ISTANBUL UNIVERSITY –
JOURNAL OF ELECTRICAL & ELECTRONICS ENGINEERING

YEAR : 2003 (699-706)
VOLUME : 3
NUMBER : 1

NEW CRITERIA FOR THE EXISTENCE OF STABLE EQUILIBRIUM POINTS IN NONSYMMETRIC CELLULAR NEURAL NETWORKS

Neyir OZCAN[1] Sabri ARIK[2] Vedat TAVSANOGLU[3]

[1, 2] Istanbul University, Department of Electrical-Electronics Engineering
34850 Avcilar, Istanbul, TURKEY

[3]Işık University, Department of Electronics, İstanbul, TURKEY

E-mail: neyir, ariks@istanbul.edu.tr

ABSTRACT

This paper presents new criteria for the existence of stable equilibrium points in the total saturation region for cellular neural networks (CNNs). It is shown that the results obtained can be used to derive some complete stability conditions for some special classes of CNNs such as positive cell-linking CNNs, opposite-sign CNNs and dominant-template CNNs. Our results are also compared with the previous results derived in the literature for the existence of stable equilibrium points for CNNs.

I. INTRODUCTION

Cellular Neural Networks introduced in [1] have been extensively used in the area of image processing. In order for a CNN to operate properly in such applications the designed CNN must be completely stable. (A CNN is said to be completely stable if every trajectory tends to converge to stable equilibrium point.). Complete stability analysis of CNNs has been one of the major problems since there are no sufficient techniques and methods to establish the complete stability conditions for CNNs. In particular, complete stability analysis of nonsymmetric CNNs is more difficult since their dynamical behaviour can exhibit various phenomena such as oscillation, periodic orbit and chaos. So far, only a few results were obtained for the complete stability of a general nonsymmetric CNN, [2][5]. Therefore, the recent results usually focus on finding stable equilibrium points for nonsymmetric CNNs. It is known that the existence of at least one stable equilibrium point is a necessary condition for the complete stability of CNNs. It has been conjectured in [6] that a CNN possessing a stable equilibrium point in the total saturation region is completely stable. This conjecture has been verified by most of the results obtained in the literature. We should also point out here that a CNN possessing a stable equilibrium may not be completely stable. A numerical example of this case is given in [7]. However, so far no one has provided a mathematical proof indicating that a CNN possessing a stable equilibrium is not completely stable. Therefore, complete stability analysis of nonsymmetric CNNs is mainly based on searching for conditions that establish the existence of at least one stable equilibrium point in the total saturation region [6]-[9].

Received Date : 07.06.2002
Accepted Date: 18.12.2002

2. PRELIMINARIES

In this paper, we will consider the model of tha CNN whose dynamical behaviour is described by the following form of state equations:

$$\dot{x} = -x + Ay(x)$$

or

$$\dot{x}_i = -x_i + \sum_{j=1}^{n} a_{ij} y(x_j) \quad \text{for } i = 1,....,n \tag{1}$$

where $x = [x_1 \ , \ . \ . \ . \ , \ x_n]^T$ is the state vector, $A = \{a_{ij}\}$ is the feedback matrix of the system, $y(x) = [y(x_1) \ , \ . \ . \ . \ , \ y(x_n)]^T$ is the output vector and

$$y(x_i) = 0.5|x_i + 1| - 0.5|x_i - 1| \quad \forall i$$

we will also assume that the self-feedback coefficients $a_{ii} > 1, \forall i$. This assumption is important as it has been proved in [10] that under the condition $a_{ii} > 1, \forall i$, a stable equilibrium point can only be in the total saturation region where $|x_i| > 1, \forall i.$ We will now establish some sufficient conditions under which there exists at least one stable equilibrium point in the total saturation region.

We first give some definitions and facts that are important in the context of the stabiliy analysis of CNNs considered in this paper.

Definition 1: Let **B** be a matrix with positive diagonal elements. **B** is called quasi-diagonally column dominant if there exist positive constants $d_i, i = 1,...,n$ such that

$$d_i b_{ii} \geq \sum_{\substack{j=1 \\ j \neq i}}^{n} d_j |b_{ji}| \quad \text{for } i = 1,....n$$

This class of matrices will be denoted by **B** $\in R_0$.

Definition 2 : Let **B** $\in R_0$ and there exists at least one index k such that

$$d_k b_{kk} > \sum_{\substack{j=1 \\ j \neq k}}^{n} d_j |b_{jk}|$$

We will denote this class of matrices by **B** $\in R_0^{\ +}$.

Definition 3 : Let B be a matrix with positive diagonal elements. **B** $\in F_0$. If

$$\sum_{i=1}^{n} b_{ii} \geq \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} |b_{ji}|$$

Definition 4 : Let B be a matrix with positive diagonal elements. **B** $\in F_0^{\ +}$. If

$$\sum_{i=1}^{n} b_{ii} > \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} |b_{ji}|$$

Definition 5 : Let B be a matrix with positive diagonal elements. The comparison matrix **S** of the matrix **B** is defined as $s_{ii} = b_{ii} > 0$ and $s_{ij} = -|b_{ij}|$ for $i \neq j$ .The comparison matrix **S** of **B** is called nonsingular M-matrix (M-matrix) if the real part of every eigenvalue of **S** is positive (nonnegative). This class of matrices is denoted by **B** $\in C, (B \in C_0)$.

Fact 1 : Let **B** be an nxn matrix with positive diagonal elements. If the comparison matrix of **B** is a nonsingular M-matrix, then there exist positive constants $d_i, i = 1,...,n$ such that

$$d_i b_{ii} > \sum_{\substack{j=1 \\ j \neq i}}^{n} d_j |b_{ji}| \quad i = 1,....,n$$

Fact 2 : Fact 1 implies that class R_0 is a strict subclass of the class C.

Fact 3 : Class R_0 is not a subclass of the class F_0.

Proof : Consider the following matrix

$$B = \begin{bmatrix} 1 & -1 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

Note that **B** $\in C$, implying that **B** $\in R_0$. It is easy to see that **B** $\notin F_0$.

Fact 4 : Class F_0 is not a subclass of the class R_0.

Proof : Consider the following matrix

$$B = \begin{bmatrix} 5 & 0 & 1.5 \\ 0 & 5 & 6 \\ 1.5 & 6 & 5 \end{bmatrix}$$

where **B** $\in F_0$. Since **B** has a negative eigenvalue, **B** $\notin R_0$.

In the following, the previous results concerning the existence of stable equilibrium points in CNNs are restated :

Theorem 1 [6] **:** The CNN defined by (1) has at least one stable equilibrium point in the total saturation region if the comparison matrix of **A-I** is a nonsingular M-matrix, **A-I** $\in C$.

In [4], the condition **A-I** $\in C$ was also proved to imply the complete stability of CNNs.

Theorem 2 [4] **:** The CNN defined by (1) is completely stable if **A-I** $\in C$.

Theorem 3 [2] **:** The CNN defined by (1) is completely stable if the matrix **A-I** is strictly diagonally row dominant, that is

$$a_{ii} - 1 > \sum_{\substack{j=1 \\ j \neq i}}^{n} \left| a_{ij} \right|, \quad \forall i$$

Theorem 4 [7] : Let the sequence $\lambda_1, \lambda_2, \ldots., \lambda_n$ be a rearrangement of the sequence $1,2,\ldots.,n$. Then, the CNN defined by (1) has at least one stable equilibrium point in the total saturation region if there exists a sequence of $\lambda_1, \lambda_2, \ldots., \lambda_n$ such that

$$a_{\lambda_i \lambda_i} - 1 > \sum_{j=i+1}^{n} \left| a_{\lambda_i \lambda_i} \right|, \quad \forall i$$

Theorem 5 [8] **:** Consider the CNN where the matrix **A-I** has the following form:

$$A - I = \begin{bmatrix} p-1 & -s_1 & 0 & . & . & 0 \\ s_2 & p-1 & -s_1 & . & . & 0 \\ 0 & s_2 & p-1 & -s_1 & . & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & s_2 & p-1 & -s_1 \\ . & . & . & 0 & s_2 & p-1 \end{bmatrix}$$

where $s_1 > 0,\ s_2 > 0,\ p > 1$. This CNN has at least one stable equilibrium point in total saturation region if $p - 1 > \min(s_1, s_2)$

Theorem 6 [9] **:** The CNN defined by (1) has a least one stable equilibrium point in the total saturation region if the following condition holds:

$$a_{ii} - 1 > \sum_{\substack{j=1 \\ j \neq i}}^{n} a_{ij}, \quad \text{for } \forall i$$

Theorem 7 [11] **:** Let the matrix **A-I** for a two-cell CNN be defined as follows:

$$A - I = \begin{bmatrix} p-1 & -s_1 \\ s_2 & p-1 \end{bmatrix}$$

Then, this two-cell CNN is completly stable if $(p-1)^2 \geq s_1 s_2$

3. NEW CONDITIONS FOR THE EXISTENCE OF A STABLE EQUILIBRIUM POINT

The following theorem presents a new condition that ensures the existence of a stable equilibrium point in the saturation region.

Theorem 8 : Under the assumption $a_{ii} - 1 > 0, \forall i$, there exists a stable equilibrium point in the the total saturation region ($|x_i| > 1, \forall i$) for the system defined by (1) if **A-I** $\in R_0^{+}$.

Proof : The condition $a_{ii} - 1 > 0, \forall i$ implies that there exists no stable equilibrium point in the linear region ($|x_i| \le 1, \forall i$) and in the partial saturation region (some states are saturated and some are not) [10]. Therefore, under the condition $a_{ii} - 1 > 0, \forall i$, a stable equilibrium point can only be in the total saturation region. We will now show that **A-I** $\in R_0^{+}$ implies the existence of a stable equlibrium in the total saturation region.

Now consider the equilibrium equations of the CNN defined by (1):

$$x_i^* = \sum_{j=1}^{n} a_{ij} y(x_i^*) \quad \forall i \qquad (2)$$

where x_i^* is the value of the ith state at the equilibrium point. Multiplying both sides of (2) by $y(x_i^*)$, we obtain

$$\sum_{j=1}^{n} a_{ij} y(x_i^*) y(x_j^*) = y(x_i^*) x_i^* \quad \forall i \qquad (3)$$

We know that $y(x_i^*) x_i^* = |x_i^*|, \forall i$ in the total saturation region, therefore (3) can be written as:

$$\sum_{j=1}^{n} a_{ij} y(x_i^*) y(x_j^*) = |x_i^*| \quad \text{for } i = 1,\dots,n \qquad (4)$$

Since $|x_i^*| > 1, \ \forall i$ defines a total saturation region, we will assume that there exists a stable equilibrium point at which $|x_i^*| = 1 + \varepsilon_i$ where ε_i are sufficiently small positive constants. Hence,(4) can be written as :

$$\sum_{j=1}^{n} a_{ij} y(x_i^*) y(x_j^*) = 1 + \varepsilon_i \quad \forall i$$

or equivalently

$$a_{ii} - 1 + \sum_{\substack{j=1 \\ j \ne i}}^{n} a_{ij} y(x_i^*) y(x_j^*) = \varepsilon_i$$

for $i = 1,\dots,n$ (5)

Now let $p_i \ge 1, \forall i.$ Multiplying both sides of (5) by p_i yields:

$$p_i (a_{ii} - 1) + \sum_{\substack{j=1 \\ j \ne i}}^{n} p_i a_{ij} y(x_i^*) y(x_j^*) = p_i \varepsilon_i \quad \forall i$$

We can write

$$\sum_{i=1}^{n} p_i (a_{ii} - 1) + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \ne 1}}^{n} p_i a_{ij} y(x_i^*) y(x_j^*) = \sum_{i=1}^{n} p_i \varepsilon_i$$

which can also be written as

$$\sum_{i=1}^{n} p_i (a_{ii} - 1) + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \ne 1}}^{n} p_j a_{ji} y(x_j^*) y(x_i^*) = \sum_{i=1}^{n} p_i \varepsilon_i$$

Since $p_i \ge 1, \forall i,$ we can write

$$\sum_{i=1}^{n} p_i (a_{ii} - 1) + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \ne 1}}^{n} p_j a_{ji} y(x_j^*) y(x_i^*) \ge \sum_{i=1}^{n} \varepsilon_i \qquad (6)$$

Inequality (6) holds if the following inequality is satisfied:

$$\sum_{i=1}^{n} p_i (a_{ii} - 1) - \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \ne 1}}^{n} p_j |a_{ji}| |y(x_j^*)| |y(x_i^*)| \ge \sum_{i=1}^{n} \varepsilon_i \qquad (7)$$

Since $|y(x_i^*)| = 1, \forall i$ in the total saturation region, (7) takes the form:

$$\sum_{i=1}^{n} p_i (a_{ii} - 1) - \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \ne 1}}^{n} p_j |a_{ji}| \ge \sum_{i=1}^{n} \varepsilon_i \qquad (8)$$

Let $\sum_{i=1}^{n} \varepsilon_i = \varepsilon > 0.$ Then, (8) can be written as:

$$\sum_{i=1}^{n} \left\{ p_i (a_{ii} - 1) - \sum_{\substack{j=1 \\ j \ne i}}^{n} p_j |a_{ji}| \right\} \ge \varepsilon \qquad (9)$$

If **A-I** $\in R_0^{+}$ then there exist positive constants $p_i \geq 1, \forall i$, such that

$$\sum_{i=1}^{n}\left\{ p_i(a_{ii}-1)-\sum_{\substack{j=1\\ j\neq i}}^{n} p_j \left|a_{ji}\right| \right\} > 0 \qquad (10)$$

Since ε is a sufficiently small positive constant, (10) can be written as

$$\sum_{i=1}^{n}\left\{ p_i(a_{ii}-1)-\sum_{\substack{j=1\\ j\neq i}}^{n} p_j \left|a_{ji}\right| \right\} \geq \varepsilon \qquad (11)$$

Therefore it follows that there exist positive constants ε_i such that $\left|x_i\right| = 1+\varepsilon_i$, $\forall i$ is a solution of the equilibrium point that is in the total saturation region. Hence, we have proved that if **A-I** $\in R_0^{+}$ then the CNN defined by (1) has a stable equilibrium point in the total saturation region.
Theorem 9, given below, directly follows from the analysis of the equilibrium equation of the CNN:

Theorem 9 : The CNN defined by (1) has at least one stable equilibrium point in the total saturation region if **A-I** $\in F_0^{+}$.

Proof : Let $p_i = 1, \forall i$. In this case, (9) can be written as :

$$\sum_{i=1}^{n}(a_{ii}-1)-\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq 1}}^{n}\left|a_{ji}\right| \geq \varepsilon$$

The choice

$$\sum_{i=1}^{n}(a_{ii}-1) > \sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq 1}}^{n}\left|a_{ji}\right|$$

implies the existence of a sufficiently small ε at the equilibrium point, Q.E.D.
In the following, we will improve the conditions given in Theorems 8 and 9 for the existance of stable equilibrium point. To this end, the following fact will be needed :

Fact 5 : Let $\hat{B}$ be a matrix of dimension (n+1)x(n+1) and **B** be the nxn leading principal submatrix of $\hat{B}$ of the following forms :

$$\hat{B} = \begin{bmatrix} b_{11} & b_{12} & . & . & . & b_{1n} & 0 \\ b_{21} & b_{22} & . & . & . & b_{2n} & 0 \\ . & . & . & . & . & . & 0 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & b_{n-1,n} & . \\ b_{n1} & b_{n2} & . & . & b_{n,n-1} & b_{nn} & 0 \\ 0 & 0 & 0 & . & . & 0 & b_{n+1,n+1} \end{bmatrix}$$

$$B = \begin{bmatrix} b_{11} & b_{12} & . & . & . & b_{1n} \\ b_{21} & b_{22} & . & . & . & b_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & b_{n-1,n} \\ b_{n1} & b_{n2} & . & . & b_{n,n-1} & b_{nn} \end{bmatrix}$$

Let $b_{n+1,n+1} > 0$. It is obvious that if **B** $\in R_0$, then $\hat{B} \in R_0^{+}$.
Now consider an (n+1)x(n+1) CNN where

$$\dot{x}_i = -x_i + \sum_{j=1}^{n} a_{ij} y(x_j) \quad \text{for } i = 1,....,n$$

and

$$\dot{x}_{n+1} = -x_{n+1} + a_{n+1,n+1} y(x_{n+1})$$

or

$$\dot{x} = -x + \hat{A} y(x) \qquad (12)$$

where $a_{ii} > 1 \quad i = 1,...,n+1$,
$x = [x_1 \;,\; . \; . \; . \; . \; . \;,\; x_n \;,\; x_{n+1}]^T$,
$y(x) = [y(x_1) \;,\; . \; . \; . \;,\; y(x_n) \;,\; y(x_{n+1})]^T$.

The matrix $\hat{A}$ is given as follows:

$$\hat{A} = \begin{bmatrix} a_{11} & a_{12} & . & . & . & a_{1n} & 0 \\ a_{21} & a_{22} & . & . & . & a_{2n} & 0 \\ . & . & . & . & . & . & 0 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & a_{n-1,n} & . \\ a_{n1} & a_{n2} & . & . & a_{n,n-1} & a_{nn} & 0 \\ 0 & 0 & 0 & . & . & 0 & a_{n+1,n+1} \end{bmatrix}$$

From Theorem 1, we can conclude that the CNN defined by (12) has a stable equilibrium point in the total saturation region if $\hat{A}-\hat{I}\in R_0^+$ [$\hat{I}$ is the (n+1)x(n+1) unity matrix]. On the other hand, by Fact 5, $\hat{A}-\hat{I}\in R_0^+$ if $A-I\in R_0$ where **A** is the nxn leading principal sub-matrix of $\hat{A}$ of the following form:

$$A=\begin{bmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & a_{n-1,n} \\ a_{n1} & a_{n2} & . & . & a_{n,n-1} & a_{nn} \end{bmatrix}$$

Note that the above matrix **A** is the feedback matrix of system (1). Since, the dynamics of system (1) is independent of the dynamical behaviour of the state x_{n+1}, the following result can be derived:

Theorem 10: The CNN defined by (1) has a stable equilibrium point in the total saturation region if $A-I\in R_0$.

The result of Theorem 9 can also be relaxed to the following :

Theorem 11: The CNN defined by (1) has at least one stable equilibrium point in the total saturation region if $A-I\in F_0$.

Now the feedback matrix given in [4] :

$$A=\begin{bmatrix} 1.1 & -0.08 \\ 2 & 1.1 \end{bmatrix}$$

where

$a_{11}=a_{22}=p=1.1, a_{12}=-s_1=-0.08$ and $a_{21}=s_2=2$. This feedback matrix **A** satisfies the conditions given in (6)-(8), thus ensuring the existence of a stable equilibrium point in the total saturation region. It is shown in [4] that (1.02, 3.1) and (-1.02, -3.1) are the stable equilibrium points of the CNN defined by (1). It is also shown in [4] that this CNN has a periodic orbit, which means that the existence of a stable equilibrium point does not directly imply the complete stability of CNNs. Hence, the conditions obtained in (6)-(8) may not always ensure the complete stability for CNNs. We should point out here that the feedback matrix **A** of this example does not satisfy the condition $A-I\in R_0$. On the other hand, we have carried out extensive simulations for CNNs with $A-I\in R_0$ and $A-I\in F_0$. We have not found any unstable CNN whose feedback matrix satisfies one of the conditions $A-I\in R_0$ and $A-I\in F_0$. Based on these simulation results and on the fact that no one, so far, has observed an unstable CNN where $A-I\in R_0$ or $A-I\in F_0$, we make the following conjectures:

Conjecture 1 : The CNN defined by (1) is completely stable if $A-I\in R_0$.

Conjecture 2 : The CNN defined by (1) is completely stable if $A-I\in F_0$.

We will now show that Conjecture 1 is true for two-cell CNNs. We first prove the following fact:

Fact : Let **B** be the 2x2 matrix with $b_{11}>0$ and $b_{22}>0$. Then, $B\in R_0$ if and only if $B\in C_0$.

Proof: Let the matrices **B** and **P** be given as follows :

$$B=\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, \quad P=\begin{bmatrix} 1 & 0 \\ 0 & \frac{b_{11}}{|b_{21}|} \end{bmatrix}$$

We obtain:

$$PB=\begin{bmatrix} b_{11} & b_{12} \\ b_{11}\frac{b_{21}}{|b_{21}|} & b_{22}\frac{b_{11}}{|b_{21}|} \end{bmatrix}$$

$B \in R_0$ if **PB** is diagonally column dominant. Since $b_{11} > 0$, the term $b_{11} - \left| b_{11} \frac{b_{21}}{|b_{21}|} \right| = 0$. Therefore, **PB** is diagonally column dominant if $b_{11}b_{22} - |b_{12}||b_{21}| \geq 0$. The comparison matrix **S** of **B**:

$$S = \begin{bmatrix} b_{11} & -|b_{12}| \\ -|b_{21}| & b_{22} \end{bmatrix}$$

Since $b_{11} > 0$ and $b_{22} > 0$, the real part of the eigenvalues of **S** is nonnegative, if and only if $\nabla(S) = b_{11}b_{22} - |b_{12}||b_{21}| \geq 0$ Hence, $B \in C_0$ implies that $B \in R_0$.

Now consider the two-cell CNN where **A-I** is of the form :

$$A - I = \begin{bmatrix} p-1 & -s_1 \\ s_2 & p-1 \end{bmatrix}$$

where $(p-1) > 0$. According to Conjecture 1, this two-cell CNN is completely stable if $(p-1)^2 \geq s_1 s_2$. In [11], the condition $(p-1)^2 \geq s_1 s_2$ was already proved to be sufficient for the complete stability of a two-cell CNN.

4. CONCLUSIONS

New conditions for the existence of stable equilibrium points for general cellular neural networks have been presented. The conditions obtained do not directly imply complete stability of CNNs. However, complete stability can be conjectured based on the remarks made in [6]. An analytical proof that clarifies the relationship between the existence of a stable equilibrium point and complete stability remains a challenging problem in the stability analysis of nonsymmetric CNNs.

REFERENCES

[1] L. O. Chua and L. Yang, “Cellular neural networks :Theory”, *IEEE Trans. Circuits Syst.*, vol. 35, pp. 1257--1272, October 1988.

[2] F. A. Savaci and J. Vandewalle, “On the stability of cellular neural networks”, *IEEE Trans. Circuits Syst.*, vol. 40, pp. 213-- 215, March 1993.

[3] Mark P. Joy and V. Tavsanoglu, “A new parameter range for the stability of oppositesign cellular neural networks”, *IEEE Trans. Circuits Syst.*, vol. 40, pp. 204207, March 1993.

[4] N. Takahashi and L. O. Chua, “On the complete stability of nonsymmetric cellular neural networks”', *IEEE Trans. Circuits Syst.*, vol. 45, pp.754758, July 1998.

[5] L. O. Chua and T. Roska, ”Stability of a class of nonreciprocal cellular neural networks”, *IEEE Trans. Circuits Syst.*, vol. 37, pp. 1520--1527, December 1990.

[6] S. Arik and V. Tavsanoglu, “Equilibrium analysis of nonsymmetric CNNs”, *International Journal of Circuit Theory and Applications*, vol. 24, pp. 269274, 1996.

[7] N. Takahashi and L. O. Chua, “A new sufficient condition for nonsymmetric CNNs to have a stable equilibrium point”, *IEEE Trans. Circuits Syst.*, vol. 44, pp.10921095, November 1997.

[8] P. P. Civalleri and M. Gilli, “Practical Stability Criteria for Cellular Neural Networks”, *Electronic Letters*, vol. 33, pp. 970971, 1997.

[9] S. Arik and V. Tavsanoglu, “A sufficient condition for the existence of stable equilibrium point in nonsymmetric cellular neural networks”, *in the Proc. 1998 IEEE International Workshop on Cellular Neural Networks and their Applications CNNA'98*, pp.7477, April 1998, London.

[10] F. Zou and J. A. Nossek, “Bifurcation and chaos in cellular neural netweorks”, *IEEE Trans. Circuits Syst.*, vol. 40, pp. 166--173, March 1993.

[11] N. Takahashi and T. Nishi, “On the stability of twocell cellular neural networks with oppositesign connections”, *in the Proc. 2001 European Conference on Circuit Theory and Design ECCTD'01*, vol. 3, pp.9396, Finland, August 2001.

Neyir OZCAN was born in Bursa, Turkey in 1978. She received the B.Sc. degree in 2001 from Istanbul University, Istanbul, Turkey. She is currently an M.Sc. student at the the same University. She is also a research assistant at the Department of Electrical and Electronics Engineering, Istanbul University. Her research interests are Neural Networks and Linear and Nonlinear Systems.