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Effects on mantel-haenszel chi-square statistic of scores in two-way contingency tables

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Abstract
Analysis of association between ordinal categorical variables has been widely studied especially in social sciences and medicine. It is known that ordinal scale has several advantages over nominal scale for researchers. Mantel-Haenszel (MH) chi-square statistic, which is known as the appropriate statistics to identify the association between ordinal categorical variables, takes into consideration the linear trend. In this study, integer, midrank, exponential, Van der Waerden, and joint scores were compared with a Monte Carlo simulation study to evaluate power of MH chi-square statistics. Although integer score has been used so far in literature, it is seen that Van der Waerden score is also preferable according to results.

Keywords: ordinal categorical data, score, Mantel-Haenszel chi-square statistic, test power.

İki-yönlü olumsallık tablolarında skorların mantel-haenszel ki-kare istatistiği üzerine etkileri

Öz
Sıralı kategorik değişkenler arasındaki ilişkinin analizi özellikle sosyal bilimlerde ve tıpta yaygın olarak çalışılmaktadır. Araştırmacılar için sıralama ölçeğinin sınıflama ölçeğine göre birçok avantajı olduğu bilinmektedir. Sıralı kategorik değişkenler arasındaki ilişkiyi belirlemek için uygun bir istatistik olarak bilinen Mantel-Haenszel ki-kare istatistiği doğrusal trendi dikkate almaktadır. Bu çalışmada, Mantel-Haenszel ki-kare istatistiğinin gücünü değerlendirmek için tamsayı, mid-rank, üstel, Van der Waerden ve ortak birikimli skorlar bir Monte Carlo benzetim çalışması ile karşılaştırıldı. Literatürde şimdiye kadar tamsayı skoru kullanılmasına rağmen, sonuçlara göre Van der Waerden skorunun da tercih edilebilir olduğu görülmektedir.

Anahtar kelimeler: sıralı kategorik veri, skor, Mantel-Haenszel ki-kare istatistiği, test gücü.

1. Introduction

The analysis of association between ordinal categorical data has received a considerable attention in practice; especially in social, medicine, and behavioral studies since ordinal scale has several advantages over nominal scale [1]. Assume an $I \times J$ contingency table which has ordered row X and column Y categories. It is known that score values should be considered in the presence of ordered contingency tables. Using this ordered categories reveals more powerful test than conventional chi-square statistic. Mantel-Haenszel (MH) chi-square (M^2) is one of the widely used statistics for categorical data analysis. However, M^2 statistic is not appropriate for nominal variables. M^2 test statistic is an ordinal measure of significance for linear association or linear by linear association chi-square, unlike ordinary and likelihood ratio chi-square [1]. It is preferred for testing the significance of linear relationship between two ordinal variables, since it is more powerful than Pearson chi-square and more likely to establish

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linear associationIf this statistic is significant, it means that an increase in a variable is associated with the increase (or decrease for negative relationship) in another [2].

An adaptive MH test for sensitivity analysis was proposed by [3]. Statistical equivalence testing approaches for MH differential item functioning analysis was studied by [4]. Extensions to the Cochran MH mean scores and correlation tests was presented by [5]. MH estimators of an assumed common odds ratio for several 2×c tables was discussed by [6]. An approach on network meta-analysis of rare events using the MH was presented by [7]. Performances of the generalized MH and a multilevel generalized MH procedure for the detection of uniform differential item functioning were compared by [8]. Although there are several studies on MH statistic in terms of different aspects in literature, score effects on this statistic is still open to be investigated. Therefore, $I \times J$ contingency tables were considered having ordinal categorical variables for M^2 statistic considering linear trend for ordinal categorical data in this study. With this purpose, a simulation study was conducted to compare integer, midrank, exponential, Van der Waerden (normal), and joint scores to evaluate the power of M^2 statistics. In addition, a real data set was studied to clarify simulation results.

2. Mantel-haenszel (MH) chi-qquare (M^2) statistic

Pearson and likelihood chi-square statistics do not consider any ordering of the rows or columns in the $I \times J$ contingency table. More powerful tests and more information can be obtained from the ordering data structure. M^2 statistic considering linear trend has been prefered for testing the independence especially in ordinal data [1]. Correlation coefficient of this statistic is known as the appropriate statistic to detect the association between ordinal variables. Assume X and Y are ordinal variables for a contingency table. Let assign the scores for row u_i and column v_j with satisfying order which is the same order for the categories [9, 10] as follow:

$$u_1 \leq u_2 \leq \cdots \leq u_I \qquad (1)$$
$$v_1 \leq v_2 \leq \cdots \leq v_J$$

The correlation coefficient ρ between X and Y is defined as in Eq (2).

$$\rho = \frac{\sum_{i=1}^{I}\sum_{j=1}^{J}(u_i-\bar{u})(v_j-\bar{v})\hat{p}_{ij}}{\sqrt{\left(\sum_{i=1}^{I}(u_i-\bar{u})^2\hat{p}_{i+}\right)\left(\sum_{j=1}^{J}(v_j-\bar{v})^2\hat{p}_{+j}\right)}} \qquad (2)$$

where $\bar{u} = \sum_{i=1}^{I} u_{i+}\,\hat{p}_{i+}$ denotes the sample mean of the row scores, $\bar{v} = \sum_{j=1}^{J} v_{+j}\,\hat{p}_{+j}$ denotes the sample mean of the column scores, $\hat{p}_{i+}$ and $\hat{p}_{+j}$ are the marginal probabilities of rows and columns, and $\hat{p}_{ij} = n_{ij}/N$ is the sample probability on the *i*th row and *j*th column [11, 12]. Hence, null and alternative hypotheses can be shown as below:

H_0: Two ordinal variables are independent, or equivalent ($\rho = 0$)
H_1: Two ordinal variables are dependent, or equivalent ($\rho \neq 0$)

M^2 statistic has components for large sample which is shown as in Eq (3).

$$M^2 = r^2(N-1) \sim \chi_1^2 \qquad (3)$$

or M can be calculated as in Eq (4).

$$M = r\sqrt{(N-1)} \sim N(0,1) \qquad (4)$$

3. The choice of scores

Even though several scores might be used using the M^2 statistic, integer scoring is generally preferred. Actually, it is often not clear how scores should be chosen. Typically, different choices of scores lead to the same results, but different scores can lead to different results. In addition, the results might be sensitive in choice of scores when margins of the table are highly unbalanced or even if some cells have considerably larger frequencies than the others. Hence, assignment of the scores is crucial. Other widely used scores are integer (equally spaced scores), midranks, exponential, Van der Waerden, and joint scores [10, 12, 13].

3.1. Integer scores

Integer scores, which are appropriate for ordinal scaled variables, usually set equal to the category order (1,2,…...).

3.2. Mid-ranks scores

A non-parametric approach to score selection is to consider the mid-ranks of each row (or column). The mid-ranks are calculated as in Eq (5) and Eq (6) for rows and columns, respectively.

$$R1_i = \sum_{k<i} n_{k+} + \frac{(n_{i+}+1)}{2}, \quad i = 1,2,\dots,I \tag{5}$$

$$C1_j = \sum_{l<j} n_{+l} + \frac{(n_{+j}+1)}{2}, \quad j = 1,2,\dots,J \tag{6}$$

where I and J are the total number of row and column categories.

3.3. Exponential scores

Expected values of the order statistics of exponential distribution are as in Eq (7) and Eq (8), respectively. Exponential scores sum to the total number of categories.

$$R_1 = \frac{1}{I}, \quad R_2 = \frac{1}{I} + \frac{1}{I-1}, \quad R_3 = \frac{1}{I} + \frac{1}{I-1} + \frac{1}{I-2}, \ \dots, \ R_I = \frac{1}{I} + \frac{1}{I-1} + \frac{1}{I-2} + \cdots + 1 \tag{7}$$

$$C_1 = \frac{1}{J}, \quad C_2 = \frac{1}{J} + \frac{1}{J-1}, \quad C_3 = \frac{1}{J} + \frac{1}{J-1} + \frac{1}{J-2}, \ \dots, \ C_J = \frac{1}{J} + \frac{1}{J-1} + \frac{1}{J-2} + \cdots + 1 \tag{8}$$

3.4. Van der Waerden scores

Let Φ^{-1} denotes inverse of the standard normal cumulative distribution function . The Van der Waerden scores are defined as in Eq (9) and Eq (10), respectively. These scores are also known as normal scores in literature.

$$R_i = \Phi^{-1}\left(\frac{i}{I+1}\right), \quad i = 1,2,\dots,I \tag{9}$$

$$C_i = \Phi^{-1}\left(\frac{j}{J+1}\right), \quad j = 1,2,\dots,J \tag{10}$$

Van der Waerden scores sum to 0.

3.5. Joint scores

Joint scores are defined as in Eq (11) and Eq (12) for rows and columns, respectively. These scores are summations of Exponential and Van der Waerden scores for each category.

$$R_{j1} = \frac{1}{I} + \Phi^{-1}\left(\frac{1}{I+1}\right), \dots, R_{jI} = \left(\frac{1}{I} + \frac{1}{I-1} + \dots + 1\right) + \Phi^{-1}\left(\frac{I}{I+1}\right),\ j = 1,2,\dots,I \quad (11)$$

$$C_{i1} = \frac{1}{J} + \Phi^{-1}\left(\frac{1}{J+1}\right), \dots, C_{i2} = \left(\frac{1}{J} + \frac{1}{J-1} + \dots + 1\right) + \Phi^{-1}\left(\frac{J}{J+1}\right),\ i = 1,2,\dots,J \quad (12)$$

4. Monte carlo simulation study

A Monte Carlo simulation study was conducted to explore the performance of M^2 statistic for testing the independence in the $I \times J$ ordinal contingency table. A random sample size of N was generated from a bivariate normal distribution with correlation ρ as in below:

$$X \sim N\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}1 & \rho\\ \rho & 1\end{bmatrix}\right)$$

Then the random numbers were transformed into equal-interval frequency tables by dividing range by class interval for each variable. Simulation scenarios for sample size, correlation rate, dimension and nominal level are considered in Table 1.

Table 1. Simulation scenarios for sample size, correlation rate, dimension and nominal level

Sample sizes (N)	Correlation rates (ρ)	Dimensions ($I \times J$)	Nominal levels
50	0	3×3, 3×4, 3×5	0.05
100	0.10	4×4, 4×5	0.10
250	0.20	5×5, 5×6	
500	0.30		
	0.40		

Monte Carlo simulation was conducted for 5000 replications (S) in Matlab2018R. The effect on statistic M^2 of scores was examined in terms of power of the statistic. Type I error probabilities of test statistics for each correlation values in terms of sample sizes, dimensions, and nominal levels were calculated as in Table 2-6. As can be seen, Type I error probabilities were close to the nominal value in all sample sizes for each scores.

Wander Waerden, joint, and integer scores are superior according to the power of M^2 statistic when there is no correlation between variables. Joint and exponential scores were superior when correlation rate was small $(\rho = 0.10, 0.20)$ for sample size 50 and square contingency tables. In addition, Van der Waerden and integer scores performed well for error rate 0.10 and dimension size increased. Van der Waerden and integer scores had the best performances when sample size increased in all nominal levels and contingency tables. Joint scores had good performance after Van der Waerden and integer scores for sample size were 50 and 100 when correlation rate increased.

Powers of M^2 test statistic increased when sample size increased for both nominal levels and correlated cases. However, the power did not regularly increase when sample size increased for both nominal levels. In addition, powers of M^2 test statistic increased when correlation rate increased as well.

Table 2. Powers of M^2 test statistic for for $\rho = 0$

	N	50		100		250		500	
r×c	Scores /α	0.05	0.10	0.05	0.10	0.05	0.10	0.05	0.10
3×3	Integer	0.0544	0.0954	0.0420	0.1020	0.0520	0.0909	0.0560	0.1060
	Mid-ranks	0.0520	0.0952	0.0418	0.1015	0.0515	0.0907	0.0500	0.0980
	Exponential	0.0514	0.0950	0.0418	0.1010	0.0510	0.0900	0.0470	0.0990
	Van der Waerden	0.0545	0.1002	0.0425	0.1080	0.0530	0.0910	0.0560	0.1060
	Joint	0.0524	0.0960	0.0423	0.1030	0.0570	0.0930	0.0520	0.1000
3×4	Integer	0.0481	0.1150	0.0571	0.1101	0.0510	0.1080	0.0610	0.1160
	Mid-ranks	0.0470	0.1110	0.0480	0.1055	0.0507	0.0910	0.0520	0.1120
	Exponential	0.0460	0.1100	0.0400	0.1050	0.0500	0.0900	0.0490	0.1070
	Van der Waerden	0.0480	0.1130	0.0560	0.1060	0.0505	0.1060	0.0480	0.1190
	Joint	0.0490	0.1140	0.0570	0.1100	0.0520	0.1000	0.0590	0.1130
3×5	Integer	0.0510	0.1010	0.0452	0.0920	0.0500	0.1120	0.0450	0.0920
	Mid-ranks	0.0450	0.0970	0.0420	0.0860	0.0490	0.1030	0.0410	0.0900
	Exponential	0.0470	0.0950	0.0410	0.0870	0.0530	0.1080	0.0415	0.0870
	Van der Waerden	0.0500	0.0960	0.0460	0.0890	0.0510	0.1080	0.0440	0.0930
	Joint	0.0480	0.1000	0.0450	0.0900	0.0540	0.1090	0.0420	0.0910
4×4	Integer	0.0500	0.0980	0.0480	0.1150	0.0650	0.0870	0.0525	0.1110
	Mid-ranks	0.0480	0.0900	0.0500	0.1100	0.0570	0.0830	0.0515	0.1100
	Exponential	0.0470	0.0920	0.0585	0.1160	0.0580	0.0840	0.0520	0.1090
	Van der Waerden	0.0523	0.0970	0.0520	0.1200	0.0640	0.0855	0.0530	0.1150
	Joint	0.0520	0.1000	0.0590	0.1170	0.0600	0.0850	0.0570	0.1160
4×5	Integer	0.0590	0.1050	0.0520	0.1058	0.0550	0.1054	0.0452	0.0996
	Mid-ranks	0.0500	0.1010	0.0500	0.1037	0.0525	0.1060	0.0449	0.0910
	Exponential	0.0515	0.1040	0.0514	0.1036	0.0530	0.1062	0.0454	0.0944
	Van der Waerden	0.0570	0.1060	0.0524	0.1068	0.0532	0.1064	0.0450	0.0982
	Joint	0.0580	0.1100	0.0506	0.1038	0.0516	0.1092	0.0492	0.0966
5×5	Integer	0.0540	0.0935	0.0460	0.1025	0.0620	0.1000	0.0565	0.1170
	Mid-ranks	0.0500	0.0900	0.0400	0.0995	0.0565	0.0925	0.0520	0.1015
	Exponential	0.0480	0.0910	0.0420	0.0970	0.0510	0.0840	0.0510	0.1010
	Van der Waerden	0.0545	0.0925	0.0470	0.1000	0.0600	0.0980	0.0545	0.1150
	Joint	0.0510	0.0920	0.0450	0.1010	0.0590	0.0950	0.0540	0.1040
5×6	Integer	0.0494	0.1028	0.0510	0.0976	0.0508	0.0962	0.0508	0.1002
	Mid-ranks	0.0436	0.1010	0.0495	0.0960	0.0502	0.0935	0.0501	0.0950
	Exponential	0.0428	0.0974	0.0494	0.0958	0.0504	0.0946	0.0484	0.0946
	Van der Waerden	0.0490	0.1046	0.0496	0.0992	0.0492	0.0948	0.0507	0.0982
	Joint	0.0442	0.1026	0.0504	0.0978	0.0496	0.0974	0.0506	0.0970

Table 3. Powers of M^2 test statistic for $\rho = 0.10$

	N	50		100		250		500	
r×c	Scores / α	0.05	0.10	0.05	0.10	0.05	0.10	0.05	0.10
3×3	Integer	0.0730	0.1260	0.1135	0.1980	0.2250	0.3210	0.3890	0.5210
	Mid-ranks	0.0700	0.1230	0.1110	0.1900	0.2220	0.3100	0.3685	0.4960
	Exponential	0.0710	0.1270	0.1090	0.1820	0.2130	0.3010	0.3440	0.4820
	Van der Waerden	0.0720	0.1220	0.1130	0.1980	0.2280	0.3215	0.3870	0.5200
	Joint	0.0740	0.1250	0.1120	0.1960	0.2260	0.3120	0.3790	0.5150
3×4	Integer	0.0905	0.1610	0.1180	0.1980	0.2410	0.3290	0.4280	0.5530
	Mid-ranks	0.0890	0.1550	0.1130	0.1820	0.2260	0.3120	0.3950	0.5240
	Exponential	0.0900	0.1560	0.1060	0.1760	0.2060	0.3030	0.3820	0.5080
	Van der Waerden	0.0810	0.1590	0.1180	0.1970	0.2400	0.3350	0.4260	0.5550
	Joint	0.0850	0.1540	0.1120	0.1840	0.2240	0.3210	0.4220	0.5420
3×5	Integer	0.0970	0.1620	0.1190	0.1970	0.2580	0.3740	0.4520	0.5900
	Mid-ranks	0.0910	0.1510	0.1100	0.1830	0.2410	0.3460	0.4070	0.5120
	Exponential	0.0960	0.1500	0.1080	0.1770	0.2330	0.3300	0.4140	0.5280
	Van der Waerden	0.0930	0.1560	0.1200	0.1980	0.2530	0.3700	0.4470	0.5940
	Joint	0.0840	0.1550	0.1070	0.1950	0.2520	0.3580	0.4400	0.5730
4×4	Integer	0.0860	0.1570	0.1680	0.2470	0.2790	0.3900	0.4700	0.5770
	Mid-ranks	0.0910	0.1530	0.1500	0.2400	0.2570	0.3650	0.4600	0.5310
	Exponential	0.0920	0.1590	0.1530	0.2330	0.2450	0.3510	0.4170	0.5400
	Van der Waerden	0.0900	0.1560	0.1630	0.2450	0.2770	0.3910	0.4680	0.5790
	Joint	0.0940	0.1540	0.1640	0.2510	0.2660	0.3720	0.4520	0.5690
4×5	Integer	0.0910	0.1680	0.1385	0.2330	0.2770	0.3980	0.4620	0.6030
	Mid-ranks	0.0905	0.1640	0.1325	0.2150	0.2450	0.3650	0.4440	0.5550
	Exponential	0.0940	0.1630	0.1270	0.2060	0.2300	0.3550	0.4080	0.5410
	Van der Waerden	0.0950	0.1690	0.1380	0.2350	0.2870	0.3940	0.4610	0.5970
	Joint	0.0920	0.1650	0.1360	0.2210	0.2650	0.3840	0.4580	0.5790
5×5	Integer	0.0930	0.1810	0.1360	0.2320	0.2660	0.3970	0.5200	0.6480
	Mid-ranks	0.0960	0.1700	0.1350	0.2230	0.2500	0.3820	0.4860	0.6030
	Exponential	0.0900	0.1930	0.1310	0.2000	0.2480	0.3590	0.4610	0.5860
	Van der Waerden	0.0980	0.1920	0.1330	0.2340	0.2710	0.3890	0.5180	0.6450
	Joint	0.0990	0.1800	0.1320	0.2250	0.2610	0.3840	0.5040	0.6340
5×6	Integer	0.1200	0.1840	0.1580	0.2620	0.2840	0.4160	0.5160	0.6260
	Mid-ranks	0.1100	0.1770	0.1550	0.2530	0.2740	0.3860	0.5030	0.5900
	Exponential	0.1040	0.1760	0.1555	0.2420	0.2550	0.3640	0.4560	0.5700
	Van der Waerden	0.1220	0.1800	0.1600	0.2600	0.2880	0.4120	0.5220	0.6200
	Joint	0.1160	0.1790	0.1560	0.2480	0.2730	0.4030	0.5050	0.6160

Table 4. Powers of M^2 test statistic for $\rho = 0.20$

	N	50		100		250		500	
r×c	Scores / α	0.05	0.10	0.05	0.10	0.05	0.10	0.05	0.10
3×3	Integer	0.1780	0.2830	0.3340	0.4640	0.6710	0.7760	0.9100	0.9530
	Mid-ranks	0.1830	0.2740	0.3150	0.4320	0.6480	0.7590	0.8860	0.9270
	Exponential	0.1810	0.2950	0.3090	0.4240	0.6110	0.7280	0.8600	0.9180
	Van der Waerden	0.1780	0.2830	0.3340	0.4640	0.6710	0.7770	0.9120	0.9520
	Joint	0.1880	0.2890	0.3300	0.4480	0.6560	0.7640	0.8950	0.9460
3×4	Integer	0.2122	0.3300	0.3642	0.5000	0.7086	0.8168	0.9364	0.9656
	Mid-ranks	0.2120	0.3130	0.3400	0.4690	0.6650	0.7890	0.9210	0.9512
	Exponential	0.2080	0.3106	0.3378	0.4590	0.6482	0.7534	0.9006	0.9460
	Van der Waerden	0.2186	0.3284	0.3628	0.5008	0.7096	0.8182	0.9370	0.9654
	Joint	0.2122	0.3260	0.3634	0.4850	0.6950	0.7992	0.9298	0.9610
3×5	Integer	0.2254	0.3422	0.4024	0.5376	0.7402	0.8314	0.9498	0.9768
	Mid-ranks	0.2150	0.3210	0.3650	0.5001	0.6890	0.8250	0.9260	0.9630
	Exponential	0.2098	0.3152	0.3530	0.4870	0.6768	0.7808	0.9146	0.9548
	Van der Waerden	0.2272	0.3374	0.4014	0.5340	0.7412	0.8316	0.9486	0.9750
	Joint	0.2208	0.3326	0.3996	0.5174	0.7220	0.8198	0.9426	0.9692
4×4	Integer	0.2530	0.3770	0.4290	0.5560	0.7750	0.8490	0.9755	0.9865
	Mid-ranks	0.2730	0.3740	0.4100	0.5130	0.7275	0.8120	0.9630	0.9830
	Exponential	0.2580	0.3810	0.3860	0.4940	0.7050	0.7900	0.9520	0.9770
	Van der Waerden	0.2470	0.3750	0.4280	0.5480	0.7700	0.8560	0.9750	0.9860
	Joint	0.2750	0.3720	0.4200	0.5320	0.7600	0.8430	0.9670	0.9850
4×5	Integer	0.2480	0.3608	0.4246	0.5510	0.7900	0.8692	0.9728	0.9870
	Mid-ranks	0.2380	0.3384	0.3968	0.5018	0.7300	0.8366	0.9502	0.9706
	Exponential	0.2360	0.3298	0.3792	0.4992	0.7188	0.8148	0.9438	0.9684
	Van der Waerden	0.2512	0.3602	0.4228	0.5494	0.7910	0.8704	0.9724	0.9868
	Joint	0.2462	0.3520	0.4172	0.5340	0.7730	0.8550	0.9644	0.9820
5×5	Integer	0.2400	0.3720	0.4380	0.5730	0.7880	0.8680	0.9870	0.9950
	Mid-ranks	0.2310	0.3360	0.4120	0.5260	0.7500	0.8470	0.9630	0.9810
	Exponential	0.2300	0.3730	0.3780	0.4960	0.7280	0.8060	0.9570	0.9750
	Van der Waerden	0.2260	0.3500	0.4320	0.5650	0.7940	0.8670	0.9850	0.9940
	Joint	0.2450	0.3530	0.4240	0.5450	0.7770	0.8500	0.9760	0.9880
5×6	Integer	0.2550	0.3668	0.4490	0.5810	0.8314	0.8974	0.9816	0.9936
	Mid-ranks	0.2410	0.3400	0.3816	0.5260	0.8006	0.8592	0.9610	0.9856
	Exponential	0.2304	0.3288	0.3732	0.4990	0.7414	0.8372	0.9604	0.9816
	Van der Waerden	0.2560	0.3708	0.4454	0.5790	0.8308	0.8844	0.9814	0.9918
	Joint	0.2460	0.3586	0.4262	0.5510	0.8040	0.8814	0.9756	0.9888

Table 5. Powers of M^2 test statistic for $\rho = 0.30$

	N	50		100		250		500	
$r \times c$	Scores / α	0.05	0.10	0.05	0.10	0.05	0.10	0.05	0.10
3×3	Integer	0.3868	0.5136	0.6610	0.7700	0.9550	0.9860	0.9980	0.9990
	Mid-ranks	0.3778	0.5002	0.6204	0.7406	0.9360	0.9600	0.9974	0.9992
	Exponential	0.3616	0.4902	0.6134	0.7238	0.9208	0.9594	0.9972	0.9986
	Van der Waerden	0.3968	0.5134	0.6620	0.7720	0.9540	0.9760	0.9984	0.9993
	Joint	0.3824	0.5126	0.6446	0.7710	0.9466	0.9728	0.9982	0.9994
3×4	Integer	0.4444	0.5662	0.7118	0.8144	0.9712	0.9868	0.9998	0.9998
	Mid-ranks	0.4162	0.5396	0.6870	0.7845	0.9560	0.9750	0.9992	0.9996
	Exponential	0.4024	0.5282	0.6536	0.7644	0.9484	0.9734	0.9984	0.9994
	Van der Waerden	0.4456	0.5668	0.7106	0.8164	0.9704	0.9870	0.9999	0.9998
	Joint	0.4292	0.5574	0.6952	0.7984	0.9656	0.9838	0.9998	0.9998
3×5	Integer	0.4460	0.5808	0.7342	0.8294	0.9790	0.9904	1.0000	1.0000
	Mid-ranks	0.4268	0.5568	0.7006	0.8006	0.9600	0.9800	0.9996	0.9997
	Exponential	0.4050	0.5406	0.6742	0.7768	0.9570	0.9794	0.9984	0.9994
	Van der Waerden	0.4444	0.5794	0.7362	0.8308	0.9788	0.9900	1.0000	1.0000
	Joint	0.4344	0.5684	0.7180	0.8130	0.9748	0.9888	0.9996	0.9998
4×4	Integer	0.4602	0.5948	0.7548	0.8446	0.9842	0.9918	1.0000	1.0000
	Mid-ranks	0.4366	0.5560	0.6900	0.8250	0.9760	0.9885	1.9995	1.000
	Exponential	0.4220	0.5480	0.6866	0.7854	0.9664	0.9830	0.9998	1.0000
	Van der Waerden	0.4632	0.5952	0.7580	0.8448	0.9840	0.9924	1.0000	1.0000
	Joint	0.4536	0.5782	0.7388	0.8304	0.9806	0.9900	1.0000	1.0000
4×5	Integer	0.4954	0.6156	0.7848	0.8608	0.9892	0.9960	1.0000	1.0000
	Mid-ranks	0.4580	0.5896	0.7000	0.8180	0.9808	0.9900	0.9998	1.0000
	Exponential	0.4350	0.5616	0.6960	0.7996	0.9758	0.9898	0.9992	1.0000
	Van der Waerden	0.4966	0.6220	0.7828	0.8614	0.9888	0.9958	1.0000	1.0000
	Joint	0.4768	0.6096	0.7540	0.8454	0.9864	0.9952	1.0000	1.0000
5×5	Integer	0.5044	0.6300	0.7974	0.8766	0.9918	0.9964	1.0000	1.0000
	Mid-ranks	0.4680	0.5895	0.7520	0.8326	0.9800	0.9928	1.0000	1.000
	Exponential	0.4486	0.5724	0.7196	0.8114	0.9756	0.9906	1.0000	1.0000
	Van der Waerden	0.5018	0.6308	0.7970	0.8768	0.9910	0.9954	1.0000	1.0000
	Joint	0.4936	0.6148	0.7728	0.8558	0.9876	0.9956	1.0000	1.0000
5×6	Integer	0.4966	0.6338	0.8054	0.8824	0.9908	0.9960	1.0000	1.0000
	Mid-ranks	0.4680	0.5730	0.7320	0.8396	0.9806	0.9902	1.0000	1.0000
	Exponential	0.4430	0.5570	0.7138	0.8116	0.9786	0.9896	1.0000	1.0000
	Van der Waerden	0.4986	0.6378	0.8052	0.8840	0.9910	0.9964	1.0000	1.0000
	Joint	0.4842	0.6042	0.7768	0.8674	0.9882	0.9944	1.0000	1.0000

Table 6. Powers of M^2 test statistic for $\rho = 0.40$

	N	50		100		250		500	
r×c	Scores / α	0.05	0.10	0.05	0.10	0.05	0.10	0.05	0.10
3×3	Integer	0.6470	0.7640	0.8980	0.9440	1.0000	1.0000	1.0000	1.0000
	Mid-ranks	0.5370	0.6670	0.8150	0.8710	0.9990	1.0000	1.0000	1.0000
	Exponential	0.5800	0.7050	0.8720	0.9230	0.9970	1.0000	1.0000	1.0000
	Van der Waerden	0.6470	0.7640	0.8980	0.9440	1.0000	1.0000	1.0000	1.0000
	Joint	0.6260	0.7320	0.8930	0.9410	1.0000	1.0000	1.0000	1.0000
3×4	Integer	0.6830	0.7904	0.9374	0.9692	1.0000	1.0000	1.0000	1.0000
	Mid-ranks	0.4690	0.5876	0.7758	0.8580	0.9904	1.0000	1.0000	1.0000
	Exponential	0.6254	0.7412	0.8966	0.9452	0.9990	1.0000	1.0000	1.0000
	Van der Waerden	0.6824	0.7926	0.9372	0.9680	1.0000	1.0000	1.0000	1.0000
	Joint	0.6640	0.7752	0.9274	0.9624	1.0000	1.0000	1.0000	1.0000
3×5	Integer	0.7026	0.8068	0.9436	0.9760	1.0000	1.0000	1.0000	1.0000
	Mid-ranks	0.6654	0.7848	0.9264	0.9678	0.9994	1.0000	1.0000	1.0000
	Exponential	0.6332	0.7492	0.9008	0.9486	0.9986	1.0000	1.0000	1.0000
	Van der Waerden	0.7032	0.8084	0.9416	0.9754	1.0000	1.0000	1.0000	1.0000
	Joint	0.6800	0.7926	0.9346	0.9728	0.9999	1.0000	1.0000	1.0000
4×4	Integer	0.7410	0.8330	0.9550	0.9820	1.0000	1.0000	1.0000	1.0000
	Mid-ranks	0.6890	0.8000	0.9300	0.9330	1.0000	1.0000	1.0000	1.0000
	Exponential	0.6530	0.7630	0.9220	0.9580	1.0000	1.0000	1.0000	1.0000
	Van der Waerden	0.7450	0.8350	0.9550	0.9810	1.0000	1.0000	1.0000	1.0000
	Joint	0.7150	0.8250	0.9430	0.9710	1.0000	1.0000	1.0000	1.0000
4×5	Integer	0.7542	0.8400	0.9566	0.9808	0.9998	1.0000	1.0000	1.0000
	Mid-ranks	0.7206	0.8130	0.9360	0.9640	0.9692	1.0000	1.0000	1.0000
	Exponential	0.6730	0.7812	0.9254	0.9594	0.9962	1.0000	1.0000	1.0000
	Van der Waerden	0.7548	0.8418	0.9570	0.9806	1.0000	1.0000	1.0000	1.0000
	Joint	0.7326	0.8258	0.9470	0.9778	0.9998	1.0000	1.0000	1.0000
5×5	Integer	0.7970	0.8840	0.9740	0.9950	1.0000	1.0000	1.0000	1.0000
	Mid-ranks	0.7500	0.8230	0.9450	0.9780	1.0000	1.0000	1.0000	1.0000
	Exponential	0.7100	0.8070	0.9360	0.9660	1.0000	1.0000	1.0000	1.0000
	Van der Waerden	0.8000	0.8800	0.9770	0.9950	1.0000	1.0000	1.0000	1.0000
	Joint	0.7730	0.8600	0.9700	0.9890	1.0000	1.0000	1.0000	1.0000
5×6	Integer	0.7818	0.8646	0.9738	0.9878	0.9999	1.0000	1.0000	1.0000
	Mid-ranks	0.7006	0.8301	0.9500	0.9706	0.9966	1.0000	1.0000	1.0000
	Exponential	0.6874	0.7950	0.9370	0.9688	0.9948	1.0000	1.0000	1.0000
	Van der Waerden	0.7800	0.8678	0.9736	0.9892	1.0000	1.0000	1.0000	1.0000
	Joint	0.7560	0.8482	0.9650	0.9842	0.9998	1.0000	1.0000	1.0000

5. Real data example

Data set represents 4x4 ordinal contingency table was used for case study (Table 7). This table shows a contingency table between the smoking level status of high-density lipoprotein (HDL) cholesterol in the blood for persons [1].

Table 7. Sample data set

Smoking level status	Level of HDL cholesterol in the blood			
	Normal	Low normal	Borderline	Abnormal
No smoking	15	3	6	1
Less than 5 fill	8	4	7	2
Less than 10 fill	11	6	15	3
More than 10 fill	5	1	11	5

The aim is to investigate the independence between smoking level status and level of HDL cholesterol in the blood. Attained scores into the row and column categories were found as in Table 8.

Table 8. Attained scores into the row and column categories

Scores	row (column) categories			
Integer	1 (1)	2 (2)	3 (3)	4 (4)
Mid-ranks	13 (20)	36 (46.5)	64 (73)	92.5 (98)
Exponential	0.2500 (0.2500)	0.5833 (0.5833)	1.0833 (1.0833)	2.0833 (2.0833)
Van der Waerden	-0.8416 (-0.8416)	-0.2533 (-0.2533)	0.2533 (0.2533)	0.8416 (0.8416)
Joint	-0.5916 (-0.5916)	0.3300 (0.3300)	1.3367 (1.3367)	2.9250 (2.9250)

M^2 statistic results of the scores were given in Table 9.

Table 9. M^2 statistic results of the scores

Scores	M^2	p-value	ρ
Integer	10.1981	0.0014	0.3162
Mid-ranks	10.1410	0.0015	0.3153
Exponential	9.8229	0.0017	0.3103
Van der Waerden	10.2485	0.0014	0.3168
Joint	10.2401	0.0014	0.3170

According to Table 9, we reject the null hypothesis with %95 confidence level for all scores in terms of M^2 statistics. In addition, integer score was so close to Van der Waerden and Joint scores in terms of M^2 statistics.

6. Discussion and Conclusion

A common application of independence test is to assess if there are significant trends (with respect to age, educational or socioeconomic status, etc.,) in the incidence or prevalence of disease. The presence or absence of the disease would define two groups and the frequencies across age bands, socioeconomic groups, educational groups, etc., would be compared. It is also used to test the association of ordinal variables with terminal events such as death in analyzing dose-response relationships. This test for trend was extended by [1] for the situation in which treatment and control units have been stratified into subgroups to eliminate possibility of confounding by one or more variables. The test result is adjusted for the strata of the potential confounder involved. This test is generally used for case–control type data and the test statistic has an approximate chi-square distribution with degrees of freedom 1. This stratified trend test is called as Mantel–Haenszel (MH) chi-square test, MH test, or extended MH test. The extended MH chi-square test is calculated that reflects the departure of a linear trend from horizontal.

Although there are several studies on MH statistic in terms of different aspects in literature, score effects on this statistic is still open to be investigated. Therefore, $I \times J$ contingency tables are studied which have ordinal categorical variables for M^2 statistic considering linear trend for ordinal categorical data in this study. It can be mentioned from the results that the power increases as the contingency table dimension and sample size increase. In general, Van der Waerden score has the highest power when sample size and ρ increase in all contingency tables. Then, integer scores has the second highest power in similar way. On the other hand, mid-rank and exponential scores have the lowest power in all scenarios of the simulation study and case study as well. Van der Waerden score is suggested instead as an alternative to integer score for ordinal categorical data to test independence.

Authors' Contributions

All authors contributed equally to the study.

Statement of Conflicts of Interest

There is no conflict of interest between the authors.

Statement of Research and Publication Ethics

The author declares that this study complies with Research and Publication Ethics.

References

[1] Mantel N., Haenszel W. 1959. Statistical Aspects of the Analysis of Data from Retrospective Studies of Disease. Journal of the National Cancer Institute, 22: 719-748.

[2] Aktaş S. 2016. Power Divergence Statistics under Quasi Independence Model for Square Contingency Tables. Sains Malaysiana, 45 (10): 1573-1578.

[3] Casabianca J.M., Lewis C. 2017. Statistical Equivalence Testing Approaches for Mantel–HaenszelDIF Analysis. Journal of Educational and Behavioral Statistics, DOI: 10.3102/1076998617742410.

[4] Rayner J.C.W., Best D.J. 2018. Extensions to the Cochran–Mantel–Haenszel Mean Scores and Correlation Tests. Journal of Statistical Theory and Practice, DOI:10.1080/15598608.2018.1439424.

[5] Rosenbaum P.R., Small D.S. 2016. An Adaptive Mantel–Haenszel Test for Sensitivity Analysis in Observational Studies. Biometrics, DOI: 10.1111/biom.12591.

[6] Suesse T., Liu I. 2018. Mantel–Haenszel Estimators of a Common Odds Ratio for Multiple Response Data. Statistical Methods and Applications, DOI: 10.1007/s10260-018-0429-z.

[7] Efthimiou O., Rücker G., Schwarzer G., Higgins J.P.T., Egger M., Salanti G. 2019. Network Meta-Analysis of Rare Events Using the Mantel-Haenszel Method. Statistics in Medicine, DOI: 10.1002/sim.8158.

[8] French B.F., Finch W.H., Immekus J.C. 2019. Multilevel Generalized Mantel-Haenszel for Differential Item Functioning Detection. Frontiers in Education, DOI:10.3389/feduc.2019.00047.

[9] Agresti A. 2007. An Introduction to Categorical Data Analysis. New Jersey: Wiley.

[10] Agresti A. 2010. Analysis of Ordinal Categorical Data. John Wiley&Sons, Inc. Publication.

[11] Gong H. 2012. Modeling and Measuring Association for Ordinal Data. University of Regina, Faculty of Graduate Studies and Research, Reginas, Canada, 1-73.

[12] Anderson C.J. 2017. Statistical Inferences for Ordinal Variables in 2-way Tables, Department of Educational Psychology. University of Illinois. http://www.coursehero.com/file/22519849/2way-ordinal-beamer-online/. (Access date: 16.09.2019).

[13] Chen H.W.N. 2014. The Assignment of Scores Procedure for Ordinal Categorical Data. Hindawi Publishing Corporation, The Scientific World Journal. http://dx.doi.org/10.1155/2014/304213.