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BLOW UP OF SOLUTIONS FOR A TIMOSHENKO EQUATION WITH DAMPING TERMS

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Abstract: In this work, we studied the following equation

$$u_{tt}+\triangle^2 u-M\left(\|\nabla u\|^2\right)\triangle u-\triangle u_t+u_t=|u|^{q-1}u$$

regard to initial and Dirichlet boundary condition. We show that the blow up of solutions with positive and negative initial energy.

Keywords: Timoshenko equation, Blow up, Damping term.

Mathematics Subject Classification (2010): 35A01.

1 Introduction

In this work, we consider the following Timoshenko equation

$$\begin{cases} u_{tt}+\triangle^2 u-M\left(\|\nabla u\|^2\right)\triangle u-\triangle u_t+u_t=|u|^{q-1}u, & (x,t)\in\Omega\times(0,T), \\ u(x,0)=u_0(x),\ u_t(x,0)=u_1(x), & x\in\Omega, \\ u(x,t)=\frac{\partial}{\partial\nu}u(x,t)=0, & x\in\partial\Omega, \end{cases} \tag{1}$$

where Ω is a bounded domain of R^n having a smooth boundary $\partial\Omega$. Also $q\geq 1$ is real numbers, outer normal is denoted by ν and $M(s)=1+s^\gamma,\ \gamma\geq 1$.

In the event of $M(s)=1$, without fourth order term $(\triangle^2 u)$ and strong damping term $(-\triangle u_t)$ the equation (1) can be recorded in the following form

$$u_{tt}-\triangle u+u_t=|u|^{q-1}u. \tag{2}$$

Georgiev and Todorova, Levine, Messaoudi, Vitillaro made further efforts to get the existence and blow up in finite time of solutions for (2).

In the event of $M(s) = 0$ and absent the strong damping term the equation (1) can be typed in the following form

$$u_{tt} + \triangle^2 u + u_t = |u|^{q-1} u. \tag{3}$$

Messaoudi [11] researched the local existence and studied blow up of the solution to the equation (3). Wu and Tsai [16] got global existence and made researches about blow up of the solution of the problem (3). Then, blow up of the solution for the problem (3) with positive initial energy was studied by Chen and Zhou [2] .

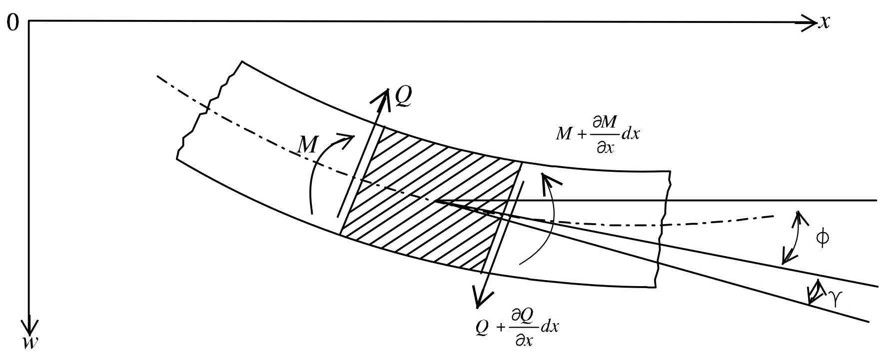
The problem (1) was researched by Esquivel-Avila [4, 5], he demonstrated blow up, unboundedness, convergence and made researches for global attractor. Pişkin [12] researched the local and global existence, asymptotic behavior also studied about blow up of the solution. Later, Pişkin and Irkıl [13] investigated blow up of the solutions (1) for positive initial energy.

In this paper, we show the blow up of solutions of the problem (1), for positive and negative initial energy.

This work is arranged as the following. In chapter 2, some lemmas and notations are given. In chapter 3, blow up of the solution is discussed.

1.1 Derivation of the Timoshenko equation

In this section, we show the derivation of the Timoshenko equation [3, 14].



In the foregoing figure, the bending moment is indicated by M and shearing force is indicated by Q. Also ϕ is the angle of bending and γ is the angle of shearing. Deflection is stated by W.

For a great number of minuscule deflections

$$\frac{\partial W}{\partial x} = \phi + \gamma \tag{4}$$

and by elementary beam theory

$$\begin{cases} M = -EI\frac{\partial \phi}{\partial x}, \\ Q = kAG\gamma. \end{cases} \tag{5}$$

Here, flexural rigidity is denoted by EI; k is a constant related to the form of cross-section of a beam; A is field of cross-section and modulus of rigidity is denoted by G.

The movements equations are:

The rotations equation is

$$-\frac{\partial M}{\partial x}dx + Qdx = \rho I \frac{\partial^2 \phi}{\partial t^2}dx. \tag{6}$$

Here, the density of the material is ρ.

In the direction of W, the equation for translation is-

$$\frac{\partial Q}{\partial x}dx = \rho A \frac{\partial^2 W}{\partial t^2}dx. \tag{7}$$

In equation (5), if the account of Q is substituted into equations (6) and (7), we get

$$-\frac{\partial M}{\partial x} + kAG\gamma = \rho I \frac{\partial^2 \phi}{\partial t^2}, \tag{8}$$

$$\frac{\partial\left(kAG\gamma\right)}{\partial x} = \rho A \frac{\partial^2 W}{\partial t^2}. \tag{9}$$

Substituting for

$$\gamma = \frac{\partial W}{\partial x} - \phi$$

in the equation (4) and

$$M = -EI\frac{\partial \phi}{\partial x}$$

in the equation (5) into equations (8) and (9), we attain

$$EI\frac{\partial^2 \phi}{\partial x^2} + kAG\left(\frac{\partial W}{\partial x} - \phi\right) - \rho I \frac{\partial^2 \phi}{\partial t^2} = 0, \tag{10}$$

$$\rho A \frac{\partial^2 W}{\partial t^2} - kAG\left(\frac{\partial^2 W}{\partial x^2} - \frac{\partial \phi}{\partial x}\right) = 0. \tag{11}$$

To eliminate ϕ from equations (10) and (11), we rearrange (12) to read

$$\frac{\partial \phi}{\partial x} = -\frac{\rho A}{kAG}\frac{\partial^2 W}{\partial t^2} + \frac{\partial^2 W}{\partial x^2}.$$

Now differentiating equation (10) accordinly to x and substituting for $\frac{\partial \phi}{\partial x}$ we attain

$$\begin{aligned} & EI\frac{\partial^2}{\partial x^2}\left[-\frac{\rho A}{kAG}\frac{\partial^2 W}{\partial t^2} + \frac{\partial^2 W}{\partial x^2}\right] \\ & +kAG\left[\frac{\partial^2 W}{\partial x^2} + \frac{\rho A}{kAG}\frac{\partial^2 W}{\partial t^2} - \frac{\partial^2 W}{\partial x^2}\right] \\ & -\rho I\frac{\partial^2}{\partial t^2}\left[-\frac{\rho A}{kAG}\frac{\partial^2 W}{\partial t^2} + \frac{\partial^2 W}{\partial x^2}\right] \\ = & \ 0. \end{aligned}$$

Simplifying the above expression we obtain

$$-\frac{EI\rho}{kG}\frac{\partial^4 W}{\partial x^2 \partial t^2}+EI\frac{\partial^4 W}{\partial x^4}+\rho A\frac{\partial^2 W}{\partial t^2}+\frac{\rho^2 I}{kG}\frac{\partial^4 W}{\partial t^4}-\rho I\frac{\partial^4 W}{\partial x^2 \partial t^2}=0,$$

therefore

$$EI\frac{\partial^4 W}{\partial x^4}-\rho I\left(1+\frac{E}{kG}\right)\frac{\partial^4 W}{\partial x^2 \partial t^2}+\rho A\frac{\partial^2 W}{\partial t^2}+\frac{\rho^2 I}{kG}\frac{\partial^4 W}{\partial t^4}=0. \quad (12)$$

This equation is termed the "Timoshenko equation".

Rotatory inertia is symbolized by

$$-\rho I\frac{\partial^4 W}{\partial x^2 \partial t^2}$$

in equation (12) and amendment related to shear by

$$-\frac{\rho IE}{kG}\frac{\partial^4 W}{\partial x^2 \partial t^2}+\frac{\rho^2 I}{kG}\frac{\partial^4 W}{\partial t^4}.$$

The Euler's equation (13) is got from the Timoshenko equation by sifting the amendments related to both shear and rotatory inertia.

$$EI\frac{\partial^4 W}{\partial x^4}+\rho A\frac{\partial^2 W}{\partial t^2}=0. \quad (13)$$

The Timoshenko beam theory can be thought like a system, such as (10) and (11) or in the one form, as equation (12).

2 Preliminaries

In this chapter, we should show some assumptions and lemmas which will be taken advantage of. Where $\|.\|$ and $\|.\|_p$ indicate the usual $L^2(\Omega)$ norm and $L^p(\Omega)$ norm, in turn.

Lemma 1 *(Sobolev-Poincare inequality) [1]. Let* p *be a number with* $2 \le p < \infty$ $(n = 1, 2)$ *or* $2 \le p \le \frac{2n}{n-2}$ $(n \ge 3)$, *and* $C_* = C_*(\Omega,\ p)$ *is a constant, such that*

$$\|u\|_p \le C_* \|\nabla u\| \ \textit{for } u \in H_0^1(\Omega).$$

We identify the energy function as follows

$$\begin{aligned} E(t) &= \frac{1}{2}\|u_t\|^2+\frac{1}{2}\left(\|\nabla u\|^2+\|\Delta u\|^2\right) \\ &\quad +\frac{1}{2(\gamma+1)}\|\nabla u\|^{2(\gamma+1)}-\frac{1}{q+1}\|u\|_{q+1}^{q+1}. \end{aligned} \quad (14)$$

Lemma 2 $E(t)$ *is a nonincreasing function also* $t \ge 0$ *and*

$$E'(t) = -\|u_t\|^2-\|\nabla u_t\|^2 \le 0. \quad (15)$$

Proof. If we multiply the equation of (1) by u_t and integrate over Ω, use integrating by parts, we attain

$$E(t) - E(0) = -\int_0^t \left(\|u_\tau\|^2 + \|\nabla u_\tau\|^2\right) d\tau \text{ for } t \geq 0. \tag{16}$$

■

Also, we remark the local existence theorem of problem (1), the proof of it can be present in [12].

Theorem 3 *(Local existence). Supposing that* $(u_0, u_1) \in H_0^2(\Omega) \times L^2(\Omega)$ *ensures, after there is an only solution* u *of (1) satisfying*

$$u \in C\left([0,T); H_0^2(\Omega)\right),$$

$$u_t \in C\left([0,T); L^2(\Omega)\right) \cap L^{p+1}(\Omega \times (0,T)).$$

Furthermore, at a minimum one of the following expressions holds:

(i) $T = \infty$,

(ii) $\|u_t\|^2 + \|\triangle u\|^2 \longrightarrow \infty$ *as* $t \longrightarrow T^-$.

3 Blow up of solutions

In this chapter, we work away the blow up of the solution for the problem (1). We should denote the following two lemmas,which will be taken advantage of then.

Lemma 4 *[9]. Let* $\delta > 0$ *and* $B(t) \in C^2(0,\infty)$ *be a nonnegative function satisfying*

$$B''(t) - 4(\delta+1)B'(t) + 4(\delta+1)B(t) \geq 0. \tag{17}$$

If

$$B'(0) > r_2 B(0) + K_0, \tag{18}$$

with $r_2 = 2(\delta+1) - 2\sqrt{(\delta+1)\delta}$*, then* $B'(t) > K_0$ *for* $t > 0$*,here* K_0 *is a constant.*

Lemma 5 *[9]. If* $H(t)$ *is a nonincreasing function on* $[t_0, \infty)$ *and satisfies the differential inequality*

$$\left[H'(t)\right]^2 \geq a + b\left[H(t)\right]^{2+\frac{1}{\delta}}, \text{ for } t \geq t_0, \tag{19}$$

where $a > 0$, $b \in R$*, then there exists a finite time* T^* *such that*

$$\lim_{t \longrightarrow T^{*-}} H(t) = 0.$$

Upper bounds for T^* are estimated as follows:

(i) If $b<0$ and $H(t_0)<\min\left\{1, \sqrt{-\frac{a}{b}}\right\}$ then

$$T^* \leq t_0 + \frac{1}{\sqrt{-b}} \ln \frac{\sqrt{-\frac{a}{b}}}{\sqrt{-\frac{a}{b}} - H(t_0)}.$$

(ii) If $b=0$, then

$$T^* \leq t_0 + \frac{H(t_0)}{\sqrt{a}}.$$

(iii) If $b>0$, then

$$T^* \leq \frac{H(t_0)}{\sqrt{a}} \text{ or } T^* \leq t_0 + 2^{\frac{3\delta+1}{2\delta}} \frac{\delta c}{\sqrt{a}} \left[1-\left(1+cH(t_0)\right)^{-\frac{1}{2\delta}}\right],$$

where $c=\left(\frac{a}{b}\right)^{\frac{\delta}{2\delta+1}}$.

Definition 6 *A solution u of (1) is termed blow up if there is a finite time T^* such that*

$$\lim_{t\longrightarrow T^{*-}} \left[\int_\Omega u^2 dx + \int_0^t \int_\Omega \left(u^2 + |\nabla u|^2\right) dx d\tau\right] = \infty. \tag{20}$$

Let

$$a(t) = \int_\Omega u^2 dx + \int_0^t \int_\Omega \left(u^2+|\nabla u|^2\right) dx d\tau, \text{ for } t \geq 0. \tag{21}$$

Lemma 7 *Assume $\frac{q-1}{4} \geq \delta \geq \frac{\gamma}{2}$, and that $\gamma \geq 0$, then we have*

$$a''(t) \geq 4(\delta+1)\int_\Omega u_t^2 dx - 4(2\delta+1)E(0) + 4(2\delta+1)\int_0^t \left(\|u_\tau\|^2 + \|\nabla u_\tau\|^2\right) d\tau. \tag{22}$$

Proof. *By differentiating (21) according to t, we have*

$$a'(t) = 2\int_\Omega u u_t dx + \|u\|^2 + \|\nabla u\|^2, \tag{23}$$

$$\begin{aligned} a''(t) &= 2\int_\Omega u_t^2 dx + 2\int_\Omega u u_{tt} dx + 2\int_\Omega u u_t dx + 2\int_\Omega \nabla u \nabla u_t dx \\ &= 2\left(\|u_t\|^2 + \|u\|_{q+1}^{q+1}\right) - 2\left(\|\nabla u\|^2 + \|\nabla u\|^{2(\gamma+1)} + \|\Delta u\|^2\right). \end{aligned} \tag{24}$$

Then from (1) and (24), we have

$$\begin{aligned} a''(t) &= 4(\delta+1)\int_\Omega u_t^2 dx - 4(2\delta+1)E(0) \\ &\quad + 4\delta\left(\|\nabla u\|^2 + \|\Delta u\|^2\right) + \left(\frac{4\delta+2}{\gamma+1} - 2\right)\|\nabla u\|^{2(\gamma+1)} \\ &\quad + \left(2 - \frac{4(2\delta+1)}{q+1}\right)\|u\|_{q+1}^{q+1} + 4(2\delta+1)\int_0^t \left(\|u_\tau\|^2 + \|\nabla u_\tau\|^2\right) d\tau. \end{aligned}$$

Since $\frac{q-1}{4} \geq \delta \geq \frac{\gamma}{2}$, we obtain (22). ■

Lemma 8 *Assume* $\frac{q-1}{4} \geq \delta \geq \frac{\gamma}{2}$, $\gamma \geq 0$ *and one of the following expressions are satisfied*

(i) $E(0) < 0$ *and* $\int_\Omega u_0 u_1 dx > 0$,

(ii) $E(0) = 0$ *and* $\int_\Omega u_0 u_1 dx > 0$,

(iii) $E(0) > 0$ *and*

$$a'(0) > r_2 \left[a(0) + \frac{K_1}{4(\delta+1)} \right] + \|u_0\|^2 + \|\nabla u_0\|^2 \tag{25}$$

holds.

Then $a'(t) > \|u_0\|^2 + \|\nabla u_0\|^2$ *for* $t > t^*$, *where* $t_0 = t^*$ *is given by (26) in case (i) and* $t_0 = 0$ *in cases (ii) and (iii).*

Where K_1 *and* t^* *are defined in (30) and (26), in turn.*

Proof. (i) If $E(0) < 0$, then by (22), we attain

$$a'(t) \geq 2 \int_\Omega u_0 u_1 dx + \|u_0\|^2 + \|\nabla u_0\|^2 - 4(2\delta+1) E(0) t, \quad t \geq 0$$

Thereby we obtain $a'(t) > \|u_0\|^2 + \|\nabla u_0\|^2$ for $t > t^*$, where

$$t^* = \max \left\{ \frac{a'(0) - \left(\|u_0\|^2 + \|\nabla u_0\|^2 \right)}{4(2\delta+1) E(0)}, \ 0 \right\}. \tag{26}$$

(ii) If $E(0) = 0$ and $\int_\Omega u_0 u_1 dx > 0$, then $a''(t) \geq 0$ for $t \geq 0$. We have $a'(t) > \|u_0\|^2 + \|\nabla u_0\|^2$, $t \geq 0$.

(iii) If $E(0) > 0$, firstly, we write down that

$$2 \int_0^t \int_\Omega u u_t dx d\tau = \|u\|^2 - \|u_0\|^2. \tag{27}$$

Utilising Hölder inequality and Young inequality, we obtain

$$\|u\|^2 \leq \|u_0\|^2 + \int_0^t \|u\|^2 d\tau + \int_0^t \|u_\tau\|^2 d\tau \tag{28}$$

From (21), (23) and (28), we attain

$$a'(t) \leq a(t) + \|u_0\|^2 + \|u_t\|^2 + \int_0^t \int_\Omega \left(u_\tau^2 + |\nabla u|^2 \right) dx d\tau. \tag{29}$$

Hence, by (22) and (29), we get

$$a''(t) - 4(\delta+1) a'(t) + 4(\delta+1) a(t) + K_1 \geq 0,$$

where

$$\begin{aligned} K_1 &= 4(2\delta+1) E(0) + 4(\delta+1) \int_\Omega u_0^2 dx \\ &\quad + 4(\delta+1) \int_\Omega |\nabla u|^2 dx - 4\delta \int_0^t \left(\|u_\tau\|^2 + \|\nabla u_\tau\|^2 \right) d\tau \end{aligned} \tag{30}$$

Let

$$b(t) = a(t) + \frac{K_1}{4(\delta+1)}, \quad t > 0.$$

After $b(t)$ provides Lemma 4. As a result, we obtain by (25) $a'(t) > \|u_0\|^2 + \|\nabla u_0\|^2$, $t > 0$, where r_2 is given in Lemma 4. ■

Theorem 9 *Assume* $\frac{q-1}{4} \geq \delta \geq \frac{\gamma}{2}$, $\gamma \geq 0$ *and one of the following expressions are satisfied*

(i) $E(0) < 0$ *and* $\int_\Omega u_0 u_1 dx > 0$,

(ii) $E(0) = 0$ *and* $\int_\Omega u_0 u_1 dx > 0$,

(iii) $0 < E(0) < \frac{\left(a'(t_0) - \left(\|u_0\|^2 + \|\nabla u_0\|^2\right)\right)^2}{8\left[a(t_0) + (T_1 - t_0)\left(\|u_0\|^2 + \|\nabla u_0\|^2\right)\right]}$ *and (25) holds.*

After the solution u *blow up in finite time* T^* *in the case of (30). In case (i),*

$$T^* \leq t_0 - \frac{H(t_0)}{H'(t_0)}. \tag{31}$$

Moreover, if $H(t_0) < \min\left\{1, \sqrt{-\frac{a}{b}}\right\}$, *we get*

$$T^* \leq t_0 + \frac{1}{\sqrt{-b}} \ln \frac{\sqrt{-\frac{a}{b}}}{\sqrt{-\frac{a}{b}} - H(t_0)}, \tag{32}$$

where

$$a = \delta^2 H^{2+\frac{2}{\delta}}(t_0) \left[\left(a'(t_0) - \|u_0\|^2\right)^2 - 8E(0) H^{-\frac{1}{\delta}}(t_0)\right] > 0, \tag{33}$$

$$b = 8\delta^2 E(0). \tag{34}$$

In case (ii),

$$T^* \leq t_0 - \frac{H(t_0)}{H'(t_0)}. \tag{35}$$

In case (iii),

$$T^* \leq \frac{H(t_0)}{\sqrt{a}} \text{ or } T^* \leq t_0 + 2^{\frac{3\delta+1}{2\delta}} \left(\frac{a}{b}\right)^{2+\frac{1}{\delta}} \frac{\delta}{\sqrt{a}} \left\{1 - \left[1 + \left(\frac{a}{b}\right)^{2+\frac{1}{\delta}} H(t_0)\right]^{-\frac{1}{2\delta}}\right\}, \tag{36}$$

where a *and* b *are given (33), (34).*

Proof. Let

$$H(t) = \left[a(t) + (T_1 - t)\left(\|u_0\|^2 + \|\nabla u_0\|^2\right)\right]^{-\delta}, \text{ for } t \in [0, T_1], \tag{37}$$

where $T_1 > 0$ is a specific constant that will be indicated then. Later, we obtain

$$\begin{aligned} H'(t) &= -\delta \left[a(t) + (T_1 - t)\left(\|u_0\|^2 + \|\nabla u_0\|^2\right)\right]^{-\delta-1} \left[a'(t) - \left(\|u_0\|^2 + \|\nabla u_0\|^2\right)\right] \\ &= -\delta H^{1+\frac{1}{\delta}}(t) \left[a'(t) - \left(\|u_0\|^2 + \|\nabla u_0\|^2\right)\right], \end{aligned} \tag{38}$$

$$\begin{aligned} H''(t) &= -\delta H^{1+\frac{2}{\delta}}(t)\, a''(t) \left[a(t) + (T_1 - t)\left(\|u_0\|^2 + \|\nabla u_0\|^2\right)\right] \\ &\quad + \delta H^{1+\frac{2}{\delta}}(t)(1+\delta) \left[a'(t) - \left(\|u_0\|^2 + \|\nabla u_0\|^2\right)\right]^2. \end{aligned} \tag{39}$$

and

$$H''(t) = -\delta H^{1+\frac{2}{\delta}}(t)\, V(t)\,, \tag{40}$$

where

$$V(t) = a''(t)\left[a(t) + (T_1 - t)\left(\|u_0\|^2 + \|\nabla u_0\|^2\right)\right] - (1+\delta)\left[a'(t) - \left(\|u_0\|^2 + \|\nabla u_0\|^2\right)\right]^2. \tag{41}$$

For simplicity of calculation, we define

$$\begin{array}{cc} P_u = \int_\Omega u^2 dx, & R_u = \int_\Omega u_t^2 dx, \\ Q_u = \int_0^t \|u\|^2 dt, & S_u = \int_0^t \|u_t\|^2 dt, \\ M_u = \int_0^t \|\nabla u\|^2 d\tau, & N_u = \int_0^t \|\nabla u_\tau\|^2 d\tau. \end{array}$$

From (23), (27) and Hölder inequality, we get

$$\begin{aligned} a'(t) &= 2\int_\Omega u u_t dx + \|u_0\|^2 + \|\nabla u\|^2 + 2\int_0^t \int_\Omega u u_t dx dt \\ &\leq 2\left(\sqrt{R_u P_u} + \sqrt{Q_u S_u}\right) + \|u_0\|^2 + \|\nabla u\|^2. \end{aligned} \tag{42}$$

If case (i) or (ii) holds, from (22) we get

$$a''(t) \geq (-4 - 8\delta)\, E(0) + 4(1+\delta)(R_u + S_u + N_u)\,. \tag{43}$$

Thus, from (41)-(43) and (37), we attain

$$\begin{aligned} V(t) \geq\; & \left[(-4-8\delta)\, E(0) + 4(1+\delta)(R_u + S_u + N_u)\right] H^{-\frac{1}{\delta}}(t) \\ & -4(1+\delta)\left(\sqrt{R_u P_u} + \sqrt{Q_u S_u} + \sqrt{M_u N_u}\right)^2. \end{aligned}$$

From (21),

$$\begin{aligned} a(t) &= \int_\Omega u^2 dx + \int_0^t \int_\Omega \left(u^2 + |\nabla u|^2\right) dx d\tau \\ &= P_u + Q_u + M_u \end{aligned}$$

and (37), we get

$$V(t) \geq (-4-8\delta)\, E(0)\, H^{-\frac{1}{\delta}}(t) + 4(1+\delta)\left[(R_u + S_u + N_u)(T_1 - t)\left(\|u_0\|^2 + \|\nabla u_0\|^2\right) + \Theta(t)\right],$$

where

$$\Theta(t) = (R_u + S_u + N_u)(P_u + Q_u + M_u) - \left(\sqrt{R_u P_u} + \sqrt{Q_u S_u} + \sqrt{M_u N_u}\right)^2.$$

Utilising the Schwarz inequality, and $\Theta(t)$ being nonnegative, we get

$$V(t) \geq (-4-8\delta)\, E(0)\, H^{-\frac{1}{\delta}}(t)\,,\ t \geq t_0. \tag{44}$$

Thus, from (40) and (44), we obtain

$$H''(t) \leq 4\delta(1+2\delta)E(0)H^{1+\frac{1}{\delta}}(t), \ t \geq t_0. \quad (45)$$

From Lemma 8, we recognise that $H'(t) < 0$ for $t \geq t_0$. Multiplying (45) by $H'(t)$ and integrating it from t_0 to t, we obtain

$$H'^2(t) \geq a + bH^{2+\frac{1}{\delta}}(t)$$

for $t \geq t_0$, we can see a, b are described in (33) and (34) in turn.

If case (iii) holds, similar to the steps of case (i), we obtain $a > 0$ if and only if

$$E(0) < \frac{\left(a'(t_0) - \left(\|u_0\|^2 + \|\nabla u_0\|^2\right)\right)^2}{8\left[a(t_0) + (T_1 - t_0)\left(\|u_0\|^2 + \|\nabla u_0\|^2\right)\right]}.$$

After, from Lemma 5, there is a finite time T^* such that $\lim_{t \longrightarrow T^{*-}} H(t) = 0$ and upper bound of T^* is estimated for the sign of $E(0)$. This implies that (20) provides. ■

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