Fundamentals of Contemporary Mathematical Sciences
doi:10.54974/fcmathsci.1561405
(**2025**) **6**(2) 108 – 122
Research Article

A New Fractional Order School Academic Performance Model and Numerical Solutions

Zafer Öztürk *
Aksaray University, Aksaray, Türkiye

Received: 04 October 2024 **Accepted:** 08 April 2025

Abstract: Academic achievement is defined as the degree to which a student has achieved a learning goal. It is typically measured through the utilisation of examinations, continuous assessments and grade point averages. The student's apprehension of failure can result in the accumulation of stress over time, which can consequently lead to a decline in academic achievement. Conversely, factors such as inadequate cognitive abilities, negative parental influence, familial circumstances and the physical and mental health of the child have been identified as the primary contributors to academic achievement. The present study proposes a novel fractional order mathematical model of academic achievement, comprising three compartments: students with above average achievement (S), students with average achievement (M) and students with below average achievement (B). The Caputo derivative definition was employed as the fractional derivative and a stability analysis of the fractional model was conducted. Numerical solutions were obtained via the Generalized Euler Method and their graphs were drawn.

Keywords: Fractional order school academic performance model, mathematical modeling, generalized Euler method, Caputo derivative, stability analysis.

1. Introduction

A substantial corpus of research has been dedicated to the study of students' behavior and learning, with a particular focus on personal characteristics such as intelligence, cognitive style, motivation, personality, self-concept, and locus of control. It has been noted by several institutions that certain factors related to students' behavior are perceived to contribute to academic failure. Consequently, the management of these institutions has adopted a serious approach to address these issues [20, 26]. Self-regulated learning strategies are important for individuals to be successful lifelong learners. It also provides them with the opportunity to manage their own learning processes [1, 17]. In particular, the impact of self-regulation skills on learners' acquisition of learning strategies merits consideration as an indisputable attribute. Self-regulated learning strategies

*Correspondence: zaferozturk@aksaray.edu.tr (Independent Researcher as an Administrative Staff with a Ph.D.)
2020 *AMS Mathematics Subject Classification*: 34A08, 92B05

refer to the capacity of individuals to exercise definitive control over their own learning processes, independently managing their learning in the absence of external influences. Consequently, self-regulation processes represent a subject of paramount significance within the domain of educational research [2, 4].

The primary issue that was addressed in the studies conducted during the 1980-1990 period pertains to the relationship between students' knowledge in a specific academic domain and their cognitive, meta-cognitive abilities, and motivation. The research undertaken during this era has yielded significant insights into the components of self-regulated learning. The cognitive and motivational strategies employed by students, the types of goals adapted for learning tasks and beliefs concerning the fulfillment of learning tasks, success and failure attributions have been identified as notable issues. The definition of self-regulatory learning strategies is generally understood to emphasize the learner's state of being active in terms of motivation, metacognition and behavior in the learning process, with the teacher playing a pivotal role in the acquisition of self-regulated learning strategies, particularly in the context of designing and implementing teaching activities in the classroom. Teachers can influence students' self-regulation skills through the strategies, methods and techniques they employ in the classroom. For instance, the development of meta-cognitive regulation skills in students who receive continuous teacher support is at a low level, underscoring the importance of teacher attention to their teaching activities. Additionally, the classroom atmosphere created by the teacher plays a pivotal role. Teachers who employ democratic, student-centered and active teaching practices in the classroom can positively influence students' motivation levels and their skills related to self-regulatory learning strategies [2, 4, 11].

Mathematical models can be defined as simplified representations of a real system or known process. They are utilised for the purpose of expressing observations or measurements of events, interactions and behaviours in a compact form, explaining them, predicting events or outcomes that have not yet been observed and designing systems that are intended to exhibit certain behaviours. The developed fractional order mathematical model will facilitate the determination of the main factors that play a role in determining the academic achievement levels of students. This will include the determination of the extent to which they are effective and their relations with each other, the level of academic failure in schools, and the obtaining of important findings on how to prevent failure [6, 7, 18, 21].

The employment of fractional order derivatives in the control theory of diverse physical and biological processes and dynamical systems has been shown to yield superior outcomes in comparison to the utilisation of integer order derivatives. One of the most significant reasons for this is that fractional order derivatives and integral definitions possess a memory property. Furthermore, the model remains identical, yet the fractional orders of the equations vary in each real-world application, thereby yielding specific and precise results for the pertinent problem. In population models, for instance, the future state of a population is contingent upon its past state, a phenomenon referred to as the 'memory effect'. The incorporation of a delay term or the utilisation of a fractional derivative within the model facilitates the analysis of the memory effect of the population. Fractional calculus, incorporating fractional derivatives and fractional integrals, has recently garnered heightened interest among researchers in the field. It has been determined that fractional operators offer a more precise and efficient characterisation of system behaviour in comparison to integer order derivatives. In view of the substantial advantages of fractional derivatives with regard to memory properties, the present system is modified by substituting the integer order time derivative with the Caputo fractional derivative [5–8, 11, 12, 18, 21].

The utilisation of fractional derivative operators, particularly non-local fractional derivatives, facilitates a more comprehensive investigation of these intricate systems. The majority of research domains pertain to supercomplex mechanisms comprising highly intricate and non-linear differential equations. To enhance comprehension, a range of fractional derivatives, encompassing singular and non-singular kernels, are employed. Through comparative analysis, the fractional derivative that yields the optimal result is identified and employed. To ensure the most accurate determination, real-life data are necessary, as the derivative exhibiting the closest behaviour to real-life data is determined as the derivative that gives the best result [3, 5, 8–10, 12–16, 19, 22–25, 27].

This paper is divided into four sections. The initial section outlines the significance of fractional mathematical modeling and the prevailing academic context within educational institutions. The second part of the paper presents the formation of a fractional order academic achievement model in schools, together with a mathematical analysis of the existence, uniqueness and non-negativity of the system, the Generalised Euler Method and a stability analysis of the model. The third section introduces a new application of the academic achievement model in fractional order schools, presents the numerical results and draws graphs. The fourth section concludes the paper.

2. Fractional Derivative and Fractional Order School Academic Performance Model

The most commonly used definitions of the fractional derivative are Riemann-Liouville, Caputo, Atangana-Baleanu and the conformable derivative. In this study, because the classical initial conditions are easily applicable and provide ease of calculation, the Caputo derivative operator was preferred and modeling was created. The definition of the Caputo fractional derivative is given below.

Definition 2.1 *[17] Let $f(t)$ be a function that is continuously differentiable n times. The value of the function $f(t)$ for α satisfying $n-1<\alpha<n$. The Caputo fractional derivative of order α*

of $f(t)$ is defined by

$$D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t (t-x)^{n-\alpha-1} f^{(n)}(x)\, dx.$$

Definition 2.2 *[17] The Riemann–Liouville (RL) fractional-order integral of a function $A(t) \in C_n$ $(n \geq -1)$ is given by*

$$J^{\gamma} A(t) = \frac{1}{\Gamma(\gamma)} \int_0^t (t-s)^{\gamma-1} A(s)\, ds, \quad J^0 A(t) = A(t).$$

Definition 2.3 *[17] The series expansion of the two-parameter Mittag–Leffler function for $a, b > 0$ is given by*

$$E_{a,b}(t) = \sum_{i=0}^{\infty} \frac{t^i}{\Gamma(ai+b)}.$$

2.1. Fractional Order School Academic Performance Model

The fractional order model of academic achievement in schools basically categorises a class into three main groups. The first one is students with above average achievement, the second one is students with average achievement and the third one is students with below average achievement. The expression of the academic achievement model in schools as a system of fractional differential equations is as follows:

$$\begin{aligned} \frac{d^{\alpha} S}{dt^{\alpha}} &= \mu N - \mu S - \beta S + \sigma S, \\ \frac{d^{\alpha} M}{dt^{\alpha}} &= \beta S - \mu M - \gamma M, \\ \frac{d^{\alpha} B}{dt^{\alpha}} &= \gamma M - \mu B - \delta B. \end{aligned} \tag{1}$$

Here $\frac{d^{\alpha}}{dt^{\alpha}}$ is the Caputo fractional derivative with respect to time t and $0 < \alpha \leq 1$. Initial values are given as,

$$S(0) = S_0, \; M(0) = M_0, \; B(0) = B_0$$

it is defined as. Since the society is divided into three compartments, $S + M + B = N$ with the derivation of all terms according to time

$$\frac{d^{\alpha} N}{dt^{\alpha}} = \frac{d^{\alpha} S}{dt^{\alpha}} + \frac{d^{\alpha} M}{dt^{\alpha}} + \frac{d^{\alpha} B}{dt^{\alpha}}.$$

In time-dependent phenomena, fractional order models are more realistic and accurate than integer order models because they possess a memory feature [3, 5, 8–10, 12–16, 19, 22–25, 27]. For $\alpha = 1$ in system (1), the fractional order differential equation is reduced to a full order differential equation. The compartment and parameters of the spell are shown in Table 1 and Table 2.

Table 1: Variables used in the systems and their meanings

Variables used in the systems	Meaning
$S(t)$	Students with above average achievement at time t
$M(t)$	Students with average achievement at time t
$B(t)$	Students with below average achievement at time t
$N(t)$	Total classroom

Table 2: Parameters and their meanings

Parameters	Meaning
β	Rate of negative teacher attitude
μ	Academic motivation rate
σ	Positive family relationship rate
δ	Negative family relationship rate
γ	Low rate of self-efficacy

The parameters defined in the model do not change with time. The N term was dimensionlessised and the new variables were created as follows:

$$s = \frac{S}{N},\ m = \frac{M}{N},\ b = \frac{B}{N}.$$

It is clear from here that $s + m + b = 1$. Thus, the new form of the academic achievement model in fractional order schools is written as follows:

$$\begin{aligned} D^{\alpha} s(t) &= \mu - \mu s(t) - \beta s(t) + \sigma s(t), \\ D^{\alpha} m(t) &= \beta s(t) - \mu m(t) - \gamma m(t), \\ D^{\alpha} b(t) &= \gamma m(t) - \mu b(t) - \delta b(t). \end{aligned} \tag{2}$$

2.2. Existence, Uniqueness and Non-Negativity of the System

We investigate the exintence and uniqueness of the solution of the fractional-order system (1) in the region $C \times [t_0, T]$ where

$$C = \{(S, M, B) \in R_+^3 : max\{| S |, | M |, | B |\} \leq \Psi, min\{| S |, | M || B | \geq \Psi_0\} \tag{3}$$

and $T < +\infty$.

Theorem 2.4 *For each $H_0 = (S_0, M_0, B_0) \in C$, there exists a unique solution $H(t) \in C$ of the fractional-order system (1) with intial condition H_0, which is defined for all $t \geq 0$.*

Proof We denote $H=(S,M,B)$ and $\bar{H}=(\bar{S},\bar{M},\bar{B})$. Consider a mapping $X(H)=(X_1(H),X_2(H),X_3(H))$ and

$$\begin{aligned} X_1(H) &= \mu N - \mu S - \beta S + \sigma S, \\ X_2(H) &= \beta S - \mu M - \gamma M, \\ X_3(H) &= \gamma M - \mu B - \delta B. \end{aligned} \tag{4}$$

For any $H, \bar{H} \in C$, it follows from (4) that

$$\| X(H) - X(\bar{H}) \| = | X_1(H) - X_1(\bar{H}) | + | X_2(H) - X_2(\bar{H}) | + | X_3(H) - X_3(\bar{H}) | \tag{5}$$

and

$$\begin{aligned} | X_1(H) - X_1(\bar{H}) | &= | \mu N - \mu S - \beta S + \sigma S - \mu N + \mu \bar{S} + \beta \bar{S} - \sigma \bar{S} | \\ &= | -\mu(S-\bar{S}) - \beta(S-\bar{S}) + \sigma(S-\bar{S}) | \\ &\leq \mu | S-\bar{S} | + \beta | S-\bar{S} | + \sigma | S-\bar{S} |, \end{aligned}$$

$$\begin{aligned} | X_2(H) - X_2(\bar{H}) | &= | \beta S - \mu M - \gamma M - \beta \bar{S} + \mu \bar{M} + \gamma \bar{M} | \\ &= | \beta(S-\bar{S}) - \mu(M-\bar{M}) - \gamma(M-\bar{M}) | \\ &\leq \beta | S-\bar{S} | + \mu | M-\bar{M} | + \gamma | M-\bar{M} |, \end{aligned}$$

$$\begin{aligned} | X_3(H) - X_3(\bar{H}) | &= | \gamma M - \mu B - \delta B - \gamma \bar{M} + \mu \bar{B} + \delta \bar{B} | \\ &= | \gamma(M-\bar{M}) - \mu(B-\bar{B}) - \delta(B-\bar{B}) | \\ &\leq \gamma | M-\bar{M} | + \mu | B-\bar{B} | + \delta | B-\bar{B} |. \end{aligned}$$

Then, (4) becomes,

$$\|X(H) - X(\bar{H})\| \leq \mu|S - \bar{S}| + \beta|S - \bar{S}| + \sigma|S - \bar{S}| + \beta|S - \bar{S}|$$
$$+ \mu|M - \bar{M}| + \gamma|M - \bar{M}| + \gamma|M - \bar{M}|$$
$$+ \mu|B - \bar{B}| + \delta|B - \bar{B}|$$
$$\leq (\mu + \sigma + 2\beta)|S - \bar{S}| + (\mu + 2\gamma)|M - \bar{M}| + (\mu + \gamma)|B - \bar{B}|,$$

$$\| X(H) - X(\bar{H}) \| \leq L \| H - \bar{H} \|,$$

where $L = max(\mu + \sigma + 2\beta, \mu + 2\gamma, \mu + \gamma)$.

Therefore, $X(H)$ obeys Lipschitz condition which implies the existence and uniqueness of solution of the fractional-order system (1). □

Theorem 2.5 *For all* $t \geq 0$, $S(0) = S_0 \geq 0$, $M(0) = M_0 \geq 0$, $B(0) = B_0 \geq 0$, *the solution of the system (1) with initial conditions* $(S(t), M(t), B(t)) \in R^3_+$ *are not negative.*

Proof (Generalized Mean Value Theorem) Let $f(x) \in C[a,b]$ and $D^\alpha f(x) \in C[a,b]$ for $0 < \alpha \leq 1$. Then, we have

$$f(x) = f(\alpha) + \frac{1}{\Gamma(\alpha)} D^\alpha f(\epsilon)(x-a)^\alpha \tag{6}$$

with $0 \leq \epsilon \leq x$ for all $x \in (a,b]$.

The existence and and uniqueness of the solution of the system (1) in $(0, \infty)$ can be obtained via Generalized Mean Value Theorem. We need to show that the domain R^3_+ is positively invariant. Since

$$D^\alpha S = \mu N - \mu S - \beta S + \sigma S \geq 0,$$
$$D^\alpha M = \beta S - \mu M - \gamma M \geq 0,$$
$$D^\alpha B = \gamma M - \mu B - \delta B \geq 0$$

on each hyperplane bounding the nonnegative orthant, the vector field points into R^3_+. □

2.3. Stability Analysis of the Fractional Order School Academic Performance Model

Definition 2.6 *That the equilibrium point of the first-order difference equation system given as*

$$X_{t+1} = F(X_t) \tag{7}$$

is the point $\bar{X}$ *that satisfies the equations* $\bar{X} = F(\bar{X})$. *Also, let us consider* $J(\bar{X})$ *to be the Jacobian matrix calculated at this equilibrium point. If the eigenvalues obtained from the equation*

$det(J(\bar{X})-\lambda I)=0$ satisfy the conditions $\lambda_i \neq 1$ for $i=1,2,\ldots,n$ then this point is called hyperbolic equilibrium, otherwise it is called non-hyperbolic equilibrium [18].

In order to find the equilibrium point in the system (2), $D^{\alpha}s=0$, $D^{\alpha}m=0$, $D^{\alpha}b=0$ it is considered to be.

$E_0=(s_0,m_0,b_0)$ including,

$$E_0=\left(\frac{\mu}{\beta+\mu+\sigma},\frac{\mu\beta}{(\beta+\mu+\sigma)(\mu+\gamma)},\frac{\mu\beta\gamma}{(\beta+\mu+\sigma)(\mu+\gamma)(\mu+\delta)}\right) \tag{8}$$

the equilibrium point of the system is obtained. Jacobian matrix of the system at the equilibrium point

$$J(E_0)=\begin{bmatrix}-\beta-\mu+\sigma & 0 & 0\\ \beta & -\mu-\gamma & 0\\ 0 & \gamma & -\mu-\delta\end{bmatrix} \tag{9}$$

it is obtained. The eigenvalues obtained from the Jacobian matrix (9) are given below:

$$\lambda_1=-\beta-\mu+\sigma,$$

$$\lambda_2=-\mu-\gamma,$$

$$\lambda_3=-\mu-\delta$$

where $\beta,\mu,\delta,\sigma,\gamma$ are the parameters of positively defined real numbers. It is clear that $\lambda_2<0$ and $\lambda_3<0$. If $\lambda_1<0$, the equilibrium point of the system is locally asymptotically stable. If $\lambda_1>0$, the equilibrium point of the system is unstable. If $-\beta-\mu+\sigma<0$, $\sigma<\beta+\mu$ is.

$R_0=\frac{\sigma}{\beta+\mu}$ is the basic threshold rate, was determined. If $R_0<1$, academic achievement in schools will increase over time. If $R_0>1$, academic achievement in schools will decrease over time. The success level of students can be taken into consideration when planning studies. In the mathematical model developed for this study, the R_0 value is affected by parameters such as individual reasons (self-efficacy, self-esteem, motivation, etc.), and family-related reasons (parents' attitudes and behaviours, their participation in education, parents' education level, family socioeconomic level, etc.).

2.4. Generalized Euler Method

Generalised Euler Method was used to solve the initial value problem with Caputo fractional derivative. A significant proportion of mathematical models comprise non-linear systems, which can present a considerable challenge in terms of identifying solutions. In the majority of cases, analytical solutions cannot be obtained, necessitating the use of a numerical approach. One such approach is the Generalised Euler Method [24]. Let $D^{\alpha}y(t)=f(t,y(t)),y(0)=y_0,0<\alpha\leq 1,0<t<\alpha$

be the initial value problem. Let $[0, a]$ the interval over which we want to find the solution of the problem. For convenience, subdivide the $[0, a]$ into n subintervals $[t_j, t_{j+1}]$. Suppose that $y(t), D^{\alpha}y(t)$ and $D^{2\alpha}y(t)$ are continuous in range $[0, a]$ and using the generalized Taylor's formula, the following equality is obtained [24]:

$$y(t_1) = y(t_0) + \frac{h^{\alpha}}{\Gamma(\alpha+1)} f(t_0, y(t_0))$$

where $h = \frac{a}{n}$ for $j = 0, 1, \ldots, n-1$.

This process will be repeated to create an array. Let $t_j = t_{j+1} + h$ such that

$$y(t_{j+1}) = y(t_j) + \frac{h^{\alpha}}{\Gamma(\alpha+1)} f(t_j, y(t_j)$$

for $j = 0, 1, .., n-1$ the generalized formula in the form is obtained. For each $k = 0, 1, \ldots, n-1$ with step size h,

$$\begin{aligned} D^{\alpha}S(t) &= \mu N - \mu S(k) - \beta S(k) + \sigma S(k), \\ D^{\alpha}M(t) &= \beta S(k) - \mu M(k) - \gamma M(k), \\ D^{\alpha}B(t) &= \gamma M(k) - \mu B(k) - \delta B(k). \end{aligned} \tag{10}$$

For $t \in [0, h)$, $\frac{t}{h} \in [0, 1)$, we have

$$\begin{aligned} D^{\alpha}S(t) &= \mu N - \mu S(0) - \beta S(0) + \sigma S(0), \\ D^{\alpha}M(t) &= \beta S(0) - \mu M(0) - \gamma M(0), \\ D^{\alpha}B(t) &= \gamma M(0) - \mu B(0) - \delta B(0). \end{aligned} \tag{11}$$

The solution of (11) reduces to

$$\begin{aligned} S(1) &= S(0) + \frac{h^{\alpha}}{\Gamma(\alpha+1)}(\mu N - \mu S(0) - \beta S(0) + \sigma S(0)), \\ M(1) &= M(0) + \frac{h^{\alpha}}{\Gamma(\alpha+1)}(\beta S(0) - \mu M(0) - \gamma M(0)), \\ B(1) &= B(0) + \frac{h^{\alpha}}{\Gamma(\alpha+1)}(\gamma M(0) - \mu B(0) - \delta B(0)). \end{aligned} \tag{12}$$

For $t \in [h, 2h)$, $\frac{t}{h} \in [1, 2)$, we get

$$S(2) = S(1) + \frac{h^\alpha}{\Gamma(\alpha+1)}(\mu N - \mu S(1) - \beta S(1) + \sigma S(1)),$$
$$M(2) = M(1) + \frac{h^\alpha}{\Gamma(\alpha+1)}(\beta S(1) - \mu M(1) - \gamma M(1)), \tag{13}$$
$$B(2) = B(1) + \frac{h^\alpha}{\Gamma(\alpha+1)}(\gamma M(1) - \mu B(1) - \delta B(1)).$$

Repeating the process n times, we obtain

$$S(n+1) = S(n) + \frac{h^\alpha}{\Gamma(\alpha+1)}(\mu N - \mu S(n) - \beta S(n) + \sigma S(n)),$$
$$M(n+1) = M(n) + \frac{h^\alpha}{\Gamma(\alpha+1)}(\beta S(n) - \mu M(n) - \gamma M(n)), \tag{14}$$
$$B(n+1) = B(n) + \frac{h^\alpha}{\Gamma(\alpha+1)}(\gamma M(n) - \mu B(n) - \delta B(n)).$$

3. Numerical Simulation of Fractional Order School Academic Performance Model

In this section, numerical simulation and graphs of the academic achievement model in fractional order schools will be presented. Let us obtain the numerical simulation of the fractional order academic achievement model in schools using the generalized Euler Method. According to the data in [22], let us consider the following parameters.

Let $S = 10$, $M = 10$, $B = 0$, $\beta = 0.001$, $\mu = 0.002$, $\gamma = 0.021$, $\sigma = 0.047$, $\delta = 0.05$ and let the step size be $h = 0.1$. Using the Euler method, the following Table 3 is obtained [22].

Table 3: The values of S, M and B at the moment t for $\alpha = 1$

t	$S(t)$	$M(t)$	$B(t)$
0	10,00	10,00	0,00
1	10,04	9,97	0,02
2	10,09	9,95	0,04
3	10,14	9,93	0,06
4	10,19	9,91	0,08
5	10,24	9,89	0,10
6	10,29	9,86	0,12
7	10,34	9,84	0,14
8	10,38	9,82	0,16
9	10,43	9,80	0,18
10	10,48	9,78	0,20
11	10,53	9,76	0,22
12	10,59	9,73	0,24
13	10,64	9,71	0,26
14	10,69	9,69	0,28

Table 4: The values of S, M and B at the moment t for $\alpha = 0.9$

t	$S(t)$	$M(t)$	$B(t)$
0	10,00	10,00	0,00
1	10,06	9,97	0,02
2	10,12	9,94	0,05
3	10,18	9,91	0,08
4	10,25	9,88	0,10
5	10,31	9,85	0,13
6	10,38	9,82	0,16
7	10,44	9,80	0,18
8	10,51	9,77	0,21
9	10,57	9,74	0,23
10	10,64	9,71	0,26
11	10,71	9,68	0,28
12	10,77	9,66	0,31
13	10,84	9,63	0,33
14	10,91	9,60	0,36

Table 5: The values of S, M and B at the moment t for $\alpha = 0.8$

t	$S(t)$	$M(t)$	$B(t)$
0	10,00	10,00	0,00
1	10,08	9,96	0,03
2	10,16	9,92	0,07
3	10,24	9,88	0,10
4	10,33	9,85	0,14
5	10,41	9,81	0,17
6	10,49	9,77	0,20
7	10,58	9,74	0,24
8	10,67	9,70	0,27
9	10,75	9,66	0,30
10	10,84	9,63	0,33
11	10,93	9,59	0,36
12	11,02	9,56	0,40
13	11,11	9,52	0,43
14	11,20	9,49	0,46

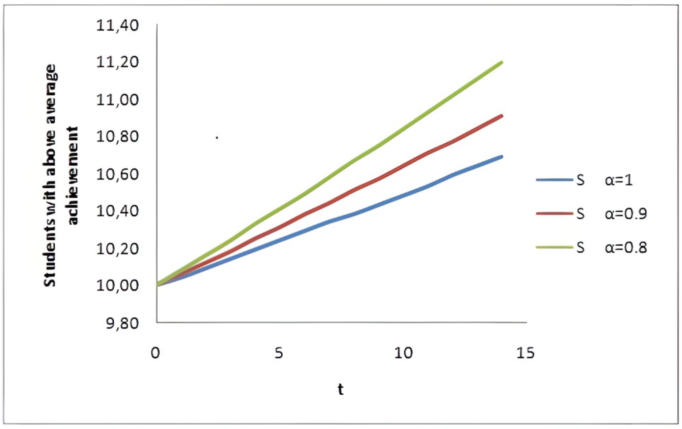

Figure 1: The graph of change of the S compartment model

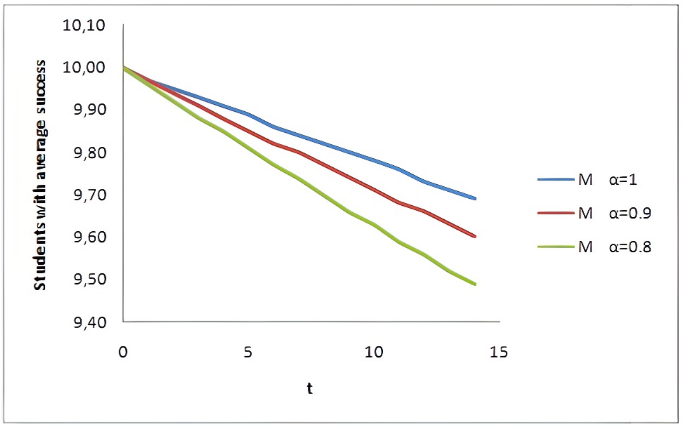

Figure 2: The graph of change of the M compartment model

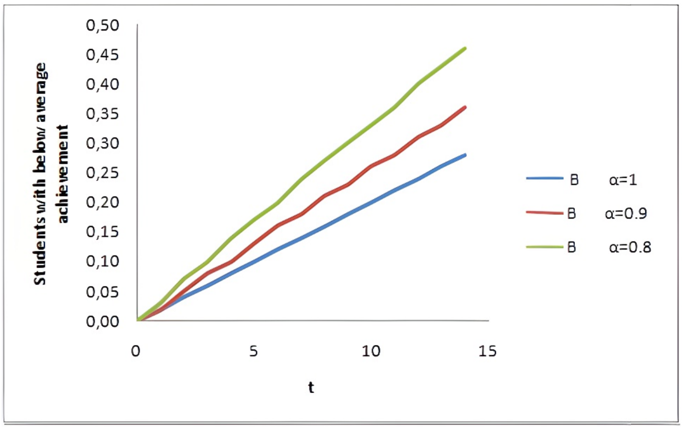

Figure 3: The graph of change of the B compartment model

Table 3, Table 4 and Table 5 show the changes of S, M and B are observed for different states of α.

By the above figures, we observe the following highlights:

* It is observed that the number of students with above average achievement progresses slowly over time (see Figure 1).

* It is observed that the average number of successful students is slowly decreasing over time (see Figure 2).

* It is observed that the number of students with below average achievement increases slowly over time (see Figure 3).

4. Conclusions and Comments

It is of great importance that students succeed academically, as this ensures that they are adequately prepared for the professional world and also has a significant impact on their social lives and future prospects. In the event of academic failure, students frequently encounter a range of emotional, cognitive and behavioural challenges. This study yielded a novel fractional order model that elucidates the factors influencing students' academic achievement levels. The model was then implemented numerically, and graphs were constructed using the numerical results obtained. The existence, uniqueness and non-negativity of the system were analysed mathematically. A stability analysis was performed by obtaining the equilibrium point of the fractional order model of Academic Achievement in Schools, and the number R_0, which is the basic threshold ratio, was found. The graphs obtained revealed that the number of students above the average achievement decreased slowly over time, the number of students with average achievement decreased slowly and the number of students below the average achievement increased slowly over time. In the subsequent models, novel characters may be incorporated into the existing framework of academic achievement, with the adjustment of grade point average score intervals. Furthermore, the variables influencing actors encompass learning speed, intelligence, gender, interests, personality traits, and readiness, among others. The mathematical model can be augmented with additional components. It is imperative to acknowledge the potential contributions of each study focusing on academic achievement, as they contribute unique values to the existing literature and lay the foundation for the development of new concepts.

Declaration of Ethical Standards

The author declares that the materials and methods used in her study do not require ethical committee and/or legal special permission.

Conflicts of Interest

The author declares no conflict of interest.

References

[1] Al Khatib S.A., *Meta-Cognitive Self-Regulated Learning and Motivational Beliefs as Predictors of College Students' Performance*, International Journal for Research in Education, 27, 57-72, 2010.

[2] An X., Hannum E., Sargent T., *Teaching quality and student outcomes: Academic achievement and educational engagement in rural northwest China*, China: An International Journal, 5(2), 309-334, 2007.

[3] Acay B., Inc M., Mustapha U.T., Yusuf A., *Fractional dynamics and analysis for a lana fever infectious ailment with Caputo operator*, Chaos, Solitons and Fractals, 153, 111605, 2021.

[4] Alderman M.K., Motivation for Achievement: Possibilities for Teaching and Learning, Routledge, 2013.

[5] Alivernini F., Lucidi F., *Relationship between social context, self-efficacy, motivation, academic achievement, and intention to drop out of high school: A longitudinal study*, The Journal of Educational Research, 104(4), 241-252, 2011.

[6] Bansal S., Thind S.K., Jaswal S., *Relationship between quality of home environment, locus of control and achievement motivation among high achiever urban female adolescents*, Journal of Human Ecology, 19(4), 253-257, 2006.

[7] Barron K.E., Harackiewicz J.M., Achievement Goals and Optimal Motivation: A Multiple Goals Approach. In C. Sansone and J.M. Harackiewicz (Eds.), Intrinsic and Extrinsic Motivation: The Search for Optimal Motivation and Performance, Academic Press, 229-254, 2000.

[8] Bilgil H., Yousef A., Erciyes A., Erdinç Ü., Öztürk Z., *A fractional-order mathematical model based on vaccinated and infected compartments of SARS-CoV-2 with a real case study during the last stages of the epidemiological event*, Journal of Computational and Applied Mathematics, 425, 115015, 2023.

[9] Bong M., *Between-and within-domain relations of academic motivation among middle and high school students: Self-efficacy, task-value, and achievement goals*, Journal of Educational Psychology, 93(1),23-34, 2001.

[10] Bouffard T., Bouchard M., Goulet G., Denoncourt I., Couture N., *Influence of achievement goals and self-efficacy on students' self-regulation and performance*, International Journal of Psychology, 40(6), 373-384, 2005.

[11] Brauer F., Castillo-Chavez C., Feng Z., Mathematical Models in Epidemiology, Springer, 2019.

[12] Kermack W.O., McKendrick A.G., *A contribution to the mathematical theory of epidemics, in: Proceedings of the Royal Society of London*, Series A, Containing Papers of a Mathematical and Physical Character, 115 (772), 700-721, 1927.

[13] Öztürk Z., Bilgil H., Sorgun S., *Stability analysis of fractional PSQ_p smoking model and application in Turkey*, New Trends in Mathematical Sciences, 10(4), 54-62, 2022.

[14] Öztürk Z., Bilgil H., Sorgun S., *Application of fractional SIQRV model for SARS-CoV-2 and stability analysis*, Symmetry, 15(5), 1048, 2023.

[15] Öztürk Z., Bilgil H., Sorgun S., A new application of fractional glucose-insulin model and numerical solutions. Sigma Journal of Engineering and Natural Sciences, 42(2), 407-413, 2024.

[16] Öztürk Z., Yousef A., Bilgil H., Sorgun S., *A Fractional-order mathematical model to analyze the stability and develop a sterilization strategy for the habitat of stray dogs*, An International Journal of Optimization and Control: Theories and Applications, 14(2), 134-146, 2024.

[17] Podlubny I., Fractional Differential Equations, Academy Press, 1999.

[18] Sharma P., Sharma M., *Relationship between self-esteem and academic achievement of secondary school students*, Elementary Education Online, 20(1), 3208-3212, 2021.

[19] Singh H., Kumar D., Baleanu D., Methods of Mathematical Modelling: Fractional Differential Equations, CRC Press, 2019.

[20] Usaini S., Mustapha U.T., Sabiu S.M., *Modelling scholastic underachievement as a contagious disease*, Mathematical Methods in the Applied Sciences, 41(18), 8603-8612, 2018.

[21] Uzun K., Karataş Z., *Predictors of academic self efficacy: Intolerance of uncertainty, positive beliefs about worry and academic locus of control*, International Education Studies, 13(6), 104-116, 2020.

[22] Uzun P.Y., Uzun K., Koca İ., *The effect of fractional order mathematical modelling for examination of academic achievement in schools with stochastic behaviors*, An International Journal of Optimization and Control: Theories and Applications, 13(2), 244-258, 2023.

[23] Veeresha P., Ilhan E., Prakasha D.G., Baskonus H.M., Gao W., *A new numerical investigation of fractional order susceptible-infected-recovered epidemic model of childhood disease*, Alexandria Engineering Journal, 61(2), 1747-1756, 2022.

[24] Yaro D., Omari-Sasu S.K., Harvim P., Saviour A.W., Obeng B.A., *Generalized Euler method for modeling measles with fractional differential equations*, International Journal of Innovative Research and Development, 4(4), 358-366, 2015.

[25] Yusuf A., Mustapha U.T., Sulaiman T.A., Hincal E., Bayram M., *Modeling the effect of horizontal and vertical transmissions of HIV infection with Caputo fractional derivative*, Chaos, Solitons and Fractals, 145, 110794, 2021.

[26] Yusuf A., Mustapha U.T., Sulaiman T.A., Hincal E., Bayram M., *Modeling the effect of horizontal and vertical transmissions of HIV infection with Caputo fractional derivative*, Chaos, Solitons and Fractals, 145, 110794, 2021.

[27] Yusuf A., Qureshi S., Mustapha U.T., Musa S.S., Sulaiman T.A., *Fractional modeling for improving scholastic performance of students with optimal control*, International Journal of Applied and Computational Mathematics, 8(1), 37, 2022.