Advances in the Theory of Nonlinear Analysis and its Applications **2** (2018) No. 4, 195–201.
https://doi.org/10.31197/atnaa.481995
Available online at www.atnaa.org
Research Article

Advances in the Theory of Nonlinear Analysis and its Applications

ISSN: 2587-2648

Peer-Reviewed Scientific Journal

Multiplicative metric spaces and contractions of rational type

Tatjana Došenović[a], Stojan Radenović[b]

[a] *University of Novi Sad, Faculty of Technology, Novi Sad, Serbia*

[b] *Department of Mathematics, College of Science, King Saud University, Riyadh 11451, Saudi Arabia*

Abstract

The main purpose of this paper is to study the fixed point theorems with contractions of rational type in multiplicative metric spaces. We analyzed whether it was possible to get better results in the context of metric spaces.

Keywords: Metric Space, Common Fixed Point, Multiplicative Metric Space, Cauchy sequence
2010 MSC: Primary 47H10; Secondary 54H25

---

1. Introduction and preliminaries

In 2008, Bashirov et al., defined new kind of spaces, called multiplicative metric spaces in the following way:

Definition 1.1. [8] Let $X \neq \emptyset$. An operator $d^* : X \times X \to \mathbb{R}$ is a multiplicative metric on X, if it satisfies:

(m1*) $d^*(x, y) \geq 1$ for all $x, y \in X$ and $d^*(x, y) = 1$ if and only if $x = y$,

(m2*) $d^*(x, y) = d^*(y, x)$ for all $x, y \in X$,

(m3*) $d^*(x, z) \leq d^*(x, y) \cdot d^*(y, z)$ for all $x, y, z \in X$ (multiplicative triangle inequality).

If operator d^* satisfies (m1*)-(m3*) then the pair (X, d^*) is called a multiplicative metric space.

The previous definition was motivation for a large number of papers where the authors proved various fixed point theorems for different contraction conditions in mentioned space (see for example [1]-[4], [8], [14] [18]-[25]).

The next definition for metric spaces is well known:

Email addresses: `tatjanad@tf.uns.ac.rs` (Tatjana Došenović), `radens@beotel.rs` (Stojan Radenović)

Received November 13, 2018, Accepted: December 7, 2018, Online: December 10, 2018

Definition 1.2. Let $X \neq \emptyset$. An operator $d : X \times X \to \mathbb{R}$ is a metric on X, if it satisfies:
(1) $d(x,y) \geq 0$ for all $x, y \in X$ and $d(x,y) = 0$ if and only if $x = y$,
(2) $d(x,y) = d(y,x)$ for all $x, y \in X$,
(3) $d(x,z) \leq d(x,y) + d(y,z)$ for all $x, y, z \in X$ (standard triangle inequality).
If operator d satisfies (1)-(3) then the pair (X, d) is called a metric space.

In ([11]) the following theorem is given.

Theorem 1.3. *Let (X, d^*) be a multiplicative metric space. Then the pair (X, d) is a metric space where $d(x,y) = \ln d^*(x,y)$ for all $x, y \in X$. Conversely, if (X, d) is a metric space then (X, d^*) is a multiplicative metric space where $d^*(x,y) = e^{d(x,y)}$ for all $x, y \in X$.*

Also, in ([6], [11], [12]) the equivalence between well-known theorems in metric and multiplicative metric spaces has been thoroughly analyzed (Banach [7], Kannan [17], Edelstein-Nemitskii [13], Boyd-Wong [9] and other).

2. Main results

Definition 2.1. [16] Two self mappings A and S of a multiplicative metric space (X, d^*) are said to be compatible on X if $\lim_{n\to\infty} d^*(ASx_n, SAx_n) = 1$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$ for some $t \in X$.

Definition 2.2. [15] Two self mappings A and S of a multiplicative metric space (X, d^*) are said to be weakly compatible on X if $Ax = Sx$ for all $x \in X$ implies $ASx = SAx$, that is, $d^*(Ax, Sx) = 1$ i.e. $d^*(ASx, SAx) = 1$.

Theorem 2.3. *([4], Theorem 3.1) Let $(X; d^*)$ be a complete multiplicative metric space. Let S; T; A, B : $X \to X$ be such that $S(X) \subset B(X)$, $T(X) \subset A(X)$ and there exists $\lambda \in \left(0, \frac{1}{2}\right)$ such that*

$$d^{*p}(Sx,Ty) \leq \left[\varphi\left(\max\left\{d^{*p}(Ax,By), \frac{d^{*p}(Ax,Sx)\, d^{*p}(By,Ty)}{1+d^{*p}(Ax,By)}, \frac{d^{*p}(Ax,Ty)\, d^{*p}(By,Ax)}{1+d^{*p}(Ax,By)}\right\}\right)\right]^{\lambda}, \quad (2.1)$$

for all $x, y \in X$ and $p \geq 1$, where $\varphi : [0,\infty) \to [0,\infty)$ is a monotone increasing function such that $\varphi(0) = 0$ and $\varphi(t) < t$ for all $t > 0$.

Suppose that one of the following conditions is satisfied:
(a) either A or S is continuous, the pair $(S; A)$ is compatible and the pair $(T; B)$ is weakly compatible;
(b) either B or T is continuous, the pair $(T; B)$ is compatible and the pair $(S; A)$ is weakly compatible.
Then $S; T; A$ and B have a unique common fixed point in X.

Remark 2.4. The function φ is superfluous, because $\varphi(t) < t$ and therefore

$$\begin{aligned} d^{*p}(Sx,Ty) &\leq \left[\varphi\left(\max\left\{d^{*p}(Ax,By), \frac{d^{*p}(Ax,Sx)\, d^{*p}(By,Ty)}{1+d^{*p}(Ax,By)}, \frac{d^{*p}(Ax,Ty)\, d^{*p}(By,Ax)}{1+d^{*p}(Ax,By)}\right\}\right)\right]^{\lambda} \\ &\leq \left[\max\left\{d^{*p}(Ax,By), \frac{d^{*p}(Ax,Sx)\, d^{*p}(By,Ty)}{1+d^{*p}(Ax,By)}, \frac{d^{*p}(Ax,Ty)\, d^{*p}(By,Ax)}{1+d^{*p}(Ax,By)}\right\}\right]^{\lambda} \\ &\leq \left[\max\left\{d^{*p}(Ax,By), d^{*p}(Ax,Sx)\, d^{*p}(By,Ty), d^{*p}(Ax,Ty)\right\}\right]^{\lambda}. \end{aligned}$$

So,

$$d^{*p}(Sx,Ty) \leq \left[\max\left\{d^{*p}(Ax,By), d^{*p}(Ax,Sx)\, d^{*p}(By,Ty), d^{*p}(Ax,Ty)\right\}\right]^{\lambda}, \quad (2.2)$$

and therefore

$$d^*(Sx,Ty) \leq \left(\max\{d^*(Ax,By), d^*(Ax,Sx)d^*(By,Ty), d^*(Ax,Ty)\}\right)^{\lambda} \quad (2.3)$$

If we apply ln on both sides of (2.3) we get

$$d(Sx,Ty) \leq \lambda \max\left\{d(Ax,By), d(Ax,Sx) + d(By,Ty), d(Ax,Ty)\right\} \quad (2.4)$$

In the next theorem we prove that condition (2.4) with assumption as in Theorem 2.3 provides existence of a common fixed point.

Remark 2.5. In previous theorem, the following condition for the function φ: $\varphi : [1,\infty) \to [1,\infty)$ is a monotone increasing function such that $\varphi(1) = 1$ and $\varphi(t) < t$ for all $t > 0$ should stay instead of the condition given in theorem.

Definition 2.6. [15] Two self-mappings A and S of a metric space (X,d) are called compatible if, $\lim_{n\to\infty} d(ASx_n, SAx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$ for some $t \in X$. It is easy to see that compatible maps commute at their coincidence points.

Definition 2.7. [16] Two self mappings A and S of a metric space (X,d) are called weakly compatible if, they commute at coincidence points. That is, if $Ax = Sx$ implies that $ASx = SAx$ for $x \in X$, i.e. $d(ASx, SAx) = 0$.

Theorem 2.8. *Let (X,d) be a complete metric space. Let S; T; A, $B : X \to X$ be such that $S(X) \subset B(X)$, $T(X) \subset A(X)$ and there exists $\lambda \in \left(0, \frac{1}{2}\right)$ such that (2.4) is satisfied for all $x, y \in X$.*

Suppose that one of the following conditions is satisfied:

(a) either A or S is continuous, the pair $(S; A)$ is compatible and the pair $(T; B)$ is weakly compatible;

(b) either B or T is continuous, the pair $(T; B)$ is compatible and the pair $(S; A)$ is weakly compatible.

Then S, T, A and B have a unique common fixed point in X.

Proof. Let $x_0 \in X$. Since $S(X) \subset B(X)$ and $T(X) \subset A(X)$, there exist $x_1, x_2 \in X$ such that $y_0 = Sx_0 = Bx_1$ and $y_1 = Tx_1 = Ax_2$. By induction, we can define the sequence $\{x_n\}$ and $\{y_n\}$ in X such that

$$y_{2n} = Sx_{2n} = Bx_{2n+1}, \;\; y_{2n+1} = Tx_{2n+1} = Ax_{2n+2}, \tag{2.5}$$

for all $n \geq 0$. Using (2.4) and (2.5) we have

$$\begin{aligned}
& d(y_{2n}, y_{2n+1}) = d\left(Sx_{2n}, Tx_{2n+1}\right) \\
\leq\; & \lambda \max\left\{d\left(Ax_{2n}, Bx_{2n+1}\right), d\left(Ax_{2n}, Sx_{2n}\right) + d\left(Bx_{2n+1}, Tx_{2n+1}\right), d\left(Ax_{2n}, Tx_{2n+1}\right)\right\} \\
=\; & \lambda \max\{d(y_{2n-1}, y_{2n}), d(y_{2n-1}, y_{2n}) + d(y_{2n}, y_{2n+1}), d(y_{2n-1}, y_{2n+1})\} \\
\leq\; & \lambda \max\{d(y_{2n-1}, y_{2n}), d(y_{2n-1}, y_{2n}) + d(y_{2n}, y_{2n+1}), d(y_{2n-1}, y_{2n}) + d(y_{2n}, y_{2n+1})\} \\
=\; & \lambda(d(y_{2n-1}, y_{2n}) + d(y_{2n}, y_{2n+1})).
\end{aligned}$$

Therefore,

$$d(y_{2n}, y_{2n+1}) \leq \frac{\lambda}{1-\lambda} d(y_{2n-1}, y_{2n}) = hd(y_{2n-1}, y_{2n}). \tag{2.6}$$

Since $\lambda \in (0, \frac{1}{2})$ we have that $h \in (0,1)$.

Also,

$$\begin{aligned}
& d(y_{2n+2}, y_{2n+1}) = d\left(Sx_{2n+2}, Tx_{2n+1}\right) \\
\leq\; & \lambda \max\left\{d\left(Ax_{2n+2}, Bx_{2n+1}\right), d\left(Ax_{2n+2}, Sx_{2n+2}\right) + d\left(Bx_{2n+1}, Tx_{2n+1}\right), d\left(Ax_{2n+2}, Tx_{2n+1}\right)\right\} \\
=\; & \lambda \max\{d(y_{2n+1}, y_{2n}), d(y_{2n+1}, y_{2n+2}) + d(y_{2n}, y_{2n+1}), d(y_{2n+1}, y_{2n+1})\} \\
\leq\; & \lambda \max\{d(y_{2n+1}, y_{2n}), d(y_{2n+1}, y_{2n+2}) + d(y_{2n}, y_{2n+1}), 0\} \\
=\; & \lambda(d(y_{2n+1}, y_{2n}) + d(y_{2n+1}, y_{2n+2})),
\end{aligned}$$

and

$$d(y_{2n+2}, y_{2n+1}) \leq \frac{\lambda}{1-\lambda} d(y_{2n+1}, y_{2n}) = hd(y_{2n+1}, y_{2n}). \tag{2.7}$$

Using (2.6) and (2.7) we have that for every $n \in \mathbb{N}$

$$d(y_n, y_{n+1}) \leq hd(y_{n-1}, y_n), \;\; h < 1.$$

So, the sequence $\{y_n\}$ is a Cauchy sequence, and since the space is complete, there exists $z \in X$ such that $\lim_{n\to\infty} y_n = z$, and since $\{y_{2n}\}$ and $\{y_{2n+1}\}$ are subsequence of $\{y_n\}$ we have

$$\lim_{n\to\infty} Sx_{2n} = \lim_{n\to\infty} Bx_{2n+1} = \lim_{n\to\infty} Tx_{2n+1} = \lim_{n\to\infty} Ax_{2n+2} = z. \tag{2.8}$$

Suppose that A is continuous. Then $A \lim_{n\to\infty} Sx_{2n} = \lim_{n\to\infty} ASx_{2n} = Az$. Using (2.8) and the assumption that the pair (S, A) is compatible we have that

$$\lim_{n\to\infty} d(SAx_{2n}, ASx_{2n}) = \lim_{n\to\infty} d(SAx_{2n}, Az) = 0,$$

which means that $\lim_{n\to\infty} SAx_{2n} = Az$. Using (2.4), we have

$$d(SAx_{2n}, Tx_{2n+1}) \leq \lambda \max\{d(A^2x_{2n}, Bx_{2n+1}), d(A^2x_{2n}, SAx_{2n}) + d(Bx_{2n+1}, Tx_{2n+1}),$$
$$d(A^2x_{2n}, Tx_{2n+1})\}.$$

Taking $n \to \infty$ in the above inequality we have

$$d(Az, z) \leq \lambda \max\{d(Az, z), d(Az, Az) + d(z, z), d(Az, z)\} = \lambda d(Az, z).$$

Therefore, $Az = z$. Using again (2.4) we have

$$d(Sz, Tx_{2n+1}) \leq \lambda \max\{d(Az, Bx_{2n+1}), d(Az, Sz) + d(Bx_{2n+1}, Tx_{2n+1}), d(Az, Tx_{2n+1})\}.$$

Letting $n \to \infty$ we have

$$d(Sz, z) \leq \lambda \max\{d(z, z), d(z, Sz) + d(z, z), d(z, z)\} = \lambda d(z, Sz),$$

i.e. $z = Sz = Az$. Since $z = Sz \in S(X) \subset B(X)$, there exist $z_1 \in X$ such that $z = Az = Sz = Bz_1$. Using (2.4) we have

$$d(z, Tz_1) = d(Sz, Tz_1) \leq \lambda \max\{d(Az, Bz_1), d(Az, Sz) + d(Bz_1, Tz_1), d(Az, Tz_1)\} =$$
$$\lambda \max\{d(z, z), d(z, z) + d(z, Tz_1), d(z, Tz_1) = \lambda d(z, Tz_1).$$

Therefore $z = Az = Sz = Bz_1 = Tz_1$. Since the pair T, B weakly compatible, we have $Tz = TBz_1 = BTz_1 = Bz$. It remains to prove that $z = Tz$. Using (2.4) we have

$$d(z, Tz) = d(Sz, Tz) \leq \lambda \max\{d(Az, Bz), d(Az, Sz) + d(Bz, Tz), d(Az, Tz)\} = \lambda d(z, Tz).$$

This implies that $z = Tz = Bz = Az = Sz$, and so z is a common fixed point of S, T, A, B. Similarly, if we suppose that S is continuous we have the same conclusion. Next we prove that S, T, A, B have a unique common fixed point. Suppose that u is another common fixed point. Then, using (2.4) we have

$$d(z, u) = d(Sz, Tu) \leq \lambda \max\{d(Az, Bu), d(Az, Sz) + d(Bu, Tu), d(Az, Tu)\} = \lambda d(z, u),$$

i.e. $z = u$. □

Theorem 2.9. *Let (X, d^*) be a complete multiplicative metric space. Let S T A, $B : X \to X$ be such that $S(X) \subset B(X)$, $T(X) \subset A(X)$ and there exists $\lambda \in \left(0, \frac{1}{2}\right)$ such that condition (2.3) is satisfied for all $x, y \in X$.*

Suppose that one of the following conditions is satisfied:
(a) either A or S is continuous, the pair (S, A) is compatible and the pair (T, B) is weakly compatible;
(b) either B or T is continuous, the pair (T, B) is compatible and the pair (S, A) is weakly compatible.
Then S, T, A and B have a unique common fixed point in X.

Theorem 2.10. *Theorem 2.8 and Theorem 2.9 are equivalent.*

Theorem 2.11. *[5] Let S and T be mappings of a complete multiplicative metric space (X, d^*) into itself satisfying the conditions $S(X) \subset X$, $T(X) \subset X$ and*

$$d^*(Sx,Ty) \le \{\max\{\frac{d^*(x,Sx)[d^*(y,Sx)+d^*(y,Ty)]}{1+d^*(Sx,Ty)}, \frac{d^*(y,Sx)d^*(x,Ty)+d^*(x,y)d^*(Sx,y)}{d^*(Sx,Ty)+d^*(Sx,y)}, \tag{2.9}$$

$$\frac{d^*(x,Sx)d^*(y,Sx)+d^*(x,y)d^*(Sx,Ty)}{d^*(y,Ty)+d^*(y,Sx)}, \frac{d^*(y,Ty)d^*(x,Ty)+d^*(x,Ty)d^*(y,Sx)}{d^*(y,Ty)+d^*(y,Sx)}\}\}^{\lambda},$$

for all $x, y \in X$, where $\lambda \in (0, \frac{1}{2})$. Then S and T have a unique common fixed point.

Remark 2.12. Let's look at each member of the right hand side of equation (2.9). Now we have the following.

$$\frac{d^*(x,Sx)[d^*(y,Sx)+d^*(y,Ty)]}{1+d^*(Sx,Ty)} \le d^*(x,Sx)d^*(y,Ty),$$

$$\frac{d^*(y,Sx)d^*(x,Ty)+d^*(x,y)d^*(Sx,y)}{d^*(Sx,Ty)+d^*(Sx,y)} \le d^*(y,Ty)d^*(x,Ty),$$

$$\frac{d^*(x,Sx)d^*(y,Sx)+d^*(x,y)d^*(Sx,Ty)}{d^*(y,Ty)+d^*(y,Sx)} \le d^*(y,Sx)d^*(x,y)$$

$$\frac{d^*(y,Ty)d^*(x,Ty)+d^*(x,Ty)d^*(y,Sx)}{d^*(y,Ty)+d^*(y,Sx)} = d^*(x,Ty).$$

Our new contractive condition is the following one:

$$d^*(Sx,Ty) \le \{\max\{d^*(x,Sx)d^*(y,Ty), d^*(y,Ty)d^*(x,Ty), d^*(y,Sx)d^*(x,y), d^*(x,Ty)\}\}^{\lambda} \tag{2.10}$$

But, we have the following:

$$d^*(Sx,Ty) \le \{\max\{d^*(x,Sx)d^*(y,Ty), d^*(y,Ty)d^*(x,Ty), d^*(y,Sx)d^*(x,y), d^*(x,Ty)\}\}^{\lambda} \tag{2.11}$$

$$\le \max\{d^*(x,Sx), d^*(y,Ty), d^*(y,Ty), d^*(x,Ty), d^*(y,Sx), d^*(x,y)\}^{2\lambda}.$$

If we apply ln on both sides of (2.11) we get

$$d(Sx,Ty) \le q\max\{d(x,Sx), d(y,Ty), d(x,Ty), d(y,Sx), d(x,y)\} \tag{2.12}$$

where $q = 2\lambda$.

The obtained contractive condition is the well known Ćirić strongly-quasi-contraction [10]. It is also well known that for $q = \frac{3}{4}$ mappings S and T do not have a common fixed point. So, additional condition is necessary. One possible solution is given in the paper [5] where the following definition is given:

Definition 2.13. A pair $\{S, T\}$ of a mapping is asymptotically regular at x_0 if $d(x_n, x_{n+1}) \to 0$ as $n \to \infty$ where $Sx_{2n} = x_{2n+1}$ and $Tx_{2n+1} = x_{2n+2}$, $n \in \mathbb{N}$.

In the same paper the following theorem was proved:

Theorem 2.14. *Let S and T be mappings of a complete metric space (X, d) into itself satisfying condition (2.12). Suppose that the pair $\{S, T\}$ asymptotically regular at x_0. Then S and T have a common fixed point.*

3. Conclusion

Multiplicative metric space was introduced by Bashirov in 2008. After that, a huge number of paper appeared where authors use a various contractive condition used in order to prove a fixed point theorem. But, in the paper [11] on Multiplicative metric space, the authors proved that various well known fixed point theorems in multiplicative metric spaces have equivalent fixed point theorem in metric space. So, natural question has appeared: Is the multiplicative metric space a generalization of the metric space? Based on that, we started to study fixed point theorems in multiplicative metric space where the contractive condition is complicated (i.e. rational type contractive condition) and at first, we conclude that there is not always equivalent theorem in metric space. We analyzed two fixed point theorems in multiplicative metric space. In the first theorem we have shown that we can find a better condition in metric space for which function has a fixed point. We proved that (2.1)⇒(2.3)⇔(2.4). So, we get better results in metric space than the ones presented in Theorem 2.3. Finally, in the second theorem we found better contractive condition for which function has a fixed point but we assume one additional condition. Open question is the following one: Is it possible to find a better condition in metric space without additional conditions? If answer is negative, we realize that in some cases multiplicative metric space is useful.

Acknowledgment The first author is thankful to Ministry of Education, Sciences and Technological Development of Serbia.

References

[1] M. Abbas, B. Ali, Yi Suleiman, *Common fixed points of locally contractive mappings in multiplicative metric spaces with applications,* Int. J., Math. Math. Sci. 2015, Article ID 218683, (2015).

[2] M. Abbas, M. De La Sen, T. Nazir, *Common fixed points of generalized rational type cocyclic mappings in multiplicative metric spaces,* Discrete Dyn. Nat. Soc. 2015, Article Id 532725, (2015).

[3] K. Abodayeh, A. Pitea, W. Shatanawi, T. Abdeljawad, *Remarks on Multiplicative Metric Spaces and Related Fixed Points,* http://arXiv.org/abs/1512.03771v1, (2015).

[4] Afrah A. N. Abdou, *Fixed point theorems for generalized contraction mappings in multiplicative metric spaces,* J. Nonlinear Sci. Appl. 9, 2347-2363, (2016).

[5] D.E. Anderson, K.L.Singh, J.H.M. Whitfield, *Common fixed point for family of mappings*, Internat. J. Math. and Math. Sci., 7(1), 1984, 89-95.

[6] R. P. Agarwal, E. Karapinar and B. Samet, *An essential remark on fixed point results on multiplicative metric spaces,* Fixed Point theory Appl., 2016:21, (2016).

[7] S. Banach, *Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales,* Fundam. Math.,3, 133-181, (1922).

[8] A. Bashirov, E. Kurpinar, A. Ozyapici, *Multiplicative calculus and its applications,* J. Math. Anal. Appl. 337 (1), 36-48, (2008).

[9] D.W. Boyd, J.S.Wong, *On linear contractions,* Proc. Amer. Maqth. Soc. 20, 458-464, (1969).

[10] Lj.B. Ćirić, *On Common Fixed Points In Uniform Spaces*, Publications de l'Institut MathÃĺmatique, 24(38), 39–43, (1978).

[11] T. Došenović, M. Postolache, S. Radenović, *On multiplicative metric spaces: Survey*, Fixed Point Theory Appl., 2016:92, (2016).

[12] T. Došenović, S. Radenović, *Some critical remarks on the paper: "An essential remark on fixed point results on multiplicative metric spaces"*, J. Adv. Math. Stud., 10(1), 20-24, (2017).

[13] M. Edelstein, *On fixed and periodic points under contractive mappings*, J. London Math. Soc., 37, 74-79, (1962).

[14] X. He, M. Song and D. Chen, *Common fixed points for weak commutative mappings on a multiplicative metric space,* Fixed Point Theory Appl., 2014:48, (2009).

[15] G. Jungck, *Compatible mappings and common fixed points*, Internat. J. Math. Math. Sci., 9, 771 - 779, (1986).

[16] G. Jungck, B. E. Rhoades, *Fixed point for set-valued functions without continuity,* Indian J. Pure Appl. Math., 29 (1998), 227âĂŞ238.

[17] R. Kannan, *Some results on fixed points*, Bull. Cal. Math., 60, 71-76, (1968).

[18] S. M. Kang, P. Kumar, S. Kumar, P. Nagpal, S.K Garg, *Common fixed points for compatible mappings and its variants in multiplicative metric spaces*, Int. J. Pure Appl. Math. 102 (2), 383-406, (2015).

[19] C. Mongkolkeha, W. Shatanawi, *Best proximity points for multiplicative proximal contraction mapping on multiplicative metric spaces,* J. Nonlinear Sci. Appl. 8, 1134-1140, (2015).

[20] M. Sarwar, B.-e-Rome, *Some Unique Fixed Point Theorems in Multiplicative Metric Space,* http://arXiv.org/abs/1410.3384v2, (2014).

[21] M. Özavsar, A. C. Cevikel, *Fixed points of multiplicative contraction mappings on multiplicative metric spacers,* http://arxiv.org/abs/1205.5131v1, (2012).

[22] S. Shukla, *Some critical remarks on the multiplicative metric spaces and fixed point results*, J. Adv. Math. Studies, 9 (3), 454-458, (2016).

[23] D. Stanley, *A multiplicative calculus,* Primus IX (4) 310326, (1999).

[24] O. Yamaod, W. Sintunavarat, *Some fixed point results for generalized contraction mappings with cyclic (α, β)-admissible mapping in multiplicative metric spaces,* J. Inequal. Appl., 2014:488, (2014).

[25] O. Zamaod, W. Sintunavarat, *Some fixed point results for generalized contraction mappings with cyclic* $(\alpha, \beta)-$*admissible mappings in multiplicative metric spaces,* J. Inequal. Appl., 2014:488, (2014).