Int. J. of Thermodynamics **ISSN 1301-9724**
Vol. 11 (No. 4), pp. 161-171, December 2008

Entropy Generation in Pin Fins of Circular and Elliptical Cross-Sections in Forced Convection with Air

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Abstract

The entropy generation of any thermodynamic system provides a useful measure of the extent of irreversibility. The irreversibility causes the loss of useful work (exergy) in the system and hence the loss of exergy has to be minimized. Entropy generation is one parameter that quantifies the loss of exergy. In this work, the entropy generation of pin fins of circular and elliptical cross sections in cross flow of air is calculated, and their performances are compared with respect to entropy generation. The optimum working parameters for a given geometry and the optimum geometry for the given working parameters are identified for the fins. The entropy generation is calculated for circular and elliptical pin fins with mass constraint.

Keywords: Pin fins, entropy generation, heat transfer coefficient, drag coefficient

1. Introduction

Extended surfaces (fins) constitute one of the most effective design features for promoting heat transfer between a solid surface and a stream of fluid. The importance of this thermal design technique in the general area of heat transfer augmentation and energy conservation is fully recognized by the heat transfer community, Bergles (1979) and Junkhan, G.H (1979).

The traditional approach to the optimization of fins consists of minimizing the consumption (investment) of fin material for the execution of a specified heat transfer task. More than a half-century ago, Schmidt stated intuitively that a two-dimensional fin must have a parabolic-law based cross-sectional profile if it is to require the least material (volume) for a certain heat transfer rate and this design principle was later proved by Duffin (1959) who worked on the formulation of variational calculus (Poulikakos, D., 1982). This design principle has been steadily brought closer to the realities of fin manufacturing and heat exchanger operation by a number of contributors who have analyzed the role of radiation, two-directional heat transfer (curvature) temperature-dependent thermal conductivity and variable heat transfer coefficient (see, for example, references Maday 1974; Guceri and Maday 1975; Razelos and Imre 1980; Kern and Kraus 1972; Kraus and Snider 1980).

Entropy generation in a thermodynamic system is defined as the difference between the entropy change of the system and the net entropy transport into the system and is a path function of the thermodynamic system. According to the second law of thermodynamics, the entropy change of a system is always greater than or equal to the net entropy transport into that system. The equality exists only in the reversible process. And in all actual (irreversible) processes, the entropy change of the system is always greater than that of the net entropy transport into the system, and hence in all actual processes, entropy generation is always a positive quantity. It is a unique parameter to measure the strength of irreversibility of thermodynamic processes.

Thermal pollution, apart from the heat rejection of the system as required by the second law of thermodynamics for any thermodynamic system, is directly proportional to entropy generation (EG). So it becomes necessary to study the mechanism of entropy generation and the influence of various parameters on it. The

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first and second laws of thermodynamics, taken together, state that the entropy generated by any engineering system is proportional to the work lost (destroyed) irreversibly by the system. This truth is expressed concisely as the Gouy-Stodola Theorem (Szargut 1980).

$$W_{lost} = T_o \sum_{\substack{\text{all} \\ \text{system} \\ \text{components}}} S_{gen} \tag{1}$$

Where W_{lost} is the lost available work (lost availability, or lost exergy) (Kestin 1980), T_o is the absolute temperature of the environment, and S_{gen} is the entropy generated in each compartment of the system. Equation (1) implies that the thermodynamic irreversibility (entropy generation) of each system component contributes to the aggregate loss of available work in the system (W_{lost}). In a heat engine the entropy generated in one component (e.g., the condenser) is responsible for a proportional share of the loss in power output from the engine cycle. Beyond that, it influences the behavior of all other components of the thermodynamic device.

The entropy production can be determined at bulk or continuum level. The bulk level has the advantage of simplicity but the information about the entropy generation mechanisms are less accurate. The continuum level is obviously more sophisticated and often it can be performed only by numerical simulation. It always offers not only a great precision of calculus, but also the possibility of a true understanding of entropy production structure. But as pointed out by Stanciu et al. (2005), in order to save a lot of computational time, it is better to combine the two methods when an optimized design is desired, meaning that the bulk method is used to quickly obtain an optimum and the continuum level is followed to refine it.

Pin fins serve as components in a wide range of heat transfer applications. Therefore, in order to conserve available work (exergy), it is necessary to approach the design of such pin fins from the point of view of entropy generation minimization. The trade-off between heat transfer and fluid friction is a classical dilemma in heat exchanger design (Junkhan 1979; and Poulikakos and Bejan 1982). The irreversibility minimization philosophy places this trade-off on a solid foundation, as heat transfer and fluid drag are both mechanisms for entropy generation.

Using the bulk level of entropy production, Taufiq et al. (2007) found the optimal design of radial fin geometry with respect to Reynolds number in both laminar and turbulent convection. In a more complete investigation, Kahn et al. (2007), emphasize the effects of tube diameter, tube length and dimensionless pitch ratio in influencing optimum design conditions and the overall performance of convection heat transfer through a bank of circular tubes.

The objective of this paper is to analyze and optimize the design of pin fin of different cross section and aspect ratios in forced convection heat transfer. As in previous cases, this approach consists of calculating the entropy generation rate of one pin fin, and minimizing it systematically. The competition between enhanced thermal contact and fluid friction is settled when the heat transfer irreversibility and the fluid friction irreversibility add to yield a minimum rate of entropy generation for the pin fins of circular and elliptical cross-sections.

A general formula for the rate of entropy generation in an arbitrary fin under the forced convection heat transfer is used. Based on this general result, it is shown how the geometric parameters of common fin shapes can be selected so that the fin saves the most exergy (available work) while performing its specified heat transfer function.

For simplicity, the classical fin heat transfer model (Gardner 1945) is adopted, whereby the fin is slender enough so that the conduction process can be regarded as unidirectional. It is further assumed that the properties of the fin material and those of the external fluid are constant. The external flow is assumed uniform and parallel to the base surface of the pin fins.

2. Entropy generation due to convective heat transfer from a single fin

The entropy generated by a single pin fin in cross flow can be evaluated based on the general model presented in Fig.1. A pin fin of arbitrary cross section is suspended in a uniform stream with velocity, U_∞, and temperature, T_∞ and a fixed control volume is selected for analysis. The heat transfer, q_B, is driven by the temperature difference between the fin base, T_B, and the free stream, T_∞. In addition, the cross flow arrangement is responsible for a net drag force F_D, which is transmitted through the fin to the control volume.

At steady state, the first and second laws of thermodynamics for the system specified in Fig. 1, respectively, yield the following equations.

$$q_B - q_\infty + F_D U_\infty = 0 \tag{2}$$

$$S_{gen} = \frac{q_\infty}{T_\infty} - \frac{q_B}{T_B} > 0 \tag{3}$$

In Equations (2) and (3), q_B is the amount of heat which is transferred from the base wall to the fin and q_∞ is the amount of heat, which is transferred from the control volume to the rest of the fluid. The free stream temperature and free stream velocity are taken as T_∞ and U_∞ respectively.

The entropy generation in the heat transfer and fluid flow arrangement is S_{gen}. By combining the Equations (2) and (3), q_∞ can be eliminated and the following equation is obtained.

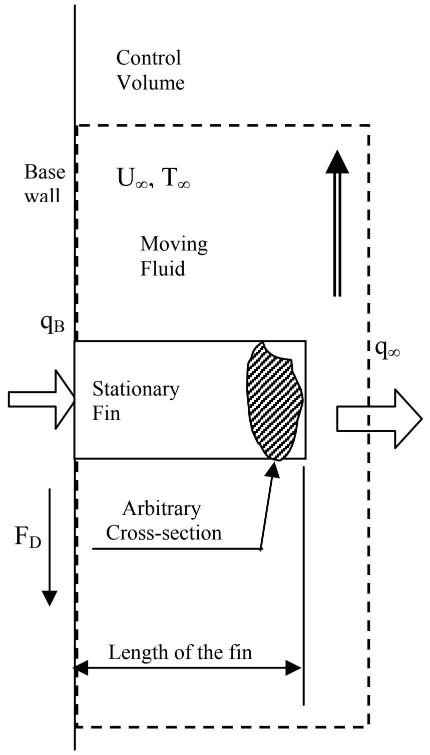

Figure 1. Schematic of a pin fin of circular cross section in a convective heat transfer arrangements

$$S_{gen} = \frac{q_B \theta_B}{T_\infty^2 \left[1 + \frac{\theta_B}{T_\infty}\right]} + \frac{F_D U_\infty}{T_\infty} \tag{4}$$

where $\theta_B = T_B - T_\infty$

If $\theta_B / T_\infty < 0.1$, the same can be neglected when compared to unity. Consequently, the entropy generation Equation (4) takes the simpler form Equation (5).

$$S_{gen} = \frac{q_B \theta_B}{T_\infty^2} + \frac{F_D U_\infty}{T_\infty} \tag{5}$$

There are two sources of entropy generation in fins. One is due to heat transfer between two streams of different temperature, and the second one is due to friction between the heat transfer surfaces and moving fluids. The first term in Equation (4) represents the entropy generation due to a heat transfer across a non-zero temperature difference, while the second term is the entropy generation associated with fluid friction.

$$S_{gen} = S_{genh} + S_{genf} \tag{6}$$

where S_{genh} and S_{genf} are entropy generations due to temperature difference and fluid friction respectively.

An examination of expression (4) leads to the conclusion that in order to minimize the heat transfer contribution to S_{gen}, one must minimize the base stream temperature difference θ_B. In practical terms, however, to minimize θ_B would imply the use of an infinitely large fin and such a fin would be impractical and thermodynamically undesirable, because an infinitely large fin would have an infinite amount of S_{genf}.

3. Analytical Relationships

In developing the analytical relationships, the classical heat transfer model is adopted. Since this paper describes the entropy generation of the pin fins in high temperature applications, θ_B / T_∞ cannot be neglected when compared to unity. Hence the entropy generation takes the complete expression, Equation (4).

3.1 Pin fin of Circular Cross section

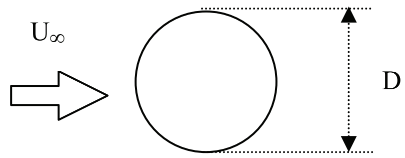

Figure 2. Flow over circular pin fins

Pin fin of circular cross section is one of the simplest geometries, since the heat transfer and drag force depend on only two dimensions, the length L and the diameter of the circular cross section D. The entropy generation Equation (4) can also be written as the following expression.

$$S_{gen} = \frac{q_B}{T_\infty \left[1 + \frac{T_\infty}{\theta_B}\right]} + \frac{F_D U_\infty}{T_\infty} \tag{7}$$

According to the unidirectional heat conduction model, the relationship between base heat flux and base stream temperature difference is (Kern 1972)

$$\theta_B = \frac{q_B}{\frac{\pi}{4} \lambda D^2 m \times \tanh(mL)} \tag{8}$$

where $m = \left(\frac{4h}{\lambda D}\right)^{1/2}$; $Nu = \frac{hD}{k}$

The objective of pin fins is to transfer heat at a specified rate, q_B, from the wall to the known stream, in a less irreversible manner (with minimum entropy generation). Therefore the entropy generation number for this design problem is constructed as (Bejan 1996)

$$N_s = S_{gen} \Big/ \left(\frac{q_B^2 U_\infty}{k\nu T_\infty^2}\right) \tag{9}$$

Substituting Equations (7) and (8) into Equation (9) and after rearranging terms, the following expression can be obtained

$$N_s = \frac{1}{Re_L\left[B_2 + \left(\frac{\pi}{2}\right)Nu^{1/2}(\Theta)^{1/2}\tanh\left[2\gamma.Nu^{1/2}\left(\frac{1}{\Theta}\right)^{1/2}\right]\right]} + \frac{1}{2}B_1.C_D.Re_L^2.\gamma \tag{10}$$

where $\Theta = \lambda/k$, the dimensionless number, is the ratio of fin and fluid thermal conductivities. The slenderness ratio ($\gamma = l/L$) is defined as the ratio between the length and characteristic dimension of the pin fins. For the slender pin fin of circular cross section, i.e.$\gamma \geq 5$, the Nusselt number and drag coefficient can be evaluated from the results developed for a single circular cylinder in cross flow and are given by Equation (11). From Zhukauskas (1972)

$$\begin{aligned} &1 \leq Re_L \leq 40 && Nu = 0.657\,Re_L^{0.4} \\ &40 < Re_L \leq 1\times10^3 && Nu = 0.447\,Re_L^{0.5} \\ &1\times10^3 < Re_L \leq 2\times10^5 && Nu = 0.228\,Re_L^{0.6} \\ &2\times10^5 < Re_L \leq 1\times10^6 && Nu = 0.067\,Re_L^{0.7} \end{aligned} \tag{11a}$$

From Gebhart (1971)

$$\begin{aligned} &1 \leq Re_L \leq 4 && C_D = 10\,Re_L^{-0.6} \\ &4 < Re_L \leq 4\times10^3 && C_D = 5.484\,Re_L^{-0.246} \\ &4\times10^3 < Re_L \leq 2\times10^5 && C_D = 1.1 \end{aligned} \tag{11b}$$

$$B_1 = \frac{\rho k \nu^3 T_\infty}{q_B^2} \; ; \qquad B_2 = \frac{q_B}{k D T_\infty} \tag{11c}$$

In the above Equation (11), the parameter B_1 is known as soon as the fluid properties, temperature, and the base heat transfer rate are specified. The parameter B_2 is a known quantity if the diameter of the cross section is specified.

The first term in Equation (10) represents the entropy generation due to heat transfer across a non-zero temperature difference, while the second term is the entropy generation associated with fluid friction.

3.2 Pin fin of Elliptical Cross section

Pin fin of elliptical cross section is a special type of cross section and the orientation of the cross section is shown in the Fig.3.

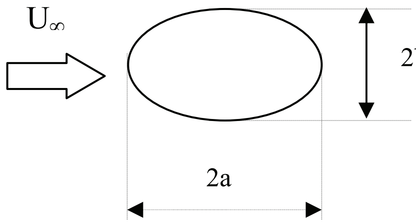

Figure 3. Flow over elliptical pin fins

The heat transfers and drag force depend on three dimensions: the length l, the major axis 2a and the minor axis 2b of the elliptical cross section. The entropy generation is given by Equation (7).

According to the unidirectional heat conduction model, the relationship between base heat flux and base stream temperature difference for elliptical pin fin is (Kern. 1972).

$$\theta_B = \frac{q_B}{\pi ab \times \lambda m \times \tanh(mL)};$$

$$m = \left(\frac{Ph}{\lambda A}\right)^{1/2} \qquad Nu = \frac{hL}{k} \tag{12}$$

Where P is the wetted perimeter and the A is the cross sectional area of the pin fin perpendicular to the direction of heat conduction. On substitution, it is possible to get the entropy generation number for elliptical fins,

For $Re_L \leq 500$,

$$N_s = \frac{1}{Re_L\left[B_2 + (1.321\,S)\left(1+\frac{1}{S^2}\right)^{1/4}(Nu\times\Theta)^{1/2}\tanh\left[1.682.\gamma.\left(1+\frac{1}{S^2}\right)^{1/4}S^{1/2}\left(\frac{Nu}{\Theta}\right)^{1/2}\right]\right]} + \frac{1}{2}B_1.C_D.Re_L^2.\gamma \tag{13}$$

$$B_1 = \frac{\rho k \nu^3 T_\infty}{q_B^2} \qquad B_2 = \frac{q_B}{k(2a)T_\infty} \tag{13a}$$

For $Re_L > 500$,

$$N_s = \frac{1}{Re_L\left[B_2 + \left(\frac{1.321}{S}\right)\left(1+S^2\right)^{1/4}\left(Nu \times \Theta\right)^{1/2}\tanh\left[1.682\,.\gamma.\left(1+S^2\right)^{1/4}\left(\frac{Nu}{\Theta}\right)^{1/2}\right]\right]} + \frac{1}{2}B_1.C_D.Re_L^2.\gamma \quad (13b)$$

$$B_1 = \frac{\rho k \nu^3 T_\infty}{q_B^2} \qquad B_2 = \frac{q_B}{k(2b)T_\infty} \quad (13c)$$

The Nusselt number and drag coefficient can be calculated from the results developed for a single elliptical cylinder in cross flow and are given by the equations for $Re_L \le 500$,

$$Nu_L = \left[0.75 - 0.16\exp\left(-0.018 S^{-3.1}\right)\right]Re_L^{0.5}\,Pr^{0.33}$$

$$C_D = \frac{1.353 + 4.43 S^{1.35}}{\sqrt{Re_L}} + \left(1.1526 + \frac{1.26}{Re_L}\right)S^{0.95} \quad (14a)$$

The Reynolds number for the elliptical cylinder is given by

$$Re_L = \frac{U_\infty(2a)}{\nu}$$

From [Zhihua Li, (2005)]

$$500 \le Re_D \le 10000$$

$$Nu = 0.37 S^{0.4}\,Re_D^{0.554}$$

$$C_D = 0.112 S^{2.6}\ln\left[Re_D\right] + 0.41 \quad (14b)$$

where S is the ratio between minor and major axis of the elliptical cross section. i.e., S=b/a. The value of S for the circular pin fin is 1. The values of B_1 and B_2 are given by Equation (13a). The geometry and working parameters (B_1, B_2, S, Re_L, γ and Θ) in the Equations (10) and (13), are the non-dimensional numbers. Entropy generation number is expressed for various values of above non-dimensional numbers.

4. Optimization

The entropy generation number, N_s, in Equations (10 and 13) emerges as a function of six dimensionless groups, three pertaining to fin geometry (S, Re_D and γ) and the remaining three (B_1, B_2, and Θ) accounting for the working fluid. Minimization of N_s with respect to γ is achieved in a straightforward manner by solving Equation (15).

$$\frac{\partial N_S}{\partial \gamma} = 0 \quad (15)$$

The optimum slenderness ratio calculated in this manner is,

$$\gamma_{opt} = \frac{1}{\alpha_2}\tanh^{-1}\left[\Omega\right] \quad (16)$$

where

$$\Omega = \frac{-\alpha_3\alpha_4 + \sqrt{\alpha_2\left[\alpha_1\alpha_3 + \alpha_2 - \left(\alpha_3/\alpha_1\right)\alpha_4^2\right]}}{\alpha_1\alpha_3 + \alpha_2}$$

For circular pin fin

$$\alpha_1 = \frac{\pi}{2}\left[Nu.\Theta\right]^{1/2}Re_L \qquad \alpha_2 = 2\left[Nu/\Theta\right]^{1/2}$$

$$\alpha_3 = \frac{1}{2}.B_1.C_D.Re_L^2 \qquad \alpha_4 = B_2.Re_L$$

For elliptical pin fin if $Re_L \le 500$

$$\alpha_1 = 1.321 \times S\left[1 + \frac{1}{S^2}\right]^{1/4}\left[Nu.\Theta\right]^{1/2}Re_L$$

$$\alpha_2 = 1.682 \times S^{1/2}\left[1 + S^2\right]^{1/4}\left[Nu/\Theta\right]^{1/2}$$

$$\alpha_3 = \frac{1}{2}.B_1.C_D.Re_L^2$$

$$\alpha_4 = B_2.Re_L$$

For elliptical pin fin $Re_L > 500$

$$\alpha_1 = \frac{1.321}{S}\left[1 + S^2\right]^{1/4}\left[Nu.\Theta\right]^{1/2}Re_L$$

$$\alpha_2 = 1.682\left[1 + S^2\right]^{1/4}\left[Nu/\Theta\right]^{1/2}$$

$$\alpha_3 = \frac{1}{2}.B_1.C_D.Re_L^2$$

$$\alpha_4 = B_2.Re_L$$

5. Results and Discussion

5.1 Optimum Slenderness Ratio

The engineering significance of Equation (16) is that the optimum pin length can be calculated immediately, provided Re_L is specified. Substituting Equation (16) into Equations (10) and (13), it is possible to obtain the minimum N_s for circular and elliptical fins, corresponding to optimum pin length, N_s ($\gamma_{opt,}$ Re_L). The optimum slenderness ratio of the circular pin fin to produce minimum entropy generation number is shown in Figure 4 for various values of Re_L. For a given fluid stream, the characteristic Reynolds number (Re_L) may be increased either due to the increment in the fluid mass flow rate or due to the increment in the characteristic dimension of the pin fin.

If the Re_L is increased due to the increment in the mass flow rate of the fluid, then the entropy generation due to fluid frictional force (S_{genf}) is also increased and at the same time the entropy generation due to heat transfer (S_{genh}) is decreased due to the high heat transfer coefficient. Due to the high heat transfer coefficient, the length of the pin fin may be decreased for the given heat load. The decrement in the length reduces S_{genf} and hence S_{gen}. This causes a reduction in the optimum slenderness ratio. If the Re_L is increased due to the increment in the characteristic dimension of the pin fin, then S_{genf} is also increased and S_{genh} is decreased due to the high heat transfer coefficient. The heat transfer area of the fin per unit length is also increased.

Due to the high heat transfer coefficient and high heat transfer area, the length of the pin fin may be decreased for the given heat load. The decrement in the length of the fin is more in this case when it compared to the previous case, because both the heat transfer coefficient and the heat transfer area are increased in this case. The decrement in the length reduces S_{genf} and hence S_{gen}. This causes the reduction in the optimum slenderness ratio in spite of the increment in the characteristic dimension.

If the optimum Re_L (the characteristic Reynolds number at which S_{gen} is minimum) is low for a particular situation, it means that the heat transfer rate is low and it would take more time to dissipate a certain quantity of heat through the pin fin for that situation.

The optimum slenderness ratio of elliptical pin fins is found numerically as shown in the Figures 5 and 6. The optimum ratio less than 5 cannot be used effectively since the "slender pin fin" model (Equations 11 and 14) loses its accuracy. An alternative approach in sizing a pin fin for minimum irreversibility consists of determining the optimum diameter, $D_{opt,}$ or in other words, Re_{Lopt} for the given value of B1, B2, S and the slenderness ratio.

5.2 Optimum Reynolds Number

Equation (10) and (13) are minimized numerically as shown in Figures 7 to 9. The entropy generation number N_s for both the circular and elliptical pin fins has a clear minimum with respect to Reynolds number Re_L.

If the Reynolds number is low, the entropy generation due to the fluid friction is also low (the second term in the Equations (10) and (13)) and the entropy generation due to finite temperature difference contributes the maximum in the total entropy generation. Hence at low Reynolds number, the effect of the term B1 in the total entropy generation number is negligible (refer to Figures 7 – 9).

The optimum Reynolds number (the Reynolds number at which the total entropy generation number is minimum) increases with the decrement in B1 for the given values of B2, Θ, S and γ.

5.3 Entropy generation due to heat transfer and fluid friction.

The equations for total entropy generation number for circular and elliptical pin fins are given by Equations (10) and (13) respectively. In these equations the first term indicates the entropy generation number due to finite temperature difference, NsT, while the second term gives the entropy generation number, NsP.

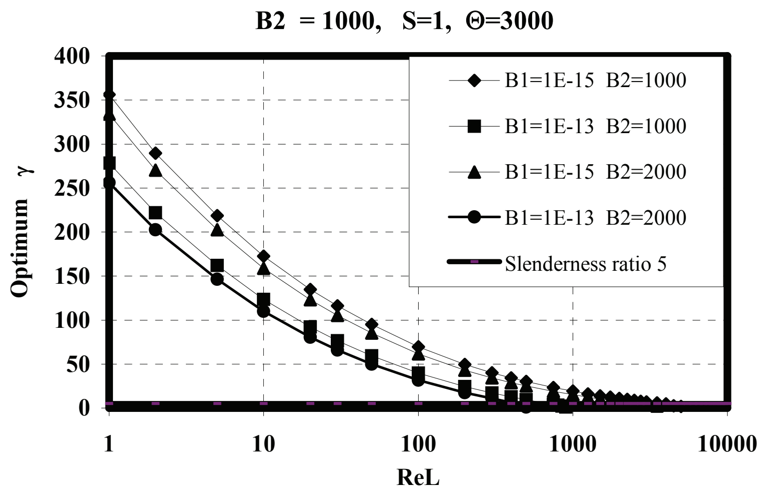

Figure 4. Optimum slenderness ratio of the circular pin fin for minimum entropy

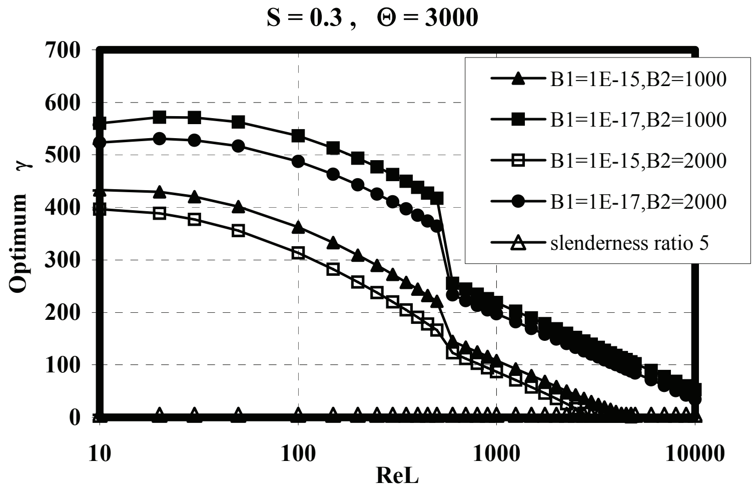

Figure 5. Optimum slenderness ratio of the elliptical pin fin of S=0.3 for minimum entropy generation

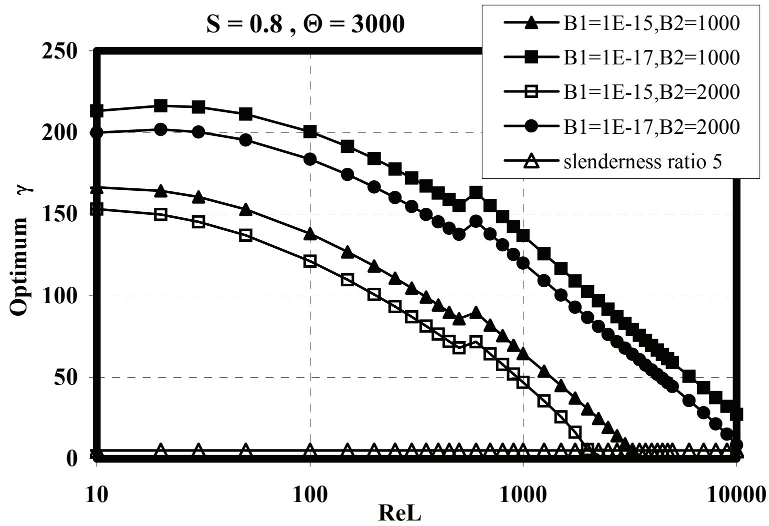

Figure 6. Optimum slenderness ratio of the elliptical pin fin of S=0.8 for minimum entropy generation

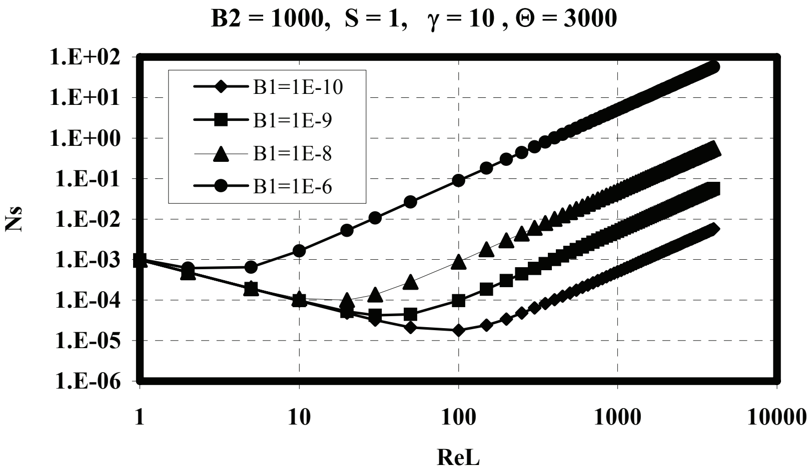

Figure 7. Variation of entropy generation number with Reynolds number of circular pin fin

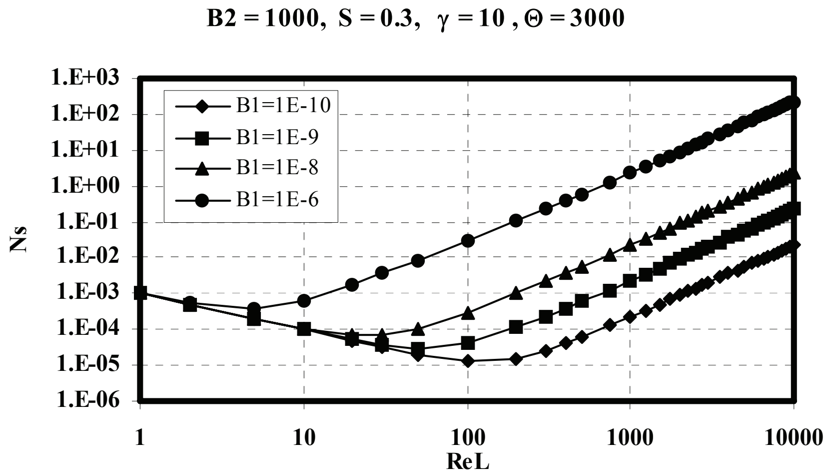

Figure 8. Variation of entropy generation number with Reynolds number of elliptical pin fin

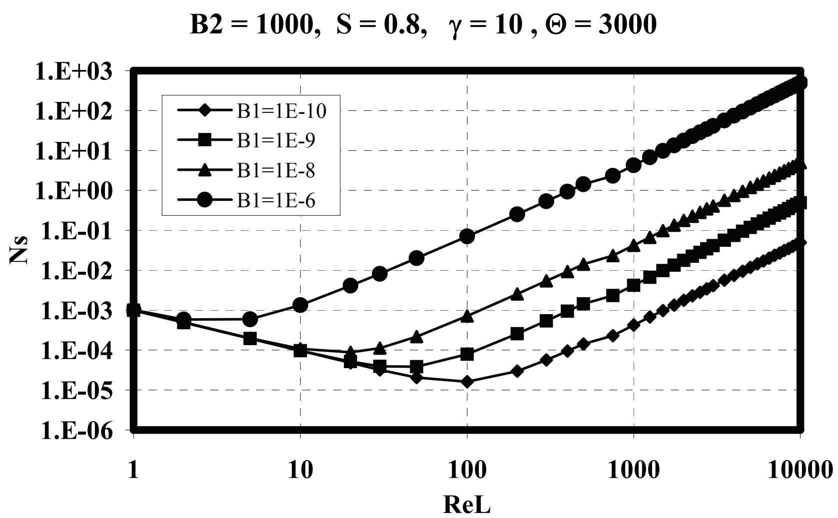

Figure 9. Variation of entropy generation number with Reynolds number of elliptical pin fin

The variation of NsT and NsP with respect to the characteristic Reynolds number for the circular and elliptical pin fins is presented in the Figures 10 and 11 respectively. At higher Reynolds number the effect of the Re_L on NsT is negligible and the variation of the NsP with respect to the Re_L is negligible at lower Reynolds number.

5.4 Mass Constraints

The entropy generation in the circular and elliptical pin fins are compared by imposing the mass constraints. The shape of the elliptical fin is selected in such a way that the mass of the elliptical and circular fins are same. Since the same material is selected, volume can be taken as equal for both the fins. Hence,

$$\frac{\pi}{4}D^2 \times L = \pi ab \times L \tag{17}$$

Equation (17) can be modified as

$$\frac{2b}{D} = CR = \sqrt{(S)_{elliptical}} \tag{18}$$

where the ratio CR is known as the circular ratio.

Hence the ratio L/2b can be modified as follows

$$\frac{L}{2b} = \frac{L}{D} \times \frac{D}{2b} \Rightarrow (\gamma)_{ell} = (\gamma)_{cir} \times \left(1/_{CR}\right) \tag{19}$$

The importance of Equation (19) is that the relationship between the slenderness ratio

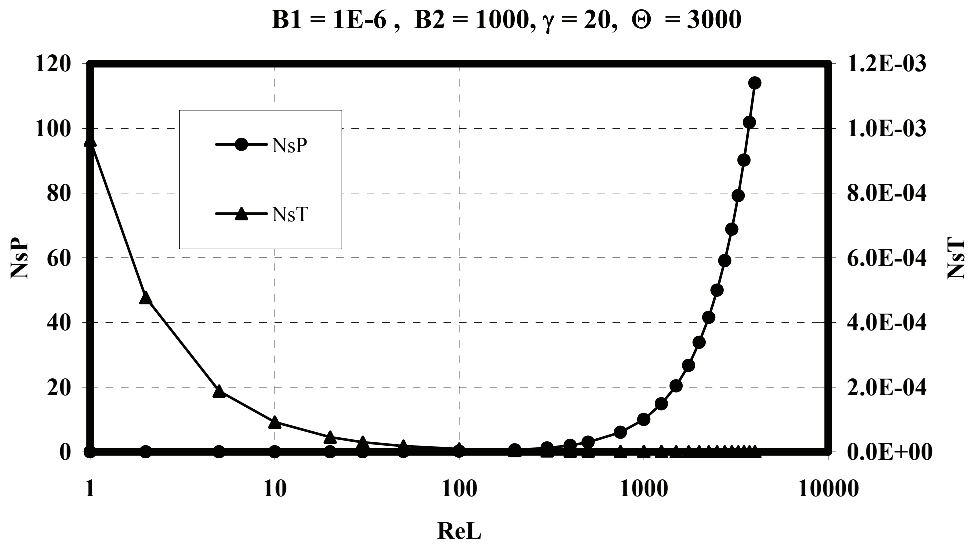

Figure 10. Variation of NsT and NsP with Reynolds number of circular pin fin

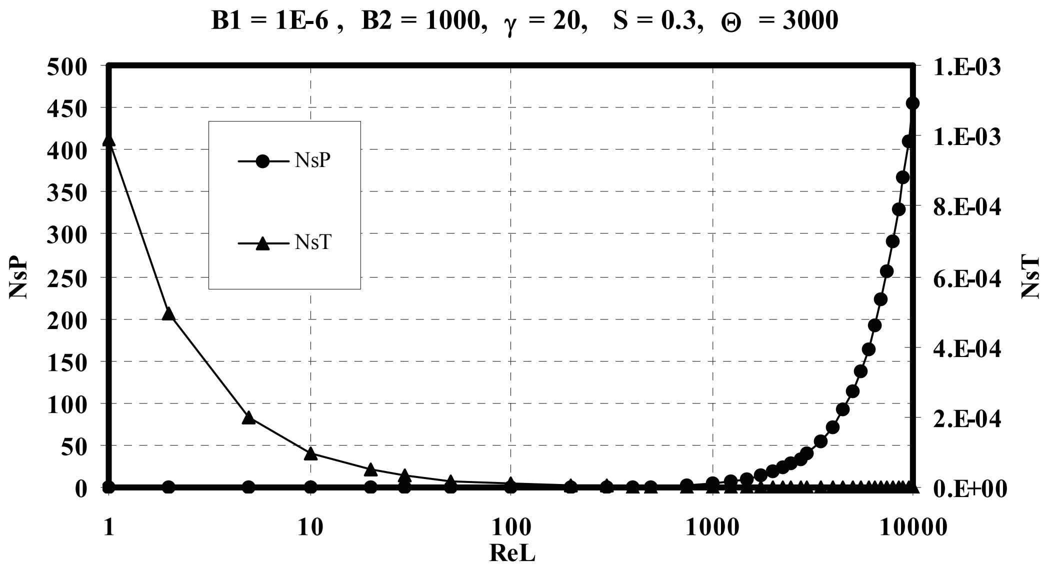

Figure 11. Variation of NsT and NsP with Reynolds number of elliptical pin fin

of elliptical and circular fins can be obtained to have the same cross sectional area for the circular and elliptical pin fins.

The entropy generation number for circular and elliptical pin fins of same volume and length is given in Figure 12 and Figure 13 respectively. The entropy generation number for elliptical pin fin is same as that of the circular pin fin for the given volume of the material if the circular ratio of the elliptical cross section is close to unity. The same can be seen in Fig.12 and Fig.13.

The entropy generation of elliptical pin fins is smaller than that of the circular pin fins at low Reynolds number ($Re_L<Re_{Ls}$) and higher than that of the circular pin fins at high Reynolds number ($Re_L> Re_{Ls}$). The Reynolds number at which the entropy generation due to circular pin fin and elliptical pin fin of same volume and length are the same is Re_{Ls}. The value of Re_{Ls} is approximately equal to 100 for a B_1 value of 1E-10 and is approximately equal to 30 for a B_1 value of $1x10^{-8}$.

6. Conclusion

The optimum ratio L/2b, for the elliptical pin fins are lower than the optimum ratio L/D of the circular fins for the given B_1 and B_2. For the high value of S, the optimum ratio L/2b is still lower than the optimum ratio L/D. The variation in the optimum ratio of circular and elliptical fins (L/D, L/2b) is negligible with respect to Θ for the given values of B_1, B_2 and S. The optimum ratio (L/D, L/2b), increases with B_1 for

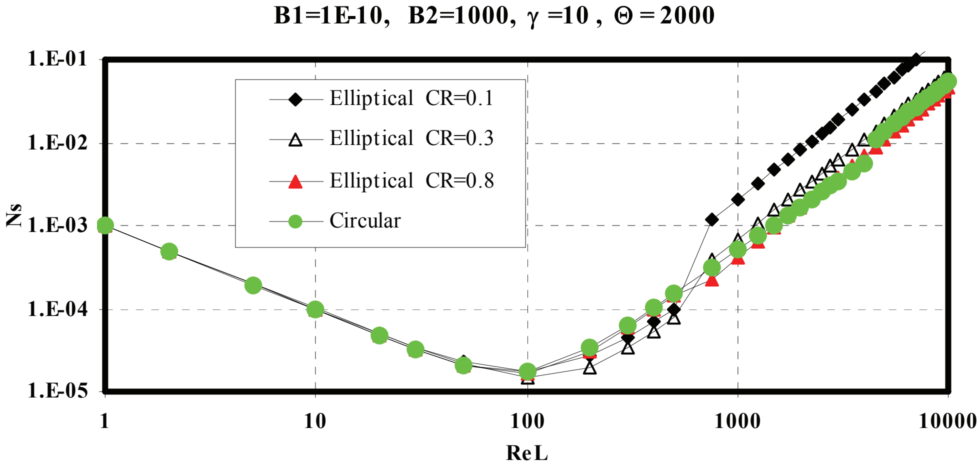

Figure 12. Entropy generation number with Re_L of circular and elliptical fins of same volume

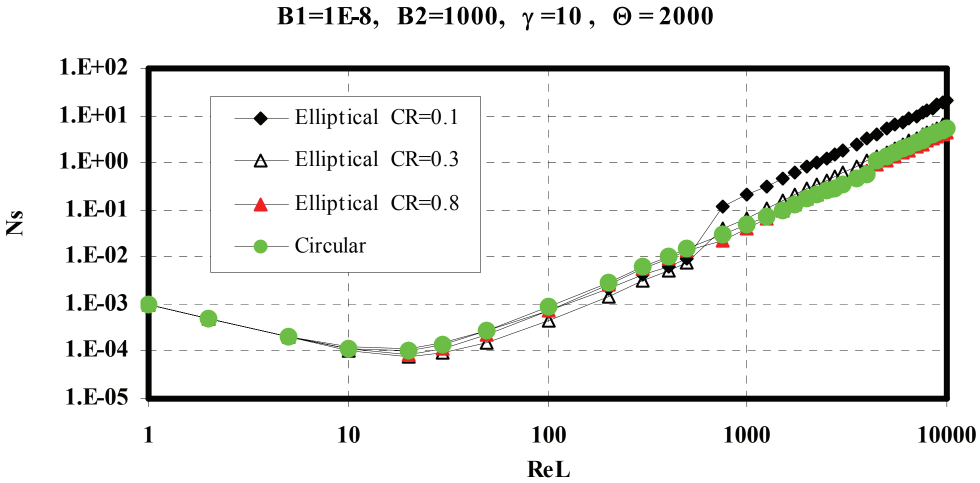

Figure 13. Entropy generation number with Re_L of circular and elliptical fins of same volume

both the circular and elliptical pin fins for the given B_2, S and Θ.

The entropy generation number N_s has a clear minimum with respect to Reynolds number Re_D. The optimum Reynolds number is less for high values of B_1 for both the circular and elliptical pin fins for the given B_2, S and Θ. The entropy generation for a given B_1 is less if β is less that is in more flat elliptical pin fins. The entropy generation number for elliptical pin fin is the same as that of the circular pin fin for the given volume of the material if the circular ratio of the elliptical cross section is close to unity.

Nomenclature

A	Cross sectional area of the fins in m^2
a	Half the length of the major axis of the elliptical cross-section in m.
B_1	Non dimensional number as given in Equation 11c
B_2	Non dimensional number as given in Equation 11c
b	Half the length of the minor axis of the elliptical cross-section in m.
C_D	Coefficient of drag
D	Diameter of the circular cross section of the fin in m.
F_D	Drag force exerted by the cross flow over the fin in N
h	Heat transfer coefficient in the cross flow over the fin in $W/m^2 \cdot K$
k	Thermal conductivity of the working fluid in $W/m \cdot K$
L	Characteristic dimension of the fins, D for circular fins and 2a or 2b for elliptical fins in m
l	Length of the fins in m
m	$\sqrt{\left(\frac{hP}{kA}\right)}$ in m^{-1}
N_s	Entropy generation number
Nu	Nusselt number
P	Wetted perimeter in m
q_B	Heat transfer through the base of the fin in w

$q_\propto$	Heat transfer to the free stream in w
S	Dimensionless number b/a
S_{gen}	Entropy generation in W/K
$T_\propto$	Free stream temperature in K
T_B	Base fin temperature in K
$U_\propto$	Free stream velocity in m/sec
W_{lost}	Power lost due to the irreversibility in W

Greek letters

Θ	Dimensionless number λ/k
β	Dimensionless number, circular ratio of elliptical fins 2b/D
	Slenderness ratio L/D for circular fin and L/2b for elliptical fin
λ	Thermal conductivity of the fin material in W/m·K
ν	Kinematic viscosity in m^2/sec
θ_B	Temperature difference T_B - $T_\propto$

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