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On the Wave Solutions of (2+1)-Dimensional Time-Fractional Zoomeron Equation

Hajar F. Ismael[1*,2] **and Hasan Bulut**[2]

[1]*Department of Mathematics, Faculty of Science, University of Zakho, Zakho, Iraq*
[2]*Department of Mathematics, Faculty of Science, Firat University, Elazig, Turkey*
[*]*Corresponding author*

Abstract

In this manuscript, we have applied the sine-Gordon expansion method and the Bernoulli sub-equation method to seek the traveling wave solutions of the (2+1)-dimensional time-fractional partial Zoomeron equation. The exact solutions of the Zoomeron equation that are obtained by the sine-Gordon method are plotted in 3D figures, as well as the effects of the fractional derivative α are illustrated in 2D figures, while the exact solutions of the Zoomeron equation that are obtained by the Bernoulli sub-equation method are plotted in 3D figures and contour plot. Bright solutions, kink soliton, singular soliton solution, and complex solutions to the studied equation are constructed. Also, different values of the fractional parameter α are tested to study the effect of the parameter. We conclude that these methods are sufficient for seeking the exact solutions.

Keywords: *Zoomeron equation; Time-fractional; Bernoulli sub-equation method; Sine-Gordon method*
2010 Mathematics Subject Classification: *Use about five key words or phrases in alphabetical order, Separated by Semicolon.*

1. Introduction

In recent years, the partial differential equations (PDEs), as well as the fractional partial differential equations (FPDEs), have been the focus of many studies according to describe several physical phenomena and its applications in various fields of mathematics, physics, biology, and engineering [1]. Finding exact solutions of PDEs and FPDEs attracted the attention of many researchers. Several techniques in the research paper have been used to succeed in this aim.

Different methods have been presented to study PDE, for example the Kudryashov method [2], the modified auxiliary expansion method [3], the modified Laplace decomposition method [4], the $\left(m+\frac{G'}{G}\right)$ −expansion method [5, 6, 7], the Bernstein approximation method [8], the modified exponential function method [9], the Kansa's collocation method [10, 11], the mesh-free method [12], the Yang-Laplace transform [13], the multiple Exp-function method [14], the $\left(\frac{1}{G'}\right)$-expansion method [15, 16], the newly modified expansion function method [17], a simple Hirota's method [18], and radial basis functions collocation method [19].

Researchers have been utilized various method to seek the solution of Zoomeron partial differential equation, such as the enhanced $\left(\frac{G'}{G}\right)$-expansion method [20], the modified Kudryashov method [21], the exponential rational function method [22], the sub-equation together with generalized Kudryashov methods [23], the Ansatz method [24], the novel exponential rational function technique [25], the Lie point symmetries [26], the bifurcation method of a dynamical system, a numerical simulation method [27], the extended tanh, the exponential function, the sechp-tanhp function methods [28], the modified simple equation method, the Exp-function method [29], the first integral method [30].

The general comparison theorems for conformable fractional differential equations have been introduced and tested in Ref. [31]. In Ref. [32], retarded conformable fractional integral inequalities utilizing non-integer order derivatives and integrals have been proposed and tested. In Refs. [33, 34], some definitions and properties of conformable derivative have been presented.

In this article, we seek new exact solutions of fractional Zoomeron that are obtained by using the sine-Gordon expansion method and Bernoulli sub-equation. The 3D, 2D, and contour plot are presented in this paper. In the two-dimensional figure, the effect of the fractional derivative α is also illustrated.

2. The Conformable Fractional Derivative

The conformable derivative has the following definitions, properties and theorems:

Email addresses: **hajar.ismael@uoz.edu.krd (Hajar Farhan Ismael), hbulut@firat.edu.tr (Hasan Bulut)**

Definition 2.1. *Suppose that $f(t)$ be a conformable fractional derivative of order α and defines as $f:(0,\infty)\rightarrow\mathbb{R}$ then*

$$D_t^{\alpha} f(t) = \lim_{\varepsilon\to 0} \frac{f\left(t+\varepsilon t^{1-\alpha}\right)-f(t)}{\varepsilon}, \ \forall\, t>0,\, 0<\alpha\leq 1.$$

Definition 2.2. *Suppose that $f(t)$ be a function defined on $(a,t]$ and $\alpha\in\mathbb{R}$, then, the α-fractional integral of the function $f(t)$ can be stated as*

$$_{t}I_a^{\alpha} f(t) = \int_a^{\alpha} \frac{f(x)}{x^{1-\alpha}}dx, \text{ where } a\geq 0 \text{ and } t\geq a,$$

Providing that the Riemann improper integral exists.

Theorem 2.3. *Suppose that $f(t)$ and $g(t)$ be α-conformable differentiable at a point $t>0$, such that $\alpha\in(0,\,1]$, then*

1. $D_t^{\alpha}(a f(t)+b g(t)) = aD_t^{\alpha} f(t)+bD_t^{\alpha} g(t)$, *for all* $a,b\in\mathbb{R}$.
2. $D_t^{\alpha}(t^{\mu}) = \mu t^{\mu-\alpha}$, *for all* $\mu\in\mathbb{R}$.
3. $D_t^{\alpha}(f(t)g(t)) = g(t)D_t^{\alpha}(f(t))+f(t)D_t^{\alpha}(g(t))$.
4. $D_t^{\alpha}\left(\frac{f(t)}{g(t)}\right) = \frac{g(t)D_t^{\alpha}(f(t))-f(t)D_t^{\alpha}(g(t))}{g(t)^2}$.

In addition, if the function $f(t)$ is a differentiable function, then

$$D_t^{\alpha}(f(t)) = t^{1-\alpha}\frac{df(t)}{dt}.$$

The chain rule for conformable fractional derivatives is set out in the following theorem [35].

Theorem 2.4. *Suppose that $f(0,\infty)\rightarrow R$ be both a α-conformable differentiable function and classic differentiable function. Assume that $g(t)$ be a classic differentiable function defined in the range of $f(t)$, then*

$$D_t^{\alpha}(fog)(t) = t^{1-\alpha}g(t)^{\alpha-1}g_t(t)D_t^{\alpha}(f(t))_{t=g(t)}.$$

3. General Forms of the SGEM

Consider the sine-Gordon equation [36, 37, 38]

$$\phi_{xx}-\phi_{tt} = -\delta^2\sin(\phi), \tag{3.1}$$

where $\phi=\phi(x,t)$ and δ is a non-zero real number. Using the wave transform

$$\phi=\phi(x,\ t)=u(\xi),\ \ \xi=x-\upsilon t,$$

on eq. (3.1) and amplify the result, we get

$$u^{''} = \frac{\delta^2}{1-\upsilon^2}\ \sin(u), \tag{3.2}$$

where $u=u(\xi)$, ξ and υ are the amplitude and speed of the travelling wave. Taking integration of eq. (3.2) and simplifying it, one can get:

$$\left(\left(\frac{u}{2}\right)^{'}\right)^2 = \frac{\delta^2}{1-\upsilon^2}\ \sin^2\left(\frac{u}{2}\right)+C, \tag{3.3}$$

where C is the constant of integration. Letting that $\frac{u}{2}=w(\xi)$, $\frac{\delta^2}{1-\upsilon^2}=\beta^2$, and putting them into eq. (3.3), the result is:

$$w'(\xi) = \sqrt{\beta^2\ \sin^2(w(\xi))+C.} \tag{3.4}$$

Taking $C=0$ and $\beta=1$, Eq. (3.4) becomes:

$$w'(\xi) = \sin(w(\xi)). \tag{3.5}$$

Using the separation method, Eq. (3.5) possess the solutions as follow:

$$sin(w) = sin(w(\xi)) = \left.\frac{2pe^{\xi}}{p^2e^{2\xi}+1}\right|_{p=1} = sech(\xi), \tag{3.6}$$

$$\cos(w) = \cos(w(\xi)) = \left.\frac{p^2e^{\xi}-1}{p^2e^{2\xi}+1}\right|_{p=1} = tanh(\xi), \tag{3.7}$$

where P is the integral constant. These two significant solutions, achieve the definition of the sine-Gordon expansion method, to get the solution of the NPDE of the form:

$$P(\phi,\phi_x,\phi_t,\phi_{tt},\phi_{xx},\phi_{xt},\ldots) = 0. \tag{3.8}$$

Now, we consider

$$u(\xi)=\sum_{i=1}^{n}\tanh^{i-1}(\xi)\left[B_i\,\mathrm{sech}(w)\pm A_i\tanh(w)\right]+A_0, \tag{3.9}$$

Eq. (3.9) can be rewritten due to Eqs. (3.6) and (3.7) as follows:

$$u(w)=\sum_{i=1}^{n}\cos^{i-1}(w)\left[B_i\sin(w)+A_i\cos(w)\right]+A_0, \tag{3.10}$$

the value of n is obtained by balancing between the highest power of nonlinear term and the highest derivative appear in the transformed NODE. Using eq. (3.10) and its derivatives along with the transformed NODE, we get a polynomial equation in the power of trigonometric functions "$w'^s sin^k(w)\, cos^j(w),\ \ (s=0,1 \text{ and } k,j=0,1,2,...)''$. After creating a few substitutions for trigonometric identities in the polynomial equation, as a result, we get a set of algebraic equations by equating each summation of the coefficients of the trigonometric functions of the same power to zero. Simplifying the set of algebraic equations and evaluates the rate of the parameters A_j, B_j and υ. Using the obtained value of A_j, B_j and υ into eq. (3.9), we get the new form of wave solutions to eq. (3.8).

4. Structures of Bernoulli Sub-Equation Function Method

The mainly modified steps of this technique can be taken as follows [39, 40, 41, 42]
Let we have a nonlinear partial differential equation:

$$P(u_x,u_t,u_{xt},u_{xx},...)=0, \tag{4.1}$$

and defining the traveling wave transformation

$$u(x,t)=q(\eta),\eta=x+\gamma t, \tag{4.2}$$

where $\gamma\neq 0$. Appliying Eq. (4.2) on Eq. (4.1) as a result, we get a nonlinear ordinary differential equation :

$$N(q,q^{'},q^{''},...)=0. \tag{4.3}$$

Using a trial equation of solution as follows:

$$q(\eta)=\sum_{i=0}^{n}a_iF^i=a_0+a_1F+a_2F^2+...+a_nF^n, \tag{4.4}$$

and

$$F^{'}=bF+dF^M,b\neq 0,d\neq 0,M\in R-\{0,1,2\}. \tag{4.5}$$

here $F(\eta)$ is Bernoulli differential polynomial. Inserting Eq. (4.4) into Eq.(4.3), as well as using Eq. (4.5) produces:

$$\Omega(F(\eta))=b_kF(\eta)^s+\cdots+b_1F(\eta)+b_0=0,$$

via the balance principle,the connection of n and M will be evaluate.
By taking all the coefficients of $\Omega(F(\eta))$ to be zero, we get an algebraic equations system:

$$b_i=0,\ \ i=0,\cdots,k, \tag{4.6}$$

solving Eq. (4.6), we will find the values of $a_0,a_1,...,a_n$.
Step 4. Solving Bernoulli Eq. (4.4), two cases are observed depending on the values of b and d:

$$F(\eta)=\left[\frac{-d}{b}+\frac{E}{e^{b(M-1)\eta}}\right]^{\frac{1}{1-M}},b\neq d, \tag{4.7}$$

$$F(\eta)=\left[\frac{(E-1)+(E+1)\tanh\left(\frac{b(1-M)\eta}{2}\right)}{1-\tanh\left(\frac{b(1-M)\eta}{2}\right)}\right]^{\frac{1}{1-M}},b=d,E\in R. \tag{4.8}$$

Where E is the non-zero constant of integration, with the help of Mathematical packages, we gain the solutions to Eq. (4.3), using a complete polynomial discrimination system.

5. Fractional Zoomeron Equation

Boomerons are described as accelerated solitons for special integrable systems of coupled wave equations. The nonlinear Zoomeron equation is one of special cases of Boomeron equation that is also a scalar nonlinear evolution equation. the phenomenon of a soliton which is a part of wave with constant shape and speed. The (2+1)-dimensional nonlinear Zoomeron equation at time fractional, can be written as:

$$D_{tt}{}^{2\alpha}\left(\frac{u_{xy}}{u}\right)-\left(\frac{u_{xy}}{u}\right)_{xx}+2D_t{}^{\alpha}\left(u^2\right)_x=0,\qquad 0<\alpha\leq 1. \tag{5.1}$$

Using the wave transformation

$$u(x,y,t)=U(\xi),\ \xi=\xi=\kappa x+\nu y-\omega\frac{t^{\alpha}}{\alpha},$$

we get the nonlinear ordinary differential equation:

$$\kappa\nu\omega^2\left(\frac{U''}{U}\right)''-\kappa^3\nu\left(\frac{U''}{U}\right)''-2\kappa\omega\left(U^2\right)''=0.$$

Now integrating the above equation two times with respect to ξ, we get

$$\kappa\nu\left(\omega^2-\kappa^2\right)U''-2\kappa\omega U^3+\varepsilon U=0, \tag{5.2}$$

where ε is the constant of integration. To find the solutions of Eq. (5.1), we implement our two mentioned methods to Eq. (5.2).

6. Results and Discussion

The soliton solutions of fractional nonlinear partial Zoomeron equation by using two methods, the sine-Gordon expansion method, and Bernoulli sub-equation are studied. The 3D figures and 2D figures with a different value of a fractional parameter that are drawn. The gained solutions via the sine-Gordon expansion method are shown as below:
Set 1 when $A_0=0, A_1=\frac{\mathrm{i}\sqrt{\varepsilon}\nu^{1/4}}{\sqrt{2}\kappa^{1/4}(2\varepsilon+\kappa^3\nu)^{1/4}}, B_1=\frac{\sqrt{\varepsilon}\nu^{1/4}}{\sqrt{2}\kappa^{1/4}(2\varepsilon+\kappa^3\nu)^{1/4}}, \omega=-\frac{\sqrt{2\varepsilon+\kappa^3\nu}}{\sqrt{\kappa}\sqrt{\nu}}$, we get

$$u_1(x,t)=\frac{\sqrt{\varepsilon}\nu^{1/4}\left(sech\left(x\kappa+y\nu-\frac{t^{\alpha}\omega}{\alpha}\right)+\mathrm{i}\tanh\left(x\kappa+y\nu-\frac{t^{\alpha}\omega}{\alpha}\right)\right)}{\sqrt{2}\kappa^{1/4}\left(2\varepsilon+\kappa^3\nu\right)^{1/4}}. \tag{6.1}$$

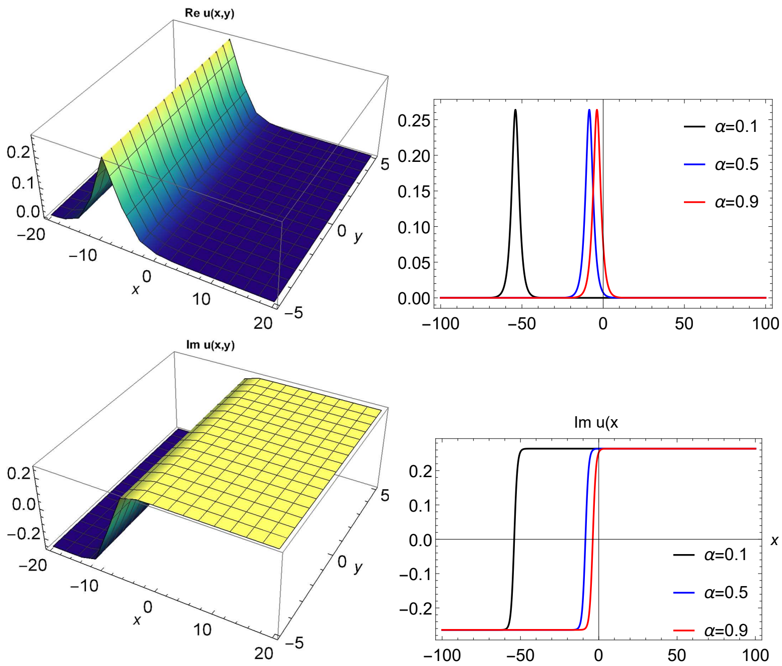

Figure 6.1: 3D and 2D figures of equation (6.1) drawn when $\varepsilon=1$, $\kappa=0.9$, $\nu=0.1$, $\alpha=0.5$, $t=1/2$ and for 2D, we choose $y=2$ with different values of α.

Set 2. When $A_0=0, A_1=\frac{\sqrt{\varepsilon}}{\sqrt{2}\sqrt{\kappa}\sqrt{\omega}}, B_1=\frac{\mathrm{i}\sqrt{\varepsilon}}{\sqrt{2}\sqrt{\kappa}\sqrt{\omega}}, \nu=-\frac{2\varepsilon}{\kappa^3-\kappa\omega^2}$, we have

$$u_2(x,y)=\frac{\sqrt{\varepsilon}\left(\mathrm{i}\,sech\left(x\kappa+y\nu-\frac{t^{\alpha}\omega}{\alpha}\right)+\tanh\left(x\kappa+y\nu-\frac{t^{\alpha}\omega}{\alpha}\right)\right)}{\sqrt{2}\sqrt{\kappa}\sqrt{\omega}}. \tag{6.2}$$

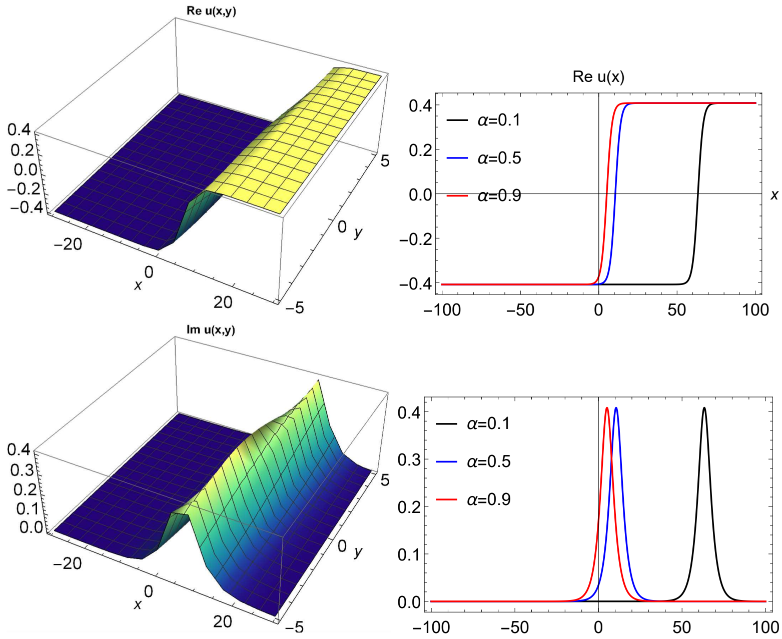

Figure 6.2: 3D and 2D graph of equation (6.2) drawn when $\varepsilon = 1$, $\kappa = 0.9$, $\omega = 0.1$, $\alpha = 0.5$, $t = 1/2$ and for 2D, we choose $y = 2$ with different values of α.

Set 3. When $A_0 = 0, A_1 = \frac{\sqrt{\varepsilon}}{\sqrt{2}\sqrt{\kappa}\sqrt{\omega}}, B_1 = \frac{\mathrm{i}\sqrt{\varepsilon}}{\sqrt{2}\sqrt{\kappa}\sqrt{\omega}}, \nu = -\frac{2\varepsilon}{\kappa^3-\kappa\omega^2}$, we get

$$u_3(x,y) = \frac{\sqrt{\varepsilon}\left(\mathrm{i}\,sech\left(x\kappa+y\nu-\frac{t^\alpha\omega}{\alpha}\right)+\tanh\left(x\kappa+y\nu-\frac{t^\alpha\omega}{\alpha}\right)\right)}{\sqrt{2}\sqrt{\kappa}\sqrt{\omega}}. \tag{6.3}$$

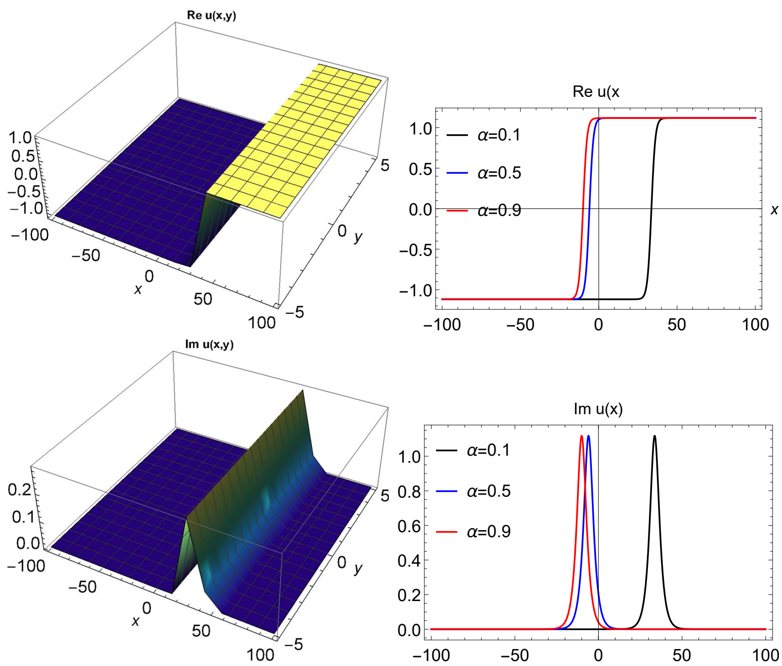

Figure 6.3: 3D and 2D graph of Eq. (6.3) drawn when $\varepsilon = 2, \omega = 2, \kappa = 0.4, \alpha = 0.1, t = 1/2$ and for 2D, we choose $y = 2$ with different values of α.

Set 4. When$A_1 = \frac{\sqrt{12\nu\omega^2 - \frac{2\,6^{1/3}\nu^3\omega^4}{\left(-9\varepsilon\nu^2+\sqrt{81\varepsilon^2\nu^4-48\nu^6\omega^6}\right)^{2/3}} - \frac{6^{2/3}\left(-9\varepsilon\nu^2+\sqrt{81\varepsilon^2\nu^4-48\nu^6\omega^6}\right)^{2/3}}{\nu}}}{6\sqrt{\omega}}, A_0 = B_1 = 0, \kappa = \frac{2\,6^{1/3}\nu^2\omega^2+\left(-9\varepsilon\nu^2+\sqrt{81\varepsilon^2\nu^4-48\nu^6\omega^6}\right)^{2/3}}{6^{2/3}\nu\left(-9\varepsilon\nu^2+\sqrt{81\varepsilon^2\nu^4-48\nu^6\omega^6}\right)^{1/3}}$, we have

$$u_4(x,y) = A_1\tanh\left(x\kappa+y\nu-\frac{t^\alpha\omega}{\alpha}\right). \tag{6.4}$$

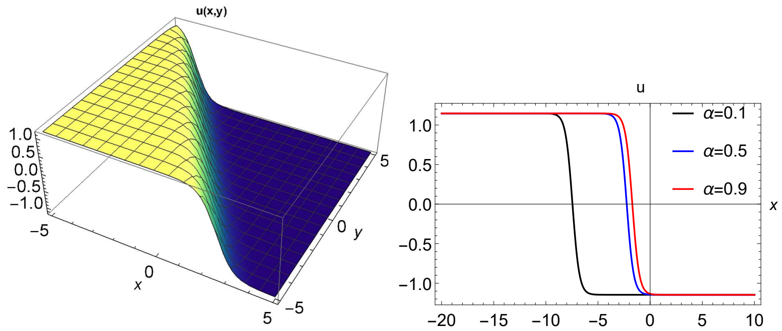

Figure 6.4: 3D and 2D graph of Eq. (6.4) drawn when$\varepsilon = -4, \omega = 1, \nu = -1, \alpha = 0.6, t = 1/2$ and for 2D, we choose $y = 2$ with a different value of α.

Set 5. When $\mathrm{A}_0 = 0, \mathrm{A}_1 = 0, \mathrm{B}_1 = \frac{\sqrt{\nu}\sqrt{(\kappa-\omega)(\kappa+\omega)}}{\sqrt{\omega}}, \varepsilon = \kappa\nu(\kappa-\omega)(\kappa+\omega)$, we get

$$u_5(x,y) = \frac{\sqrt{\nu}\sqrt{(\kappa-\omega)(\kappa+\omega)}Sech\left[x\kappa + y\nu - \frac{t^\alpha \omega}{\alpha}\right]}{\sqrt{\omega}}. \tag{6.5}$$

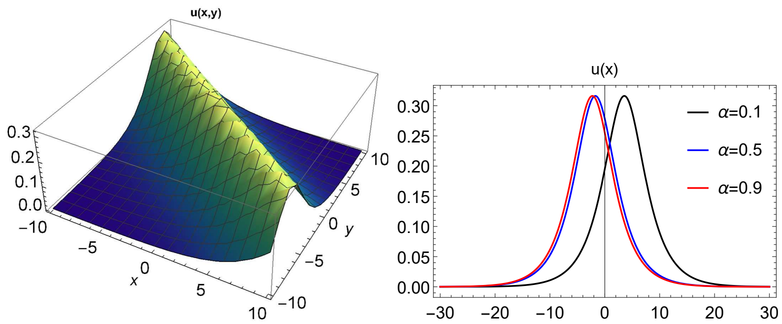

Figure 6.5: 3D and 2D graph of Eq. (6.5) drawn when $\omega = 0.2, \kappa = 0.3, \nu = 0.4, \alpha = 0.5, t = 1/2$ and for 2D, we choose$y = 2$ with different values of α.

Set 6. When $A_0 = 0, A_1 = -\frac{\sqrt{\nu(-\kappa^2+\omega^2)}}{2\sqrt{\omega}}, B_1 = \frac{\mathrm{i}\sqrt{\nu}\sqrt{-\kappa^2+\omega^2}}{2\sqrt{\omega}}, \varepsilon = \frac{1}{2}\kappa\nu\left(-\kappa^2+\omega^2\right)$, we get

$$u_6(x,y) = \frac{\mathrm{i}\left(\sqrt{\nu}\sqrt{-\kappa^2+\omega^2}\,sech\left(x\kappa + y\nu - \frac{t^\alpha\omega}{\alpha}\right) + \mathrm{i}\sqrt{\nu\left(-\kappa^2+\omega^2\right)}\tanh\left(x\kappa + y\nu - \frac{t^\alpha\omega}{\alpha}\right)\right)}{2\sqrt{\omega}}. \tag{6.6}$$

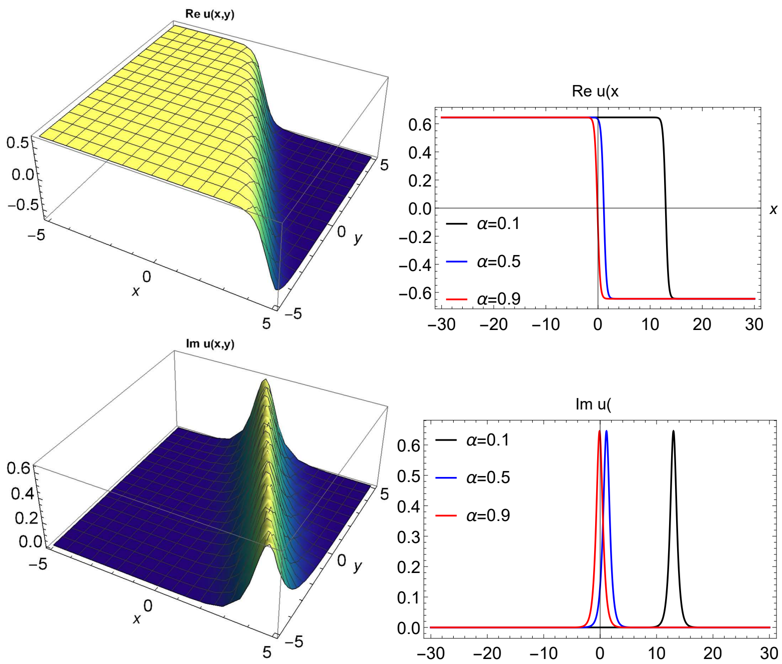

Figure 6.6: 3D and 2D graph of Eq. (6.6) drawn when $\omega = 3, \kappa = 2, \nu = 1, \alpha = 0.5, t = 1/2$ and for 2D, we choose $y = 2$ with different values of α.

The 3D figures and contour plot for some cases obtained by Bernoulli sub-equation are showed as:

Set 7. When $a_0=\frac{\sqrt{\varepsilon}\sqrt{\nu}\sqrt{\omega}}{\sqrt{2}\sqrt{\frac{\omega^2}{-\kappa^2+\omega^2}}\sqrt{\kappa\nu(-\kappa^2+\omega^2)}}, a_1=0, a_2=-\frac{2d\sqrt{\nu}\sqrt{\omega}}{\sqrt{\frac{\omega^2}{-\kappa^2+\omega^2}}}$, $b=-\frac{\sqrt{\varepsilon}}{\sqrt{2}\sqrt{\kappa\nu(-\kappa^2+\omega^2)}}$,

$$u_7(x,y)=-\frac{2d\sqrt{\nu}\sqrt{\omega}}{\left(-\frac{d}{b}+ce^{-2b\left(x\kappa+y\nu-\frac{t^\alpha\omega}{\alpha}\right)}\right)\sqrt{\frac{\omega^2}{-\kappa^2+\omega^2}}}+\frac{\sqrt{\varepsilon}\sqrt{\nu}\sqrt{\omega}}{\sqrt{2}\sqrt{\frac{\omega^2}{-\kappa^2+\omega^2}}\sqrt{\kappa\nu\left(-\kappa^2+\omega^2\right)}}. \tag{6.7}$$

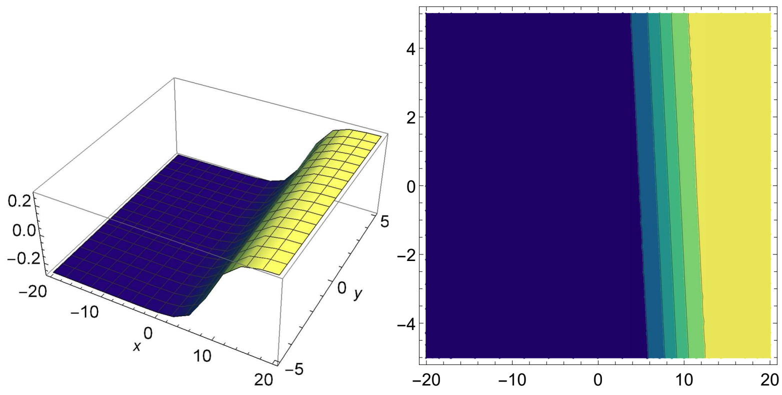

Figure 6.7: 3D graph and contour plot of Eq. (6.7) drawn when $\varepsilon=0.2, \kappa=0.5, \nu=1, \alpha=0.9, t=1/2, c=0.2, \omega=0.9, a_2=1, d=1, b=-2$.

Set 8. When $a_0=\frac{\sqrt{\varepsilon}\sqrt{\nu(-\kappa^2+\omega^2)}}{\sqrt{2}\sqrt{\omega}\sqrt{\kappa\nu(-\kappa^2+\omega^2)}}, a_1=0, b=\frac{\sqrt{\varepsilon}}{\sqrt{2}\sqrt{\kappa\nu(-\kappa^2+\omega^2)}}, d=\frac{a2\sqrt{\omega}}{2\sqrt{\nu(-\kappa^2+\omega^2)}}$, we set up

$$u_8(x,y)=\frac{a_2}{-\frac{d}{b}+ce^{-2b\left(x\kappa+y\nu-\frac{t^\alpha\omega}{\alpha}\right)}}+\frac{\sqrt{\varepsilon}\sqrt{\nu\left(-\kappa^2+\omega^2\right)}}{\sqrt{2}\sqrt{\omega}\sqrt{\kappa\nu\left(-\kappa^2+\omega^2\right)}}. \tag{6.8}$$

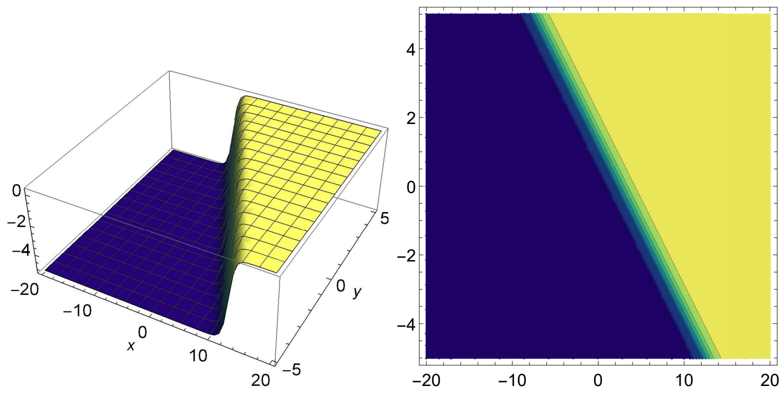

Figure 6.8: 3D graph and contour plot of Eq. (6.8) drawn when $\varepsilon=0.2, \kappa=0.5, \nu=2, \alpha=0.5, t=1/2, c=0.2, \omega=2, a_2=1, d=1, b=-1$.

Set 9. When $a_0=-\frac{b\sqrt{d^2\nu(-\kappa^2+\omega^2)}}{d\sqrt{\omega}}, a_1=0, a_2=-\frac{2\sqrt{d^2\nu(-\kappa^2+\omega^2)}}{\sqrt{\omega}}, \varepsilon=2b^2\kappa\nu\left(-\kappa^2+\omega^2\right)$, we have

$$u_9(x,y)=-\frac{b\sqrt{d^2\nu\left(-\kappa^2+\omega^2\right)}}{d\sqrt{\omega}}-\frac{2\sqrt{d^2\nu\left(-\kappa^2+\omega^2\right)}}{\left(-\frac{d}{b}+ce^{-2b\left(x\kappa+y\nu-\frac{t^\alpha\omega}{\alpha}\right)}\right)\sqrt{\omega}}. \tag{6.9}$$

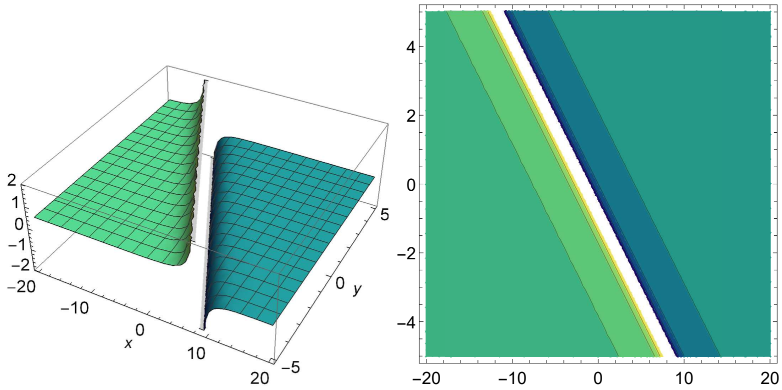

Figure 6.9: 3D graph and contour plot of Eq. (6.9) drawn when $\varepsilon = 2, \kappa = 0.5, \nu = -1, \alpha = 0.5, t = 1/2, c = 0.2, \omega = 0.2, a_2 = -1, d = 1, b = 2$.

7. Conclusion

In this paper, some novel soliton solutions of nonlinear time-fractional Zoomeron equation are studied. The sine-Gordon and Bernoulli sub-equation methods with using the Mathematica program are utilized to get some new solution of the partial Zoomeron equation. Bright, kink-type, singular, and complex solutions to the studied equations are presented. As a result, the figures in three dimensions, two dimensions but with different values of the fractional parameter α, as well as the contour plot for some cases are introduced. We are confident that the solutions will be useful for scientists to work in this area. All the solutions obtained in this paper are verified by putting them back to original PDE and all of them satisfy the PDE.

References

[1] E.M.E. Zayed, Y.A. Amer, The First Integral Method and its Application for Deriving the Exact Solutions of a Higher-Order Dispersive Cubic-Quintic Nonlinear Schrödinger Equation, Comput. Math. Model. 27 (2016), 80-94. https://doi.org/10.1007/s10598-015-9305-y.
[2] M. Eslami, Exact traveling wave solutions to the fractional coupled nonlinear Schrodinger equations, Appl. Math. Comput. 285 (2016), 141–148.
[3] W.G. and H.F.I. and H.B. and H.M. Baskonus, Instability modulation for the (2+1)-dimension paraxial wave equation and its new optical soliton solutions in Kerr media, Phys. Scr. 95 (2019), 035207. http://iopscience.iop.org/10.1088/1402-4896/ab4a50.
[4] M. Yavuz, T.A. Sulaiman, F. Usta, H. Bulut, Analysis and numerical computations of the fractional regularized long-wave equation with damping term, Math. Methods Appl. Sci. (2020).
[5] H.F. Ismael, H. Bulut, H.M. Baskonus, Optical soliton solutions to the Fokas–Lenells equation via sine-Gordon expansion method and $(m+(G'/G))$ -expansion method, Pramana - J. Phys. 94 (2020), 35. https://doi.org/10.1007/s12043-019-1897-x.
[6] W. Gao, H.F. Ismael, A.M. Husien, H. Bulut, H.M. Baskonus, Optical Soliton Solutions of the Cubic-Quartic Nonlinear Schrödinger and Resonant Nonlinear Schrödinger Equation with the Parabolic Law, Appl. Sci. 10 (2019), 219. https://doi.org/10.3390/app10010219.
[7] H.F. Ismael, H.M. Baskonus, H. Bulut, Abundant novel solutions of the conformable Lakshmanan-Porsezian-Daniel model, Discret. Contin. Dyn. Syst. - S. 0 (n.d.) 0.https://doi.org/10.3934/dcdss.2020398.
[8] F. Usta, Numerical analysis of fractional Volterra integral equations via Bernstein approximation method, J. Comput. Appl. Math. 384 (2021), 113198.
[9] W. Gao, H.F. Ismael, S.A. Mohammed, H.M. Baskonus, H. Bulut, Complex and real optical soliton properties of the paraxial nonlinear Schrödinger equation in Kerr media with M-fractional, Front. Phys. 7 (2019), 197.
[10] F. Usta, Fractional type Poisson equations by radial basis functions Kansa approach, J. Inequalities Spec. Funct. 7 (2016), 143–149.
[11] F. Usta, Numerical solution of fractional elliptic PDE's by the collocation method, Appl. Appl. Math. An Int. J. 12 (2017), 470–478.
[12] F. Usta, A mesh-free technique of numerical solution of newly defined conformable differential equations, Konuralp J. Math. 4 (2016), 149–157.
[13] F. Usta, H. Budak, M. Sarikaya, Yang-Laplace transform method Volterra and Abel's integro-differential equations of fractional order, Int. J. Nonlinear Anal. Appl. 9 (2018), 203–214.
[14] P. Wan, J. Manafian, H.F. Ismael, S.A. Mohammed, Investigating One-, Two-, and Triple-Wave Solutions via Multiple Exp-Function Method Arising in Engineering Sciences, Adv. Math. Phys. 2020 (2020), 8018064. https://doi.org/10.1155/2020/8018064.
[15] K.K. Ali, A.R. Seadawy, A. Yokus, R. Yilmazer, H. Bulut, Propagation of dispersive wave solutions for (3+ 1)-dimensional nonlinear modified Zakharov–Kuznetsov equation in plasma physics, Int. J. Mod. Phys. B. 34 (2020), 2050227.
[16] K.K. Ali, H. Dutta, R. Yilmazer, S. Noeiaghdam, On the new wave behaviors of the Gilson-Pickering equation, Front. Phys. 8 (2020), 54.
[17] H.F. Ismael, H. Bulut, H.M. Baskonus, W. Gao, Newly modified method and its application to the coupled Boussinesq equation in ocean engineering with its linear stability analysis, Commun. Theor. Phys. 72 (2020), 115002. https://doi.org/10.1088/1572-9494/aba25f.
[18] H.F. Ismael, H. Bulut, C. Park, M.S. Osman, M-lump, N-soliton solutions, and the collision phenomena for the (2+1)-dimensional Date-Jimbo-Kashiwara-Miwa equation, Results Phys. 19 (2020), 103329. https://doi.org/10.1016/j.rinp.2020.103329.
[19] F. Usta, A conformable calculus of radial basis functions and its applications, An Int. J. Optim. Control Theor. Appl. 8 (2018), 176–182.
[20] K. Khan, M. Ali Akbar, M. Abdus Salam, M. Hamidul Islam, A note on enhanced (G'/G)-expansion method in nonlinear physics, Ain Shams Eng. J. 5 (2014), 877-884. https://doi.org/10.1016/j.asej.2013.12.013.
[21] K. Hosseini, A. Korkmaz, A. Bekir, F. Samadani, A. Zabihi, M. Topsakal, New wave form solutions of nonlinear conformable time-fractional Zoomeron equation in (2 + 1)-dimensions, Waves in Random and Complex Media. (2019), 1-11. https://doi.org/10.1080/17455030.2019.1579393.
[22] N. Ahmed, S. Bibi, U. Khan, S.T. Mohyud-Din, A new modification in the exponential rational function method for nonlinear fractional differential equations, Eur. Phys. J. Plus. 133 (2018), 1-11. https://doi.org/10.1140/epjp/i2018-11896-0.
[23] E. Aksoy, A.C. Çevikel, A. Bekir, Soliton solutions of (2+1)-dimensional time-fractional Zoomeron equation, Optik, 127 (2016), 6933-6942. https://doi.org/10.1016/j.ijleo.2016.04.122.
[24] O. Guner, A. Bekir, Bright and dark soliton solutions for some nonlinear fractional differential equations, Chinese Phys. B. 25 (2016), 030203. https://doi.org/10.1088/1674-1056/25/3/030203.
[25] D. Kumar, M. Kaplan, New analytical solutions of (2+1)-dimensional conformable time fractional Zoomeron equation via two distinct techniques, Chinese J. Phys. 56 (2018), 2173-2185. https://doi.org/10.1016/j.cjph.2018.09.013.
[26] T. Motsepa, C. Khalique, M. Gandarias, Symmetry Analysis and Conservation Laws of the Zoomeron Equation, Symmetry, 9 (2017), 27. https://doi.org/10.3390/sym9020027.
[27] Y. Zhou, S. Cai, Q. Liu, Bounded Traveling Waves of the (2+1)-Dimensional Zoomeron Equation, Math. Probl. Eng. 2015 (2015), 163597. https://doi.org/10.1155/2015/163597.
[28] M. Alquran, K. Al-Khaled, Mathematical methods for a reliable treatment of the (2+1)-dimensional Zoomeron equation, Math. Sci. 6 (2013), 11. https://doi.org/10.1186/2251-7456-6-11.
[29] K. Khan, M. Ali Akbar, Traveling wave solutions of the (2 + 1)-dimensional Zoomeron equation and the Burgers equations via the MSE method and the Exp-function method, Ain Shams Eng. J. 5 (2014), 247-256. https://doi.org/10.1016/j.asej.2013.07.007.

[30] A. Bekir, F. Taşcan, Ö. Ünsal, Exact solutions of the Zoomeron and Klein-Gordon-Zakharov equations, J. Assoc. Arab Univ. Basic Appl. Sci. 17 (2015), 1-5. https://doi.org/10.1016/j.jaubas.2013.12.002.
[31] M.Z. Sarikaya, F. Usta, On comparison theorems for conformable fractional differential equations, Int. J. Anal. Appl. 12 (2016), 207–214.
[32] F. Usta, M.Z. Sarıkaya, The analytical solution of Van der Pol and Lienard differential equations within conformable fractional operator by retarded integral inequalities, Demonstr. Math. 52 (2019), 204–212.
[33] R. Khalil, M. Al Horani, A. Yousef, M. Sababheh, A new definition of fractional derivative, J. Comput. Appl. Math. 264 (2014), 65–70.
[34] S.G. Samko, A.A. Kilbas, O.I. Marichev, Fractional integrals and derivatives: Theory and applications, 1993, Gordan and Breach, Amsterdam. (1993).
[35] Z. Zheng, W. Zhao and H. Dai, A new definition of fractional derivative, Int. J. Non. Linear. Mech., 108 (2019), 1-6.
[36] H.M. Baskonus, H. Bulut, New complex exact travelling wave solutions for the generalized-Zakharov equation with complex structures, An Int. J. Optim. Control Theor. Appl. 6 (2016), 141-150. https://doi.org/10.11121/ijocta.01.2016.00295.
[37] K.K. Ali, R. Yilmazer, H.M. Baskonus, H. Bulut, New wave behaviors and stability analysis of the Gilson–Pickering equation in plasma physics, Indian J. Phys. 94 (2020), 1-6. https://doi.org/10.1007/s12648-020-01773-9.
[38] K.K. Ali, R. Yilmazer, H. Bulut, Analytical Solutions to the Coupled Boussinesq–Burgers Equations via Sine-Gordon Expansion Method, in: 4th Int. Conf. Comput. Math. Eng. Sci., Springer Nature, n.d.: p. 233.
[39] H.M. Baskonus, H. Bulut, On the complex structures of Kundu-Eckhaus equation via improved Bernoulli sub-equation function method, Waves in Random and Complex Media. 25 (2015), 720-728. https://doi.org/10.1080/17455030.2015.1080392.
[40] H.H. Abdulkareem, H.F. Ismael, E.S. Panakhov, H. Bulut, Some Novel Solutions of the Coupled Whitham-Broer-Kaup Equations, in: Int. Conf. Comput. Math. Eng. Sci., Springer, 2019: pp. 200–208.
[41] H.F. Ismael, H. Bulut, On the Solitary Wave Solutions to the (2+ 1)-Dimensional Davey-Stewartson Equations, in: Int. Conf. Comput. Math. Eng. Sci., Springer, 2019: pp. 156–165.
[42] K.K. Ali, R. Yilmazer, H.M. Baskonus, H. Bulut, Modulation instability analysis and analytical solutions to the system of equations for the ion sound and Langmuir waves, Phys. Scr. 95 (2020), 065602.