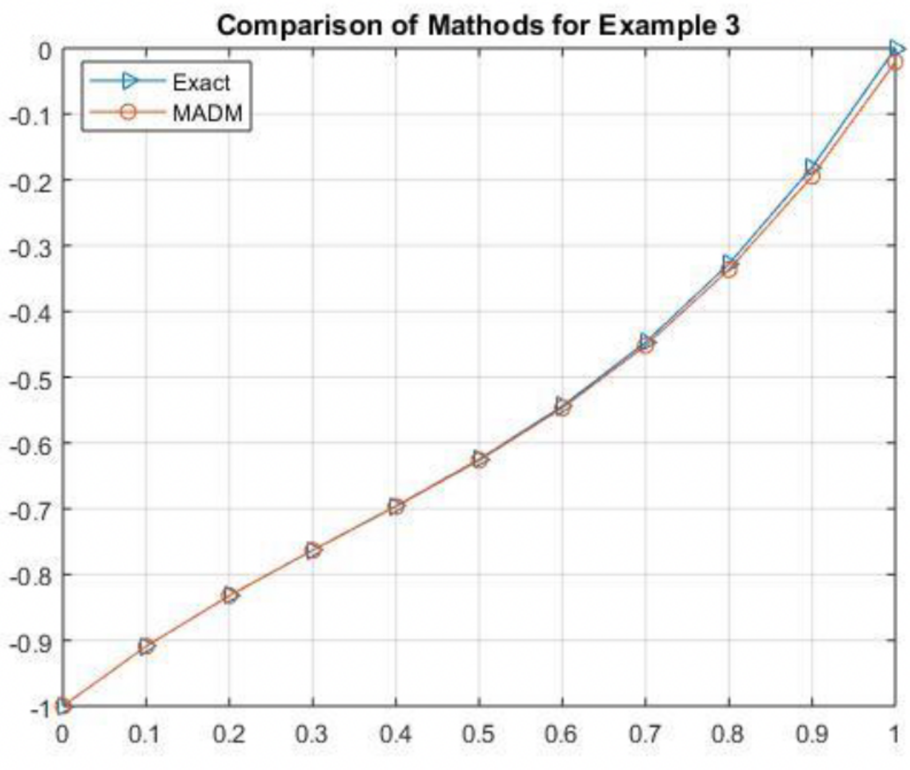

Figure 3. Graph of Comparison for Example 3

6. DISCUSSION OF RESULTS

The study of LFIDE findings using the Modified Adomian Decomposition Method demonstrates its efficacy in resolving problems associated with older methodologies. The method's improved convergence, stability, and adaptability make it an appealing tool for academics and practitioners working on linear Fredholm integro-differential equation issues. The findings given here add to the expanding body of knowledge on appropriate numerical strategies for solving complicated mathematical models in a variety of scientific and engineering disciplines.

7. CONCLUSION

The use of MADM to LFIDEs yields encouraging results, with enhanced accuracy and stability over standard approaches. The method's capacity to manage a wide spectrum of LFIDEs seen in scientific and engineering applications is emphasized, highlighting its adaptability and dependability. This study's numerical studies and comparisons give solid proof of the MADM's effectiveness. The convergence evaluations validate the method's resilience in addressing LFIDEs, giving it a viable option for academics and practitioners looking for accurate and efficient solutions to complicated issues in a variety of domains.

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