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Some Traveling Wave Solutions of (3+1)-Dimensional Jimbo-Miwa Equation

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Abstract: In this paper, we implemented a $\tan\left(\frac{F(z)}{2}\right)$ -Expansion Method for some traveling wave solutions of (3+1)-dimensional Jimbo-Miwa equation. We find some traveling wave solutions such as trigonometric function, hyperbolic function, exponential function solutions. Then, we show the two and three dimensional surface for some traveling wave solutions obtained in this study by the Mathematica.

Keywords: (3+1)-dimensional Jimbo-Miwa equation, $\tan\left(\frac{F(z)}{2}\right)$ -Expansion Method, Traveling wave solutions.

1. Introduction

Nonlinear partial differential equations (NPDEs) have an important place in applied mathematics and physics [1,2]. Many analytical methods have been found in literature [3,4,5,6,7,8,9,10,11]. Besides these methods, there are many methods which reach to solution by using an auxiliary equation. Using these methods, partial differential equations are transformed into ordinary differential equations. These nonlinear partial differential equations are solved with the help of ordinary differential equations. These methods are given in [12,13,14,15,16,17,18,19,20,21,22,23,24,25,26].

We used the $\tan\left(\frac{F(z)}{2}\right)$ expansion method for finding the some traveling wave solutions of (3+1)-dimensional Jimbo-Miwa equation. This method is presented by Manafian and Lakestani [25].

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2. Analysis of Method

Let's introduce the method briefly. Consider a general partial differential equation of four variables,

$$\varphi\left(v, v_t, v_y, v_z, v_x, v_{xx}, \ldots\right) = 0. \tag{1}$$

Using the wave variable $(x, y, z, t) = v(z)$, $z = (x + \alpha y + \beta z - kt)$, the equation (1) turns into an ordinary differential equation,

$$\varphi' = (v', v'', v''', \ldots) = 0. \tag{2}$$

here k, α, β are constants. With this conversion, we obtain a nonlinear ordinary differential equation for $v(z)$. We can express the solution of equation (2) as below,

$$v(x, y, z, t) = v(z) = \sum_{i=0}^{m} A_i \left[p + tan\left(\frac{F(z)}{2}\right)\right]^i + \sum_{i=1}^{m} B_i \left[p + tan\left(\frac{F(z)}{2}\right)\right]^{-i}, A_i \neq 0, B_i \neq 0$$

(3)

here m is a positive integer and is found as the result of balancing the highest order linear term and the highest order nonlinear term found in the equation, the coefficients A_i $(0 \leq i \leq m)$, $B_i (1 \leq i \leq m)$, are constant. If we write these solutions in equation (2), we obtain a system of algebraic equations for $tan\left(\frac{F(z)}{2}\right)^i, cot\left(\frac{F(z)}{2}\right)^i$ after, if the coefficients of $tan\left(\frac{F(z)}{2}\right)^i, cot\left(\frac{F(z)}{2}\right)^i$ are equal to zero, we can find the $k, \alpha, \beta, p, A_0, A_1, B_1, \ldots, A_m, B_m$ constants, where $F = F(z)$ satisfies the first order nonlinear ODE:

$$F'(z) = asin\big(F(z)\big) + bcos\big(F(z)\big) + c \tag{4}$$

and a, b and c are constants. The expressed the solutions by Manafian and Lakestani [25].

3. Application of Method

3.1. The (3+1)-dimensional Jimbo-Miwa Equation

We consider the (3+1)-dimensional Jimbo-Miwa equation,

$$u_{xxxy} + 3u_y u_{xx} + 3u_x u_{xy} + 2u_{yt} - 3u_{xz} = 0 \tag{5}$$

Let us consider the traveling wave solutions $u(x, y, z, t) = u(z)$, $u(z) = x + \alpha y + \beta z - kt$ then equation (5) becomes

$$\alpha u^{(4)} + 6\alpha u' u'' - 2\alpha k u'' - 3\beta u'' = 0, \tag{6}$$

When balancing $u^{(4)}$ with $u'u''$ then gives *m=1*. The solution is as follows.

$$u(z) = A_0 + A_1 \left[p + tan\left(\frac{F(z)}{2}\right)\right] + B_1 \left[p + tan\left(\frac{F(z)}{2}\right)\right]^{-1}, \ A_i \neq 0, \ B_i \neq 0 \tag{7}$$

(7) is substituted in equation (6), a system of algebraic equations for $k, \alpha, \beta, A_0, A_1, B_1$ are obtained. The obtained systems of algebraic equations are as follows

$$a^3A_1\alpha + 12aA_1^2b\alpha - 11aA_1b^2\alpha - a^3B_1\alpha + 11ab^2B_1\alpha - 12abB_1^2\alpha - 6aA_1^2c\alpha + 18aA_1bc\alpha - 12aA_1B_1c\alpha + 18abB_1c\alpha - 6aB_1^2c\alpha - 7aA_1c^2\alpha + 7aB_1c^2\alpha - 2aA_1k\alpha + 2aB_1k\alpha + 30a^2A_1^2p\alpha - 35a^2A_1bp\alpha - 30A_1^2b^2p\alpha + 25A_1b^3p\alpha + 12a^2A_1B_1p\alpha - 29a^2bB_1p\alpha - 12A_1b^2B_1p\alpha + 7b^3B_1p\alpha + 6a^2B_1^2p\alpha - 6b^2B_1^2p\alpha + 35a^2A_1pc\alpha + 30A_1^2bcp\alpha - 55A_1b^2cp\alpha - 15B_1a^2cp\alpha + 36A_1B_1bcp\alpha + 3b^2B_1cp\alpha + 6bB_1^2pc\alpha + 35A_1bc^2p\alpha - 7bB_1c^2p\alpha - 5A_1c^3p\alpha - 3B_1c^3p\alpha + 10A_1bkp\alpha - 2B_1bkp\alpha - 10A_1ckp\alpha - 6B_1ckp\alpha - 10a^3A_1p^2\alpha - 120aA_1^2bp^2\alpha + 110aA_1b^2p^2\alpha + 11a^3B_1p^2\alpha - 48abA_1B_1p^2\alpha - 25ab^2B_1p^2\alpha + 60aA_1^2cp^2\alpha - 180abcA_1p^2\alpha + 36acA_1B_1p^2\alpha + 18abcB_1p^2\alpha + 70aA_1c^2p^2\alpha + 7aB_1c^2p^2\alpha + 20aA_1kp^2\alpha + 2aB_1kp^2\alpha - 60a^2A_1^2p^3\alpha + 70a^2A_1bp^3\alpha + 60A_1^2b^2p^3\alpha - 50A_1b^3p^3\alpha - 12a^2A_1B_1p^3\alpha + \cdots \quad (8)$$

If the system is solved, the coefficients are found as:

Case 1:

$$\mathrm{a} = -\mathrm{bp} + \mathrm{cp}, \mathrm{b} = \mathrm{b}, \mathrm{c} = \mathrm{c}, A_1 = 0, B_1 = b + c + bp^2 - cp^2, B_1\alpha \neq 0, k = \frac{b^2\alpha - c^2\alpha + b^2p^2\alpha - 2bcp^2\alpha + c^2p^2\alpha - 3\beta}{2\alpha}, bp - cp \neq 0. \quad (9)$$

Case 2:

$$\mathrm{a} = -\mathrm{bp} + \mathrm{cp}, \mathrm{b} = \mathrm{b}, \mathrm{c} = \mathrm{c}, A_1 = b - c, B_1 = b + c + bp^2 - cp^2, A_1\alpha \neq 0, k = \frac{b^2\alpha - c^2\alpha + b^2p^2\alpha - 2bcp^2\alpha + c^2p^2\alpha - 3\beta + 3bB_1\alpha - 3B_1c\alpha}{2\alpha}, bp - cp \neq 0, b + c + bp^2 - cp^2 \neq 0. \quad (10)$$

with the help of the Mathematica program. After these operations, The solutions of equation (5) for case 1 and case 2 are as follows:

For Case 1:

Solution 1: $\mathrm{a}^2 + \mathrm{b}^2 - c^2 < 0$ and $\mathrm{b} - \mathrm{c} \neq 0$,

$$u_1 = \sqrt{-\mathrm{a}^2 - \mathrm{b}^2 + c^2}\,Cot\left[\frac{1}{2}\sqrt{-\mathrm{a}^2 - \mathrm{b}^2 + c^2}\left(x + \alpha y + \beta z - \frac{t(a^2\alpha + b^2\alpha - c^2\alpha - 3\beta)}{2\alpha}\right)\right] \quad (11)$$

Solution 2: $\mathrm{a}^2 + \mathrm{b}^2 - c^2 > 0$ and $\mathrm{b} - \mathrm{c} \neq 0$,

$$u_2 = \sqrt{\mathrm{a}^2 + \mathrm{b}^2 - c^2}\,Coth\left[\sqrt{\mathrm{a}^2 + \mathrm{b}^2 - c^2}\left(x + \alpha y + \beta z - \frac{t(a^2\alpha + b^2\alpha - c^2\alpha - 3\beta)}{2\alpha}\right)\right] \quad (12)$$

Solution 3: $a^2 + b^2 - c^2 > 0, b \neq 0, c = 0,$

$$u_3 = \sqrt{a^2 + b^2}Coth\left[\sqrt{a^2 + b^2}\left(x + \alpha y + \beta z - \frac{t(a^2\alpha + b^2\alpha - 3\beta)}{2\alpha}\right)\right] \tag{13}$$

Solution 4: $a^2 + b^2 - c^2 > 0, c \neq 0, b = 0,$

$$u_4 = \sqrt{-a^2 + c^2}Cot\left[\frac{1}{2}\sqrt{-a^2 + c^2}\left(x + \alpha y + \beta z - \frac{t(a^2\alpha - c^2\alpha - 3\beta)}{2\alpha}\right)\right] \tag{14}$$

Solution 5: $a = c = wa$ and $b = -wa,$

$$u_5 = \frac{a\left(-1 + e^{aw\left(-\frac{1}{2}a^2w^2t + x + \alpha y + \beta z + \frac{3\beta t}{2\alpha}\right)}\right)w}{1 + e^{aw\left(-\frac{1}{2}a^2w^2t + x + \alpha y + \beta z + \frac{3\beta t}{2\alpha}\right)}} \tag{15}$$

Solution 6: $c = a,$

$$u_6 = \frac{b^2}{(b-a)\left(\frac{a}{a-b} - \frac{-1 + (a+b)e^{b\left(-\frac{b^2t}{2} + x + \alpha y + \beta z + \frac{3\beta t}{2\alpha}\right)}}{-1 + (a-b)e^{b\left(-\frac{b^2t}{2} + x + \alpha y + \beta z + \frac{3\beta t}{2\alpha}\right)}}\right)} \tag{16}$$

Solution 7: $a = c,$

$$u_7 = \frac{b^2}{(b-c)\left(\frac{c}{c-b} + \frac{1 + (b+c)e^{b\left(-\frac{b^2t}{2} + x + \alpha y + \beta z + \frac{3\beta t}{2\alpha}\right)}}{-1 + (b-c)e^{b\left(-\frac{b^2t}{2} + x + \alpha y + \beta z + \frac{3\beta t}{2\alpha}\right)}}\right)} \tag{17}$$

Solution 8: $c = -a,$

$$u_8 = -\frac{b\left(a + b - e^{b\left(-\frac{1}{2}b^2t + x + \alpha y + \beta z + \frac{3\beta t}{2\alpha}\right)}\right)}{a + b + e^{b\left(-\frac{1}{2}b^2t + x + \alpha y + \beta z + \frac{3\beta t}{2\alpha}\right)}} \tag{18}$$

Solution 9: $b = -c,$

$$u_9 = \frac{a\left(-1 + ce^{a\left(-\frac{1}{2}a^2t + x + \alpha y + \beta z + \frac{3\beta t}{2\alpha}\right)}\right)}{1 + ce^{a\left(-\frac{1}{2}a^2t + x + \alpha y + \beta z + \frac{3\beta t}{2\alpha}\right)}} \tag{19}$$

For Case 2:

Solution 1: $a^2 + b^2 - c^2 < 0$ and $b - c \neq 0,$

$$u_1 = 2\sqrt{-a^2 - b^2 + c^2}Cot\left[\sqrt{-a^2 - b^2 + c^2}\left(\left(-2a^2 - 2b^2 + 2c^2 + \frac{3\beta}{2\alpha}\right)t + x + \alpha y + \beta z\right)\right] \tag{20}$$

Solution 2: $a^2 + b^2 - c^2 > 0$ and $b - c \neq 0,$

$$u_2 = 2\sqrt{a^2 + b^2 - c^2}Coth\left[\sqrt{a^2 + b^2 - c^2}\left(\left(-2a^2 - 2b^2 + 2c^2 + \frac{3\beta}{2\alpha}\right)t + x + \alpha y + \beta z\right)\right] \tag{21}$$

Solution 3: $a^2 + b^2 - c^2 > 0, b \neq 0, c = 0,$

$$u_3 = \sqrt{a^2 + b^2} Coth\left[\frac{1}{2}\sqrt{a^2 + b^2}\left(\left(-2a^2 - 2b^2 + \frac{3\beta}{2\alpha}\right)t + x + \alpha y + \beta z\right)\right] + Tanh\left[\frac{1}{2}\sqrt{a^2 + b^2}\left(\left(-2a^2 - 2b^2 + \frac{3\beta}{2\alpha}\right)t + x + \alpha y + \beta z\right)\right] \quad (22)$$

Solution 4: $a^2 + b^2 - c^2 > 0, c \neq 0, b = 0,$

$$u_4 = \sqrt{c^2 - a^2} Cot\left[\frac{1}{2}\sqrt{c^2 - a^2}\left(\left(-2a^2 + 2c^2 + \frac{3\beta}{2\alpha}\right)t + x + \alpha y + \beta z\right)\right] - Tan\left[\frac{1}{2}\sqrt{c^2 - a^2}\left(\left(-2a^2 + 2c^2 + \frac{3\beta}{2\alpha}\right)t + x + \alpha y + \beta z\right)\right] \quad (23)$$

Solution 5: $a = c = wa$ and $b = -wa$,

$$u_5 = \frac{2aw\left(1 + e^{2aw\left(-2a^2tw^2 + x + \alpha y + \beta z + \frac{3\beta t}{2\alpha}\right)}\right)}{-1 + e^{2aw\left(-2a^2tw^2 + x + \alpha y + \beta z + \frac{3\beta t}{2\alpha}\right)}} \quad (24)$$

Solution 6: $c = a,$

$$u_6 = \frac{2b\left(e^{4b^3t} + (a-b)^2 e^{b\left(2x + 2\alpha y + 2\beta z + \frac{3\beta t}{\alpha}\right)}\right)}{-e^{4b^3t} + (a-b)^2 e^{b\left(2x + 2\alpha y + 2\beta z + \frac{3\beta t}{\alpha}\right)}} \quad (25)$$

Solution 7: $a = c,$

$$u_7 = \frac{b^2}{(b-c)\left(\frac{c}{c-b} + \frac{1 + (b+c)e^{b\left(-2b^2t + x + \alpha y + \beta z + \frac{3\beta t}{2\alpha}\right)}}{-1 + (b-c)e^{b\left(-2b^2t + x + \alpha y + \beta z + \frac{3\beta t}{2\alpha}\right)}}\right)} + (b-c)\left(\frac{c}{c-b} + \frac{1 + (b+c)e^{b\left(-2b^2t + x + \alpha y + \beta z + \frac{3\beta t}{2\alpha}\right)}}{-1 + (b-c)e^{b\left(-2b^2t + x + \alpha y + \beta z + \frac{3\beta t}{2\alpha}\right)}}\right) \quad (26)$$

Solution 8: $c = -a,$

$$u_8 = -\frac{2b\left(a^2 + 2ab + b^2 + e^{2b\left(-2b^2t + x + \alpha y + \beta z + \frac{3\beta t}{2\alpha}\right)}\right)}{a^2 + 2ab + b^2 - e^{2b\left(-2b^2t + x + \alpha y + \beta z + \frac{3\beta t}{2\alpha}\right)}} \quad (27)$$

Solution 9: $b = -c,$

$$u_9 = \frac{2a\left(1 + c^2 e^{2a\left(-2a^2t + x + \alpha y + \beta z + \frac{3\beta t}{2\alpha}\right)}\right)}{-1 + c^2 e^{2a\left(-2a^2t + x + \alpha y + \beta z + \frac{3\beta t}{2\alpha}\right)}} \quad (28)$$

4. Figures

4.1. The Graphs of Some of the Solutions of Equation

The graphs of some of the solutions of Equation (5) are as follows:

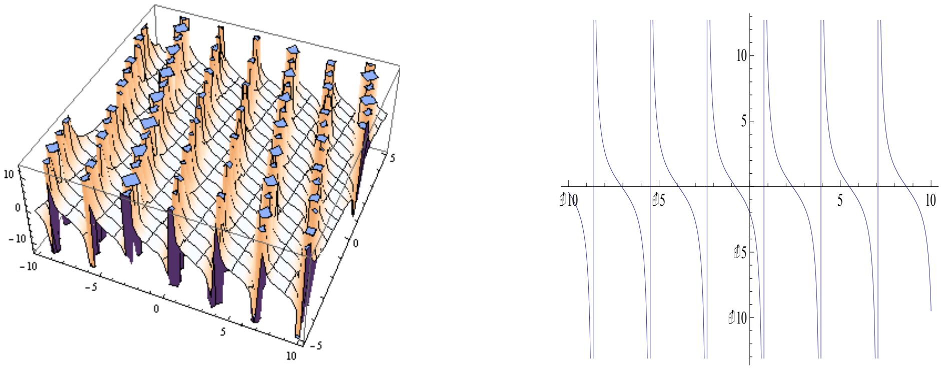

FIGURE 1. The 3D surfaces of Equation (11) for the values a=1, b=2, c=3, y=1, z=0, $\alpha = 5$ and $\beta = -5$ within the interval $-10 \leq x \leq 10, -5 \leq t \leq 5$. The 2D surfaces of Equation (11) for the values a=1, b=2, c=3, y=1,z=0, $\alpha = 5$, $\beta = -5$ and t=1 within the interval $-10 \leq x \leq 10$.

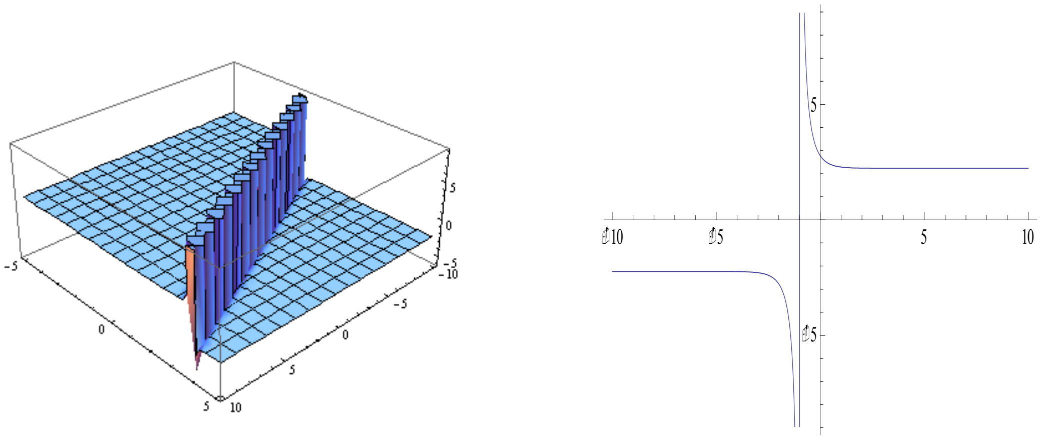

FIGURE 2. The 3D surfaces of Equation (13) for the values a=1, b=2, y=1,z=0, $\alpha = 5$ and $\beta = -5$ within the interval $-10 \leq x \leq 10, -5 \leq t \leq 5$. The 2D surfaces of Equation (13) for the values a=1, b=2, y=1, z=0,$\alpha = 5$, $\beta = -5$ and t=1 within the interval $-10 \leq x \leq 10$.

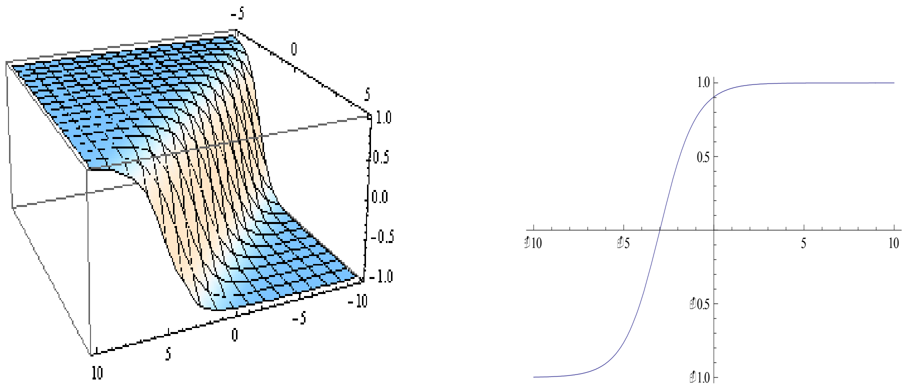

FIGURE 3.The 3D surfaces of Equation (15) for the values a=1, y=1, z=0,$\alpha = 5$, $\beta = -5$ and w=1 within the interval $-10 \leq x \leq 10, -5 \leq t \leq 5$. The 2D surfaces of Equation (15) for the values a=1, y=1, z=0,$\alpha = 5$, $\beta = -5$, w=1 and t=1 within the interval $-10 \leq x \leq 10$.

5. Conclusion

We used the $\tan\left(\frac{F(z)}{2}\right)$ expansion method for find some traveling wave solutions of (3+1)-dimensional Jimbo-Miwa equation. This method has been successfully applied to solve some nonlinear wave equations and can be used to many other nonlinear equations or coupled ones. Moreover, this method is also computerizable, which lets us to perform confused and oppressive algebraic calculation on a computer by the aid of symbolic programs such as Mathematica, Maple, Matlab, and so on.

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