Celal Bayar University Journal of Science

Hybrid of Thevenin and Norton Equivalent Circuits Analogous to a Source Equivalence Theorem in Electromagnetics

Ömer Işık[1*], Lokman Erzen[2], Ali Uzer[3]

[1]Department of Electrical and Electronics Engineering, Beykent University, Istanbul, Turkey
[2]Applied Electronics and Systems Engineering, Regent University College of Science and Technology, Accra, Ghana
[3]Vocational School, Program of Electricity, Beykent University, Istanbul, Turkey
* omeri@beykent.edu.tr
*Orcid: 0000-0002-0726-9446

Received: 13 February 2020
Accepted: 14 December 2020
DOI: 10.18466/cbayarfbe. 689110

Abstract

According to the conventional perception among engineers, once a circuit is reduced to its Thevenin or Norton equivalent, the voltage and current may be determined only at the load, but not in the remaining parts. The other voltages and currents that exist in the remaining parts of circuit should be determined by returning to the original circuit; substituting the solutions obtained at the load location; and then employing the rules of circuit theory. In this paper, we presented a source equivalence theorem wherein such a back-substitution is never need. It splits an original circuit into two sub-circuits that can be solved separately by using different techniques. Then the voltages and currents everywhere in the circuit can be obtained as a sum of the solutions of those two sub-circuits without making any back-substitution.

Keywords: Nonlinear circuits, Source equivalence, Thevenin theorem, Norton theorem, Equivalent circuits, Integrated circuit interconnections.

1. Introduction

The Thevenin and Norton theorems were developed by Hermann Von Helmholtz, Léon Charles Thévenin, Edward Lawry Norton, and Hans Ferdinand Mayer in the period 1853-1926, [1, 2]. These theorems are described in a lot of books and papers such as [3- 7] and also used in the analysis of practical problems as in [8-10].

The Thevenin and Norton theorems can be used for determining the responses across a load in a circuit. The voltages and currents in the remaining parts should be determined by making back substitutions. Consider a circuit as seen in Figure 1, where N_{ext} stands for a linear network, N_{load} stands for a (possibly) nonlinear network, and *S* means a surface enclosing N_{load}. Its Thevenin or Norton equivalents will be as shown in Figure 2 or Figure 3 respectively. The sources $v_T(t)$ and $i_N(t)$ are determined conventionally by replacing N_{load} in the original circuit by an open-circuit or a short-circuit. Note that all sources within N_{ext} in the equivalent circuit are deactivated (switched off), therefore, N_{ext} may be reduced to a single impedance $Z_{eq}(\omega)$ since it is a linear network.

In case N_{load} is nonlinear, the analysis cannot be carried out in the frequency domain but an impulse response $z_{eq}(t)$ should be used instead of $Z_{eq}(\omega)$. That may be written as $z_{eq}(t) = F^{-1}\{Z_{eq}(\omega)\}$, where F^{-1} means the inverse Fourier transform operator.

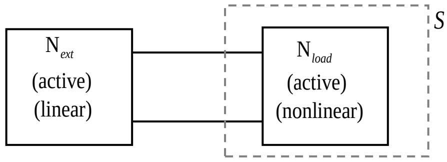


Figure 1. The original circuit. It is made up of linear components except some components inside *S*, which may be nonlinear.

Celal Bayar University Journal of Science
Volume 16, Issue 4, 2020, p 361-365
Doi: 10.18466/cbayarfbe. 689110

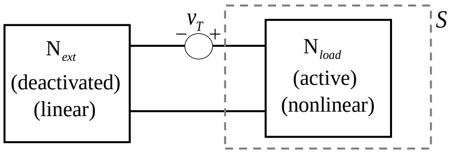


Figure 2. The Thevenin equivalent of the original circuit.

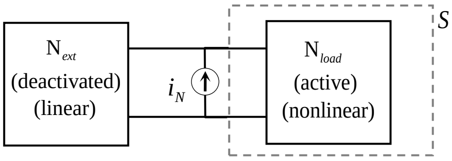


Figure 3. The Norton equivalent of the original circuit.

After that the equivalent circuit may be solved by employing a suitable technique such as finite difference time domain method. But in this manner, one can obtain solutions for voltages and currents only within N_{load}. For determining voltages and currents in the remaining parts, i.e., within N_{ext}, one should substitute the results into the original circuit in Figure 1 and then proceed to calculate the voltages and currents in N_{ext}. The overall procedure will be lengthy due to this back substitution procedure.

On the other hand, such back-substitutions will not be needed according to a new theorem that will be given in the next section. It was postulated during an investigation of electromagnetic source equivalence principles in [11]. In that paper, a newly given electromagnetic source equivalence theorem was shown to be analogous to a hybrid form of Thevenin and Norton theorems of circuit theory.

2. The New Equivalence Theorem

Theorem: Assume a circuit, called Circuit A, is made up of linear components except for one or more nonlinear components located inside a closed surface *S* as seen in Figure 4. Any voltage or current in this circuit can be written as

$$\begin{array}{l} v_A(t) = v_B(t) + v_C(t) \\ i_A(t) = i_B(t) + i_C(t) \end{array}, \text{ outside } S, \quad \textbf{(2.1)}$$

$$\begin{array}{l} v_A(t) = v_C(t) \\ i_A(t) = i_C(t) \end{array}, \text{ within } S, \quad \textbf{(2.2)}$$

where $v_B(t)$ and $i_B(t)$ denote responses in Circuit B and $v_C(t)$ and $i_C(t)$ denote responses in Circuit C; which are described below.

*Circuit B***:** It is the same as Circuit A except for N_{load} is replaced by an arbitrarily chosen network N_{test} as seen in Figure 5.

*Circuit C***:** It is the same as Circuit A except for: all sources in N_{ext} are switched off; a Thevenin source $v_T(t)$ is inserted across terminals at the interface; and a Norton source $i_N(t)$ is connected in parallel across terminals at the interface as seen in Figure 6. The sources $v_T(t)$ and $i_N(t)$, respectively, are equal to the voltage and current observed at the indicated location in Figure 5.

3. Comments

i. The proof of theorem (with a slightly different notation) can be found in an appendix in [11] and so we do not repeat it.

ii. Note that the expressions are given in the time domain since the circuits are nonlinear in the most general sense. The expressions will be literally valid in the phasor domain in the case where N_{load} is also a linear network.

iii. The solution in a nonlinear circuit may not be unique. For instance, when a nonlinear resistor is connected to a DC source, there may be multiple solutions for the current depending on the current-voltage characteristic of the resistor. In case Circuit A has multiple solutions, then each solution can still be written as in (2.1) and (2.2) since Circuit C in that case will also have multiple solutions.

Circuit B and Circuit C are standalone circuits. They can be analyzed using different methods. For instance, Circuit B may be solved in the phasor domain (provided N_{test} is linear) by using analytical methods, whereas Circuit C may be solved in the time domain by using finite differences methods.

The analysis can be simplified greatly if the test network N_{test} in Circuit B is chosen intuitively. For example, the terminals may be short-circuited or open-circuited for simplifying the analysis. In both cases Circuit B may be solved in the phasor domain by using analytical methods. But in the general case, N_{test} may be a linear or nonlinear resistor, capacitor, inductor, a voltage source, current source e.t.c., or even an interconnection of a number of such components as demonstrated in an example in the following section.

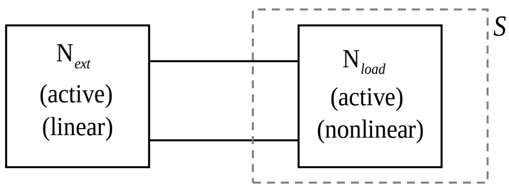


Figure 4. Circuit A (the original circuit). *S* stands for a closed surface enclosing a network N_{load}, which may be nonlinear; whereas a network N_{ext} outside *S* is linear.

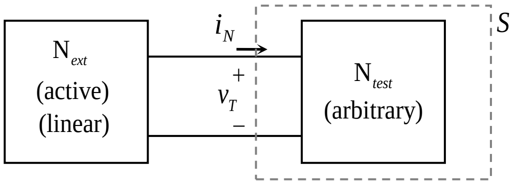


Figure 5. Circuit B (a test setup). It is for determining the sources $v_T(t)$ and $i_N(t)$. The network N_{load} has been replaced by an arbitrarily chosen N_{test}.

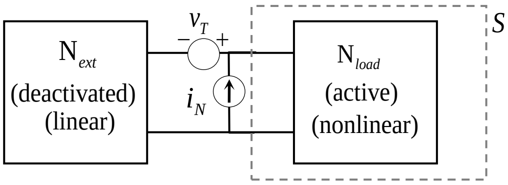


Figure 6. Circuit C (a hybrid equivalent circuit). All sources outside *S* are switched off, while the Thevenin and Norton sources $v_T(t)$ and $i_N(t)$ are placed in the terminals. The time variables are dropped for simplicity.

4. Numerical Example

We consider a circuit as shown in Figure 7, which is used in practice for suppressing multiple reflections of logic signals in interconnected digital systems. It is the same circuit analyzed in [12] (in Example 3.2) and we have chosen it for validating accuracy of our results. The given circuit involves a 5V step source with a 1ns rise time as shown in the inset. The is an air filled transmission line connecting the source to two identical diodes each of which is equivalent to a 3Ω resistor plus a nonlinear capacitor and nonlinear resistor as shown in the inset. The capacitance of nonlinear capacitor is defined by an equation

$$C(v_d) = 38.8\times10^{-12} / \sqrt{1 - v_d / 0.7531} \text{ Farad} \qquad \textbf{(4.1)}$$

so that the charge of capacitor is to be written as

$$Q_C = v_d C(v_d) \qquad \textbf{(4.2)}$$

and the current of capacitor is to be calculated from

$$i_C = \frac{d}{dt}\{Cv_d\} = C\frac{dv_d}{dt} + v_d\frac{dC}{dv_d}\frac{dv_d}{dt} \qquad \textbf{(4.3)}$$

The terminal equation of nonlinear resistor is given by

$$i_d = 2.58327\times10^{-14}\left(e^{v_d/0.026} - 1\right) \text{Amper} \qquad \textbf{(4.4)}$$

We employ the theorem in calculation of a current $i_{A,ext}(t)$ as indicated in the figure. Let the given circuit be Circuit A. It is possible to split Circuit A into a sub-circuit as seen in Figure 8. containing an arbitrarily selected resistor R_{test} inside a surface *S* and another sub-circuit containing equivalent sources placed across the surface *S* as seen in Figure 9. The theorem says that $i_{A,ext}(t)$ can be written as a sum $i_{B,ext}(t) + i_{C,ext}(t)$ of currents indicated in the figures.

Consider the resistor R_{test} seen in Figure 8. It can be chosen arbitrarily and the choice affects the strengths of Thevenin and Norton sources $v_T(t)$ and $i_N(t)$. If we use an open circuit, $R_{test} = \infty$, then $i_N(t)$ will be zero and the voltage $v_T(t)$ can be determined simply by employing a conventional bouncing diagram method. Alternatively, if we use $R_{test} = 0$ then $v_T(t)$ will be zero and the current $i_N(t)$ can be determined by employing bouncing diagram method. On the other hand, if we use $R_{test} = 45\Omega$, that is a matched-load according to the transmission line in the problem, the reflections will not occur along the transmission line. The voltage and current at the end of transmission line, $v_T(t)$ and $i_N(t)$, simply equal to

$$v_T(t) = \frac{45}{55} v_{ext}(t - t_0) \qquad \textbf{(4.5)}$$

$$i_N(t) = \frac{1}{55} v_{ext}(t - t_0) \qquad \textbf{(4.6)}$$

where t_0 stands for the time delay, that is, the ratio of the line length and the speed of light

$$t_0 = \frac{1.5}{3\times10^8} = 5\times10^{-9} = 5\text{ns} \qquad \textbf{(4.7)}$$

Any choice (open-circuit, short-circuit, or matched-load) for R_{test} affects only $i_{B,ext}(t)$ and $i_{C,ext}(t)$. But the sum $i_{B,ext}(t) + i_{C,ext}(t)$ always equals to $i_{A,ext}(t)$ according to the theorem. The same is true for any voltage or current anywhere along the transmission line.

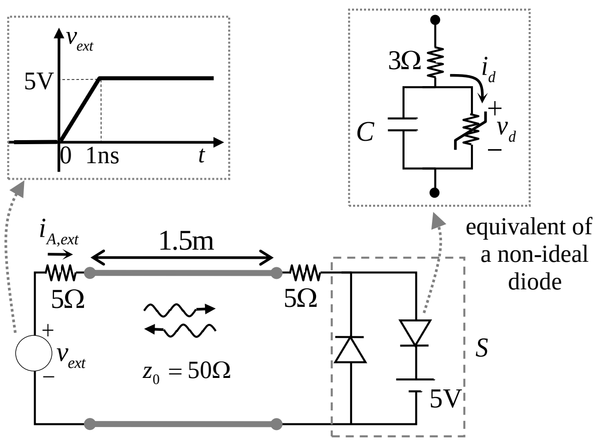


Figure 7. Circuit A: a voltage source as shown in the inset is feeding a nonlinear load, which is made up of two diodes and a DC source, via an air filled

Celal Bayar University Journal of Science
Volume 16, Issue 4, 2020, p 361-365
Doi: 10.18466/cbayarfbe. 689110

transmission line. The diodes are identical and each has an equivalent as indicated.

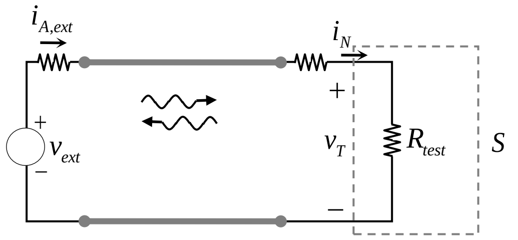


Figure 8. Circuit B: a resistor R_{test} is used to determine $v_T(t)$ and $i_N(t)$, which are to become equivalent sources in an equivalent circuit.

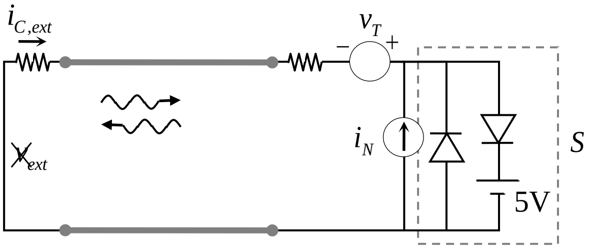


Figure 9. Circuit C: the equivalent circuit with the sources $v_T(t)$ and $i_N(t)$ at the interface.

According to the given theorem, we don't need to place a resistor R_{test} inside S in Circuit B but any number of linear and nonlinear components as well as sources may be placed inside S. For example, we may place some capacitors, voltage sources, and diodes inside S as seen in Figure 10. There is no restriction in selection of the types and connections of components. We make solution of this circuit to demonstrate the validity of our theorem and arbitrariness in its utilization. Otherwise the capacitors, voltage sources, and diodes placed inside S as seen in Figure 10. are not for simplifying the solution of problem. We employ a finite difference time domain scheme as described in [13] for the numerical solution of circuit in Figure 10. We use a time step $\Delta t = 10\text{ps}$ up to 100nS and a segment size $\Delta z = 3\text{mm}$ along the transmission line of length 1.5m. The selected values satisfy the criteria

$$\frac{\Delta z}{\Delta t} \geq 3\times 10^{8}\,\text{m/s} \tag{4.8}$$

for the stability of solutions along the air filled transmission line.

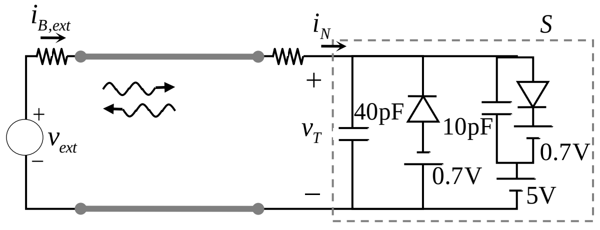
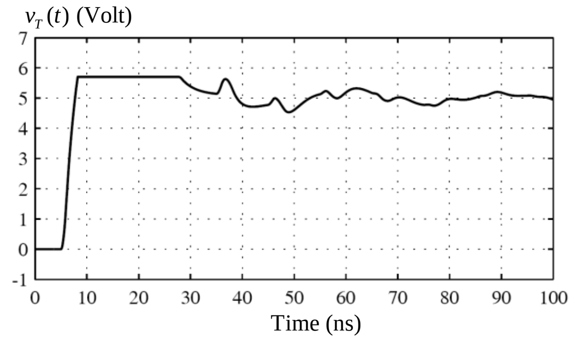


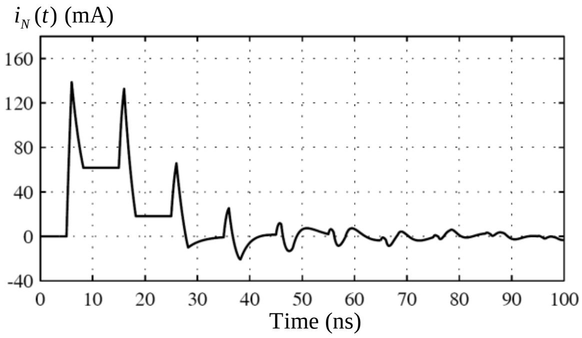
Figure 10. Circuit B: some arbitrarily chosen capacitors, sources, and diodes are placed inside S.

We calculate $v_T(t)$ and $i_N(t)$ as shown in Figure 11. They are calculated and saved into a memory for a later use in each time step. Then the saved values are substituted into Circuit C as independent sources in shown Figure 9. Circuit C, in turn, is solved by employing the same computational scheme and the same parameters (Δt and Δz) that were used when solving Circuit B. We also exploit Newton-Raphson iteration method in each time step for solving nonlinear equations of diodes. The iterations are carried out until the changes in diode voltages become less than $10^{-7}\,\text{V}$.

The original problem, Circuit A, is also solved using the same computational scheme and parameters (Δt and Δz) mentioned above. Specifically the current $i_{A,ext}(t)$ is calculated for making comparisons with other calculated values. Calculations have shown that the sum $i_{B,ext}(t)+i_{C,ext}(t)$ exactly equals to $i_{A,ext}(t)$ for all $t \geq 0$ and attests the statement of theorem in (2.1). It is also obvious from the separate plots of $i_{A,ext}(t)$, $i_{B,ext}(t)$, and $i_{C,ext}(t)$ shown Figure 12. Hence the plots also validate visually the validity of theorem. Meanwhile the curve of $i_{A,ext}(t)$ given in this paper matches with the previously calculated results given in [12].



(a)



(b)

Figure 11. (a) The Thevenin voltage $v_T(t)$ versus t according to the case inside S in Figure 10. **(b)** The Norton current $i_N(t)$ versus t according to the case inside S in Figure 10.

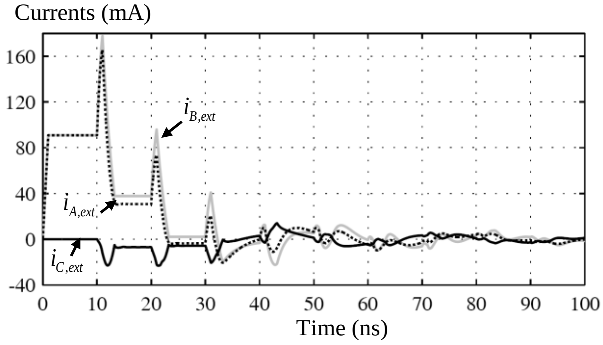


Figure 12. The numerical results for the individual currents $i_{A,ext}(t)$, $i_{B,ext}(t)$, and $i_{C,ext}(t)$ in circuits in Figure 7, Figure 10, and Figure 9, respectively. It displays the sum $i_{B,ext}(t)+i_{C,ext}(t)$ is equal to $i_{A,ext}(t)$ as claimed by the theorem.

5. Conclusion

In circuit theory, the Thevenin and Norton theorems are two of the important theorems that may be used to simplify analysis of complex circuits. It is believed that these source equivalence theorems can be used to calculate voltages and currents only across a specific component called the load in a circuit, but those in the remaining parts should be calculated via back substitutions. Sometimes, this back substitution procedure can be very lengthy.

We proposed a new circuit equivalence theorem, which eliminates necessity of any back-substitution procedure. The theorem splits any bulky circuit into two sub-circuits each of which can be solved by using a different method. Then the solutions of sub-circuits can be added simply to obtain the overall solution. The theorem is applied to a practical problem involving a transmission line and nonlinear components.

Author's Contributions

Ömer Işık: Made literature search, prepared computer codes for numerical solutions of the sample problem, made simulations, and wrote the manuscript.

Lokman Erzen: Made literature search, decided for a sample problem for testing the given theorem, supervised the simulation process, and made comparisons.

Ali Uzer: Extended an electromagnetic equivalence theorem in [11] into the circuit theory and also helped in manuscript preparation.

Ethics

There are no ethical issues after the publication of this manuscript.

References

1. Johnson, DH. 2002. Origins of the equivalent circuit concept: the voltage-source equivalent. *Proceedings of the IEEE*, 91(4): 636-640.

2. Johnson, DH. 2002. Origins of the equivalent circuit concept: the current-source equivalent. *Proceedings of the IEEE*, 91(5): 817-821.

3. Haus, HA, Adler RB. Circuit theory of linear noisy circuits; Cambridge, Massachusetts, New York, Wiley, 1959.

4. Carlin, HJ, Giordano, AB. Circuit theory-An introduction to reciprocal and nonreciprocal circuits; Englewood Cliffs, New Jersey, Prentice-Hall, 1964.

5. Corazza, GC, Someda, CG, Longo, G. 1969. Generalized Thevenin's theorem for linear n-port circuits. *IEEE Transactions on Circuit Theory*, 16(4): 564-566.

6. Hashemian, R. Hybrid equivalent circuit, an alternative to Thevenin and Norton equivalents, its properties and applications. *Midwest Symposium on Circuits and Systems (MWSCAS 2009)*, Mexico, 2009, pp 800-803.

7. Rathore, TS. 2017. Generalized Maximum Power Transfer Theorem. *IETE Journal of Education*, 58(1): 39-41.

8. Femia, N. 1996. A fast method to find the state of switches after forced commutations in switching converters. *Proceedings of IEEE Power Electronics Specialists Conference (ESC'96), Atlanta*, (2): 1356-1362.

9. Femia, N. 2003. Understanding commutations in switching converters-Part I: Basic theory and application of the compensation theorem. *IEEE Transactions on Aerospace and Electronic Systems*, 39(1): 282-297.

10. Heydt, GT. 2017. G Thevenin's Theorem applied to the analysis of polyphase transmission circuits. *EEE Transactions on Power Delivery*, 32(1): 72-77.

11. Uzer, A. 2012. Some electromagnetic equivalence theorems unified by a single theorem and generalized to a nonlinear case. *IEEE Antennas and Propagation Magazine*, 54(3): 86-99.

12. Chang, FY. 1996. Transient analysis of diode switching circuits including charge storage effect. *IEEE Transactions on Circuits and Systems-I: Fundamental Theory and Applications*, 43(3): 177-190.

13. Dommel, HW. 1969. Digital computer solution of electromagnetic transients in single and multiphase circuits. *IEEE Trans. Power Apparatus and Systems*, 88(4): 388-399.

.