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# A Research on the Spherical Indicatrices of Directional Space Curve

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**Abstract** In this paper, we give a parametrization of a directional space curve by using adapted frame called q-frame.. The spherical images of both directional normal indicatrix and directional tangent indicatrix of this space curve with q-frame are studied. Later using the geodesic curvature of the spherical image of the directional normal indicatrices, we work on the condition that a space curve to be slant helix. Finally, we give an application of the results.

**Keywords:** q-frame, Slant Helix, Spherical Indicatrix.

## Yönlü Uzay Eğrisinin Küresel Göstergeleri Üzerine Bir Çalışma

**Öz** Bu çalışmada, q-çatı olarak adlandırılan adapte edilmiş çatı kullanılarak yönlü uzay eğrisinin parametrik denklemini verdik. q-çatılı bu uzay eğrisinin Yönlü normaller ve teğetler göstergesinin küresel resimleri çalışıldı. Sonrasında yönlü normaller göstergelerinin küresel resminin jeodezik eğriliği kullanılarak uzay eğrisinin slant helis olma şartını çalıştık. Son olarakta, elde edilen sonuçların uygulaması verdik.

**Anahtar Kelimeler:** q-çatı, Slant Helis, Küresel Gösterge.

## 1. Introduction

The most well-known adapted frame is the Frenet frame. The Frenet frame plays an important role in classical differential geometry. Let $\alpha(t)$ be a regular space curve Bloomenthal (1975), Chouaieb et al. (2006), then the Frenet frame is defined as follows

$$\boldsymbol{t} = \frac{\alpha'}{\|\alpha'\|}, \boldsymbol{b} = \frac{\alpha' \wedge \alpha''}{\|\alpha' \wedge \alpha''\|}, \boldsymbol{n} = \boldsymbol{b} \wedge \boldsymbol{t}.$$

The curvature κ and the torsion τ are given by

$$\kappa = \frac{\|\alpha' \wedge \alpha''\|}{\|\alpha'\|^3}, \boldsymbol{\tau} = \frac{det(\alpha', \alpha'', \alpha''')}{\|\alpha' \wedge \alpha''\|^2}.$$

The well-known Frenet formulas are given by

$$\begin{bmatrix} \boldsymbol{t}' \\ \boldsymbol{n}' \\ \boldsymbol{b}' \end{bmatrix} = \upsilon \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{t} \\ \boldsymbol{n} \\ \boldsymbol{b} \end{bmatrix},$$

However, Frenet frame has several disadvantages in applications. For instance, Frenet frame is undefined wherever the curvature vanishes. Moreover, the main drawback of the Frenet frame is that it has undesirable rotation about tangent vector (Bloomenthal 1975). Therefore, Bishop (1975) introduced a new frame along a space curve which is more suitable for applications. But, it is well known that Bishop frame calculations is not a easy work (Wang et al., 2008). In order to construct the 3D curve offset, Coquillart (1987) introduced the quasi-normal vector of a space curve. The quasi-normal vector is defined for each point

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of the curve, and lies in the plane perpendicular to the tangent of the curve at this point (Shin et al., 2003). Then by using the quasi-normal vector Dede et al. (2015) introduced the q-frame along a space curve used in (Dede and Ekici 2018, Ekici et al., 2017, Kaymanli 2018). Given a space curve α(t) the q-frame consists of three orthonormal vectors, these being the unit tangent vector t, the quasi-normal nq and the quasi-binormal vector bq. The q-frame $\{\mathbf{t}, \mathbf{n_q}, \mathbf{b_q}\}$ is given by

$$\boldsymbol{t} = \frac{\alpha'}{\|\alpha'\|}, \boldsymbol{n_q} = \frac{\boldsymbol{t} \wedge \boldsymbol{k}}{\|\boldsymbol{t} \wedge \boldsymbol{k}\|}, \boldsymbol{b_q} = \boldsymbol{t} \wedge \boldsymbol{n_q} \tag{1}$$

where k is the projection vector (Dede et al., 2015). The q-frame has many advantages versus other frames (Frenet, Bishop). For instance the q-frame can be defined even along a line (κ = 0) and the construction of the q-frame doesn't change if the space curve has unit speed or not. Moreover the q-frame can be easily calculated. For simplicity, we have chosen the projection vector k = (0, 0, 1) in this paper. However, the q-frame is singular in all cases where t and k are parallel. Thus, in those cases where t and k are parallel the projection vector k can be chosen as k = (0, 1, 0) or k = (1, 0, 0). A q-frame along a space curve is shown in Figure1.

The variation equations of the directional q-frame is given by

$$\begin{bmatrix} \boldsymbol{t}' \\ \boldsymbol{n}'_q \\ \boldsymbol{b}'_q \end{bmatrix} = \|\alpha'\| \begin{bmatrix} 0 & k_1 & k_2 \\ -k_1 & 0 & k_3 \\ -k_2 & -k_3 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{t} \\ \boldsymbol{n_q} \\ \boldsymbol{b_q} \end{bmatrix},$$

where the q-curvatures are expressed as follows

$$k_1 = -\frac{\langle \boldsymbol{t}, \boldsymbol{n}'_q \rangle}{\|\alpha'\|}, k_2 = -\frac{\langle \boldsymbol{t}, \boldsymbol{b}'_q \rangle}{\|\alpha'\|}, k_3 = -\frac{\langle \boldsymbol{n}', \boldsymbol{b_q} \rangle}{\|\alpha'\|}$$

Recently, since helices are widely encountered in nature, they are worked by many researchers (Ali 2012, Altunkaya and Kula , 2016, Bukcu and Karacan, 2009, Kula et al., 2009). Izumiya and Takeuchi (2004) give the characterization of helix and slant helix by using the geodesic curvature of the spherical image of the principal normal indicatrix. It is known that the curve γ is a helix if and only if $\frac{\tau}{\kappa}$ is constant. A curve γ with κ≠0 is called a slant helix if and only if the geodesic curvature of the spherical image of the principal normal indicatrix of γ

$$\kappa_g(s) = \left( \frac{\kappa^2}{(\kappa^2 + \tau^{\,2})^{\frac{3}{2}}}, \left(\frac{\tau}{\kappa}\right)' \right)(s)$$

is a constant function. While Körpınar (2018) worked on the principal normal spherical image, Kula and Yayli (2005) worked on spherical images of the tangent and binormal indicatrices of a slant helix, and showed spherical images of them are slant helix.

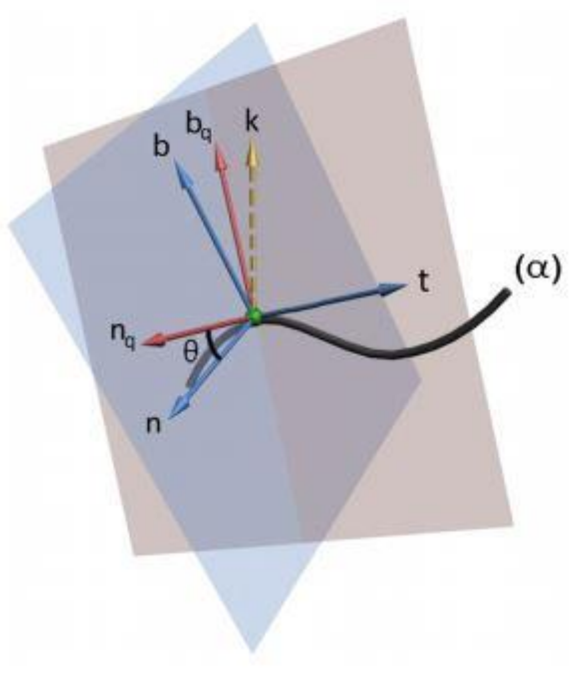


Figure 1: The q-frame and Frenet frame

## 2. Directional Tangent Indicatrix

**Definition 2.1**. Let $\alpha = \alpha(t)$ be a regular space curve with q-frame vectors $\{\mathbf{t}, \mathbf{n_q}, \mathbf{b_q}\}$. Then the unit tangent vectors along the curve $\alpha$ generates a curve $\phi = \phi\ (\boldsymbol{t_\phi})$ on the unit sphere. We call the curve $\phi$ as the directional tangent indicatrix of the curve.

**Theorem 2.2.** Let $\phi = \phi\ (\boldsymbol{t_\phi})$ be directional tangent indicatrix of the curve. Then, the Frenet frame vectors $(\mathbf{t}_\phi, \mathbf{n}_\phi, \mathbf{b}_\phi)$ of $\phi$ can be given by in terms of q-frame $(\mathbf{t}, \mathbf{n_q}, \mathbf{b_q})$ of $\alpha$ in the following form

$$\mathbf{t}_\phi = \frac{\boldsymbol{n_q} + f\boldsymbol{b_q}}{\sqrt{1+f^2}}$$

$$\mathbf{n}_\phi = \frac{-\sqrt{1+f^2}\boldsymbol{t} - \sigma f\boldsymbol{n_q} + \sigma\boldsymbol{b_q}}{\sqrt{1+f^2}\sqrt{1+\sigma^2}}$$

and

$$\mathbf{b}_\phi = \frac{1}{\sqrt{1+\sigma^2}}\left(\sigma\boldsymbol{t} - \frac{f}{\sqrt{1+f^2}}\boldsymbol{n_q} + \frac{1}{\sqrt{1+f^2}}\boldsymbol{b_q}\right)$$

where

$$\sigma = \frac{(1+f^2)k_3 + f'}{(1+f^2)^{3/2}k_1}$$

The curvature and the torsion of $\phi$ are

$$\kappa_\phi = k_1\sqrt{1+f^2}\sqrt{1+\sigma^2},$$

$$\tau_\phi = \frac{\sigma'}{1+\sigma^2}$$

**Proof:** By differentiating the curve $\phi$ with respect to t we get

$$\phi' = \frac{d\phi}{d\boldsymbol{t_\phi}}\frac{d\boldsymbol{t_\phi}}{dt} = \left(k_1\boldsymbol{n_q} + k_2\boldsymbol{b_q}\right)\frac{d\boldsymbol{t_\phi}}{dt}$$

where prime means differentiation respect to $t$. Simple calculation implies that

$$\frac{d\boldsymbol{t_\phi}}{dt} = k_1\sqrt{1+f^2}$$

Where $f = \frac{k_2}{k_1}$. The unit tangent vector $\mathbf{t}_\phi$ is obtained by

$$\mathbf{t}_\phi = \frac{\boldsymbol{n_q} + f\boldsymbol{b_q}}{\sqrt{1+f^2}}$$

Differentiating $\mathbf{t}_\phi$ with respect to $s$, then substituting (1) into the result gives

$$\begin{aligned}\mathbf{n}_\phi &= -\frac{\boldsymbol{t'}_\phi}{\|\boldsymbol{t_\phi}'\|} \\ &= \frac{1}{\sqrt{1+f^2}\sqrt{1+\sigma^2}}\left(-\sqrt{1+f^2}\boldsymbol{t} - \sigma f\boldsymbol{n_q} + \sigma\boldsymbol{b_q}\right)\end{aligned}$$

where

$$\sigma = \frac{(1+f^2)k_3 + f'}{(1+f^2)^{3/2}k_1}$$

The unit binormal vector $\boldsymbol{b_\phi} = \boldsymbol{t_\phi} \wedge \mathbf{n}_\phi$ is given by

$$\mathbf{b}_\phi = \frac{1}{\sqrt{1+\sigma^2}}\left(\sigma\boldsymbol{t} + \frac{f}{\sqrt{1+f^2}}\boldsymbol{b_q} - \frac{1}{\sqrt{1+f^2}}\boldsymbol{n_q}\right)$$

The curvature $\kappa = \|\boldsymbol{t_\phi}'\|$ is obtained by

$$\kappa = k_1\sqrt{1+f^2}\sqrt{1+\sigma^2}$$

By differantiating $\mathbf{n}_\phi$ we have

$$\begin{aligned}\mathbf{n}'_\phi = \frac{1}{\sqrt{1+f^2}\sqrt{1+\sigma^2}}\Bigg[&\left(\frac{\sigma\sigma'\sqrt{1+f^2}}{1+\sigma^2}\right)\boldsymbol{t} \\ &+ \left(fW - k_1\sqrt{1+f^2} - f'\sigma - f\sigma' - k_3\sigma\right)\boldsymbol{n_q} \\ &- \left(W + k_2\sqrt{1+f^2} + k_3 f\sigma - \sigma'\right)\boldsymbol{b_q}\Bigg]\end{aligned}$$

where

$$W = \frac{ff'\sigma(1+\sigma^2) + \sigma^2\sigma'(1+f^2)}{(1+f^2)(1+\sigma^2)}$$

Finally, the torsion $\tau_\phi = \langle \boldsymbol{n'}_\phi, \boldsymbol{b}_\phi \rangle$ is obtained by

$$\tau_\phi = \frac{\sigma'}{1+\sigma^2}$$

**Corollary 2.3.** Let $\phi = \phi(s_\phi)$ be the directional tangent spherical image of a curve $\alpha = \alpha(s)$. If $f$ and $h$ are constant, then D-tangent spherical image $\phi = \phi(s_\phi)$ is a circle in the osculating plane.

**Proof:** Since $f$ and $h$ are constant, one can easily find $\sigma = \frac{h}{\sqrt{1+f^2}}$ and therefore,

$$\kappa_\phi = k_1\sqrt{1+f^2+h^2}$$

is constant too.

$$\tau_\phi = \frac{\sigma'}{(1+\sigma^2)} = 0$$

Therefore, the tangent image is a circle in the osculating plane.

**Theorem 2.4.** Let $\theta = \theta(\mathbf{t}_\theta)$ be directional tangent indicatrix of the curve with $q$ frame ($\mathbf{t}$, $\mathbf{n}_{q\theta}$, $\mathbf{b}_{q\theta}$). Then, the q-frame vectors of $\theta$ can be given by in the following form

$$\mathbf{t} = \frac{\boldsymbol{n}_q + f\boldsymbol{b}_q}{\sqrt{1+f^2}}$$

$$\boldsymbol{n}_{q\phi} = \frac{\boldsymbol{t}\wedge\boldsymbol{k}}{\|\boldsymbol{t}\wedge\boldsymbol{k}\|} = \frac{\mu\boldsymbol{k} - \boldsymbol{t} + f\mu\boldsymbol{n}_q}{\Delta}$$

and

$$\boldsymbol{b}_{q\phi} = \boldsymbol{t}\wedge\boldsymbol{n}_{q\phi} = \mu\boldsymbol{n}_q + \frac{f\mu}{\Delta}\boldsymbol{b}_q$$

where $\Delta = \sqrt{1-\mu^2+f^2\mu^2}$ and $\mu$ is the third component of $\mathbf{t}$, found in (Dede and Ekici, 2018).

Using definition of q-curvatures of $\phi$ they are calculated by

$$k_{1\phi} = \frac{1}{\Delta}\begin{pmatrix}\frac{1}{\sqrt{1+f^2}}(k_1+fk_2)(1-\mu^2) \\ +k_1\sigma\mu(\eta-f^2)\end{pmatrix}$$

$$k_{2\phi} = \frac{1}{\Delta}\begin{pmatrix}\frac{1}{\sqrt{1+f^2}}(k_1+fk_2)(1-\mu^2) \\ +k_1\sigma\mu(\eta-f^2)\end{pmatrix}$$

$$k_{3\phi} = \mu' - \frac{k_3 f\mu}{\Delta}$$

where $\eta$ is the third component of $\mathbf{b_q}$.

The geondesic curvature of the spherecial image of the directional tangent indicatrix of a curve $\alpha$ is calculated by

$$\kappa_g = \frac{\langle \mathbf{t}'_\phi, \mathbf{n_q}\wedge\mathbf{t}_\phi\rangle}{\|\mathbf{t}_\phi\|^3}$$

By definition, one can find easily geodesic curvature of the spherical image of the directional tangent indicatrix with respect to $q$-curvatures

$$\kappa_g = \frac{1}{(k_1^2+k_2^2)^{3/2}}\left[k_1^2\left(\frac{k_2}{k_1}\right)' + k_3(k_1^2+k_2^2)\right]$$

## 3. Directional Normal Indicatrix

In this section, we give the characterization of the directional slant helix by using the geodesic curvature of the spherical image of the directional normal indicatrix.

**Theorem 3.1.** Let $\phi = \phi(\mathbf{n}_{q\phi})$ be directional tangent indicatrix of the curve. Then, the Frenet frame vectors ($\mathbf{t}_\phi$, $\mathbf{n}_\phi$, $\mathbf{b}_\phi$) of $\phi$ can be given by in terms of q-frame ($\mathbf{t}$, $\mathbf{n}_q$, $\mathbf{b}_q$) of $\alpha$ in the following form

$$\mathbf{t}_\phi = \frac{\boldsymbol{t} + g\boldsymbol{b}_q}{\sqrt{1+g^2}}$$

$$\mathbf{n}_\phi = \frac{-\beta g\boldsymbol{t} - \sqrt{1+g^2}\boldsymbol{n}_q - \beta\boldsymbol{b}_q}{\sqrt{1+g^2}\sqrt{1+\beta^2}}$$

and

$$\mathbf{b}_{\phi} = \frac{1}{\sqrt{1+\beta^2}}\left(\frac{g}{\sqrt{1+g^2}}\boldsymbol{t} - \beta \boldsymbol{n_q} - \frac{1}{\sqrt{1+g^2}}\boldsymbol{b_q}\right)$$

where $\beta = \frac{(1+g^2)k_2+g\prime}{(1+g^2)^{3/2}k_1}$ and $g = -\frac{k_3}{k_1}$.

The curvature and the torsion of $\phi$ are

$$\kappa = k_1\sqrt{1+g^2}\sqrt{1+\beta^2}, \tau_{\phi} = -\frac{\beta'}{1+\beta^2}$$

**Proof:** By differentiating the curve $\phi$ with respect to *t* we get

$$\phi' = \frac{d\phi}{d\boldsymbol{t_\phi}}\frac{d\boldsymbol{t_\phi}}{d\boldsymbol{t}} = \left(-k_1\boldsymbol{t} + k_3\boldsymbol{b_q}\right)\frac{d\boldsymbol{t_\phi}}{d\boldsymbol{t}}$$

where prime means differentiation with respect to *t*. Simple calculation implies that

$$\frac{d\boldsymbol{t_\phi}}{d\boldsymbol{t}} = k_1\sqrt{1+g^2}$$

where $g=\frac{k_3}{k_1}$. The unit tangent vector $\mathbf{t}_\phi$ is obtained by

$$\mathbf{t}_{\boldsymbol{\phi}} = \frac{\boldsymbol{t} + g\boldsymbol{b_q}}{\sqrt{1+g^2}}$$

Differentiating $\mathbf{t}_\phi$ with respect to *s*, then substituting (1) into the result gives

$$\mathbf{n}_{\boldsymbol{\phi}} = \frac{-\beta g\boldsymbol{t} - \sqrt{1+g^2}\boldsymbol{n_q} - \beta\boldsymbol{b_q}}{\sqrt{1+g^2}\sqrt{1+\beta^2}}$$

where $\beta = \frac{(1+g^2)k_2+g\prime}{(1+g^2)^{3/2}k_1}$.

The unit binormal vector $\boldsymbol{b}_\phi = \boldsymbol{t}_\phi \wedge \mathbf{n}_\phi$ is calculated by

$$\mathbf{b}_{\phi} = \frac{1}{\sqrt{1+\beta^2}}\left(\frac{g}{\sqrt{1+g^2}}\boldsymbol{t} - \beta \boldsymbol{n_q} - \frac{1}{\sqrt{1+g^2}}\boldsymbol{b_q}\right)$$

The curvature $\kappa = \|\boldsymbol{t_\phi}'\|$ is obtained by

$$\kappa = k_1\sqrt{1+g^2}\sqrt{1+\beta^2}$$

By differantiating $\mathbf{n}_\phi$ we have

$$\begin{aligned}\mathbf{n'}_{\boldsymbol{\phi}} = \frac{1}{\sqrt{1+g^2}\sqrt{1+\beta^2}}\Big[&\Big(g\beta\overline{W} - g'\beta - g\beta' \\ &+ k_1\sqrt{1+g^2} - k_2\beta\Big)\boldsymbol{t} \\ &+ \left(\sqrt{1+g^2}\overline{W} - \frac{gg'}{\sqrt{1+g^2}}\right)\boldsymbol{n_q} \\ &- \left(\beta\overline{W} + k_3\sqrt{1+g^2} + k_2 g\beta - \beta'\right)\boldsymbol{b_q}\Big]\end{aligned}$$

where

$$\overline{W} = \frac{gg'(1+\beta^2) + \beta\beta'(1+\beta^2)}{(1+\beta^2)(1+\beta^2)}$$

Finally, the torsion $\tau_{\boldsymbol{\phi}} = \langle \boldsymbol{n'}_{\boldsymbol{\phi}}, \boldsymbol{b}_{\boldsymbol{\phi}}\rangle$ is obtained by

$$\tau_{\boldsymbol{\phi}} = \frac{\beta'}{1+\beta^2}$$

**Corollary 3.2.** Let $\phi = \phi(s_\phi)$ be D-normal spherical image of a curve $\alpha = \alpha(s)$. If *f* and *g* are constant, then directional normal spherical image $\phi = \phi(s_\phi)$ is a circle in the osculating plane.

**Proof:** Since *f* and *g* are constant, one can easily find $\beta = \frac{f}{\sqrt{1+g^2}}$ and therefore,

$$\kappa_{\boldsymbol{\phi}} = k_1\sqrt{1+f^2+g^2}$$

is constant too.

$$\tau_{\boldsymbol{\phi}} = \frac{\beta'}{(1+\beta^2)} = 0$$

Therefore, the tangent image is a circle in the osculating plane.

**Theorem 3.3.** Let $\theta = \theta(\mathbf{t}_\theta)$ be directional normal indicatrix of the curve with *q* frame ($\mathbf{t}$, $\mathbf{n}_{q\theta}$, $\mathbf{b}_{q\theta}$). Then, the q-frame vectors of $\theta$ can be given by in the following form

$$\mathbf{t}_{\phi} = \frac{\boldsymbol{t} + g\boldsymbol{b}_{\boldsymbol{q}}}{\sqrt{1+g^2}}$$

$$\boldsymbol{n}_{\boldsymbol{q\theta}} = \boldsymbol{n}_{\boldsymbol{q}}$$

and

$$\boldsymbol{b}_{\boldsymbol{q\phi}} = \boldsymbol{t} \wedge \boldsymbol{n}_{\boldsymbol{q\phi}} = \frac{1}{\sqrt{1+g^2}}\left(\boldsymbol{b}_{\boldsymbol{q}} - g\boldsymbol{t}\right)$$

Using definition of q-curvatures of $\phi$ they are calculated by

$$\mathrm{k}_{1\phi} = \frac{1}{\sqrt{1+g^2}}(k_3 g - k_1)$$

$$\mathrm{k}_{2\phi} = \sqrt{1+g^2}k_1\beta$$

and

$$\mathrm{k}_{3\phi} = 0$$

The geondesic curvature of the spherecial image of the directional normal indicatrix of a curve $\alpha$ is calculated by

$$\kappa_g = \frac{\langle \mathbf{t'}_{\boldsymbol{\phi}}, \mathbf{n}_{\mathbf{q}} \wedge \mathbf{t}_{\boldsymbol{\phi}} \rangle}{\left\|\mathbf{t}_{\boldsymbol{\phi}}\right\|^3}$$

By definition, one can find easily geodesic curvature of the spherical image of the directional normal indicatrix with respect to $q$-curvatures

$$\kappa_g = \frac{1}{(k_1^2 + k_3^2)^{3/2}}\left[k_1^2\left(\frac{k_3}{k_1}\right)' - k_2(k_1^2 + k_3^2)\right] \qquad (2)$$

Therefore, $\phi = \phi(s_\phi)$ is a slant helix if and only if the geodesic curvature of the spherical image of the directional normal indicatrix (2) is a constant function.

**Corollary 3.3.** If the second q-curvature vanishes, one can derive the result found in (Izumiya and Tkeuchi, 2004).

## 4. Example

Let us consider a rectifying slant helix given by

$$\alpha(u) = \begin{pmatrix} -\dfrac{\sqrt{u^2+1}\cos(3arc\tan(u))}{3}, \\ -\dfrac{\sqrt{u^2+1}\sin(3arc\tan(u))}{3}, \\ \dfrac{2\sqrt{2}\sqrt{1+u^2}}{3} \end{pmatrix}$$

The $q$-curvatures are calculated by

$$k_1 = sgn(u)\frac{6\sqrt{2}}{(u^2+1)\sqrt{9u^2+17}}$$

$$k_2 = -sgn(u)\frac{8}{(u^2+1)\sqrt{u^2+1}\sqrt{9u^2+17}}$$

and

$$k_3 = \frac{(24u^3+40u)\sqrt{2}}{(u^2+1)\sqrt{u^2+1}(9u^2+17)}$$

Then substituting $q$-curvatures into (2), we have the geodesic curvature of the spherical image of the directional normal indicatrix is a constant function, shown in Figure 2.

Figure 2: Spherical Image of the directional normal indicatrix

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