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Non-dominated sorting genetic algorithm (NSGA-II) approach to the multi-objective economic statistical design of variable sampling interval T^2 control charts

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Abstract

T^2 control charts are used to primarily monitor the mean vector of quality characteristics of a process. Recent studies have shown that using variable sampling interval (VSI) schemes results in charts with more statistical power for detecting small to moderate shifts in the process mean vector. In this study, we have presented a multiple-objective economic statistical design of VSI T^2 control chart when the in-control process mean vector and process covariance matrix are unknown. Then we exert to find the Pareto-optimal designs in which the two objectives are minimized simultaneously by using the Non-dominated sorting genetic algorithm. Through an illustrative example, the advantages of the proposed approach is shown by providing a list of viable optimal solutions and graphical representations, thereby bolding the advantage of flexibility and adaptability.

2000 AMS Classification:

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1. Introduction

Control charts have been used widely to monitor industrial processes. Nowadays, in industry, there are many situations in which the simultaneous monitoring or control of two or more related quality process characteristics is necessary. Unfortunately, the current practice in industry toward these multivariate and highly correlated variables is usually to have one set of univariate control charts for each variable. This approach creates many control charts that could easily overwhelm the operator. Also, this approach produces misleading results.

Important literature on multivariate process control include Jackson [1, 2], Alt [3] and Mason, Tracy and Young [4]. Lowry and Montgomery [5] wrote an excellent literature review on multivariate control charts. Extensive discussions on multivariate statistical process control can be found in Mason and Young [6], as well as in Fuchs and Kenett [7].

A common statistical method to monitor multivariate processes is to use the Hotelling T^2 control chart. The Hotelling T^2 control chart, an extension of the univariate Shewhart control chart, was developed by Hotelling [8]. However, because computing the T^2 statistic requires a lot of computations and requires some knowledge of matrix algebra, acceptance of multivariate control charts by industry was slow and hesitant.

Nowadays, with rapid progress in sensor technology and computing power, we are getting more and more data in production, manufacturing, and business operation. Most of these data are correlated multivariate data. The need to implement multivariate process control is growing. Also, with the increasing capability of modern computers, most of the laborious computational work can be accomplished in a split second, and it is getting easier and easier to implement multivariate process control.

The reduction of defective products and non-conformities is a fundamental principle of any quality improvement program and control charts are a powerful statistical tool to reach this goal. Duncan [9] was the first who evaluates the economic consequences of control charts which are affected by the choice of the control chart parameters such as the selection of the sample size (n), the control limits (k), and the time interval between samples (h). Consequently, Duncan [9] showed that statistical control charts may not be cost-effective and may increase the cost of production. Therefore, a wise attention should be given to economic objectives while designing control charts, i.e. selecting the control chart parameters.

Woodall [10] criticized economic designs by their poor statistical performance or their high Type I error rates. Saniga [11] developed a new approach named Economic Statistical Design (ESD) by adding statistical constraints on an economic model to combine the benefits of both pure statistical and economic designs. The ESD approach is very popular in the academic literature; in fact, Montgomery and Woodall [12] mentioned that the trend in economic modeling and design for control charts is to incorporate statistical constraints.

The traditional implementation of control charts is to apply a fixed ratio sampling (FRS) scheme in which samples of fixed size n_0 are obtained at constant intervals h_0 to monitor a process. Taylor [13] noted that economic control charts using the FRS scheme are non-optimal.

Accordingly, some researchers studied the ESDs of control charts with adaptive sampling schemes such as: Variable Sampling Intervals (VSI) (e.g. Chen [14] and Chao et al. [15]), Variable sample sizes (VSS) (e.g. Burr [16], Daudin [17] and Prabhu et al. [18]), Variable Sample Sizes and Sampling Intervals $(VSSI)$ (e.g. Chen [19]), Variable Sampling Intervals and control limits ($VSIC$) (e.g. Torabian et al. [20]), Variable Sample sizes and Control limits ($VSSC$) (e.g. Seif et al. [21, 22]) and Variable Parameters (VP) (e.g. Costa et al. [23]).

One major problem with any of the above mentioned designs is that they may not be flexible and adaptive. Faraz and Saniga [24] addressed the control chart design problem in a way that users are provided with a set of optimal designs which can be tailored to the temporal imperatives of the specific industrial situation. They showed that the proposed approach has the advantages of flexibility and thus adaptability when compared to the traditional economic statistical designs and yet preserve the statistical strengths and economic optimality of traditional designs.

Different solution algorithms are developed to obtain the optimal solution of the multi-objective optimization models. However, the quality of a Pareto optimal set can be evaluated based on three desirable properties, namely, diversity (a wide range of non-dominated solutions), uniformity (a uniform distribution of non-dominated solutions), and cardinality (a large number of non-dominated solutions) ([25, 26]).

The Pareto optimal solutions with the abovementioned properties can be obtained through the evolutionary algorithms such as multi-objective tabu search [27], vector evaluated genetic algorithm [25], multi-objective genetic algorithm [28], and non-dominated sorting genetic algorithm (NSGA and NSGA II) [27]. Unlike most of aforesaid methods that use one elite preservation strategy, NSGA II finds much spread solutions over the Pareto optimal set. It is one of the most popular multi-objective evolutionary algorithms known for its capacity to promote the quality of solutions [27].

Hence, NSGA-II that is an efficient method to identify the Pareto optimal set has been utilized in this research. The proposed Pareto optimization method searches for non- dominated solutions; optimization through the Pareto dominance compares each objective only with itself which remove the need for standardization of objectives.

In this paper, we develop the double objective ESD design of the VSI T^2 control chart, a study that hasn't been found in the literature yet. First we apply the Non-dominated Sorting Genetic Algorithm (NSGA-II) as a solution method. It's been proven that NSGA-II has a better capability in multi-objective optimization problems (see, Deb et al. [29]). Second, we theoretically develop an adaptive sampling intervals scheme with two sampling intervals. We also compare the results with the classical economic statistical designs through an illustrative example.

This paper is organized as follows: In Section 2, the VSI T^2 control scheme and Markov chain approach are briefly reviewed. In Section 3, the cost model proposed by Costa and Rahim [23] is described for our situation then double-objective optimization problem of the ESD VSI T^2 are presented in Section 3.3. Section 4 provides a brief introduction to the principle of the Non-dominated Sorting Genetic Algorithm (NSGAII). Numerical illustrations and comparisons are made in Section 5. Finally, concluding remarks make up the last section.

2. VSI T^2 Control Scheme and Markov Chain Approach

In order to control a process with p correlated characteristics using the T^2 scheme, it is first assumed that the joint probability distribution of the quality characteristics is a p-variate normal distribution with in-control mean vector $\mu_0' = (\mu_{01}, ..., \mu_{0p})$ and variance-covariance matrix $\sum$. Then the subgroups (each of size n) statistics $T_i^2 = n(\bar{X}_i - \mu_0)' \sum^{-1}(\bar{X}_i - \mu_0)$ are plotted in sequential order to form the T^2 control chart. The chart signals as soon as $T_i^2 \geqslant k$.

In statistical design methodology, If the process parameters(μ_0, Σ) are known, k is given by the upper α percentage point of chi-square variable with p degrees of freedom. However μ_0 and Σ are generally unknown and have to be estimated through m initial samples when the process is in control. In this case, the parameter k is obtained upon the $1-\alpha$ percentage point F distribution with p and ν degrees of freedom as follows:

$$k = c(m, n, p)F_\alpha(p, \nu) \tag{2.1}$$

$c(m, n, p) = \frac{p(m+1)(n-1)}{m(n-1)-p+1}$ and $\nu = m(n-1) - p + 1$. Note that if $n = 1$ then we have $c(m, n, p) = \frac{p(m+1)(n-1)}{m(m-p)}$ and $\nu = m(m-p)$.

In this paper, it is assumed that the process starts in a state of statistical control with mean vector μ_0 and covariance matrix Σ and then after a while assignable causes occur resulting in a shift in the process mean (μ_1). The magnitude of the shift is measured by $d = n(\mu_1 - \mu_0)' \sum^{-1}(\mu_1 - \mu_0)$. Further it is assumed that the time before the assignable cause occurs has an exponential distribution with parameter λ.Thus, the mean time that the process remains in state of statistical control is λ^{-1}.

When an $FRST^2$ chart is used to monitor a multivariate process, a sample of size n_0 is drawn every h_0 hour, and the value of the T^2 statistic (sample point) is plotted on a control chart with $k_0 = c(m, n_0, p)F_\alpha(p, \nu_0)$ as the control limit or action limit. One procedure to improve the statistical performance of the FRS control schemes is Variable Sampling Interval (VSI) scheme that varies the sampling interval between successive samples as a function of prior sample results. In this procedure, the area between the control limits and the origin has been divided into two zones by a warning line w for the use of two different sampling intervals$(h_1 > h_2)$. If the current sample value falls in a particular zone, then the next sample is to be drawn from the process after according to corresponding sampling interval. The use of the VSI control schemes requires the user to select five design parameters: the long and short sampling intervals h_1 and h_2, the fixed sample size n, the warning limit w and the control limit k.

In the literature, the most commonly used measure for comparing control schemes with different sampling strategies is the adjusted average time to signal $(AATS)$. This is also the average time from a process mean shift until the chart produces a signal and is defined as follows::

$$AATS = ATC - \lambda^{-1} \tag{2.2}$$

where ATC (the average time of the cycle) is the average time from the beginning of the process until the first signal after the process shift. One method of calculating ATC is using Markov chains. Readers are referred to Cinlar [30] for the fundamental ideas behind the Markov chain approach we use. Now, upon the VSI scheme, each sampling stage can be considered as one of the following five transient states:
State 1: $0 \leq T^2 < w$ and the process is in control;
State 2: $w \leq T^2 < k$ and the process is in control;
State 3: $T^2 \geq k$ and the process is in control (false alarm);
State 4: $0 \leq T^2 < w$ and the process is out of control;
State 5: $w \leq T^2 < k$ and the process is out of control;

The control chart produces a signal when $T^2 \geq k$. If the current state is 3, the signal is a false alarm; the absorbing state (state 6) is reached when the true alarm occurs. The transition probability matrix is given by

$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} \\ p_{21} & p_{22} & p_{23} & p_{24} & p_{25} & p_{26} \\ p_{31} & p_{32} & p_{33} & p_{34} & p_{35} & p_{36} \\ 0 & 0 & 0 & p_{44} & p_{45} & p_{46} \\ 0 & 0 & 0 & p_{54} & p_{55} & p_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{2.3}$$

Where p_{ij} denotes the probability of transitioning from state i to j state . In what follows, $F(x, p, \nu, \eta)$ will denote the cumulative probability distribution function of a non-central F distribution with p and ν degrees of freedom and non-centrality parameter $\eta = nd^2$.

$$p_{11} = p(T^2 < w) \times e^{-\lambda h_1} = F(\frac{w}{c(m,n,p)}, p, \eta = 0) \times e^{-\lambda h_1} \tag{2.4}$$

$$p_{12} = p(w \leq T^2 < k) \times e^{-\lambda h_1} = [F(\frac{k}{c(m,n,p)}, p, \eta = 0) - F(\frac{w}{c(m,n,p)}, p, \eta = 0)] \times e^{-\lambda h_1} \tag{2.5}$$

$$p_{13} = p(T^2 \geq k) \times e^{-\lambda h_1} = [1 - F(\frac{k}{c(m,n,p)}, p, \eta = 0)] \times e^{-\lambda h_1} \tag{2.6}$$

$$p_{14} = p(T^2 < w) \times (1 - e^{-\lambda h_1}) = F(\frac{w}{c(m,n,p)}, p, \eta = nd^2) \times (1 - e^{-\lambda h_1}) \tag{2.7}$$

$$p_{15} = p(w \leq T^2 < k) \times (1 - e^{-\lambda h_1}) = [F(\frac{k}{c(m,n,p)}, p, \eta = nd^2) - F(\frac{w}{c(m,n,p)}, p, \eta = nd^2)] \times (1 - e^{-\lambda h_1}) \tag{2.8}$$

$$p_{16} = p(T^2 \geq k) \times (1 - e^{-\lambda h_1}) = [1 - F(\frac{k}{c(m,n,p)}, p, \eta = nd^2)] \times (1 - e^{-\lambda h_1}) \tag{2.9}$$

$$p_{21} = p_{31} = p(T^2 < w) \times e^{-\lambda h_2} = F(\frac{w}{c(m,n,p)}, p, \eta = 0) \times e^{-\lambda h_2} \tag{2.10}$$

$$p_{22} = p_{32} = p(w \leq T^2 < k) \times e^{-\lambda h_2} = [F(\frac{k}{c(m,n,p)}, p, \eta = 0) - F(\frac{w}{c(m,n,p)}, p, \eta = 0)] \times e^{-\lambda h_2} \tag{2.11}$$

$$p_{23} = p_{33} = p(T^2 \geq k) \times e^{-\lambda h_2} = [1 - F(\frac{k}{c(m,n,p)}, p, \eta = 0)] \times e^{-\lambda h_2} \tag{2.12}$$

$$p_{24} = p_{34} = p(T^2 < w) \times (1 - e^{-\lambda h_2}) = F(\frac{w}{c(m,n,p)}, p, \eta = nd^2) \times (1 - e^{-\lambda h_2}) \tag{2.13}$$

$$p_{25} = p_{35} = p(w \leq T^2 < k) \times (1 - e^{-\lambda h_2}) = [F(\frac{k}{c(m,n,p)}, p, \eta = nd^2) - F(\frac{w}{c(m,n,p)}, p, \eta = nd^2)] \times (1 - e^{-\lambda h_2}) \tag{2.14}$$

$$p_{26} = p_{36} = p(T^2 \geq k) \times (1 - e^{-\lambda h_2}) = [1 - F(\frac{k}{c(m,n,p)}, p, \eta = nd^2)] \times (1 - e^{-\lambda h_2}) \tag{2.15}$$

(2.16) $p_{44} = p_{54} = p(T^2 < w) = F(\frac{w}{c(m,n,p)}, p, \eta = nd^2)$

(2.17) $p_{45} = p_{55} = p(w \le T^2 < k) = F(\frac{k}{c(m,n,p)}, p, \eta = nd^2) - F(\frac{w}{c(m,n,p)}, p, \eta = nd^2)$

(2.18) $p_{46} = p_{56} = p(T^2 \ge k) = 1 - F(\frac{k}{c(m,n,p)}, p, \eta = nd^2)$

Now, ATC is calculated as follows:

(2.19) $ATC = \mathbf{b}'(\mathbf{I} - \mathbf{Q})^{-1}\mathbf{h}$

where $\mathbf{h}' = (h_1, h_2, h_2, h_1, h_2)$ is the vector of sampling time intervals, $\mathbf{Q}$ is the 5×5 matrix obtained from $\mathbf{P}$ by deleting the elements corresponding to the absorbing state, $\mathbf{I}$ is the identity matrix of order 5 and $\mathbf{b}' = (p_1, p_2, p_3, p_4, p_5)$ is a vector of initial probabilities, with $\sum_{i=1}^{5} p_i = 1$. In this paper, the vector $\mathbf{b}'$ is set to $(0, 1, 0, 0, 0)$ to provide extra protection and prevent problems that are encountered during start-up.

3. The cost model

3.1. Assumptions. In building our model of a process controlled by a $VSIT^2$ control chart we make the usual assumptions about the process, namely:

1. The p quality characteristics follow a multivariate normal distribution with mean vector μ and covariance matrix Σ.
2. The process is characterized by an in-control state $\mu = \mu_0$.
3. A single assignable cause produces "step changes" in the process mean from $\mu = \mu_0$ to a known $\mu = \mu_1$. This results in a known value of the Mahalanobis distance.
4. "Drifting processes" are not a subject of this research. That is, assignable causes that affect process variability are not addressed, and hence it is assumed that the covariance matrix Σ is constant over time.
5. Before the shift, the process is considered to be in a state of statistical control.
6. The assignable cause is assumed to occur according to a Poisson distribution with intensity λ occurrences per hour.
7. The process is not self-correcting.
8. The quality cycle starts with the in-control state and continues until the process is repaired after an out-of-control signal. It is assumed that the quality cycle follows a renewal reward process.
9. During the search for an assignable cause, the process is shut down.

3.2. The loss function. The process cycle consists of the following four phases: in control, out of control, assignable cause detection, and repair. Therefore, the expected length of a production cycle is given by

(3.1) $E(T) = ATC + T_0 ANF + T_1$

where T_0 is the average amount of time wasted searching for the assignable cause when the process is in control, T_1 is the average time to find and remove the assignable cause, and ANF is the expected number of false alarms per cycle. The expected number of false alarms per cycle is given by

(3.2) $ANF = \mathbf{b}'(\mathbf{I} - \mathbf{Q})^{-1}(0, 0, 1, 0, 0)$

The expected net profit from a production cycle is given by

$$E(C) = V_0 \times (\frac{1}{\lambda}) + V_1 \times (ATC - \frac{1}{\lambda}) - C_0 \times ANF - C_1 - S \times ANI \tag{3.3}$$

where V_0 is the average profit per hour earned when the process is operating in control, V_1 is the average profit per hour earned when the process is operating out of control, C_0 is the average cost of a false alarm, C_1 is the average cost for detecting and removing the assignable cause, S the cost per inspected item, and ANI is the average number of inspected items per cycle. The average number of inspected items per cycle is given by

$$ANI = \mathbf{b}'(\mathbf{I} - \mathbf{Q})^{-1}(n, n, n, n, n) \tag{3.4}$$

and the loss function $E(L)$ is given by

$$E(L) = V_0 - \frac{E(C)}{E(T)} \tag{3.5}$$

3.3. Double-objective ESD of the VSIT^2 chart. Equations (2), (21) and (24) give the three important objectives for designing a control chart. By minimizing ANF, a practitioner can reduce false alarm rates. In a similar fashion minimizing $AATS$ guarantees detecting assignable causes as quickly as possible and minimizing the quality cycle cost, or $E(L)$, satisfies the firm's economic objectives. Saniga's [11] ESD approach considers all of the above mentioned criteria but it lacks flexibility and adaptability. This approach provides the practitioners with solutions that consider the trade offs between the statistical and economic objectives.

Let $\overrightarrow{x} = (k, w, h_1, h_2, n)$ be the VSI design vector comprising control limit k, warning line w, and sampling frequencies h_1 and h_2 and sample size n. The most plausible approach to determine the optimal values of the design vector $\overrightarrow{x}$ is that proposed by Saniga [11], called the ESD approach. This approach considers the design problem as an economic single-objective problem with several statistical constraints which has a major focus on reducing the cost of applying control charts. However, in designing control charts, there are three objectives: the expected loss per hour $E(L)$ and the two statistical objectives Type II and Type I error rates, or equivalently $AATS$ and ANF, which should be traded off in some way.

Usually the Type I error rate is somewhat fixed by the practitioners but there is no clear relative preference of the other two objectives. Hence, in this paper, we consider two objectives $E(L)$ and $AATS$ which are of the minimization type and tackle the Type I error issue in constraints. The goal of the double-objective ESD of the $VSIT^2$ scheme is to find $\overrightarrow{x}$ to simultaneously minimize both $E(L)$ and $AATS$ objectives subject to some constraints. Therefore, the double-objective problem is defined as follows:

$$\begin{aligned} &Min\ \ (E(L); AATS) \\ &\qquad s.t.: \\ &ANF \leq ANF_0 \\ &\quad k < w \\ &\quad 1 \leq n \in Z^+ \\ &h_2 \leq h_1 \leq h_{max} \end{aligned} \tag{3.6}$$

In the above double-objective model, the constraint $ANF \leq ANF_0$ is added to form the best protection against false alarms; in this paper, without loss of generality, the value of $ANF_0 = 0.05$

shall be used. The parameter h_{max} is added to keep the chart more practical; in particular, we use the values of $h_{max} = 15h$ to eliminate other solutions that may prove problematic in a work shift. The goal of Double-objective ESD of the $VSIT^2$ control chart is to find the seven chart parameters (k, w, h_1, h_2, n) which optimization problem (25), given the five process parameters $(p, \lambda, d, T_0, T_1)$ and the five cost parameters (V_0, V_1, C_0, C_1, S).

4. Elitist non-dominated sorting genetic algorithm (NSGA-II)

A solution to the optimization problem (25) can be described by a decision vector $\overrightarrow{x} = (x_1, x_2, \ldots, x_5)$ in the design space X. The objective functions (2) and (24) define the function f which assigns an objective vector $\overrightarrow{y} = (y_1, y_2)$ in the objective space Y to each solution vector $\overrightarrow{x}$, i.e. f is a vector map of the form $f : X \to Y$. In the multi-objective optimization the optimal solutions form a dominant boundary which is defined as follows:
Suppose $(\overrightarrow{x}_1)$ and $(\overrightarrow{x}_2)$ are two arbitrary and viable solutions in X. we say:
• $(\overrightarrow{x}_1)$ dominates $(\overrightarrow{x}_2)$ $(\overrightarrow{x}_1 < \overrightarrow{x}_2)$ if the two components $\overrightarrow{y}_1 = f(\overrightarrow{x}_1)$ are less than or equal to their corresponding components in $\overrightarrow{y}_2 = f(\overrightarrow{x}_2)$.
• A solution $\overrightarrow{x}$ in X belongs to the dominant boundary if there is no other solution in X that dominates $\overrightarrow{x}$.

Dominant boundary includes all non-dominated optimal solutions to the problem. The set of these solutions is named Pareto set while its image in objective space is named Pareto front. A generic multi-objective optimization solver searches for non-dominated solutions that correspond to trade-offs between all the objectives. The genetic algorithms (GA) are semi-stochastic methods, based on an analogy with Darwin's laws of natural selection. The first multi-objective genetic algorithm (MOGA), called vector-evaluated GA (or VEGA), was proposed by Schaffer [31]. Recently, more advanced MOGA approaches are proposed, for example: the Niched Genetic Algorithm (NPGA) [32], the Non-dominated Sorting Genetic Algorithm (NSGA). Through a comparative case study, Zitzler and Thiele [33] showed that the NSGA has a better capability in multi-objective optimization problems than the VEGA and NPGA. Deb et al. [29] presented a fast and elitist NSGA algorithm called NSGA-II which is proven to have a better capability than the NSGA algorithm. Its main features are as follows:
• A sorting non-dominated procedure where all the individuals are sorted according to the level of non-dominance.
• It implements elitism which stores all non-dominated solutions and enhances convergence properties.
• It adapts a suitable automatic mechanism based on the crowding distance in order to guarantee the diversity of solutions.
• Constraints are implemented using a modified definition of dominance without the use of penalty functions.

In the NSGA-II procedure we have used the following settings of the control parameters: population size (N_{pop}) is set to 100; crossover percentage (p_c) is set to 0.2; mutation rate (r_m) is set to 0.1; mutation percentage (r_p) is set to 0.9; and the maximum number of iterations is set to 500.

5. Numerical analysis

In this section, the model application is illustrated through an industrial example. Consider a product with two important quality characteristics that should be monitored jointly $(p = 2)$. The estimated fixed and variable cost of sampling is \$5 $(S = 5)$ per item. The process is subject to several different types of assignable causes. However, on the average, when the process goes out of control, the magnitude of the mean shift is approximately 0.5 $(d = 0.5)$ and the process mean shift occurs every 100 hours of operation which reasonably can be modeled with an exponential distribution with parameter $\lambda = 0.01$. The average time to investigate an out-of-control signal and repairing the process is 60 minute ($T_1 = 1$), while the time spent to investigate a false alarm is 5 hours ($T_0 = 5$). The cost of detecting and removing the assignable cause is \$500 ($C_1 = 500$), while the cost of investigating a false alarm is \$500 ($C_0 = 500$). The average profit per hour earned when the process is operating in-control is \$500 per hour ($V_0 = 500$), while the average profit per hour earned when the process is operating out-of-control is \$50 per hour ($V_1 = 50$).

In the following, Hotelling's T^2 control charts with the VSI scheme (Table2), and the FRS scheme (Table1), for $d = 0.5$, are compared with respect to the loss function. For example, approximately 16% more savings per hour can be achieved by applying the VSI scheme than the FRS scheme and better statistical properties are also obtained. Consider the process working 8 h a day, 5 days a week and 22 days a month; here, the VSI scheme results in more than \$111724 savings annually. The VSI scheme is also able to detect the process shift $d = 0.5$ after 367-399 min with $AATS$ close to 6.5, but if someone is interested in detecting that shift sooner (around 294-304 min, say) the bolded designs with $AATS$ close to 5 are the good choices, costing 7.73-8.77 dollars per hour more than the economic ones.

Table 1

The optimal design of FRS scheme.

k	h	n	ANF	$AATS$	$Loss$
11.07	9.57	50	0.05	9.70	75.53

In Table 2, we list 20 designs on the Pareto optimal contour or Pareto front. Note that the first design is the least costly, and we see a consistent increase in cost as the $AATS$ becomes smaller, an expected result because Pareto optimal designs, unlike pure statistical design, are cost optimal for these prescribed constraints on $AATS$ and ANF. As illustrated in Figure 1, the multiple-objective economic statistical design(MOESD), using NSGA-II approach, gives a visual indication of how the $AATS$ and $E(L)$ trade off; this easily allow users to consider the costs of improved quality monitoring; that is, tighter control costs more. The advantage of the MOESD using NSGA-II approach is apparent in this example; by providing a set of designs, including graphical representations, each with its own cost, $AATS$, and ANF, the user can tailor the design to the temporal imperative of the industrial process, thereby having the advantage of flexibility and adaptability. Several findings from Tables (1-2) are spelled out as follows.

- The Loss values of the VSI control schemes are consistently smaller than that of the FRS control scheme.
- Compared with the FRS schemes, the corresponding VSI scheme requires more often sampling with a wider upper control limit and a smaller sample size.
- All the cases from the tables indicate that the optimal value of h_2 is close to zero, which means the process should be sampling immediately if T^2 falls into the warning region.
- Smaller $AATS$ implies the VSI control schemes offering a quicker speed for detecting a mean shift.
- The multi-objective solution has the added advantage of demonstrating the tradeoffs between the statistical and economic objectives.

Finally, we point out some more advantages of the proposed multi-objective model in a comparison with the traditional ESD designs introduced by Saniga [11]. Table 3 gives the classical ESDs for the example with two constraints on $AATS$, i.e, $AATS \leq 7$ and $AATS \leq 6$. First, setting these constraints is subjective. Second, in the classical ESDs practitioners have no clear idea about the trade-off between the cost function and statistical constraints. Third, a good guess can be setting the statistical constraints close to the Pareto optimal contour ($AATS \leq 6.65$) of the MOESD approach, i.e., $AATS \leq 7$ or $AATS \leq 6$. Please note that this would not be true in general because setting proper statistical constraints as one does in classical ESDs is different than optimizing the statistical constraints (such as that on $AATS$) as one does in the MOESD approach. Note also the lack of flexibility of the classical ESDs versus the MOESDs. In the latter case, the user has a choice of 20 designs each of which is Pareto optimal, whereas in the former case, only a single design is provided. Reducing control, as the second ESD example shows (larger $AATS$ constraint), results in a decrease in cost.

Table 2
The MOESD $VSIT^2$ chart.

$No.$	k	w	h_1	h_2	n	ANF	$AATS$	$Loss$
1	**11.58**	**3.46**	**10.32**	**0.0001**	**39**	**0.05**	**6.65**	**64.95**
2	**11.70**	**3.48**	**9.91**	**0.0001**	**39**	**0.05**	**6.42**	**64.99**
3	**11.83**	**3.45**	**9.90**	**0.0001**	**40**	**0.05**	**6.28**	**65.14**
4	**11.85**	**3.52**	**9.36**	**0.0001**	**39**	**0.05**	**6.11**	**65.16**
5	11.85	3.21	9.36	0.0001	39	0.05	5.90	65.36
6	11.98	3.47	9.03	0.0001	39	0.05	5.88	65.38
7	11.98	3.47	9.03	0.0001	40	0.05	5.75	65.53
8	12.15	3.60	8.70	0.0001	40	0.04	5.65	65.75
9	12.16	3.50	8.50	0.0001	40	0.05	5.45	66.01
10	12.08	3.47	8.27	0.0001	40	0.05	5.28	66.26
11	12.17	3.57	7.92	0.0001	40	0.05	5.12	66.64
12	**12.18**	**3.47**	**7.93**	**0.0001**	**40**	**0.05**	**5.07**	**66.76**
13	**12.18**	**3.47**	**7.72**	**0.0001**	**40**	**0.05**	**4.93**	**67.10**
14	**12.46**	**3.36**	**8.17**	**0.0001**	**43**	**0.04**	**4.90**	**67.80**
15	12.44	3.46	7.04	0.0001	38	0.05	4.75	67.89
16	12.43	3.48	7.02	0.0001	40	0.05	4.52	68.66
17	13.39	3.27	7.00	0.0001	40	0.03	4.51	69.73
18	13.17	3.46	6.64	0.0001	40	0.04	4.35	70.16
19	12.98	3.41	6.65	0.0001	41	0.04	4.21	70.63
20	12.67	3.51	6.49	0.0001	41	0.05	4.12	70.78

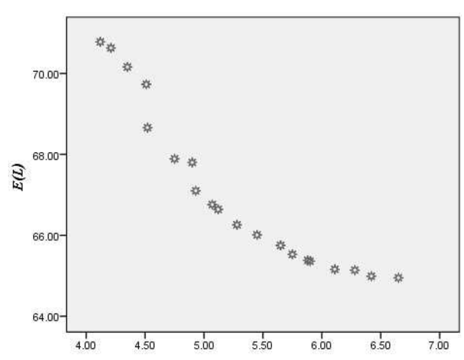

Figure1. Pareto front graph of Table 2.

6. Concluding remark

In this study we have presented a multiple-objective economic statistical design of $VSIT^2$ control chart when the in-control process mean vector and process covariance matrix are unknown. Therefore, a cost model was derived by the Markov Chain approach, and NSGA-II approach was applied to find the optimal design parameters. These solutions define a Pareto optimal set of solutions which greatly increase the flexibility and adaptability of control chart design in practical applications. Using the VSI scheme has been shown to give substantially faster detection of most process shifts than the conventional FRS scheme.

Table 3
The ESD $VSIT^2$ chart.

$Design$	$Constraints$	k	w	h_1	h_2	n	ANF	$AATS$	$Loss$
ESD	$ANF \leq 0.05 \& AATS \leq 7$	11.58	3.52	10.23	0.0001	39	0.05	6.6	64.95
ESD	$ANF \leq 0.05 \& AATS \leq 6$	11.56	4.02	9.81	0.0001	45	0.05	6	65.6

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