Sakarya University Journal of Science

ISSN 1301-4048 | e-ISSN 2147-835X | Period Bimonthly | Founded: 1997 | Publisher Sakarya University | http://www.saujs.sakarya.edu.tr/

Title: Estimations For The Odd Weibull Distribution under Progressive Type-II Right Censored Samples

Authors: Gülcan Gencer, Kerem Gencer

Recieved: 2019-01-03 09:16:15

Accepted: 2019-09-20 09:53:04

Article Type: Research Article
Volume: 24
Issue: 1
Month: February
Year: 2020
Pages: 29-36

How to cite
Gülcan Gencer, Kerem Gencer; (2020), Estimations For The Odd Weibull Distribution under Progressive Type-II Right Censored Samples. Sakarya University Journal of Science, 24(1), 29-36, DOI: 10.16984/saufenbilder.507300
Access link
http://www.saujs.sakarya.edu.tr/tr/issue/49430//507300

Sakarya University Journal of Science 24(1), 29-36, 2020

Estimations For The Odd Weibull Distribution under Progressive Type-II Right Censored Samples

Gülcan Gencer[1*], Kerem Gencer[2]

Abstract

In this study, We introduced performances Maximum Likelihood (ML) and Bayes estimation under LINEX, GENTROPY and SQUARED loss functions results concerning a progressively type-II censored samples for parameters of Odd Weibull distribution. We used obtain the Tierney Kadane's approximation to obtain Bayesian estimates. The Mean Squared Error (MSE)s of MLEs and (MSE)s of Bayes estimates under LINEX, GENTROPY and SQUARED loss functions for unknown parameters are computed using Monte Carlo simulation.

Key words: Bayes estimator, Odd Weibull distribution, Loss Functions, Maximum likelihood estimation, Progressively type-II censoring, Tierney-Kadane's approximation.

1.INTRODUCTION

OddW distribution based on the opinion of appraising the distribution of the 'odds of death' of a lifetime variable has been used life time models. To read in literature about this distribution, the readers may look are Cooray [1], Cooray [2] and Jiang et al. [3] Many authors studied about estimation for some distributions under progressive type II right censoring. For some of this studies Balakrishnan et al. [4], Wu [5], Wu et al. [6], Balakrishnan and Aggarwala [7], Balakrishnan [8]. Generalized Weibull distribution is one of the distributions used in the literature to estimate the parameter. A few of study are Lai [9], Mudholkar and Srivastava [10], Gencer and Saraçoğlu [11], Nadarajah et.al. [12], Salem and Abo-Kasem [13], Nassar and Eissa [14], Edwin et al., [15], Abdelal [16], Korkmaz et.al. [17] and Alizadeh et al. [18]. This distribution is only one of the generalized distribution has been presented by Cooray [19]. Odd Weibull (OddW) distribution with λ,θ and β parameters is given with $\mathrm{OddW}(\lambda,\theta,\beta)$.The cumulative distribution function (cdf), probability density function (pdf), hazard function of a X random variable having $\mathrm{OddW}(\lambda,\theta,\beta)$ are as,

$$f(x;\lambda,\theta,\beta)=\left(\frac{\beta\theta}{x}\right)\left(\frac{x}{\lambda}\right)^{\lambda} e^{\left(\frac{x}{\lambda}\right)^{\beta}}\left(e^{\left(\frac{x}{\lambda}\right)^{\beta}}-1\right)^{\theta-1}\left(1+\left(e^{\left(\frac{x}{\lambda}\right)^{\beta}}-1\right)^{\theta}\right)^{-2} \quad (1)$$

[1*]Corresponding author:gulcangencer@kmu.edu.tr.
[1] Selcuk University, Department of Statistics, Konya, TURKEY, ORCID: 0000-0002-3543-041X,
[2] Karamanoglu Mehmetbey University, Department of Computer Programming, , Karaman, TURKEY, ORCID: 0000-0002-2914-1056

$$F(x;\lambda,\theta,\beta)=1-(1+(e^{(x/\lambda)^{\beta}}-1)^{\theta})^{-1}$$
$$,x>0,\ \beta\theta>0,\ \lambda>0 \tag{2}$$

$$h(x;\lambda,\theta,\beta)=\left(\frac{\beta\theta}{x}\right)\left(\frac{x}{\lambda}\right)^{\beta}e^{\left(\frac{x}{\lambda}\right)^{\beta}}\left(e^{\left(\frac{x}{\lambda}\right)^{\beta}}-1\right)^{\theta-1}\left[1+\left(e^{\left(\frac{x}{\lambda}\right)^{\beta}}-1\right)^{\theta}\right]^{-1} \tag{3}$$

Type-I and type-II censoring are favorite censoring schemes. Progressive censoring scheme presents the observer to remove active units during the test. A generalization of classic type-II right censoring is referred to as progressive type II right censoring.

And note that the classic Type-II right censoring is a particular status of the progressive Type-II right censoring, and it can be attained by using $R=(0,0,...,n-m)$.

In this process, $X^{R}{}_{1:m:n},X^{R}{}_{2:m:n},...,X^{R}{}_{m:m:n}$ are refer to as progressive Type-II right censored order statistics with $R=(R_1,R_2,...,R_m)$ scheme.

The joint pdf of these statistics is given;

$$f_{X^R_{1:m:n},X^R_{2:m:n},...,X^R_{m:m:n}}(x_1,x_2,...,x_m)=K\prod_{i=1}^{m}f(x_i)\left[1-F(x_i)\right]^{R_i},$$
$$-\infty<x_1\le x_2...\le x_m<\infty \tag{4}$$

Here,

$$K=n(n-R_1-1)\times...\times(n-R_1-R_2-...-R_{m-1}-m+1)$$

For more elaborations see Balakrishnan [8] and Balakrishnan and Aggarwala [7].

The purpose of this article is to compare the Mean Squared Error (MSE)s of MLEs and (MSE)s of Bayes estimates under LINEX, GENTROPY and SQUARED loss functions for unknown parameters of the OddW distribution under the progressive type II right censoring using Monte Carlo simulation method. The rest of the paper is organized as the maximum likelihood estimators, Bayesian estimators under Tierney Kadane' approximation parameters of the OddW distribution, These estimators are computed using Monte Carlo simulation and results is taken place.

2.MAXIMUM LIKELIHOOD ESTIMATION

Let $X^R=X^{R_1}_{1:m:n},X^{R_2}_{2:m:n},...,X^{R_m}_{m:m:n}$ with $X^{R_1}_{1:m:n}<X^{R_2}_{2:m:n}<...<X^{R_m}_{m:m:n}$ denote progressive type-II right cencored sample taken from OddW distribution with λ, θ and β parameters. Then the log- likelihood function is given by

$$\begin{aligned}\ell\left(\lambda,\theta,\beta\middle|x^R\right)&=\ln(K)+m\ln(\beta)+m\ln(\theta)\\&+\beta\left(\sum_{i=1}^{m}\ln\left(\frac{x_{i:m:n}}{\lambda}\right)\right)+\left(\sum_{i=1}^{m}\left(\frac{x_{i:m:n}}{\lambda}\right)^{\beta}\right)\\&-\left(\sum_{i=1}^{m}\ln\left(x_{i:m:n}\right)\right)+\\&(\theta-1)\left(\sum_{i=1}^{m}\ln\left(\left(e^{\left(\frac{x_{i:m:n}}{\lambda}\right)}\right)^{\beta}-1\right)\right)\\&-2\left(\sum_{i=1}^{m}\ln\left(1+\left(\left(e^{\left(\frac{x_{i:m:n}}{\lambda}\right)}\right)^{\beta}-1\right)^{\theta}\right)\right)\\&-\sum_{i=1}^{m}R_i\ln\left(1+\left(\left(e^{\left(\frac{x_{i:m:n}}{\lambda}\right)}\right)^{\beta}-1\right)^{\theta}\right)\end{aligned} \tag{5}$$

Differentiating the log-likelihood function $\ell\left(\lambda,\theta,\beta\middle|\underline{x}\right)$ partially in connection with λ, θ and β parameters and then equating to zero, following non-linear equations is attained and may be resolved by Newton-Raphson method.

$$\frac{\partial\ell(\lambda,\theta,\beta/x^R)}{\partial\lambda}=0\Rightarrow -\frac{\beta(\theta-1)}{\lambda}\sum_{i=1}^{m}\left(\frac{\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}e^{\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}}}{\left(e^{\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}}-1\right)}\right)$$

$$-\frac{m\beta}{\lambda}-\frac{\beta}{\lambda}\sum_{i=1}^{m}\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}+$$

$$+2\frac{\beta\theta}{\lambda}\sum_{i=1}^{m}\left(\frac{\left(e^{\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}}-1\right)^{\theta-1}\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}e^{\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}}}{\left(1+\left(e^{\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}}-1\right)^{\theta}\right)}\right)$$

$$+\frac{\beta\theta}{\lambda}\sum_{i=1}^{m}\left(\frac{R_i\left(e^{\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}}-1\right)^{\theta-1}\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}e^{\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}}}{\left(1+\left(e^{\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}}-1\right)^{\theta}\right)}\right) \quad (6)$$

$$\frac{\partial\ell(\lambda,\theta,\beta/x^R)}{\partial\theta}=0\Rightarrow\frac{m}{\theta}+\sum_{i=1}^{m}\ln\left(e^{\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}}-1\right)-2\sum_{i=1}^{m}\left(\frac{\left(e^{\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}}-1\right)^{\theta}\ln\left(e^{\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}}-1\right)}{1+\left(e^{\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}}-1\right)^{\theta}}\right) \quad (7)$$

$$-\sum_{i=1}^{m}\left(\frac{R_i\left(e^{\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}}-1\right)^{\theta}\ln\left(e^{\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}}-1\right)}{1+\left(e^{\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}}-1\right)^{\theta}}\right)$$

$$\frac{\partial\ell(\lambda,\theta,\beta/x^R)}{\beta}=0\Rightarrow\frac{m}{\beta}+(\theta-1)\sum_{i=1}^{m}\left(\frac{\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}\ln\left(\frac{x_{i:mn}}{\lambda}\right)e^{\left(\frac{x_{i:mn}}{\alpha}\right)^{\beta}}}{\left(e^{\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}}-1\right)}\right)$$

$$+\sum_{i=1}^{m}\ln\left(\frac{x_{i:mn}}{\lambda}\right)+\sum_{i=1}^{m}\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}\ln\left(\frac{x_{i:mn}}{\lambda}\right)-$$

$$-2\theta\sum_{i=1}^{m}\left(\frac{\left(e^{\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}}-1\right)^{\theta-1}\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}\ln\left(\frac{x_{i:mn}}{\lambda}\right)e^{\left(\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}\right)}}{\left(1+\left(e^{\left(\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}\right)}-1\right)^{\theta}\right)}\right)$$

$$-\theta\sum_{i=1}^{m}\left(\frac{R_i\left(e^{\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}}-1\right)^{\theta-1}\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}\ln\left(\frac{x_{i:mn}}{\lambda}\right)e^{\left(\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}\right)}}{\left(1+\left(e^{\left(\left(\frac{x_{i:mn}}{\lambda}\right)^{\beta}\right)}-1\right)^{\theta}\right)}\right)=0 \quad (8)$$

3.APPROXIMATE BAYESIAN ESTIMATION

Let $X^R = X^{R_1}_{1:m:n}, X^{R_2}_{2:m:n}, \ldots, X^{R_m}_{m:m:n}$ with $X^{R_1}_{1:m:n} < X^{R_2}_{2:m:n} < \ldots < X^{R_m}_{m:m:n}$ denote progressive type-II right cencored sample taken from OddW distribution. For Bayesian estimation of the parameters, it is needed to prior distributions for these parameters. It is accepted that the λ, θ and β parameters have the following independent prior densities;

$$\pi_1(\lambda)=\frac{\lambda^{d_1-1}e_1^{d_1}\exp(e_1\lambda)}{\Gamma(d_1)} \quad (9)$$

$$\pi_2(\theta)=\frac{\theta^{d_2-1}e_2^{d_2}\exp(e_2\theta)}{\Gamma(d_2)} \quad (10)$$

$$\pi_3(\beta)=\frac{\beta^{d_3-1}e_3^{d_3}\exp(e_3\beta)}{\Gamma(d_3)} \quad (11)$$

3.1.Bayesian Estimation under Tierney Kadane's Approximation for Different Loss Functions

Tierney and Kadane (TK) [20] is one of the methods to procured the approximate value of the mathematical explanations as the ratio of two integrals. This approximation can be written as follows for case with three parameter.

$$l(\lambda,\theta,\beta)=\frac{1}{m}\{\rho(\lambda,\theta,\beta)+\ell(\lambda,\theta,\beta)\} \quad (12)$$

$$l^*(\lambda,\theta,\beta)=\frac{1}{m}\log s(\lambda,\theta,\beta)+l(\lambda,\theta,\beta) \quad (13)$$

where $s(\lambda,\theta,\beta)$ is any function of λ, θ and β, $\ell\left(\lambda,\theta,\beta\middle|x^R\right)$ is defined in Eq., (5), $\rho(\lambda,\theta,\beta)$ is logarithm joint prior distribution and defined as follows.

$$\rho(\lambda,\theta,\beta)=\ln\left(\begin{array}{c}\frac{e_1^{d_1}e_2^{d_2}e_3^{d_3}}{\Gamma(d_1)\Gamma(d_2)\Gamma(d_3)}\lambda^{d_1-1}\\ \exp(e_1\lambda)\theta^{d_2-1}\exp(e_2\theta)\beta^{d_3-1}\exp(e_3\beta)\end{array}\right) \quad (14)$$

Approximate Bayes estimators of $s(\lambda,\theta,\beta)$ under different loss functions for OddW distribution is obtained as follows; TK Bayes estimator of $s(\lambda,\theta,\beta)$ under LINEX loss function is defined as by,

$$\hat{s}_{LINEX}(\lambda,\theta,\beta) = -\frac{1}{t}\ln E\left[\exp\left(-ts(\lambda,\theta,\beta)\right)\middle|x^R\right]$$
$$= -\frac{1}{t}\ln\left[\left(\frac{\det\Sigma^*_{LINEX}}{\det\Sigma}\right)^{1/2}\exp\left[m\begin{pmatrix} l^*_{LINEX}\left(\hat{\lambda}_{l^*_{LINEX}},\hat{\theta}_{l^*_{LINEX}},\hat{\beta}_{l^*_{LINEX}}\right) \\ -l\left(\hat{\lambda}_l,\hat{\theta}_l,\hat{\beta}_l\right)\end{pmatrix}\right]\right] \quad (15)$$

Where,

$$l^*_{LINEX}(\lambda,\theta,\beta) = \frac{1}{m}\log\left[\exp\left(-ts(\lambda,\theta,\beta)\right)\right] + l(\lambda,\theta,\beta) \quad (16)$$

TK Bayes estimator of $s(\lambda,\theta,\beta)$ under GENTROPY loss function is defined as by,

$$\hat{s}_{GENTROPY}(\lambda,\theta,\beta) = \left\{E\left(\left[s(\lambda,\theta,\beta)\right]^{-a}\middle|x^R\right)\right\}^{-\frac{1}{a}}$$
$$= \left[\left(\frac{\det\Sigma^*_{GENTROPY}}{\det\Sigma}\right)^{1/2}\exp\left[m\begin{pmatrix} l^*_{GENTROPY}\left(\hat{\lambda}_{l^*_{GENTROPY}},\hat{\theta}_{l^*_{GENTROPY}},\hat{\beta}_{l^*_{GENTROPY}}\right) \\ -l\left(\hat{\lambda}_l,\hat{\theta}_l,\hat{\beta}_l\right)\end{pmatrix}\right]\right]^{-\frac{1}{a}} \quad (17)$$

where

$$l^*_{GENTROPY}(\lambda,\theta,\beta) = \frac{1}{m}\log\left[\left[s(\lambda,\theta,\beta)\right]^{-a}\right] + l(\lambda,\theta,\beta) \quad (18)$$

TK Bayes estimator of $s(\lambda,\theta,\beta)$ under SQUARED (Squared error) loss function is defined as follows.

$$\hat{s}_{SQUARED}(\lambda,\theta,\beta) = E\left(s(\lambda,\theta,\beta)\middle|x^R\right) = \frac{\int e^{ml^*(\lambda,\theta,\beta)}d(\lambda,\theta,\beta)}{\int e^{ml(\lambda,\theta,\beta)}d(\lambda,\theta,\beta)}$$
$$= \left(\frac{\det\Sigma^*}{\det\Sigma}\right)^{1/2}\exp\left[m\begin{pmatrix} l^*\left(\hat{\lambda}_{l^*},\hat{\theta}_{l^*},\hat{\beta}_{l^*}\right) \\ -l\left(\hat{\lambda}_l,\hat{\theta}_l,\hat{\beta}_l\right)\end{pmatrix}\right] \quad (19)$$

where l^* is defined in Eq. (13). Moreover, $\hat{\lambda}_{l^*},\hat{\theta}_{l^*},\hat{\beta}_{l^*}$ and $\hat{\lambda}_l,\hat{\theta}_l,\hat{\beta}_l$ maximize $l^*(\lambda,\theta,\beta)$ and $l(\lambda,\theta,\beta)$,respectively $\Sigma^*$and Σ are minus the inverse Hessians of $l^*(\lambda,\theta,\beta)$ and $l(\lambda,\theta,\beta)$ at $\hat{\lambda}_{l^*},\hat{\theta}_{l^*},\hat{\beta}_{l^*}$ and $\hat{\lambda}_l,\hat{\theta}_l,\hat{\beta}_l$ respectively.

4.SIMULATION STUDY

The following algorithm has been introduced to generate numbers from progressive type-II right censored sample for any continuous distribution using Uniform Distribution (Balakrishnan and Sandu) [21]. This algorithm for OddW distribution is as follows.

1. $W_1, W_2, ..., W_m$ are generated sample with m size getting from Uniform (0,1) distribution.
2. $W_i^{\left(i+\sum_{j=m-i+1}^{m} R_j\right)^{-1}} = V_i$, is described for i=1,2,...m .
3. $U^R_{i:m:n} = 1 - V_m V_{m-1} ... V_{m-i+1}$ is procured for i=1,2,...,m.
4. Eventually, $U^R_{1:m:n} < U^R_{2:m:n}, ..., < U^R_{m:m:n}$ are progressively Type-II censored samples with censoring scheme $R = (R_1, R_2, ... R_m)$ taken from Uniform (0,1) distribution.

We reported the MSEs parameters $(\lambda = 1.2,\ \theta = 0.9,\ \beta = 0.9)$ over 1000 replications.

The results are consituted. As seen from Table 1 and Table 2, generally mean squared errors obtained under LINEX and GENTROPY loss functions are better than others. It is observed that for the same and all censoring schemes as m/n increases, the MSEs of estimators tend to decrease in general. Moreover, MSEs all estimates for (n-m,,0,…,0) censoring scheme are usually smaller than MSEs under LINEX, GENTROPY and SQUARED loss functions Bayes estimates and MLE's estimates for other censoring schemes.

5.CONCLUSION

In this report, approximate Bayes estimators under squared error, Linex and General entropy loss function obtained by using TK approximation and MSEs MLE's for OddW distribution with λ, θ and β parameters under progressive type II censored sample. As the number of failure units for the same n increases, MLE's performances and performances of the LINEX, GENTROPY and SQUARED loss functions Bayes estimates are getting better. Also in the case of n = 20, the linex loss function has been generally found to have a minimum MSE, whereas in the case of n = 80 the general entropy loss function has mostly minimum MSE. The best estimates are obtained for n = m (complete sample status).

REFERENCES

[1] K. Cooray, "Analyzing grouped, censored, and truncated data using the odd weibull family", *Communications in Statistics - Theory and Methods*, vol. 41 no.15, pp. 2661-2680, 2012.

[2] K. Cooray, "A study of moments and likelihood estimators of the odd Weibull distribution", *Statistical Methodology*, vol. 26, pp. 72-83, 2015.

[3] H. Jiang , M. Xie and L.C. Tang, "On the odd Weibull distribution", *J. Risk and Reliability*, vol. 222, pp. 583-594, 2008.

[4] N. Balakrishnan, N. Kannan, C.T. Lin, and S. J. S. Wu, "Inference for the extreme value distribution under progressive type-II censoring", *J. Statist. Comput. Simul.*, vol. 74, pp. 25–45, 2004.

[5] S.J. Wu, "Estimations of the Parameters of the Weibull Distributions With Progressively Censored Data*", J. Japan Statist. Soc.*, vol. 32 no. 2, pp. 155-163, 2002.

[6] S.J. Wu, C.T. Chang and T.R. Tsai, "Point and Interval Estimations For The Gompertz Distribution Under Progressive Type II Censoring", *Metron-International Journal Of Statistics LXI*, vol.3, pp. 409-418, 2003.

[7] N. Balakrishnan, and R. Aggarwala, "Progressive Censoring: Theory, Methods and Applications", Birkhauser, Boston, 2000.

[8] N. Balakrishnan, "Progressive censoring methodology: an appraisal, (with discussions) Test", vol. 16, no. 2, 211 – 296, 2007.

[9] C.D. Lai, "Generalized Weibull Distributions", London, Springer, pp. 63-65, 2014.

[10] G.S. Mudholkar and D.K. Srivastava, "Exponentiated Weibull family for analyzing bathtub failure rate data", *IEEE Transactions on Reliability*, vol.42, pp. 299-302, 1993.

[11] G. Gencer and B. Saraçoğlu, "Comparison of approximate Bayes Estimators under different loss functions for parameters of Odd Weibull Distribution", *Journal of Selçuk University Natural and Applied Science*, vol. 5, no.1, pp. 18-32, 2016.

[12] S. Nadarajah, G.M. Cordeiro and E.M.M. Ortega, "Exponentiated Weibull Distribution: a survey", *Statistical Papers*, vol. 54, no.3, pp.839-877, 2013.

[13] A.M. Salem and O.E. Abo-Kasem, "Estimation for the Parameters of the Exponentiated Weibull Distribution Based on Progressive Hybrid Censored Samples", *Int. J. Contemp. Math. Sciences*, vol. 6, no. 35, pp. 1713-1724, 2011.

[14] M.M. Nassar and F.H. Eissa, "Bayesian estimation for the exponentiated Weibull model", *Communications in Statistics – Theory and Methods*, vol.33, pp. 2343- 2362, 2004.

[15] E.M.M. Ortega, G.M. Cordeiro, E.M. Hashimoto and K. Cooray, “A log-linear regression model for the odd Weibull distribution with censored data”, *Journal of Applied Statistics*, vol.41 no.9, pp.1859–1880, 2014.

[16] Y.Y Abdelall, “The Odd Generalized Exponential Modified Weibull Distribution”, ınternational Mathematical Forum, vol.11, no.16, pp. 943-959, 2016.

[17] M.Ç. Korkmaz, M. Alizadeh, H.M. Yousof and N.S. Butt, “The Generalized Odd Weibull Generated Family of Distributions: Statistical Properties and Applications”, *Pakistan Journal of Statistics and Operation Research*, vol.14, no.3, pp. 541-556, 2018.

[18] M. Alizadeh, E. Altun, A.Z. Afify and G. Ozel, “The Extended Odd Weibull-G Family: Properties And Applıcatıons”, *Commun. Fac. Sci. Univ. Ank. Ser. A1 Math. Stat.*, vol.68 no.1, pp. 161-186, 2018.

[19] K. Cooray, “Generalization of theWeibull distribution: the odd Weibull family”, *Statistical Modelling*, vol. 6, pp.265–277, 2006.

[20] L. Tierney and J. B. Kadane, "Accurate Approximations for Posterior Moments and Marginal Densities", *Journal of the American Statistical Association*, vol.81, no.393, pp. 82-86, 1986.

[21] N. Balakrishnan and R.A. Sandhu, “A Simple Simulation Algorithm for Generating Progressively Type-II Censored Sample”, *American Statistician*, vol.49, pp.229-230, 1995.

Table 1. MSEs MLEs and MSEs Bayesian Estimation loss functions for parameters $\lambda = 1.2, \theta = 0.9$ and $\beta = 0.9$ $(n = 20)$

n	m	Scheme	$\hat{\lambda}_{MSE}, \hat{\theta}_{MSE}, \hat{\beta}_{MSE}$	MLE	Squared	General Entropy	Linex
20	5	0,0,0,0,15	$\hat{\lambda}_{MSE}$	0,352332	0,177452	0,066781	0,032058
			$\hat{\theta}_{MSE}$	0,227924	0,578193	0,067784	0,091324
			$\hat{\beta}_{MSE}$	0,652410	0,09858	0,010094	0,013226
		15,0,0,0,0	$\hat{\lambda}_{MSE}$	0,197128	0,115519	0,037168	0,019645
			$\hat{\theta}_{MSE}$	0,388298	0,706185	0,083379	0,112060
			$\hat{\beta}_{MSE}$	0,355582	0,048763	0,005598	0,006603
		5,5,5,0,0	$\hat{\lambda}_{MSE}$	0,253206	0,183954	0,078249	0,032756
			$\hat{\theta}_{MSE}$	0,298850	0,647531	0,073276	0,098197
			$\hat{\beta}_{MSE}$	0,334272	0,023094	0,004127	0,002988
		0,15,0,0,0	$\hat{\lambda}_{MSE}$	0,224745	0,149638	0,058804	0,026184
			$\hat{\theta}_{MSE}$	0,328061	0,620910	0,072019	0,095432
			$\hat{\beta}_{MSE}$	0,327578	0,032390	0,004271	0,004180
		10,5,0,0,0	$\hat{\lambda}_{MSE}$	0,207780	0,130976	0,047812	0,022755
			$\hat{\theta}_{MSE}$	0,341740	0,670287	0,078414	0,104665
			$\hat{\beta}_{MSE}$	0,358961	0,038563	0,004504	0,005033
		5,10,0,0,0	$\hat{\lambda}_{MSE}$	0,198666	0,121776	0,04433	0,021415
			$\hat{\theta}_{MSE}$	0,340749	0,651316	0,075964	0,10095
			$\hat{\beta}_{MSE}$	0,345278	0,034082	0,004111	0,004403
	10	0,0,0,0,0,0,0,0,0,10	$\hat{\lambda}_{MSE}$	0,162100	0,211899	0,020498	0,032423
			$\hat{\theta}_{MSE}$	0,074010	0,293996	0,03576	0,044908
			$\hat{\beta}_{MSE}$	0,644509	0,104622	0,011299	0,014344
		10,0,0,0,0,0,0,0,0,0	$\hat{\lambda}_{MSE}$	0,157821	0,086665	0,018305	0,014311
			$\hat{\theta}_{MSE}$	0,255109	0,306523	0,035049	0,043919
			$\hat{\beta}_{MSE}$	0,234564	0,020189	0,005786	0,003019
		0,0,0,0,5,5,0,0,0,0	$\hat{\lambda}_{MSE}$	0,232217	0,179526	0,066235	0,032333
			$\hat{\theta}_{MSE}$	0,278772	0,299030	0,031632	0,040939
			$\hat{\beta}_{MSE}$	0,199352	0,014533	0,009888	0,002724
	15	0,0,..,5	$\hat{\lambda}_{MSE}$	0,185511	0,147606	0,017225	0,026713
			$\hat{\theta}_{MSE}$	0,488503	0,74762	0,100736	0,130004
			$\hat{\beta}_{MSE}$	0,254237	0,081270	0,011463	0,012550
		0,0,…,5,.,.0,0	$\hat{\lambda}_{MSE}$	0,120401	0,081689	0,026427	0,015624
			$\hat{\theta}_{MSE}$	0,209708	0,141338	0,013565	0,017422
			$\hat{\beta}_{MSE}$	0,1652089	0,019112	0,014432	0,003951
		5,0,..,0	$\hat{\lambda}_{MSE}$	0,176583	0,140813	0,016783	0,025302
			$\hat{\theta}_{MSE}$	0,484108	0,686497	0,093169	0,118550
			$\hat{\beta}_{MSE}$	0,224042	0,067467	0,009427	0,010306

Table 2. MSE's MLEs and MSE's Bayesian Estimation loss functions for parameters $\lambda = 1.2, \theta = 0.9$ and $\beta = 0.9$ $(n = 80)$

n	m	Scheme	$\hat{\lambda}_{MSE}, \hat{\theta}_{MSE}, \hat{\beta}_{MSE}$	MLE	Squared	General Entropy	Linex
80	20	60,0,…,0,0	$\hat{\lambda}_{MSE}$	0,187969	0,152193	0,018115	0,028129
			$\hat{\theta}_{MSE}$	0,523966	0,902353	0,122723	0,163913
			$\hat{\beta}_{MSE}$	0,363778	0,139487	0,021346	0,022781
		0,0,0…,20,20,20,…,0,0	$\hat{\lambda}_{MSE}$	0,193584	0,198240	0,063298	0,033165
			$\hat{\theta}_{MSE}$	0,303006	0,735036	0,087453	0,110216
			$\hat{\beta}_{MSE}$	0,203677	0,019796	0,003455	0,002484
		0,0,…,0,60	$\hat{\lambda}_{MSE}$	0,167187	0,164954	0,031056	0,036007
			$\hat{\theta}_{MSE}$	0,076405	0,487393	0,082484	0,105272
			$\hat{\beta}_{MSE}$	0,657842	0,254215	0,038819	0,043959
	40	40,0,…,0,0	$\hat{\lambda}_{MSE}$	0,138785	0,121687	0,014482	0,022070
			$\hat{\theta}_{MSE}$	0,435785	0,434851	0,063440	0,073701
			$\hat{\beta}_{MSE}$	0,079726	0,022688	0,004938	0,003581
		0,0,0…,20,20,…,0,0	$\hat{\lambda}_{MSE}$	0,081622	0,070789	0,018361	0,012594
			$\hat{\theta}_{MSE}$	0,438219	0,379009	0,051926	0,059660
			$\hat{\beta}_{MSE}$	0,065438	0,021359	0,012497	0,004171
		0,0,…,0,40	$\hat{\lambda}_{MSE}$	0,314368	0,451161	0,047355	0,083016
			$\hat{\theta}_{MSE}$	0,054685	0,410287	0,058563	0,069152
			$\hat{\beta}_{MSE}$	0,911926	0,267410	0,038057	0,042117