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Double Fuzzifying Topogenous Space, Double Fuzzifying Quasi-Uniform Spaces and Applications of Dynamics Fuzzifying Topology in Breast Cancer

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Abstaract − The main motivation behind this work is to introduce the notion of $(2, L)$-double fuzzifying topology which is a generalization of the notion of $(2, L)$-fuzzifying topology and classical topology. We define the notions of $(2, L)$-double fuzzifying preproximity and $(2, L)$-fuzzifying syntopogenous structures. Some fundamental properties are also established. These concepts will help in verifying the existing characterizations and also help in achieving new and generalized results. Finally we study a model as an application of fuzzifying topology in biology.

Keywords − *$(2, L)$-double fuzzifying topology, $(2, L)$-doublefuzzifying preproximity, L-fuzzifying dynamice topology, breast cancer.*

1 Introduction

A lattice is a poset $L = (L, \leq)$ in which every finite subset has both join $\vee$ and a meet $\wedge$ with the smallest element $\perp_L$ and the largest element $\top_L$. We assume that $\top_L \neq \perp_L$, *i.e.*, L has at least two elements. A distibutive lattice is a lattice which satisfies the distributive laws. A lattice is said to be complete if it has arbitrary joins and meets, *i.e.*, for every subset $A \subseteq L$ the join $\bigvee A$ and the meet $\bigwedge A$ are defined. In particular, $\bigvee L = \top_L$ and $\bigwedge L = \perp_L$. Throughout this work L always denote a complete residuated lattice intoduced by [7,14] used L as a complete MV-algebra but [17,18,19] used L as a complete residuated lattice, $L_0 = L - \{\perp_L\}$ and $I = [0, 1]$. We say a is a wedge below b, in a symbol, $b \rhd a$, if for every subset $D \subseteq L$, $\bigvee D \geq b$ implies $a \leq d$ for some $d \in D$. The concept of $(2, L)$-fuzzifying topology appeared in [7] under the name " $(2, L)$-fuzzy topology" (cf. Definition 4.6, Proposition 4.11 in [8] where L is a completely distributive complete lattice. In the case of $L = [0, 1]$ this terminology traces back to [18,19], where it was studied the fuzzifying topology and elementarily it was developed fuzzy topology from a new direction with semantic method of continuous valued logic. Fuzzifying topology (resp. L-Fuzzifying topology) in the sense of M. S. Ying (resp. U. Höhle) was introduced as a fuzzy subset (resp. an L-Fuzzy subset) of the power set of an ordinary set. On the other hand, in topology a proximity space is an axiomatization of notions of "nearness" that hold set-to-set, as opposed to the better known point-to-set notions that characterize topological spaces, in this regard. [3,4] gave a new method for the foundation of general topology based on the theory of syntopogenous structure to develop a unified approach to the three main structures of set-theoretic topology: topologies, uniformities and proximities. This helped him to develop a theory including the basis of the three classical theories of topological spaces, uniform spaces and proximity spaces. In the case of the fuzzy structures there are at least two notions of fuzzy syntopogenous structures Motivated by their works, we continue investigating the properties $(2, L)-$double fuzzifying preproximity. We show that each $(2, L)$-double fuzzifying preproximity on X induces $(2, L)$-double fuzzifying topology on the same set.Also, we define the notion of $(2, L)$-Double fuzzifying semi topogenous order and obtain a few results analogous to the ones that hold for $(2, L)$-double fuzzifying topology, he relation between a L-double fuzzifying preproximity structures is also investigated $(2, L)$-Double fuzzifying semi topogenous order, double fuzzifying topogenous order on X, double fuzzifying topogenous continuous, $(2, L)$-double fuzzifying preproximity, double quasi proximity spaces, double fuzzifying quasi uniform space. This work arranged by: In section 1 and 2 introduction and more survay results in the subject. In section 3, we give a new notion of $(2, L)$-Double fuzzifying semi topogenous order, double fuzzifying topogenous order on X,double fuzzifying topogenous continuous, $(2, L)$-double fuzzifying preproximity, double quasi proximity spaces, double fuzzifying quasi uniform space, we study the relations between them and relations between $(2, L)$-double fuzzifying topology. In section 4 Mathematical models have been used in biology. In fact, dramatic developments in biology and in pure mathematics together, may have led to the interpretation of many natural phenomena in life, Also, it has been creatively described in the analysis and diagnosis of multiple diseases dynamically. However, there are many phenomena that are still in the interest of scientists. This work shows that using dynamic physiological topology we can describe many natural phenomena dynamically and identify the appropriate times in which scientists intervene to the subject of human solutions to the distortions of the situation. We will shed light on breast cancer at the five-stage and determine the possibility of conformation and therapeutic intervention. We will show how the dynamical topologies [5]. can develop the diagnostic mechanism and time analysis of the situation and determine the appropriate time to avoid distortions in the stages of the case. The present article demonstrates an application of L-fuzzifying dynamice topology clarify a model describing biological phenomena, This model allow to know all levels of development of an breast cancer. from 0-level (infection outside cells) until 5-level (infection liver).

2 Preliminary

Definition 2.1. [16] Let (X, τ) be an L-fuzzifying topological space, and let $Y \subseteq X$. Define the map $\tau_Y : P(Y) \to L$ as follows: $\tau_Y(U) = \bigvee_{H \cap Y = U} \tau(H)$.

Definition 2.2. [9] The double negation law in a complete residuated lattice L is given as follows: $\forall a, b \in L,\ (a \to \bot) \to \bot = a$.

Definition 2.3. [9] A structure $(L, \vee, \wedge, *, \to, \perp, \top)$ is called a strictly two-sided commutative quantale iff

(1) $(L, \vee, \wedge, \perp, \top)$ is a complete lattice whose greatest and least element are $\top, \perp$ respectively,

(2) $(L, *, \top)$ is a commutative monoid,

(3)(a) $*$ is distributive over arbitrary joins, i.e.,
$a * \bigvee_{j\in J} b_j = \bigvee_{j\in J} (a * b_j)\ \forall a \in L, \forall \{b_j \mid j \in J\} \subseteq L,$

(b) $\to$ is a binary operation on L defined by: $a \to b = \bigvee_{\lambda * a \leq b} \lambda \quad \forall a, b \in L.$

Definition 2.4. [8,9] Let X be a nonempty set and let $P(X)$ be the family of all ordinary subsets of X. An element $\mathcal{T} \in L^{P(X)}$ is called an L-fuzzifying topology on X iff it satisfies the following axioms:

(1) $\mathcal{T}(X) = \mathcal{T}(\phi) = \top$,

(2) $\forall A, B \in P(X), \mathcal{T}(A \cap B) \geq \mathcal{T}(A) \wedge \mathcal{T}(B)$,

(3) $\forall \{A_j \mid j \in J\} \subseteq P(X),\ \mathcal{T}(\bigcup_{j\in J} A_j) \geq \bigwedge_{j\in J} \mathcal{T}(A_j)$.

The pair $(X, \mathcal{T})$ is called an L-fuzzifying topological space.

Definition 2.5. [11] Let X be a set and let $\delta \in L^{P(X)\times P(X)}$, i.e., $\delta : P(X) \times P(X) \to L$. Assume that for every $A, B, C \in P(X)$, the following axioms are satisfied:

$(LFP1)$ $\delta(X, \phi) = \perp$,

$(LFP2)$ $\delta(B, A) = \delta(A, B)$,

$(LFP3)$ $\delta(A, B \cup C) = \delta(A, B) \vee \delta(A, C)$,

$(LFP4)$ For every $A, B \in P(X), \exists C \in P(X)\ s.t.\ \delta(A, B) \geq \delta(A, C) \vee \delta(B, X - C)$,

$(LFP5)$ $\delta(\{x\}, \{y\}) = CE(\{x\}, \{y\})$.

Then δ is called an L-fuzzifying proximity on X and (X, δ) is called an L-fuzzifying proximity space.

Definition 2.6. [4] A uniform structure U on a set X is a family of subsets of $X \times X$, called entourage, which satisfies the following properties:

$(U1)$ If $u \in U$, then $\triangle \subseteq u$, where $\triangle$ is the diagonal: $\triangle = \{(x, x) \mid x \in X\}$

$(U2)$ If $v \subseteq u$, and $v \in U$ then $u \in U$,

$(U3)$ for every $u, v \in U,\ u \cap v \in U$,

$(U4)$ If $u \in U$, then $u^{-1} \in U$, where $u^{-1} = \{(x, y) \mid (y, x) \in u\}$.

$(U5)$ for every $u \in U$, there exists $v \subseteq U$ such that $v \circ v \subseteq u$, where $v \circ v \subseteq u$, where $v \circ u$ is defined by:

$v \circ u = \{(x, y) \mid \exists z \in X$ such that $(x, z) \in u$ and $(z, y) \in u\},\ \forall x, y \in X.$

The pair (X, U) is said to be a uniform space.

3. $(2, L)$-Double Fuzzifying Semi Topogenous Order Spaces

Definition 3.1. Let X be a non-empty set. The pair $(\mathcal{T}, \mathcal{T}^*)$ of maps $\mathcal{T}, \mathcal{T}^* : 2^X \times 2^X \to L$ is called an $(2, L)$-double fuzzifying semi topogenous order on X if it satisfies the following conditions:

(LST1) $\mathcal{T}(A, B) \leq \mathcal{T}^*(A, B) \to \perp$, for each $(A, B) \in 2^X \times 2^X$, (LST2) $\mathcal{T}(X,X) = \mathcal{T}(\phi,\phi) = \top$ and $\mathcal{T}^*(X,X) = \mathcal{T}^*(\phi,\phi) = \bot$,

(LST3) If $\mathcal{T}(A,B) \neq \bot$, $\mathcal{T}^*(A,B) \neq \top$, then $A \subseteq B$,

(LST4) If $A_1 \subseteq A$, $B_1 \subseteq B$, then $\mathcal{T}(A_1,B_1) \leq \mathcal{T}(A,B)$ and $\mathcal{T}^*(A_1,B_1) \geq \mathcal{T}^*(A,B)$.

The pair $(X,\mathcal{T},\mathcal{T}^*)$ is called an $(2,L)$-double fuzzifying semi topogenous order on X.

The complement of a double fuzzifying semi topogenous order $(\mathcal{T},\mathcal{T}^*)$ is the double fuzzifying semi topogenous order $(\widehat{\mathcal{T}},\widehat{\mathcal{T}^*})$ defined by $\widehat{\mathcal{T}}(A,B) = \widehat{\mathcal{T}}(A^-,B^-)$ and $\widehat{\mathcal{T}^*}(A,B) = \widehat{\mathcal{T}^*}(A^-,B^-)$. such that A^-, B^- are the complement of $A$and B respectively.

A double fuzzifying semi topogenous order $(\mathcal{T},\mathcal{T}^*)$ is called:

(S) symmetrical if $(\mathcal{T},\mathcal{T}^*) = (\widehat{\mathcal{T}},\widehat{\mathcal{T}^*})$

(T) topogenous if $\mathcal{T}((A_1 \cup A_2,B) \geq \mathcal{T}(A_1,B) \wedge \mathcal{T}(A_1,B)$ and $\mathcal{T}^*(A_1 \cup A_2,B) \leq \mathcal{T}^*(A_1,B) \vee \mathcal{T}^*(A_1,B)$, (PF) perfect if $\mathcal{T}(\bigcup_{i\in\Gamma} A_i,B) \geq \bigwedge_{i\in\Gamma} \mathcal{T}((A_i,B))$ and $\mathcal{T}^*((\bigcup_{i\in\Gamma} A_i,B)) \leq \bigvee_{i\in\Gamma} \mathcal{T}^*((A_i,B)$, for each $\{(A_i,B) : i \in \Gamma\} \subseteq 2^X \times 2^X$.

(BP) biperfect if it is perfect and $\mathcal{T}(A, \bigcap\limits_{i\in\Gamma} B_i) \geq \bigwedge_{i\in\Gamma} \mathcal{T}(A,B_i)$ and $\mathcal{T}^*(A, \bigcap\limits_{i\in\Gamma} B_i) \leq \bigwedge_{i\in\Gamma} \mathcal{T}^*(A,B_i)$.

Double fuzzifying semi topogenous order $(\mathcal{T}_1,\mathcal{T}_1^*)$ is said to be finer than another one $(\mathcal{T}_2,\mathcal{T}_2^*)$ if $\mathcal{T}_1(A,B) \geq \mathcal{T}_2(A,B)$ and $\mathcal{T}_1^*(A,B) \leq \mathcal{T}_2^*(A,B)$ for each $(A,B) \in 2^X \times 2^X$.

Definition 3.2. Let X be a nonempty set. The pair (δ,δ^*) of maps $\delta,\delta^* : 2^X \to L$ is called an $(2,L)$-double fuzzifying topology on X if it satisfies the following conditions:

(DO1) $\delta(A) \leq \delta^*(A) \to \bot$, for each $A \in 2^X$,

(DO2) $\delta(X) = \delta(\emptyset) = \top$ and $\delta^*(X) = \delta^*(\emptyset) = \bot$,

(DO3) $\delta(A \cap B) \geq \delta(A) \wedge \delta(B)$ and $\delta^*(A \cap B) \leq \delta^*(A) \vee \delta^*(B)$, for each $A,B \in 2^X$,

(DO4) $\delta(\bigcup_{i\in\Gamma} A_i) \geq \bigwedge_{i\in\Gamma} \delta(A_i)$ and $\delta^*(\bigcup_{i\in\Gamma} A_i) \leq \bigvee_{i\in\Gamma} \delta^*(A_i)$, for each $\{A_i : i \in \Gamma\} \subseteq 2^X$.

The pair (X,δ,δ^*) is called an $(2,L)$-double fuzzifying topological space.

Definition 3.3. Let (X,δ_1,δ_1^*) and (Y,δ_2,δ_2^*) be two $(2,L)$-double fuzzifying topological spaces. Then the map $f : (X,\mathcal{T}_1,\mathcal{T}_1^*) \to (Y,\mathcal{T}_2,\mathcal{T}_2^*)$ is called double fuzzifying continuous, if $\delta_2(B) \leq \delta_1(f^{-1}(B))$ and $\delta_2^*(B) \geq \delta_1^*(f^{-1}(B))$, for each $B \in 2^Y$.

Theorem 3.1. Let $(\mathcal{T}_1,\mathcal{T}_1^*)$ and $(\mathcal{T}_2,\mathcal{T}_2^*)$ be perfect (resp. double fuzzifying topogenous, biperfect) double fuzzifying semi topogenous order on X . Define the compositions $\mathcal{T}_1 \circ \mathcal{T}_2$ and $\mathcal{T}_1^* \circ \mathcal{T}_2^*$ on X by $\mathcal{T}_1 \circ \mathcal{T}_2(A,B) = \bigvee_{h\in 2^X} [\mathcal{T}_1(A,h) \wedge (\mathcal{T}_2(h,B)]$ and $\mathcal{T}_1^* \circ \mathcal{T}_2^*(A,B) = \bigwedge_{h\in 2^X} [\mathcal{T}_1^*(A,h) \vee (\mathcal{T}_2^*(h,B)]$. Then $(\mathcal{T}_1 \circ \mathcal{T}_2\ ,\ \mathcal{T}_1^* \circ \mathcal{T}_2^*)$ is perfect (resp. double fuzzifying topogenous, biperfect) double fuzzifying semi topogenous order on X.

Proof Let $(\mathcal{T}_1,\mathcal{T}_1^*)$ and $(\mathcal{T}_2,\mathcal{T}_2^*)$ be perfect double fuzzifying semi topogenous order on X. Then (LST3) If $\mathcal{T}_1 \circ \mathcal{T}_2(A,B) \neq \bot$ and $\mathcal{T}_1^* \circ \mathcal{T}_2^*(A,B) \neq \top$. Then $\exists\, h \in 2^X$ such that $\mathcal{T}_1 \circ \mathcal{T}_2(A,B) \geq \mathcal{T}_1(A,h) \wedge (\mathcal{T}_2(h,B)) \neq \bot$ and $\mathcal{T}_1^* \circ \mathcal{T}_2^*(A,B) \leq \mathcal{T}_1^*(A,h) \vee (\mathcal{T}_2^*(h,B)) \neq \top$, It implies $A \subseteq h \subseteq B$. Easily only prove (PF) from

$$\begin{aligned}\mathcal{T}_1\circ\mathcal{T}_2(\bigcup_{i\in\Gamma}A_i,B) &= \bigvee_{h\in 2^X}\left[\mathcal{T}_1(\bigcup_{i\in\Gamma}A_i,h)\wedge(\mathcal{T}_2(h,B)\right]\\ &\geq \bigwedge_{i\in\Gamma}\left[\bigvee_{h\in 2^X}\left[\mathcal{T}_1(A_i,h)\wedge(\mathcal{T}_2(h,B))\right]\right]\\ &= \bigwedge_{i\in\Gamma}\mathcal{T}_1\circ\mathcal{T}_2(A_i,B)\end{aligned}$$

and

$$\begin{aligned}\mathcal{T}_1^*\circ\mathcal{T}_2^*((\bigcup_{i\in\Gamma}A_i,B)) &= \bigwedge_{h\in 2^X}\left[\mathcal{T}_1^*(\bigcup_{i\in\Gamma}A_i,h)\vee(\mathcal{T}_2^*(h,B)\right]\\ &\leq \bigvee_{i\in\Gamma}\left[\bigwedge_{h\in 2^X}\left[\mathcal{T}_1^*(A_i,h)\vee(\mathcal{T}_2^*(h,B))\right]\right]\\ &= \bigvee_{i\in\Gamma}\mathcal{T}_1^*\circ\mathcal{T}_2^*(A_i,B).\end{aligned}$$

Definition 3.4. A double fuzzifying syntopogenous structure on $X\Psi$ is a non-empty family $\Upsilon_X\Psi$ of double fuzzifying topogenous orders on X. If it satisfies the following conditions:
$(LS1)$ $\Upsilon_X\Psi$ is directed, i.e. a two double fuzzifying topogenous orders $(\mathcal{T}_1,\mathcal{T}_1^*)$, $(\mathcal{T}_2,\mathcal{T}_2^*)\in\Upsilon_X$, $\exists$ double fuzzifying topogenous orders $(\mathcal{T}_1,\mathcal{T}^*)\in\Upsilon_X$ such that $\mathcal{T}\geq\mathcal{T}_1,\mathcal{T}_2$, and $\mathcal{T}^*\leq\mathcal{T}_1^*,\mathcal{T}_2^*$,
$(LS2)$ For every $(\mathcal{T},\mathcal{T}^*)\in\Upsilon_X$, $\exists$ $(\mathcal{T}_1,\mathcal{T}_1^*)\in\Upsilon_X$ such that $\mathcal{T}\leq\mathcal{T}_1\circ\mathcal{T}_2$, and $\mathcal{T}^*\geq\mathcal{T}_1^*\circ\mathcal{T}_2^*$.

Definition 3.5. (1) A double fuzzifying syntopogenous structure $\Upsilon_X\Psi$ is called double fuzzifying topogenous orders If $\Upsilon_X\Psi$ cosists of a single element. denoted by $\Upsilon_X\Psi=\{(\mathcal{T},\mathcal{T}^*)\}$, and (X,Υ_X) double fuzzifying topogenous space.

(2) A double fuzzifying syntopogenous structure $\Upsilon_X\Psi$ is called perfect (resp. biperfect, symmetric) if each double fuzzifying topogenous order $(\mathcal{T},\mathcal{T}^*)\in\Upsilon_X$ is perfect (resp. biperfect, symmetric).

Theorem 3.2. Let $(\mathcal{T},\mathcal{T}^*)$ be a double fuzzifying topogenous order on X. The mapping $f_{(\mathcal{T},\mathcal{T}^*)}:2^X\times L_0\times L_1\to 2^X$, is defined by $f_{(\mathcal{T},\mathcal{T}^*)}(A,\alpha,\beta)=\bigcap\{B^-\in 2^X:\mathcal{T}(B,A^-)>\alpha,\ \mathcal{T}^*(B,A^-)<\beta\}$ for each $A,A_1,A_2\in 2^X$ and $\alpha,\alpha'\in L_0,\ \beta,\beta'\in L_1$.

Then it has the following properties:

- (i) $\mathrm{f}_{(\mathcal{T},\mathcal{T}^*)}(X,\alpha,\beta)=X$,
- (ii) $A\subseteq f_{(\mathcal{T},\mathcal{T}^*)}(A,\alpha,\beta)$,
- (iii) $If\quad A_1\subseteq A_2$ then $f_{(\mathcal{T},\mathcal{T}^*)}(A_1,\alpha,\beta)\subseteq f_{(\mathcal{T},\mathcal{T}^*)}(A_2,\alpha,\beta)$,
- (iv) $f_{(\mathcal{T},\mathcal{T}^*)}(A_1\cup A_2,\alpha\wedge\alpha',\beta\vee\beta')\subseteq f_{(\mathcal{T},\mathcal{T}^*)}(A_1,\alpha,\beta)\cap f_{(\mathcal{T},\mathcal{T}^*)}(A_2,\alpha',\beta')$.
- (v) $If\ \alpha\leq\alpha',\ \beta\geq\beta'$, then $f_{(\mathcal{T},\mathcal{T}^*)}(A_1,\alpha,\beta)\subseteq f_{(\mathcal{T},\mathcal{T}^*)}(A_2,\alpha',\beta')$,
- (vi) $If\ (\mathcal{T},\mathcal{T}^*)$ be a double fuzzifying topogenous order on X, $\mathrm{f}_{(\mathcal{T},\mathcal{T}^*)}(\mathrm{f}_{(\mathcal{T},\mathcal{T}^*)}(X,\alpha,\beta))\subseteq\mathrm{f}_{(\mathcal{T},\mathcal{T}^*)}(X,\alpha,\beta)$.

Proof (i) since $\mathcal{T}(X,X)=\top$ and $\mathcal{T}^*(X,X)=\perp$, $f_{(\mathcal{T},\mathcal{T}^*)}(X,\alpha,\beta)=X$.

(ii) Since $\mathcal{T}(B,A^-)\neq\perp$ and $\mathcal{T}^*(B,A^-)\neq\top$, then $B\subseteq A^-$. Then $A\subseteq f_{(\mathcal{T},\mathcal{T}^*)}(A,\alpha,\beta)$

(iii) For $A_1\subseteq A_2$, since $\mathcal{T}(B,A_2^-)\subseteq\mathcal{T}(B,A_1^-)>\alpha$ and $\mathcal{T}^*(B,A_2^-)\geq\mathcal{T}^*(B,A_1^-)<\beta$, we have $f_{(\mathcal{T},\mathcal{T}^*)}(A_2,\alpha,\beta)\supseteq f_{(\mathcal{T},\mathcal{T}^*)}(A_1,\alpha,\beta)$,

(iv) Suppose taht there exist $A_1, A_2 \in 2^X$ such that $f_{(\mathcal{T},\mathcal{T}^*)}(A_1 \cup A_2, \alpha \wedge \alpha', \beta \vee \beta') \subsetneq f_{(\mathcal{T},\mathcal{T}^*)}(A_1, \alpha, \beta) \cap f_{(\mathcal{T},\mathcal{T}^*)}(A_2, \alpha', \beta')$, by the definition of $f_{(\mathcal{T},\mathcal{T}^*)}(A, \alpha, \beta)$ there exist $B_1, B_2 \in 2^X$ with $\mathcal{T}(B_1, A_1^-) > \alpha$, $\mathcal{T}^*(B_1, A_1^-) < \beta$, $\mathcal{T}(B_2, A_2^-) > \alpha'$, $\mathcal{T}^*(B_2, A_2^-) < \beta'$, such that $f_{(\mathcal{T},\mathcal{T}^*)}(A_1 \cup A_2, \alpha \wedge \alpha', \beta \vee \beta') \subsetneq B_1^- \cap B_2^-$.

On the other hand, by (T), and $(LST4)$ $\mathcal{T}((B_1 \cap B_2, (A_1 \cup A_2)^-) \geq \mathcal{T}((B_1 \cap B_2, A_1^-) \wedge \mathcal{T}(B_1 \cap B_2, A_2^-) \geq \mathcal{T}((B_1, A_1^-) \wedge \mathcal{T}(B_2, A_2^-) > \alpha \wedge \alpha'$ and $\mathcal{T}^*((B_1 \cap B_2, (A_1 \cup A_2)^-) \leq \mathcal{T}^*((B_1 \cup B_2, A_1^-) \vee \mathcal{T}^*(B_1 \cap B_2, A_2^-) \leq \mathcal{T}^*((B_1, A_1^-) \vee \mathcal{T}(B_2, A_2^-) < \beta \wedge \beta'$. It is implies $f_{(\mathcal{T},\mathcal{T}^*)}(A_1 \cup A_2, \alpha \wedge \alpha', \beta \vee \beta') \subseteq (B_1 \cap B_2)^- = B_1^- \cup B_2^-$. This is a contradiction.

(v) and (vi) by the fashion.

Theorem 3.3. Let $(\mathcal{T}, \mathcal{T}^*)$ be a double fuzzifying topogenous order on X. The mapping $f_{(\mathcal{T},\mathcal{T}^*)} : 2^X \times L_0 \times L_1 \to 2^X$, is defined by.

$$f_{(\mathcal{T},\mathcal{T}^*)}(A, \alpha, \beta) = \bigcup \{Q \in 2^X : \mathcal{T}(Q, A^-) > \alpha \to \perp,\ \mathcal{T}^*(Q, A^-) < \beta \to \perp\}.$$

Then it has the following properties:

(i) $\mathrm{f}_{(\mathcal{T},\mathcal{T}^*)}(X, \alpha, \beta) = \phi$,
(ii) $f_{(\mathcal{T},\mathcal{T}^*)}(A, \alpha, \beta) \subseteq A^-$,
(iii) $If \quad \alpha \geq \alpha'\ and\ \beta \leq \beta'$, then $f_{(\mathcal{T},\mathcal{T}^*)}(A, \alpha', \beta') \subseteq f_{(\mathcal{T},\mathcal{T}^*)}(A, \alpha, \beta)$,
(iv) $f_{(\mathcal{T},\mathcal{T}^*)}(A_1 \cap A_2, \alpha \wedge \alpha', \beta \vee \beta') \supseteq f_{(\mathcal{T},\mathcal{T}^*)}(A_1, \alpha, \beta) \cap f_{(\mathcal{T},\mathcal{T}^*)}(A_2, \alpha', \beta')$.

Proof (i) From (LST2) and since $\mathcal{T}(Q, A^-) > \alpha \to \perp$ and $\mathcal{T}^*(Q, A^-) < \beta \to \perp$, $f_{(\mathcal{T},\mathcal{T}^*)}(X, \alpha, \beta) = \phi$.

(ii) From (LST3) and since $\mathcal{T}(Q, A^-) < \alpha \to \perp$ and $\mathcal{T}^*(Q, A^-) > \beta \to \perp$, then, $Q \subseteq A^-$. Thus $f_{(\mathcal{T},\mathcal{T}^*)}(A, \alpha, \beta) \subseteq A^-$

(iii) For $\alpha \geq \alpha'$, $\beta \leq \beta'$, since $\mathcal{T}(Q, A^-) > \alpha' \to \perp > \alpha \to \perp$ and $\mathcal{T}^*(Q, A^-) < \beta' \to \perp < \beta \to \perp$, we have $f_{(\mathcal{T},\mathcal{T}^*)}(A, \alpha, \beta) \subseteq f_{(\mathcal{T},\mathcal{T}^*)}(A, \alpha', \beta')$.

(iv) $f_{(\mathcal{T},\mathcal{T}^*)}(A, \alpha, \beta) \cap f_{(\mathcal{T},\mathcal{T}^*)}(B, \alpha', \beta') = \bigcup\{Q_1 \in 2^X : \mathcal{T}(Q_1, A^-) > \alpha \to \perp, \mathcal{T}^*(Q_1, A^-) < \beta \to \perp\} \cap (\bigcup\{Q_2 \in 2^X : \mathcal{T}(Q_2, B^-) > \alpha' \to \perp, \mathcal{T}^*(Q_2, A^-) < \beta' \to \perp\}) = \bigcup\{Q_1 \cap Q_2 \in 2^X : \mathcal{T}(Q_1, A^-) > \alpha \to \perp, \mathcal{T}(Q_2, B^-) > \alpha' \to \perp, \mathcal{T}^*(Q_1, A^-) < \beta \to \perp, \mathcal{T}^*(Q_2, B^-) < \beta' \to \perp\}$
$\subseteq \bigcup\{Q_1 \cap Q_2 \in 2^X : \mathcal{T}(Q_1, A^-) \vee \mathcal{T}(Q_2, B^-) > (\alpha \to \perp) \vee (\alpha' \to \perp), \mathcal{T}^*(Q_1, A^-) \wedge \mathcal{T}^*(Q_2, B^-) < (\beta \to \perp) \wedge (\beta' \to \perp)\}$
$\subseteq \bigcup\{Q_1 \cap Q_2 \in 2^X : \mathcal{T}(Q_1 \cap Q_2, A^- \cup B^-) < (\alpha \wedge \alpha') \to \perp\}$,
$\geq \mathcal{T}(Q_1, A^-) \wedge \mathcal{T}(Q_2, B^-) > (\alpha \to \perp) \wedge (\alpha' \to \perp) = (\alpha \vee \alpha') \to \perp, \mathcal{T}^*(Q_1 \cup Q_2, A^- \cup B^-) > (\beta \vee \beta') \to \perp\}$
$= \mathcal{T}^*(Q_1, A^-) \vee \mathcal{T}^*(Q_2, B^-) < (\beta \to \perp) \vee (\beta' \to \perp) = (\beta \vee \beta') \to \perp\}$
$= \mathcal{T}^*(Q_1, A^-) \vee \mathcal{T}^*(Q_2, B^-) < (\beta \to \perp) \vee (\beta' \to \perp) = (\beta \vee \beta') \to \perp$
$= \bigcup\{Q \in 2^X : \mathcal{T}(Q, (A \cap B)^-) > (\alpha \vee \alpha') \to \perp, \mathcal{T}^*(Q, (A \cap B)^-) < (\beta \wedge \beta') \to \perp\}$

$= f_{(\mathcal{T},\mathcal{T}^*)}(A \cap B, \alpha \vee \alpha', \beta \wedge \beta')$

Theorem 3.4. Let $(\mathcal{T},\mathcal{T}^*)$ be a double fuzzifying topogenous order on X, and L be a chain The mapping $\delta_{\mathcal{T}}, \ \delta^*_{\mathcal{T}^*} : 2^X \to L$, is defined by $\delta_{\mathcal{T}}(A) = \bigvee \{\alpha \in L_0 : f_{(\mathcal{T},\mathcal{T}^*)}(A,\alpha,\beta) = A, \ \alpha \leq \beta \to \perp\}$ and $\delta^*_{\mathcal{T}^*}(A) = \bigwedge \{\beta \in L_1 : f_{(\mathcal{T},\mathcal{T}^*)}(A,\alpha,\beta) = A, \ \alpha \leq \beta \to \perp\}$. Then the pair (X, δ, δ^*) is an $(2, L)$- double fuzzifying topology on X.

Proof For each $A \in 2^X$, we have

$$\begin{aligned}(DO1)\ \delta^*_{\mathcal{T}^*}(A) \to \perp &= \bigwedge \{\beta \in L_1 : f_{(\mathcal{T},\mathcal{T}^*)}(A,\alpha,\beta) = A, \ \alpha \leq \beta \to \perp\} \to \perp \\ &= \bigvee \{\beta \to \perp : f_{(\mathcal{T},\mathcal{T}^*)}(A,\alpha,\beta) = A, \ \alpha \leq \beta \to \perp\} \\ &\geq \bigvee \{\alpha \in L_0 : f_{(\mathcal{T},\mathcal{T}^*)}(A,\alpha,\beta) = A, \ \alpha \leq \beta \to \perp\} \\ &= \delta_{\mathcal{T}}(A)\end{aligned}$$

$(DO2)$ It is clear.

$(DO3)$ Suppose taht there exist $A, B \in 2^X$ such that $\delta_{\mathcal{T}}(A \cap B) \not\geq \delta_{\mathcal{T}}(A) \wedge \delta_{\mathcal{T}}(B)$ and $\delta^*_{\mathcal{T}^*}(A \cap B) \not\leq \delta^*_{\mathcal{T}^*}(A) \vee \delta^*_{\mathcal{T}^*}(B)$. since L is chain and by the defintion of $\delta_{\mathcal{T}}(A)$ and $\delta^*_{\mathcal{T}^*}(A)$, there exist $\alpha_1 \in L_0$, $\beta_1 \in L_1$ with $\alpha_1 \leq \beta_1 \to \perp$ and $f_{(\mathcal{T},\mathcal{T}^*)}(A,\alpha_1,\beta_1) = A$ such that $\delta_{\mathcal{T}}(A \cap B) \not\geq \alpha_1 \wedge \delta_{\mathcal{T}}(B)$ and $\delta^*_{\mathcal{T}^*}(A \cap B) \not\leq \beta_1 \vee \delta^*_{\mathcal{T}^*}(B)$. Again, by the definition of $\delta_{\mathcal{T}}(B)$ and $\delta^*_{\mathcal{T}^*}(A)$, there exist $\alpha_2 \in L_0$, $\beta_1 \in L_1$ with $\alpha_2 \leq \beta_2 \to \perp$ and $f_{(\mathcal{T},\mathcal{T}^*)}(B,\alpha_2,\beta_2) = B$ such that $\delta_{\mathcal{T}}(A \cap B) \not\geq \alpha_1 \wedge \alpha_2$ and $\delta^*_{\mathcal{T}^*}(A \cap B) \not\leq \beta_1 \vee \beta_2$. By Theorem 3.2 (v) , we have $f_{(\mathcal{T},\mathcal{T}^*)}(A \cap B, \alpha_1 \wedge \alpha_2, \beta_1 \vee \beta_2) \supseteq f_{(\mathcal{T},\mathcal{T}^*)}(A,\alpha_1,\beta_1) \cap f_{(\mathcal{T},\mathcal{T}^*)}(B,\alpha_2,\beta_2) = A \cap B$. Then, we have $f_{(\mathcal{T},\mathcal{T}^*)}(A \cap B, \alpha_1 \wedge \alpha_2, \beta_1 \vee \beta_2) = A \cap B$. Thus, $\delta_{\mathcal{T}}(A \cap B) \geq \alpha_1 \wedge \alpha_2$ and $\delta^*_{\mathcal{T}^*}(A)(A \cap B) \leq \beta_1 \vee \beta_2$. This is a contradiction. Hence $\delta_{\mathcal{T}}(A \cap B) \geq \delta_{\mathcal{T}}(A) \wedge \delta_{\mathcal{T}}(B)$ and $\delta^*_{\mathcal{T}^*}(A)(A \cap B) \leq \delta^*_{\mathcal{T}^*}(A) \vee \delta^*_{\mathcal{T}^*}(A)$ $\forall A, B \in 2^X$.

$(DO4)$ Suppose that there exist $A = \bigcup_{i\in\Gamma} A_i \in 2^X$ and $\alpha \in L_0$, $\beta \in L_1$ with $\alpha \leq \beta \to \perp$ such that $\delta_{\mathcal{T}}(A) < \alpha \leq \bigwedge_{i\in\Gamma} \delta_{\mathcal{T}}(A_i)$ and $\delta^*_{\mathcal{T}^*}(A) > \beta \geq \bigvee_{i\in\Gamma} \delta^*_{\mathcal{T}^*}(A_i)$. Then $\delta_{\mathcal{T}}(A_i) \geq \alpha$ and $\delta^*_{\mathcal{T}^*}(A_i) \leq \beta$ for each $i \in \Gamma$. This implies that $f_{(\mathcal{T},\mathcal{T}^*)}(A_i,\alpha,\beta) = A_i \ \ \forall i \in \Gamma$. Since $A_i \subseteq A \ \ \forall i \in \Gamma$. Then, $f_{(\mathcal{T},\mathcal{T}^*)}(A_i,\alpha,\beta) \subseteq f_{(\mathcal{T},\mathcal{T}^*)}(A,\alpha,\beta)$. Then $A_i = f_{(\mathcal{T},\mathcal{T}^*)}(A_i,\alpha,\beta) \subseteq f_{(\mathcal{T},\mathcal{T}^*)}(A,\alpha,\beta)$. Therefore $A = \bigcup_{i\in\Gamma} A_i \subseteq f_{(\mathcal{T},\mathcal{T}^*)}(A,\alpha,\beta)$. Then, $f_{(\mathcal{T},\mathcal{T}^*)}(A,\alpha,\beta) = A$. Then $\delta_{\mathcal{T}}(A) \geq \alpha$ and $\delta^*_{\mathcal{T}^*}(B) \leq \beta$. It is a contradiction. Hence, $\delta_{\mathcal{T}}(\bigcup_{i\in\Gamma} A_i) \geq \bigwedge_{i\in\Gamma} \delta_{\mathcal{T}}(A_i)$ and $\delta^*_{\mathcal{T}^*}(\bigcup_{i\in\Gamma} A_i) \leq \bigvee_{i\in\Gamma} \delta^*_{\mathcal{T}^*}(A_i)$, for each $\{A_i : i \in \Gamma\} \subseteq 2^X$.

Definition 3.6. Let $(X, \mathcal{T}_1, \mathcal{T}_1^*)$ and $(Y, \mathcal{T}_2, \mathcal{T}_2^*)$ be two double fuzzifying topogenous order spaces. Then the map $\phi_L : (X, \mathcal{T}_1, \mathcal{T}_1^*) \to (Y, \mathcal{T}_2, \mathcal{T}_2^*)$ is called double fuzzifying topogenous continuous, if $\mathcal{T}_2(A,B) \leq \mathcal{T}_1(\phi_L^{\leftarrow}(A), \phi_L^{\leftarrow}(B))$ and $\mathcal{T}_2^*(A,B) \geq \mathcal{T}_1^*(\phi_L^{\leftarrow}(A), \phi_L^{\leftarrow}(B))$, for each $A, B \in 2^Y$.

Theorem 3.5. Let $(X, \mathcal{T}_1, \mathcal{T}_1^*)$ and $(Y, \mathcal{T}_2, \mathcal{T}_2^*)$ be two double fuzzifying topogenous order spaces, Let $\phi_L : (X, \mathcal{T}_1, \mathcal{T}_1^*) \to (Y, \mathcal{T}_2, \mathcal{T}_2^*)$ be double fuzzifying topogenous continuous. Then:

(*i*) $f_{(\mathcal{T},\mathcal{T}^*)}(\phi_L^{\leftarrow}(Q),\alpha,\beta) \supseteq \phi_L^{\leftarrow}(\ f_{(\mathcal{T}_2,\mathcal{T}_2^*)}(Q,\alpha,\beta))$, for each $Q \in 2^Y$, $\alpha \in L_0$, $\beta \in L_1$.

(*ii*) $\phi_L : (X, \delta_{\mathcal{T}_1}, \delta^*_{\mathcal{T}_1^*}) \rightarrow (Y, \delta_{\mathcal{T}_2}, \delta^*_{\mathcal{T}_2^*})$ is double fuzzifying continuous.

Pr**oof** (i) From the definition of $f_{(\mathcal{T},\mathcal{T}^*)}$ in Theorem 3.3 and since $\phi_L : (X,\mathcal{T}_1,\mathcal{T}_1^*) \rightarrow (Y,\mathcal{T}_2,\mathcal{T}_2^*)$ is double fuzzifying continuous, then

$$\begin{aligned}
&\phi_L^{\leftarrow}(f_{(\mathcal{T}_2,\mathcal{T}_2^*)}((Q),\alpha,\beta)\\
&= \phi_L^{\leftarrow}\left[\bigcup\left\{D \in 2^Y : \mathcal{T}_2(D,Q^-) > \alpha \rightarrow \bot,\ \mathcal{T}_2^*(D,Q^-) < \beta \rightarrow \bot\right\}\right]\\
&\subseteq \bigcup\left\{\phi_L^{\leftarrow}(D) \in 2^X : \mathcal{T}_1(\phi_L^{\leftarrow}(D),\phi_L^{\leftarrow}(Q^-)) > \alpha \rightarrow \bot,\ \mathcal{T}_1^*(\phi_L^{\leftarrow}(D),\phi_L^{\leftarrow}(Q^-)) < \beta \rightarrow \bot\right\}\\
&\subseteq \bigcup\left\{A \in 2^X : \mathcal{T}_1(A,(\phi_L^{\leftarrow}(Q)^-) > \alpha \rightarrow \bot,\ \mathcal{T}_1^*(A,(\phi_L^{\leftarrow}(Q))^-)) < \beta \rightarrow \bot\right\}\\
&= f_{(\mathcal{T}_2,\mathcal{T}_2^*)}(\phi_L^{\leftarrow}(Q),\alpha,\beta).
\end{aligned}$$

(ii) For each $A \in 2^Y$. If $\delta_{\mathcal{T}_2}(A) = \bot$ and $\delta^*_{\mathcal{T}_2^*}(A) = \top$, the prove is trivial. So let $\delta_{\mathcal{T}_2}(A) \neq \bot$ and $\delta^*_{\mathcal{T}_2^*}(A) \neq \top$.

Since $\delta_{\mathcal{T}_2}(A) \neq \bot$, by the defintion of $\delta_{\mathcal{T}_2}(A)$ there exist $\alpha_0 \in L_0$, $\beta_0 \in L_1$ with $\alpha_0 \leq \beta_0 \rightarrow \bot$ such that $\delta_{\mathcal{T}_2}(A) = \alpha_0$ and $f_{(\mathcal{T}_2,\mathcal{T}_2^*)}(A,\alpha_0,\beta_0) = A$. Thus $\phi_L^{\leftarrow}(A) = \phi_L^{\leftarrow}(f_{(\mathcal{T}_2,\mathcal{T}_2^*)}(A,\alpha_0,\beta_0)) \subseteq f_{(\mathcal{T}_2,\mathcal{T}_2^*)}(\phi_L^{\leftarrow}(A),\alpha_0,\beta_0)$ (by (i))., we have $\phi_L^{\leftarrow}(A) = f_{(\mathcal{T}_2,\mathcal{T}_2^*)}(\phi_L^{\leftarrow}(A),\alpha_0,\beta_0)$ since $\alpha_0 \leq \beta_0 \rightarrow \bot$, $\delta_{\mathcal{T}_1}(\phi_L^{\leftarrow}(A)) \geq \alpha_0 = \delta_{\mathcal{T}_2}(A)$.
similarly, when $\mathcal{T}^*_{\delta_2^*} \neq \top$, $\delta^*_{\mathcal{T}_1^*}(\phi_L^{\leftarrow}(A)) \leq \delta^*_{\mathcal{T}_2^*}(A)$. Hence $\phi_L : (X, \delta_{\mathcal{T}_1}, \delta^*_{\mathcal{T}_1^*}) \rightarrow (Y, \delta_{\mathcal{T}_2}, \delta^*_{\mathcal{T}_2^*})$ is double fuzzifying continuous.

Theorem 3.6. Let $(X,\mathcal{T}_1,\mathcal{T}_1^*)$, $(Y,\mathcal{T}_2,\mathcal{T}_2^*)$ and $(Y,\mathcal{T}_3,\mathcal{T}_3^*)$ be double fuzzifying topogenous order spaces, if $\phi_L : (X,\mathcal{T}_1,\mathcal{T}_1^*) \rightarrow (Y,\mathcal{T}_2,\mathcal{T}_2^*)$, and $\Psi_L : (X,\mathcal{T}_2,\mathcal{T}_2^*) \rightarrow (Y,\mathcal{T}_3,\mathcal{T}_3^*)$ are double fuzzifying topogenous continuous, then $\Psi \circ \phi : (X,\mathcal{T}_1,\mathcal{T}_1^*) \rightarrow (Y,\mathcal{T}_3,\mathcal{T}_3^*)$ is double fuzzifying topogenous continuous

Pr**oof** For each $A, B \in 2^Z$

$$\begin{aligned}
\mathcal{T}_1((\Psi\circ\phi)_L^{\leftarrow}(A),(\Psi\circ\phi)_L^{\leftarrow}(B)) &= \mathcal{T}_1((\phi_L^{\leftarrow}(\Psi_L^{\leftarrow}(A)),(\phi_L^{\leftarrow}(\Psi_L^{\leftarrow}(B))\\
&\geq \mathcal{T}_2((\Psi_L^{\leftarrow}(A)),(\Psi_L^{\leftarrow}(B)))\\
&\geq \mathcal{T}_3((A),(B))),\\
\mathcal{T}_1^*((\Psi\circ\phi)_L^{\leftarrow}(A),(\Psi\circ\phi)_L^{\leftarrow}(B)) &= \mathcal{T}_1^*((\phi_L^{\leftarrow}(\Psi_L^{\leftarrow}(A)),(\phi_L^{\leftarrow}(\Psi_L^{\leftarrow}(B))\\
&\leq \mathcal{T}_2^*((\Psi_L^{\leftarrow}(A)),(\Psi_L^{\leftarrow}(B)))\\
&\leq \mathcal{T}_3^*((A),(B))).
\end{aligned}$$

Theorem 3.7. Let $(X,\mathcal{T}_1,\mathcal{T}_1^*)$, $(Y,\mathcal{T}_2,\mathcal{T}_2^*)$ be double fuzzifying topogenous order spaces, if $\phi_L : (X,\mathcal{T}_1,\mathcal{T}_1^*) \rightarrow (Y,\mathcal{T}_2,\mathcal{T}_2^*)$, is double fuzzifying topogenous continuous.Then it has the following properties:

(1) $\phi_L^{\rightarrow}(f_{(\mathcal{T}_1,\mathcal{T}_1^*)}(A,\alpha,\beta)) \leq f_{(\mathcal{T}_2,\mathcal{T}_2^*)}(\phi_L^{\rightarrow}(A),\alpha,\beta))$ for each $A \in 2^X, \alpha \in L_0$, $\beta \in L_1$

(2) $f_{(\mathcal{T}_1,\mathcal{T}_1^*)}(\phi_L^{\leftarrow}(B),\alpha,\beta)) \leq \phi_L^{\leftarrow}(f_{(\mathcal{T}_2,\mathcal{T}_2^*)}(B,\alpha,\beta))$ for each $B \in 2^Y, \alpha \in L_0$, $\beta \in L_1$

(3) $\phi_L : (X, \delta_{\mathcal{T}_1}, \delta^*_{\mathcal{T}_1^*}) \rightarrow (Y, \delta_{\mathcal{T}_2}, \delta^*_{\mathcal{T}_2^*})$ is double fuzzifying topogenous continuous.

Pr**oof** (2) for each $B \in 2^Y, \alpha \in L_0$, $\beta \in L_1$, Put $A = \phi_L^{\leftarrow}(B)$, From (1), then

$$\begin{aligned}
\phi_L^{\rightarrow}(f_{(\mathcal{T}_1,\mathcal{T}_1^*)}(\phi_L^{\leftarrow}(B),\alpha,\beta)) &\leq (f_{(\mathcal{T}_2,\mathcal{T}_2^*)}(\phi_L^{\rightarrow}(\phi_L^{\leftarrow}(B)),\alpha,\beta))\\
&\leq f_{(\mathcal{T}_2,\mathcal{T}_2^*)}(B,\alpha,\beta)
\end{aligned}$$

It implies

$$\begin{aligned} f_{(\mathcal{T}_1,\mathcal{T}_1^*)}(\phi_L^{\leftarrow}(B),\alpha,\beta)) &\le \phi_L^{\leftarrow}(\phi_L^{\rightarrow}(f_{(\mathcal{T}_1,\mathcal{T}_1^*)}(\phi_L^{\leftarrow}(B),\alpha,\beta))) \\ &\le \phi_L^{\leftarrow}((f_{(\mathcal{T}_1,\mathcal{T}_1^*)}(\phi_L^{\rightarrow}(\phi_L^{\leftarrow}(B),\alpha,\beta))) \\ &\le \phi_L^{\leftarrow}(f_{(\mathcal{T},\mathcal{T}^*)}(B,\alpha,\beta)) \end{aligned}$$

(3) It is easily from Theorem 3.5 and $f_{(\mathcal{T}_2,\mathcal{T}_2^*)}(B,\alpha,\beta) = B$ implies $f_{(\mathcal{T}_1,\mathcal{T}_1^*)}(\phi_L^{\leftarrow}(B),\alpha,\beta)) = \phi_L^{\leftarrow}(B)$.

Definition 3.7. The pair (Ω,Ω^*) of maps $\Omega,\Omega^*: 2^X \times 2^X \to L$ is called $(2,L)$-double fuzzifying preproximity. If it is satisfies the following conditions:
(DP1) $\Omega(\mathrm{A},\mathrm{B}) \ge \Omega^*(\mathrm{A},\mathrm{B}) \to \bot, \ \forall \mathrm{A},\mathrm{B} \in 2^X$,
(DP2) $\Omega(\mathrm{X},\phi) = \Omega(\phi,\mathrm{X}) = \bot$, $\Omega^*(\mathrm{X},\phi) = \Omega^*(\phi,\mathrm{X}) = \top$,
$(DP3)$ If $\Omega(A,B) \ne \top$ and $\Omega^*(A,B) \ne \bot$, then $A \subseteq B^-$,
$(DP4)$ If $A_1 \subseteq A_2$, then $\Omega(A_1,C) \le \Omega(A_2,C)$, and $\Omega^*(A_1,C) \ge \Omega^*(A_2,C)$,
$(DP5)$ $\Omega(A_1\cap A_2, B_1\cup B_2) \le \Omega(A_1,B_1)\vee\Omega(A_2,B_2)$, and $\Omega^*(A_1\cap A_2, B_1\cup B_2) \ge \Omega^*(A_1,B_1)\wedge\Omega^*(A_2,B_2)$.
The pair $(X, \ \Omega,\Omega^*)$ is said to be an $(2,L)$-double fuzzifying preproximity space.

$(2,L)$ double fuzzifying preproximity space is called $(2,L)$-double fuzzifying quasi proximity provided that

$(DP6)$ $\Omega(A,B) \ge \bigwedge_{D\in 2^X} \{\Omega(A,D)\vee\Omega(D^c,B)\}$,

and

$\Omega^*(A,B) \le \bigvee_{D\in 2^X} \{\Omega^*(A,D)\wedge\Omega^*(D^c,B)\}$

$(2,L)$ double fuzzifying quasi-proximity is called $(2,L)$-double fuzzifying proximity provided taht
(DP) $\Omega(A,B) = \Omega(B,A)$ and $\Omega^*(A,B) = \Omega^*(B,A)$.

$(2,L)$ double fuzzifying preproximity space is called $(2,L)$-double fuzzifying prinicipal provided that:

$(DP7)$ $\Omega(\bigcup_{i\in\Gamma} A_i, B) \le \bigvee_{i\in\Gamma} \Omega(A_i,B)$, and $\Omega^*(\bigcup_{i\in\Gamma} A_i, B) \ge \bigwedge_{i\in\Gamma} \Omega^*(A_i,B)$.

Let (Ω_1,Ω_1^*) and (Ω_2,Ω_2^*) be $(2,L)$-double fuzzifying proximities on X . (Ω_1,Ω_1^*) is coarser than (Ω_2,Ω_2^*) if $\Omega_1(A,B) \le \Omega_2(A,B)$ and $\Omega_1^*(A,B) \ge \Omega_2^*(A,B)$, for each $A,B \in 2^X$ and we write $(\Omega_1,\Omega_1^*) \le (\Omega_2,\Omega_2^*)$.

Theorem 3.8. (1) Let $(X,\mathcal{T},\mathcal{T}^*)$ be double fuzzifying (resp. symmetric) topogenous order spaces, and let the map $\phi_L: \ \Omega_{\mathcal{T}}, \ \Omega^*_{\mathcal{T}^*}: 2^X \times 2^X \to L$ defined by $\Omega_{\mathcal{T}}(A,B) = (\mathcal{T}(A,B^-)) \to \bot$ and $\Omega^*_{\mathcal{T}^*}(A,B) = (\mathcal{T}^*(A,B^-)) \to \bot \ \forall A,B \in 2^X$. Then $(\Omega_{\mathcal{T}},\Omega^*_{\mathcal{T}^*})$ is double fuzzifying quasi proximity space (resp. double fuzzifying proximity space) on X.

(2) Let (Ω,Ω^*) be an $(2,L)$-double fuzzifying quasi proximity space (resp. $(2,L)$-double fuzzifying quasi proximity space) on X. $\mathcal{T}_\Omega, \ \mathcal{T}^*_{\Omega^*}: 2^X \times 2^X \to L$ defined

by $\mathcal{T}_\Omega(A,B) = (\Omega(A,B^-)) \to \perp)$ and $\mathcal{T}^*_{\Omega^*}(A,B) = (\Omega^*(A,B^-)) \to \perp\ \forall A,B \in 2^X$. Then $(\mathcal{T}_\Omega,\ \mathcal{T}^*_{\Omega^*})$ is double fuzzifying (resp. symmetric) topogenous order spaces.

(3) $(\Omega,\Omega^*) = (\Omega_{\mathcal{T}_\Omega},\ \Omega^*_{\mathcal{T}^*_{\Omega^*}})$ and $(\mathcal{T},\mathcal{T}^*)(\mathcal{T}_{\Omega_\mathcal{T}}, \mathcal{T}^*_{\Omega^*_{\mathcal{T}^*}})$

Proof (1) Since $\mathcal{T}\circ\mathcal{T} \geq \mathcal{T}$ and $\mathcal{T}^*\circ\mathcal{T}^* \leq \mathcal{T}^*$.

$$\begin{aligned}
\Omega_\mathcal{T}(A,B) &= (\mathcal{T}(A,B^-)) \to \perp \\
&\geq ((\mathcal{T}\circ\mathcal{T})(A,B^-)) \to \perp \\
&\geq \left[\bigvee_{h\in 2^X}[\mathcal{T}(A,h)\wedge(\mathcal{T}(h,B^-)]\right] \to \perp \\
&= \bigwedge_{h\in 2^X} [[[\mathcal{T}(A,h)] \to \perp] \vee [[\mathcal{T}(h,B^-)] \to \perp]] \\
&= \bigwedge_{h\in 2^X} \{\Omega_\mathcal{T}(A,h^-) \vee \Omega_\mathcal{T}(h,B)\}, \\
\Omega^*_{\mathcal{T}^*}(A,B) &= (\mathcal{T}^*(A,B^-)) \to \perp \\
&\leq ((\mathcal{T}^*\circ\mathcal{T}^*)(A,B^-)) \to \perp \\
&\geq \left[\bigwedge_{h\in 2^X} [\mathcal{T}^*(A,h)\vee(\mathcal{T}^*(h,B^-)]\right] \to \perp \\
&= \bigvee_{h\in 2^X} [[[\mathcal{T}^*(A,h)] \to \perp] \wedge [[\mathcal{T}^*(h,B^-)] \to \perp]] \\
&= \bigvee_{h\in 2^X} \{\Omega^*_{\mathcal{T}^*}(A,h^-) \wedge \Omega^*_{\mathcal{T}^*}(h,B)\},
\end{aligned}$$

(2) and (3) are easily proved

Theorem 3.9. Let (Ω,Ω^*) be a double quasi proximity.The mapping $f_{(\Omega,\Omega^*)} : 2^X \to L$, is defined by.

$$f_{(\Omega,\Omega^*)}(A,\alpha,\beta) = \bigcap \{Q^- \in 2^X :\ \Omega(Q,A) < \alpha \to \perp,\ \Omega^*(Q,A) > \beta \to \perp\}.$$

Then it has the following properties:

(*i*) $\mathrm{f}_{(\Omega,\Omega^*)}(\phi,\alpha,\beta) = \phi$,
(*ii*) $f_{(\Omega,\Omega^*)}(A,\alpha,\beta) \supseteq A$,
(*iii*) *If* $A \subseteq B$, *then* $f_{(\Omega,\Omega^*)}(A,\alpha,\beta) \subseteq f_{(\Omega,\Omega^*)}(B,\alpha,\beta)$,
(*iv*) $f_{(\Omega,\Omega^*)}(A\vee B,\alpha\wedge\alpha_1,\beta\vee\beta_1) \subseteq f_{(\Omega,\Omega^*)}(A,\alpha,\beta) \vee f_{(\Omega,\Omega^*)}(B,\alpha_1,\beta_1)$
(*v*) *If* $\alpha \leq \alpha_1$ *and* $\beta \geq \beta_1$, then $f_{(\Omega,\Omega^*)}(A,\alpha,\beta) \subseteq f_{(\Omega,\Omega^*)}(A,\alpha_1,\beta_1)$,
(*v*) $f_{(\Omega,\Omega^*)}(f_{(\Omega,\Omega^*)}(A,\alpha,\beta),\alpha,\beta) \subseteq f_{(\Omega,\Omega^*)}(A,\alpha,\beta)$.

Theorem 3.10. Let (Ω,Ω^*) be a double quasi proximity.Define the mas $\delta_\otimes(A) = \bigvee \{\alpha \in L_0 : f_{(\Omega,\Omega^*)}(A,\alpha,\beta) = A\}$ and $\delta^*_{\Omega^*}(A) = \bigwedge \{\beta \in L_1 : f_{(\mathcal{T},\mathcal{T}^*)}(A^-,\alpha,\beta) = A^-\}$. Then the pair (X,Ω,Ω^*) is an $(2,\ L)$- double fuzzifying topology induced by (Ω,Ω^*).

Definition 3.8. Let (X,Ω_1,Ω_1^*) and (Y,Ω_2,Ω_2^*) be a double quasi proximity spaces. A maps $\phi_L : (X,\Omega_1,\Omega_1^*) \to (Y,\Omega_2,\Omega_2^*)$ is said to be quasi proximity continuous if $\Omega_2(A,B) \geq \Omega_1(\phi_L^\leftarrow(A),\phi_L^\leftarrow(B))$ and $\Omega_2^*(A,B) \leq \Omega_1^*(\phi_L^\leftarrow(A),\phi_L^\leftarrow(B))$, for each $A,B \in 2^Y$.

Theorem 3.11. Let (X,Ω_1,Ω_1^*) and (Y,Ω_2,Ω_2^*) be a double quasi proximity spaces, A map $\phi_L : (X,\Omega_1,\Omega_1^*) \to (Y,\Omega_2,\Omega_2^*)$ is quasi proximity continuous iff $\phi_L : (X,\mathcal{T}_{\Omega_1},\mathcal{T}^*_{\Omega_1^*}) \to (Y,\mathcal{T}_{\Omega_2},\mathcal{T}^*_{\Omega_2^*})$ is topogenous continuous.

Proof For each $A, B \in 2^Y$.

$$
\begin{aligned}
\Omega_2(A,B) &\geq \Omega_1(\phi_L^{\leftarrow}(A), \phi_L^{\leftarrow}(B)) \\
&\Leftrightarrow \mathcal{T}_{\Omega_2}((A, B^-) \to \bot) \\
&\geq \mathcal{T}_{\Omega_1}((\phi_L^{\leftarrow}(A), \phi_L^{\leftarrow}(B^-)))) \to \bot) \\
&\Leftrightarrow \mathcal{T}_{\Omega_2}((A, B^-)) \\
&\leq \mathcal{T}_{\Omega_1}((\phi_L^{\leftarrow}(A), \phi_L^{\leftarrow}(B^-)), \\
\Omega_2^*(A,B) &\leq \Omega_1^*(\phi_L^{\leftarrow}(A), \phi_L^{\leftarrow}(B)) \\
&\Leftrightarrow \mathcal{T}^*_{\Omega_2^*}((A, B^-) \to \bot) \\
&\leq \mathcal{T}^*_{\Omega_1^*}((\Omega_1(\phi_L^{\leftarrow}(A), \phi_L^{\leftarrow}(B^-)))) \to \bot) \\
&\Leftrightarrow \mathcal{T}^*_{\Omega_2^*}(A, B^-) \\
&\geq \mathcal{T}^*_{\Omega_1^*}((\phi_L^{\leftarrow}(A), \phi_L^{\leftarrow}(B^-)).
\end{aligned}
$$

Definition 3.9. Let X be a nonempty set and let , $\mathcal{U}, \mathcal{U}^* \in L^{P(X\times X)}$. Assume that the following statments are satisfied:

$(LU1)$ $\mathcal{U}(A) \leq (\mathcal{U}^*(A)) \to \bot$ for all $A \in P(X \times X)$,

$(LU2)$ $\mathcal{U}(A\cap B) \geq \mathcal{U}(A) \wedge \mathcal{U}(A)$ and $\mathcal{U}^*(A\cap B) \leq \mathcal{U}^*(A) \vee \mathcal{U}^*(B)$,

$(LU3)$ There exists $A \in P(X \times X)$ s.t. $\mathcal{U}(A\ ) = \top$, and $\mathcal{U}^*(A\ ) = \bot$,

$(LU4)$ For any $A \in P(X \times X)$, $\exists\ B \in P(X \times X)$ s.t. $B \circ B \subseteq A$ and $\mathcal{U}(B) \geq \mathcal{U}(A\ )$ and $\mathcal{U}^*(B) \leq \mathcal{U}^*(A)$. where $B \circ A$ is defined by $B\circ A = \{(x,y)|\, \exists z \in X$ such that $(x,z) \in A$ and $(z,y) \in A\}$, $\forall x, y \in X$. Then $(X, \mathcal{U}, \mathcal{U}^*)$ is called an double fuzzifying quasi uniform space.

An double fuzzifying quasi uniform space $(X, \mathcal{U}, \mathcal{U}^*)$ is said to be a double fuzzifying uniform space if it satisfies.

(LU) For any $A, B \in P(X\times X)$, $\mathcal{U}(A\ ) \leq \mathcal{U}(B^{\leftarrow}\ )$, and $\mathcal{U}^*(A) \geq \mathcal{U}^*(B^{\leftarrow})$, where $B^{\leftarrow} = \{(x,y)\,|(y,x) \in P(X \times X)\}$

Definition 3.10. Let X be a nonempty set and let ,$\Xi, \Xi^* \in L^{P(X\times X)}$. Assume that the following statments are satisfied:

$(LUB1)$ $\Xi(A) \leq (\Xi^*(A)) \to \bot$ for all $A \in P(X \times X)$,

$(LUB2)$ $\bigvee_{B\in P(X\times X)} \Xi(B) \leq \Xi(A) \wedge \Xi(A)$ and $\overset{*}{\bigwedge}_{B\in P(X\times X)} \Xi(B) \geq \Xi^*(A) \vee \Xi^*(B)$,

$(LUB3)$ There exists $A \in P(X \times X)$ s.t. $\Xi(A\ ) = \top$, and $\Xi^*(A\ ) = \bot$,

$(LUB4)$ For any $A \in P(X \times X)$, $\exists\ B \in P(X \times X)$ s.t. $B \circ B \subseteq A$ and $\Xi(B) \geq \Xi(A\ )$ and $\Xi^*(B) \leq \Xi^*(A)$.

Then (X,Ξ, Ξ^*) is called an double fuzzifying quasi uniform base.A double fuzzifying quasi uniform base (X,Ξ, Ξ^*) is said to be a double fuzzifying uniform base if it satisfies.

(LUB) For any $A, B \in P(X \times X)$, $\Xi(A\ ) \leq \Xi(B^{\leftarrow}\ )$, and $\Xi^*(A) \geq \Xi^*(B^{\leftarrow})$.

Theorem 3.12. Let $(\Xi, \Xi^*) \in L^{P(X\times X)}$. Define $(\mathcal{U}_\Xi, \mathcal{U}^*_{\Xi^*}) \in L^{P(X\times X)}$ as $\mathcal{U}_\Xi(A) = \bigvee_{B\in P(X\times X)} \{\Xi(B) : B \subseteq A\}$ and $\mathcal{U}^*_{\Xi^*}(A) = \bigwedge_{B\in P(X\times X)} \{\Xi^*(B) : B \subseteq A\}$. Then $(\mathcal{U}_\Xi, \mathcal{U}^*_{\Xi^*})$ is a double fuzzifying uniformity on X.

Proof Because prove the cases are easily so only prove (LU). For any $A, B \in P(X\times X)$. Since $A= (A^{\leftarrow})^{\leftarrow}$, we have $\mathcal{U}_{\Xi}(A^{\leftarrow}) \le \mathcal{U}_{\Xi}(A)$ and $\mathcal{U}^{*}_{\Xi^*}(A^{\leftarrow}) \ge \mathcal{U}^{*}_{\Xi^*}(A)$. and

$$\begin{aligned}
\mathcal{U}_{\Xi}(A) &= \bigvee_{B\in P(X\times X)} \{\Xi(B) : B \subseteq A\} \\
&\le \bigvee_{B\subseteq A} \left\{\bigvee \Xi(Q) : Q \subseteq B^{\leftarrow}\right\} \quad \text{by } (LUB) \\
&\le \bigvee_{B\subseteq A} \mathcal{U}_{\Xi}(B^{\leftarrow}) \\
&= \bigvee_{B^{\leftarrow}\subseteq A^{\leftarrow}} \mathcal{U}_{\Xi}(B^{\leftarrow}), \\
&\le \mathcal{U}_{\Xi}(A^{\leftarrow}) \\
\mathcal{U}^{*}_{\Xi^*}(A) &= \bigwedge_{B\in P(X\times X)} \{\Xi^*(B) : B \subseteq A\} \\
&\ge \bigwedge_{B\subseteq A} \{\bigwedge \Xi^*(Q) : Q \subseteq B^{\leftarrow}\} \quad \text{by } (LUB) \\
&\ge \bigwedge_{B\subseteq A} \mathcal{U}^{*}_{\Xi^*}(B^{\leftarrow}) \\
&= \bigwedge_{B^{\leftarrow}\subseteq A^{\leftarrow}} \mathcal{U}^{*}_{\Xi^*}(B^{\leftarrow}), \\
&\ge \mathcal{U}^{*}_{\Xi^*}(A^{\leftarrow}).
\end{aligned}$$

4 Fuzzifying Topology and Dynamics of Breast Cancer

In this section we will show how the dynamical topologies [CsaszarA.(1978)]. can develop the diagnostic mechanism and time analysis of the situation and determine the appropriate time to avoid distortions in the stages of the case. The present article demonstrates an application of L-fuzzifying dynamice topology clarify a model describing biological phenomena, This model allow to know all levels of development of an breast cancer. from 0-level (infection outside cells) until 5-level (infection liver).

Definition 4.1. Let X be compact metric space, T is a time L is a chain, then the function $\mathcal{T} : 2^X \times T \to L$ is called an L-fuzzifying dynamice topology on X (T-dynamic topologies) iff it satisfies the following axioms:

(1) $\mathcal{T}(X, t) = \top,\ \mathcal{T}(\phi, t) = \bot$

(2) $\forall A, B \in 2^X, \mathcal{T}((A \cap B), t) \ge \mathcal{T}(A, t) \wedge \mathcal{T}(B, t),$

(3) $\forall\{A_j\,|j \in J\} \subseteq 2^X,\ \mathcal{T}((\bigcup_{j\in J} A_j), t) \ge \bigwedge_{j\in J} \mathcal{T}((A_j), t).$

We also write $\mathcal{T} = \mathcal{T}_d(T)$. such that and $\mathcal{T}_d(T)$ can be viewed as parametric or dynamic sets of X, say that $(X, L, \mathcal{T}_d(T))$ is an L-fuzzifying T-dynamice topological space. The inductive dimension of a fuzzifying dynamice topology X is either of two values, the small inductive dimension $ind(X)$ or the large inductive dimension $Ind(X)$. We want the dimension of a point to be $\bot$, and a point has empty boundary, so we start with $ind(\phi) = Ind(\phi) = \bot$. If $L = I = [0, 1]$ a fuzzifying dynamice topological space has dimension $\le n$. $n \ge 0$ iff for any point $p \in X$, each neighborhood of p contains a neighborhood of p whose boundary has dimension $\le n - 1$.

Definition 4.2. A riemannian manifold is a smooth manifold equipped with a riemannian metric. A map $f : (X, \mathcal{T}) \to (Y, \gamma)$, where X and Y are riemannian manifolds. is said to be a topological folding if and only if for any piecewise geodesic path, α, in X, the induced path, $f \circ \alpha$ is a piecewise geodesic in Y . It is possible $f(X) = Y$ or $f(X) \neq Y$; accordingly, a topological folding f of (X, τ) into itself satisfies $f(X) \subseteq X$ and for each $\beta \in \tau$, we have $f(\beta) \subseteq \beta$. The contrary definition to the folding of (X, τ) into itself is the unfolding: a map $f : (X, \mathcal{T}) \to (Y, \gamma)$ is called unfolding iff $f(\beta) \supseteq \beta$ for each $\beta \in \tau$ [4].

From these topological concepts we can form templates to form the biological structures of the course of breast cancer progression as follows:

Molding (I) : 0-$level$(infection outside cells normal cells) until $1 - level$ (very slow growing cancer cells)

Molding (II) : 2-$level$ (Slow grwoing cancer cells) , 3-level (Moderately growing cancer cells) , $4 - level$ the arrival of cancer of the liver (Fast growing cancer cells)

Molding (III) : 5-$level$ (Infection spreads to liver)

2 Main Results

when begins infection outside cells $(0 - level)$, we suppose that an $0 - level$ at time $t_0 = 0$, after certain time and constant rate of differentiation of tumor is 2 cm in size and the lymph nodes under the armpit are intact from the cancer cells $(1 - level)$, then $(1 - level)$ differentiate into The size of the tumor is 2 cm, and may have moved under the control but not spread to the rest of the body $(2 - level)$, which differentiate into system, The tumor is adherent to the skin of the breast and muscles and the size of the tumor is greater than 5 cm and has moved under the armpit $(3 - level)$, and $(3 - level)$ differentiates to the arrival of cancer of the liver $(4 - level)$, and finally $(4 - level)$ differentiates to infection liver and mastectomy $(5 - level)$ at time $t = 1$. Thus

$$\begin{gathered}(\mathbf{0} - \mathbf{level})_{t_0=0} \Rightarrow (\mathbf{1} - \mathbf{level}) \Rightarrow (\mathbf{2} - \mathbf{level}) \Rightarrow (\mathbf{3} - \mathbf{level}) \\ \Rightarrow (\mathbf{4} - \mathbf{level}) \Rightarrow (\mathbf{5} - \mathbf{level})_{t=1}\end{gathered}$$

Now we can define a L-fuzzifying dynamice topology (T-dynamic topologies) as follows:

$$\mathcal{T}(A, t) = \begin{cases} 0 & A = (0 - level)_{t_0=0} \\ \alpha_1 & A = ((1 - level), (0 < t < t_1)) \\ \alpha_2 & A = ((2 - level), (t_1 < t < t_2)) \\ \alpha_3 & A = ((3 - level), (t_2 < t < t_3)) \\ \alpha_4 & A = ((4 - level) , (t_3 < t < t_4)) \\ 1 & A = (5 - level)_{t=1} \end{cases}$$

such taht $(t_0 = 0) < t_1 < t_2 < t_3 < t_4 < (t_5 = 1)$, and $\alpha_1 < \alpha_2 < \alpha_3 < \alpha_4$ in L.

where, $(\mathbf{0} - \mathbf{level})_{t_0=0}$ at $t_0 = 0$ and $(\mathbf{5} - \mathbf{level})$ happens at $t = 1$. It is obvious that $((\mathbf{5} - \mathbf{level}), \mathcal{T})$ forms a L-fuzzifying dynamice topology (T-dynamic topologies) as the growth's rate of breast cancer. from $(\mathbf{0-level})_{t_0=0}$(infection outside cells) until $(\mathbf{5-level})$ (infection liver) depends on time. Perhaps over time there is no differentiation for example

$$\begin{gathered}(\mathbf{0-level})_{t_{01}=0} \Rightarrow (\mathbf{0-level})_{\mathbf{t_{02}=0}} \Rightarrow (\mathbf{0-level})_{\mathbf{t_{03}=0}} \Rightarrow (\mathbf{1-level})_{t_{11}} \\ \Rightarrow (\mathbf{1-level})_{t_{12}} \Rightarrow (1-level)_{t=2} \Rightarrow (\mathbf{2-level})_{t=3} \Rightarrow (\mathbf{3-level})_{t=4} \\ \Rightarrow (\mathbf{4-level})_{t=5} \Rightarrow (\mathbf{5-level})_{t=\max imum}\end{gathered}$$

Here, from $t_{01} = 0$ up to t_{11} only the infection outside cells without a real development and from t_{11} up to t_{12}. constant rate of differentiation of tumor is 2 cm in size and the lymph nodes under the armpit are intact from the cancer cells is without real expansion this is a topological invariant. In these fixed stages with the passage of time may take the development of the disease different aspects of the injury and may lead to injury in other areas. Using precise time scales such as femtoseconds, we can identify the inaccurate stages of the disease as natural time evolves treatment is therefore necessary. In fact, cognitive method depend on synchronization of abnormality step during cells development. We assume that $\lambda(t)$ is the shape of cells as we reach a specific time, t. Then, a chain of T-dynamic topologies can be given

$$((\lambda_0(t_0), \mu_0(t_0)),\ ((\lambda_1(t_1), \mu_1(t_1)),\ ((\lambda_2(t_2), \mu_2(t_2)),\ ...,\ (\max(\lambda_i(t_i), \max \mu_i(t_i))$$

With the attributes

$$\lambda_0(t_0) \subseteq \lambda_1(t_1) \subseteq ... \subseteq \max(\lambda_i(t_i) \ \text{ and } \ \mu_0(t_0) \subseteq \mu_1(t_1) \subseteq ... \subseteq \max \mu_i(t_i)$$

and $f_n(\lambda_{n+1}) = \lambda_n$, $n = 0, 1, ..., i-1$, where f_n is a folding from λ_{n+1} into λ_n.

It is also satisfying $\mu_{n+1}(t_n) = f_n(\mu_n), n = 0, 1, ..., i-1$.

In the same path, $\lambda = \phi$ at $t = 0$ and after a limit of time the maximum of measurement formation of cancer cells.

This gives us the increasing chain to determine a cancer at a limit time.

$$\phi \overset{t=1}{\Rightarrow} \lambda_1 \subseteq .\lambda_2 \subseteq ... \text{ with } \ \mu_0 \subseteq \mu_1 \subseteq \mu_2 \subseteq ...$$

Or otherwise we get another decreasing chain can not determine a cancer at a limit time.

$$\lambda_1 \supseteq \lambda_2 \supseteq ... \supseteq \lambda_\infty \text{ with } \ \mu_0 \supseteq \mu_1 \supseteq \mu_2 \supseteq ... \supseteq \mu_i \to \phi$$

Some times in some steps fluctuation happens in the growth cancer , for example

$$\lambda_1 \subseteq \lambda_2 = \lambda_3 \subseteq ... \text{ with } \ \mu_0 \subseteq \mu_1 \subseteq \mu_2 = \mu_3 \subseteq ...$$

This causes a delay of the growth cancer at specific time. Giving the opportunity for treatment at this time. Based on the properties of local topological subspaces for the dynamical topology a demand for a medical treatment should be started to stop cognitive anomalies at any step of growth, and a positive result may be achieved as we use a femto second as a measurement unit.

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