DOI: 10.17065/huniibf.463143

WORKLOAD FORECASTING UNDER ERRATIC DEMAND USING FAST FOURIER TRANSFORM ALGORITHM

Hacettepe University Journal of Economics and Administrative Sciences
Vol. 38, Issue 1, 2020
pp. 97-114

Onur KOYUNCU
Assist.Prof.Dr., Hacettepe University
Faculty of Economics and Administrative Sciences
Department of Business Administration
okoyuncu@hacettepe.edu.tr

Erdem TURFAN
Program Manager
Dassault Systémes
erdemturfan@yahoo.com

Abstract: Forecasting process may become cumbersome under erratic demand structures where no conventional methodology seems to provide plausible outcomes. The company investigated in this paper is subject to this condition and needs to forecast the workload in order to plan the indispensable process enhancement activities. Two parameters to be forecasted are the total production volume and the ratio of in-house production ratio with respect to the total production. The required forecasts for relatively regular structured in-house production to total production ratio are computed via static forecasting. The more important erratic structured total production volume forecasts are computed using Fast Fourier Transform algorithm. Total production is further observed during the next year to check the reliability of the forecasts. The resulting respective Mean Absolute Percentage Errors of approximately 11% (in-house ratio) and 14 - 22% (total production) are considered acceptable under the erratic demand structure.

Keywords: *Forecasting, erratic demand, discrete fourier transform, fast fourier transform.*

Makale Başvuru Tarihi/Manuscript Received: 24/09/2018
Makale Kabul Tarihi/Manuscript Accepted: 17/02/2020

DOI: 10.17065/huniibf.463143

HIZLI FOURIER DÖNÜŞÜMÜ ALGORİTMASI İLE DÜZENSİZ TALEP ALTINDA İŞ YÜKÜ ÖNGÖRÜSÜ

Hacettepe Üniversitesi
İktisadi ve İdari Bilimler Fakültesi Dergisi
Cilt 38, Sayı 1, 2020
s. 97-114

Onur KOYUNCU
Dr.Öğr.Üyesi, Hacettepe Üniversitesi
İktisadi ve İdari Bilimler Fakültesi
İşletme Bölümü
okoyuncu@hacettepe.edu.tr

Erdem TURFAN
Program Yöneticisi
Dassault Systémes
erdemturfan@yahoo.com

Öz: Düzensiz talep altında öngörüde bulunma oldukça zahmetli bir süreç olup, klasik öngörü yöntemlerinden verim almak oldukça zordur. Bu çalışmada incelenen firmanın üretim talep yapısı da düzensiz seyretmektedir ve firma vazgeçilmez süreç geliştirme faaliyetlerini planlayabilmek için iş yükü öngörüsüne ihtiyaç duymaktadır. Öngörülecek iki parametre toplam üretim hacmi ve fason üretim miktarının toplam üretime oranı şeklinde belirlenmiştir. Göreceli daha düzenli bir yapı sergileyen fason üretim oranı öngörüleri durağan öngörü yöntemiyle gerçekleştirilmiştir. Daha kritik olan düzensiz toplam üretim miktarı ise Hızlı Fourier Dönüşüm algoritmasıyla hesaplanmıştır. Toplam üretim parametresi ayrıca tahmin güvenilirliği ölçümü amacıyla bir yıl boyunca gözlenmiştir. Gözlenen Ortalama Mutlak Hata Yüzdeleri %11 (fason üretim oranı) ve %14-%22 (toplam üretim miktarı) olarak hesaplanmış ve düzensiz talep göz önüne alındığında kabul edilebilir olarak değerlendirilmiştir.

Anahtar Sözcükler: *Öngörü, düzensiz talep, kesikli fourier dönüşümü, hızlı fourier dönüşümü.*

Makale Başvuru Tarihi/Manuscript Received: 24/09/2018
Makale Kabul Tarihi/Manuscript Accepted: 17/02/2020

INTRODUCTION

Management science has been providing answers to many planning problems faced in production discipline including those about inventory management, lot sizing and scheduling, workforce and aggregate planning. Most of the proposed approaches either start with the assumption of deterministic demand or that the researcher actually has an idea of the underlying probability distribution of a possible stochastic demand. Unless the demand is deterministic, it must be forecasted. Even in the absence of these assumptions, it is always possible to utilize simple approaches like demand levelling where an excess demand of one period may be transferred to the next in order to smooth out the general demand structure.

Obviously, well behaving data sets containing dominant explainable components are easier to forecast, whereas data sets which display random or erratic behaviour are harder to be represented by a functional form. As noted by Treharne and Sox (2002) and Shang (2012), it is very rare to deal with a real life demand data which does not show nonstationary behaviour. It is common practice to resort to regression or time series based analyses when they are applicable. Problems start to emerge under the absence of a set of explaining variables or when certain stationarity assumptions such as mean, variance and autocorrelation structure are violated. Using conventional methods, it is even harder to obtain a plausible forecast with an added fact of randomly fluctuating data or an erratic behaviour. Whereas the first option is to bind the problem with assumptions instead of dealing with the seemingly untameable data (Shang, 2012), depending on the case tackling the data as it is an inevitable task. In such cases, the remedy would be any methodology with sufficiently acceptable outcomes.

This study examines such a case. The company in question is a branch of one of the main actors of defence industry settled in Turkey. They produce their own products along with in-house products as a subcontractor. Process enhancement activities are considered indispensable whereas they must be planned for periods with relatively lower workload for both internal and external products. For this purpose, the company decided to benefit from demand forecasting. By identifying the periods which they were likely to face low production volumes as well as a low in-house production ratio, they would have more admissible conditions for the purpose.

The remainder of this study is organised as follows. Section one provides a literature review along with information on data and methodology. Section two consists of the theoretical framework, analyses and findings, followed by a conclusion.

1. LITERATURE REVIEW AND METHODOLOGY

This paper aims to provide consistent forecasts of total amount of monthly production and of a reliable ratio of in-house production over total production for a leading defence industry company from Turkey. Based on these objectives two research problems are constructed:

- Analysis and forecast of the in-house production ratio
- Analysis and forecast of the total production

The past data of interest was obtained from the company. These included the total and contracted amounts of in-house production for the convenient January 2011–December 2015 period. The problems were tackled with a time dependent manner since the total production volume did not present any structural shape and showed no evidence of a possible causal model approach that could have been attempted to carry out. For the first problem, the time frame was narrowed down to January 2014–December 2015 period. The time frame for the second problem was February 2012–December 2015.

First problem was rather easily solved via static forecasting which is basically a time dependent regression model utilising a seasonal effect component. The method is similar to those presented by Ferbar Tratar *et al*. (2016) and Ferbar Tratar and Strmčnik (2016) except that the parameters are not updated with each newly observed demand.

Second problem could not be forecasted using conventional forecasting approaches due to its random and erratic behaviour, so FFT (Fast Fourier Transform) (Van Loan, 1992), which provided the most acceptable outcome, was employed. The second research problem, estimating the total production volumes which present erratic behaviour, is the main target of the study. Some possible reasons for the erratic behaviour are considered to be exchange rate volatility and changes in economic conditions (Song, Zipkin, 1993).

Even though FTs (Fourier Transforms) are widely utilized for forecasting purposes, this is not the case in operations management. In Table 1, we present a summary of studies which involve Fourier transform for forecasting purposes. Most of these cases deal with data sets with obvious trend and/or seasonal structures (ie, González-Romera *et.al*., 2008; Beiraghi, Ranjbar, 2011, Dong *et.al*., 2011; Ichinose *et. al*., 2012, Zong-chang, 2013) and use FT along with other methods as noted. Nevertheless, FTs have not been very popular in industrial demand forecasting literature where they might be used as a solution to forecast erratic behaviour which is common to these type of data sets.

Table 1. A Summary of Forecasting Literature Using FTs

Problem focus	**Methodology**	**Reference**
Electricity demand	FT and regression	Yukseltan *et al.*, 2017
Electricity demand	FT and neural networks	González-Romera *et al*., 2008
Electricity demand	FT and ARIMA	Beiraghi, Ranjbar, 2011
Stock market volatility	FT and time series analysis	Malliavin, Mancino, 2002
Stock market volatility	FT and non-harmonic analysis	Ichinose *et al.*, 2012
Real time urban water demand	FT and support vector regression	Brentan *et al.*, 2017
Wind power	FT and discrete wavelet transform	Bitaraf *et al.*, 2015
Traffic flow	FT and AR and neighbourhood regression	Dong *et al*., 2011
Air temperature movement	FT	Zong-chang, 2013
Monthly reservoir inflows	FT	Yu *et al.*, 2018
Fashion	FT	Fumi *et al.*, 2013

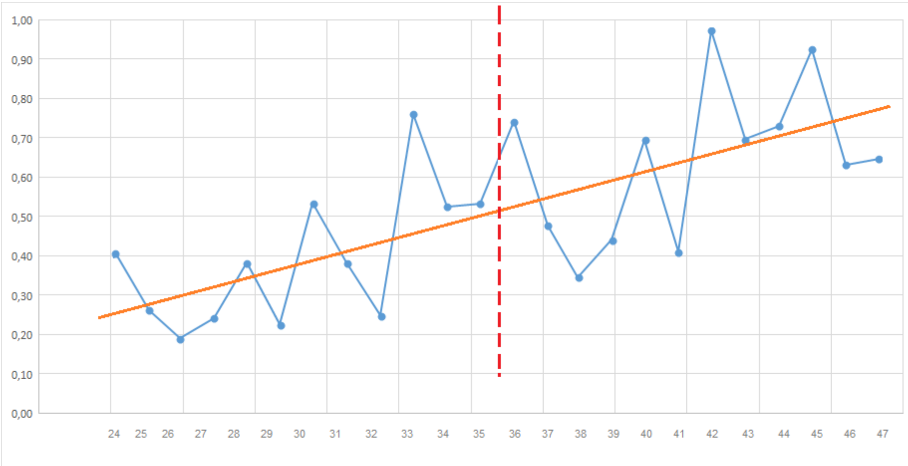
Forecasting smooth demand is relatively easy. There are many studies on lumpy and intermittent demand. Researchers also focused on forecasting erratic demand but the volume of these studies are considered less than expected whereas erratic demand is as common as the other types. The methodologies vary, and FT is not one of the most popular methods. Zhang *et al*. (2010) utilized support vector machines. Amjady *et al*. (2010) practised a two level forecast approach where the lower level consists of neural networks and evolutionary algorithms and the upper level optimises the performance via an enhanced differential evolution algorithm. Onyeocha *et al*. (2014) studied the erratic demand's underlying statistical distributions via discrete event simulations. Molina *et al*. (2016) compared artificial neural networks and ARIMA performances under erratic demand in medicine industry. We propose using FFT directly for forecasting erratic demand.

2. ANALYSIS AND DISCUSSIONS

2.1. Forecasting the in-House Production Ratio

At the start of the analysis, total production amount and in-house production ratio were both investigated. It was noticed at this point that the in-house production ratio exhibited a characteristic pattern, that of a typical run including a level, a trend and seasonality at the same time. Figure 1 presents the proportioned data. Along with the ratio run, linear regression line and a border showing the periodicity for the seasonal analysis is also added. The periodicity, 12, was observed as a monthly change through a whole year. The seasonal similarity of the two years can readily be seen around the dashed line. On all the graphs, months are presented as numbers starting from 1 denoting February, 2012.

Figure 1. The Proportion of In-house Production over Total Production



To forecast the in-house production ratio CPF_p, we propose an additive static forecast model;

$$CPF_p = L + T * p + SF_p + u_p \tag{1}$$

where p denotes the period, L is the basic level, T is the trend, SF_p is the seasonal factor of period p and u_p is the random error term.

After smoothing the data with respect to periodicity using moving averages to obtain deseasonalised proportions, a linear regression was carried out where p is the explaining variable. The resulting equation where DS_p denotes the deseasonalised data was found to be;

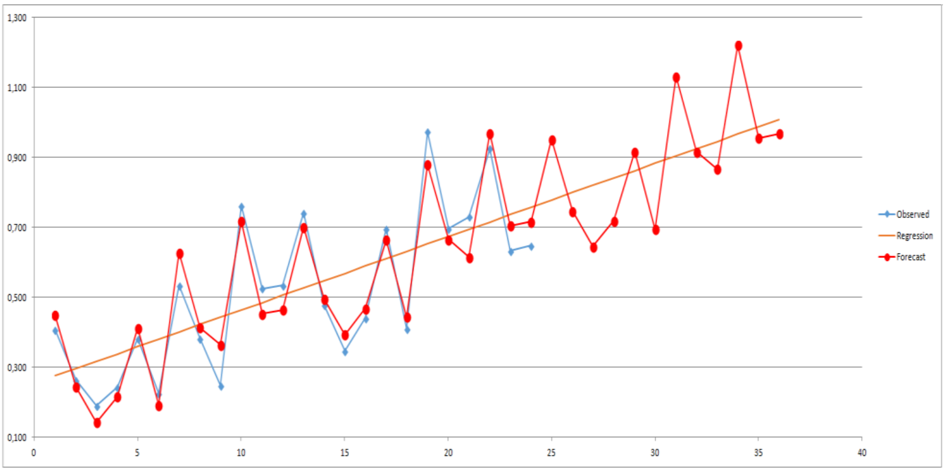
$$\widehat{DS}_p = 0.02097 + 0.25427 * p$$

The differences of real proportions and the proportions estimated via regression are found and averaged period wise (with lag 12) to obtain the seasonal factors to be added. The seasonal factor array for the 12 month period is;

$$SF_p = \{0.174 - 0.052 - 0.175 - 0.122 * 0.054, -0.188 * 0.227, -0.008 - 0.079 * 0.254 - 0.032 - 0.041\}$$

Finally the seasonal factors are added to the regression outcomes to get the final forecasts. The resulting errors have a standard deviation of 0.06 and a MAPE of 0.11 which is considered acceptable. Forecasts are presented in Figure 2, along with the original data and the regression line.

Figure 2. Static Forecast of In-house Production Ratio



The seasonal structure is promising in terms of better future planning of process enhancement activities. It may be expected that the in-house production will consume most of the production capacity in July and October; therefore process enhancement activities should take place during August and September where the in-house production ratios are relatively lower. Because of the linear structure, it may be argued that the in-house production ratio may, at some point, exceed 100 % which would then imply a possible need for a capacity expansion. Furthermore, the linearity assumption is not always realistic and recalculations are required as new data accumulates.

2.2. Forecasting the Total Production

The only factor affecting the process enhancement activities is not the in-house production ratio; the company is also interested in forecasting the total production volume. The total production volume has a slight upward trend, yet it does not fit into any conventional category for forecasting due to its erratic nature. Owing to the lack of any causal variables to explain the high volatility and the fact that data does not respond well to other regression, smoothing or time based methodologies, the effort was made to obtain a deterministic mathematical procedure. A data set should not include a noise term if it needs to be described mathematically. In this sense, first we examined the data to verify an obvious lack of random change.

2.2.1. Cross-Period Correlation Analysis

It is common to propose that unless two variables are correlated, at least one of them is regarded as noise and the two cannot be bound using a mathematical model. With the aid of this fact, 28 subsets of the 47-period data set were constructed in a rolling horizon approach (i.e. the first set consisted of periods 1 to 20, the second set consisted of periods 2 to 21 and so on). Pearson correlations of these 28 subsets were calculated. Figure 3 shows the matrix correlation heat map and Figure 4 shows the 3D correlation change graph both of which were analysed in Matlab. P values for significance of the calculated correlations are given in Figure 5.

Figure 3. Correlation Heat Map Graph of Period Subsets

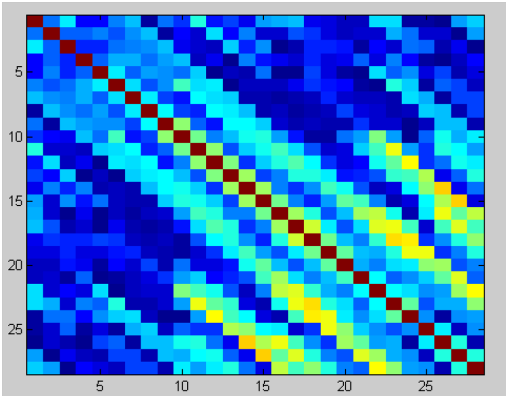


Figure 4. Pearson Correlation Change Graph of Period Subsets

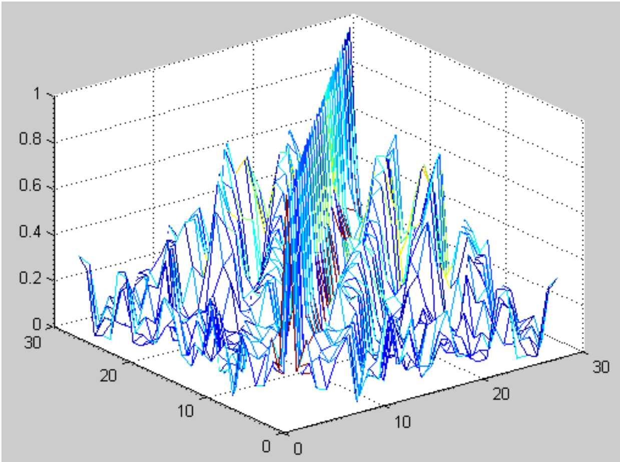


Figure 5. Correlation Matrix of the Samples

Set	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
1	0,00	0,34	0,13	0,59	0,63	0,30	0,18	0,91	0,32	0,46	0,08	0,56	0,46	0,66	0,25	0,21	0,18	0,67	0,68	0,81	0,81	0,14	0,15	0,92	0,91	0,97	0,24	0,17
2	0,34	0,00	0,35	0,35	0,31	0,39	0,26	0,20	0,93	0,59	0,76	0,15	0,63	0,29	0,74	0,77	0,38	0,49	0,71	0,80	0,74	0,77	0,22	0,34	0,75	0,75	0,96	0,24
3	0,13	0,35	0,00	0,50	0,51	0,32	0,29	0,27	0,18	0,67	0,77	0,83	0,17	0,52	0,25	0,98	0,85	0,54	0,51	0,58	0,92	0,59	0,74	0,33	0,37	0,83	0,68	0,95
4	0,59	0,35	0,50	0,00	0,36	0,21	0,20	0,34	0,23	0,20	0,79	0,65	0,96	0,30	0,55	0,66	0,76	0,51	0,70	0,64	0,37	0,71	0,31	0,62	0,92	0,93	0,78	0,63
5	0,63	0,31	0,51	0,36	0,00	0,12	0,36	0,24	0,28	0,31	0,17	0,93	0,80	0,87	0,31	0,98	0,95	0,44	0,69	0,88	0,93	0,13	0,37	0,25	0,72	0,45	0,86	0,72
6	0,30	0,39	0,32	0,21	0,12	0,00	0,15	0,42	0,26	0,05	0,13	0,12	0,98	0,78	0,79	0,81	0,64	0,46	0,58	0,63	0,91	0,96	0,07	0,16	0,89	0,38	0,45	0,82
7	0,18	0,26	0,29	0,20	0,36	0,15	0,00	0,12	0,44	0,57	0,11	0,15	0,16	0,86	0,84	0,90	0,87	0,80	0,50	0,75	0,83	0,82	0,86	0,14	0,22	0,89	0,44	0,44
8	0,91	0,20	0,27	0,34	0,24	0,42	0,12	0,00	0,11	0,46	0,52	0,11	0,13	0,18	0,88	0,80	0,91	0,86	0,79	0,45	0,69	0,75	0,85	0,81	0,17	0,20	0,86	0,43
9	0,32	0,93	0,18	0,23	0,28	0,26	0,44	0,11	0,00	0,02	0,37	0,56	0,11	0,11	0,17	0,74	0,86	0,82	0,86	0,84	0,48	0,73	0,75	0,95	0,78	0,18	0,18	0,86
10	0,46	0,59	0,67	0,20	0,31	0,05	0,57	0,46	0,02	0,00	0,03	0,29	0,18	0,09	0,08	0,22	0,58	0,92	0,92	0,34	0,23	0,02	0,24	0,97	0,35	0,09	0,21	0,13
11	0,08	0,76	0,77	0,79	0,17	0,13	0,11	0,52	0,37	0,03	0,00	0,02	0,40	0,16	0,10	0,06	0,15	0,40	0,76	0,72	0,16	0,06	0,01	0,23	0,50	0,08	0,11	0,24
12	0,56	0,15	0,83	0,65	0,93	0,12	0,15	0,11	0,56	0,29	0,02	0,00	0,03	0,50	0,16	0,07	0,05	0,12	0,46	0,72	0,62	0,09	0,04	0,00	0,47	0,32	0,09	0,12
13	0,46	0,63	0,17	0,96	0,80	0,98	0,16	0,13	0,11	0,18	0,40	0,03	0,00	0,02	0,45	0,30	0,09	0,08	0,13	0,56	0,54	0,42	0,07	0,06	0,01	0,64	0,36	0,09
14	0,66	0,29	0,52	0,30	0,87	0,78	0,86	0,18	0,11	0,09	0,16	0,50	0,02	0,00	0,02	0,29	0,46	0,26	0,06	0,09	0,40	0,78	0,69	0,11	0,02	0,00	0,68	0,33
15	0,25	0,74	0,25	0,55	0,31	0,79	0,84	0,88	0,17	0,08	0,10	0,16	0,45	0,02	0,00	0,04	0,28	0,54	0,23	0,04	0,06	0,31	0,87	0,68	0,11	0,01	0,00	0,69
16	0,21	0,77	0,98	0,66	0,98	0,81	0,90	0,80	0,74	0,22	0,06	0,07	0,30	0,29	0,04	0,00	0,02	0,09	0,39	0,18	0,02	0,01	0,09	0,85	0,13	0,01	0,04	0,01
17	0,18	0,38	0,85	0,76	0,95	0,64	0,87	0,91	0,86	0,58	0,15	0,05	0,09	0,46	0,28	0,02	0,00	0,01	0,13	0,39	0,14	0,01	0,01	0,06	0,74	0,05	0,01	0,05
18	0,67	0,49	0,54	0,51	0,44	0,46	0,80	0,86	0,82	0,92	0,40	0,12	0,08	0,26	0,54	0,09	0,01	0,00	0,03	0,18	0,28	0,07	0,00	0,00	0,50	0,28	0,08	0,02
19	0,68	0,71	0,51	0,70	0,69	0,58	0,50	0,79	0,86	0,92	0,76	0,46	0,13	0,06	0,23	0,39	0,13	0,03	0,00	0,04	0,19	0,22	0,08	0,01	0,01	0,50	0,30	0,08
20	0,81	0,80	0,58	0,64	0,88	0,63	0,75	0,45	0,84	0,34	0,72	0,72	0,56	0,09	0,04	0,18	0,39	0,18	0,04	0,00	0,22	0,30	0,15	0,12	0,02	0,02	0,40	0,30
21	0,81	0,74	0,92	0,37	0,93	0,91	0,83	0,69	0,48	0,23	0,16	0,62	0,54	0,40	0,06	0,02	0,14	0,28	0,19	0,06	0,00	0,10	0,36	0,27	0,16	0,04	0,01	0,37
22	0,14	0,77	0,59	0,71	0,13	0,96	0,82	0,75	0,73	0,02	0,06	0,09	0,42	0,78	0,31	0,01	0,01	0,07	0,22	0,30	0,10	0,00	0,11	0,14	0,27	0,25	0,02	0,00
23	0,15	0,22	0,74	0,31	0,37	0,07	0,86	0,85	0,75	0,24	0,01	0,04	0,07	0,69	0,87	0,09	0,01	0,00	0,08	0,15	0,36	0,11	0,00	0,04	0,38	0,21	0,21	0,02
24	0,92	0,34	0,33	0,62	0,25	0,16	0,14	0,81	0,95	0,97	0,23	0,00	0,06	0,11	0,68	0,85	0,06	0,00	0,01	0,12	0,27	0,14	0,04	0,00	0,27	0,89	0,25	0,24
25	0,91	0,75	0,37	0,92	0,72	0,89	0,22	0,17	0,78	0,35	0,50	0,47	0,01	0,02	0,11	0,13	0,74	0,50	0,01	0,02	0,16	0,27	0,38	0,27	0,00	0,13	0,81	0,27
26	0,97	0,75	0,83	0,93	0,45	0,38	0,89	0,20	0,18	0,09	0,08	0,32	0,64	0,00	0,01	0,01	0,05	0,28	0,50	0,02	0,04	0,25	0,21	0,89	0,13	0,00	0,08	0,84
27	0,24	0,96	0,68	0,78	0,86	0,45	0,44	0,86	0,18	0,21	0,11	0,09	0,36	0,68	0,00	0,04	0,01	0,08	0,30	0,40	0,01	0,02	0,21	0,25	0,81	0,08	0,00	0,09
28	0,17	0,24	0,95	0,63	0,72	0,82	0,44	0,43	0,86	0,13	0,24	0,12	0,09	0,33	0,69	0,01	0,05	0,02	0,08	0,30	0,37	0,00	0,02	0,24	0,27	0,84	0,09	0,00

In Figure 5, the red cells the null hypothesis of no correlation is rejected at a 95% confidence level and thus reveal strong correlations (greater than 0.50). The yellow cells indicate marginal significance at a 90% confidence level and are therefore moderately strong correlations (0.50 to 0.30). The remaining green cells indicate low or no correlations. Even though the green cells are higher in frequency, there is sufficient evidence of correlation. Therefore we proceed to the FFT in order to model the total production volumes.

2.2.2. Fast Fourier Transform

Fourier Transform (FT) (Bracewell, 1965) is a method to map and transform a function in time domain to frequency domain. Discrete Fourier Transform (DFT) is a special case of Fourier Transform where the function consists of equally spaced samples. Computing an N-point DFT for N samples of a FT means applying a computation of N points on a unit circle on z plane N equally spaced samples ($w_k = 2\pi k/N$). FFT's (Van Loan, 1992) are efficient algorithms used for computing N-point DFTs which provide less computational complexity.

DFT computation for a series $\{x_k\}_{k=1}^{N}$ is as follows;

$$c_k = \sum_{j=0}^{N-1} x_k w_N^{jk}, k = 0, \ldots, N-1 \quad (2)$$

where $w_N = e^{-j(2\pi/N)}$.

The idea is to convert the total production data from time domain to frequency domain using FFT and to analyse the dominant frequencies. For this purpose, the previously constructed 28 samples of 20 elements were converted to frequency domain via FFT. As long as the resulting distribution has a dominant frequency in most of the samples, it may be put forward that the data set can be modelled by a mathematical function. A dominant sinusoidal frequency observed in the main function must also be dominant in other functions which derive from the main function.

$$f = A_1 * sin(B_1 * t - C_1) + A_2 * sin(B_2 * t - C_2) + \cdots + A_n * sin(B_n * t - C_n) \quad (3)$$

For the generic function (3), suppose $A_1 * sin(B_1 * t - C_1)$ function is as shown on Figure 6 and $A_2 * sin(B_2 * t - C_2)$ function is as shown on Figure 7. Then function f which is a summation of these two terms will be as Figure 8.

Figure 6. $A_1 * sin(B_1 * t - C_1)$ **Function**

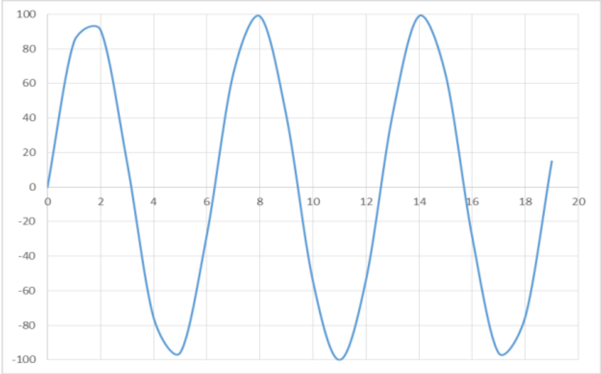


Figure 7. $A_2 * sin(B_2 * t - C_2)$ **Function**

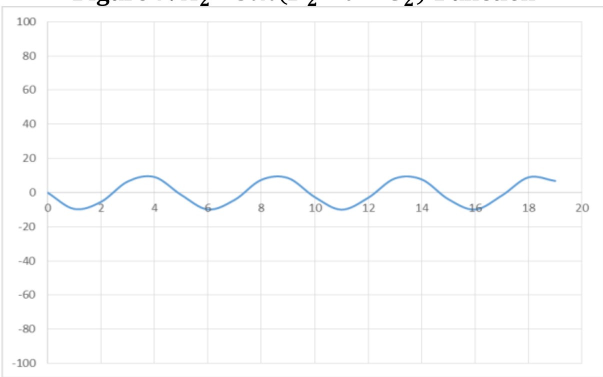
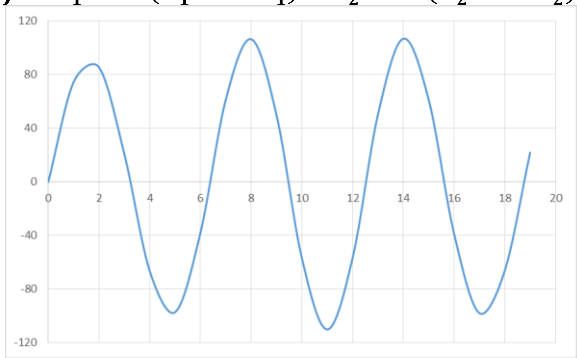
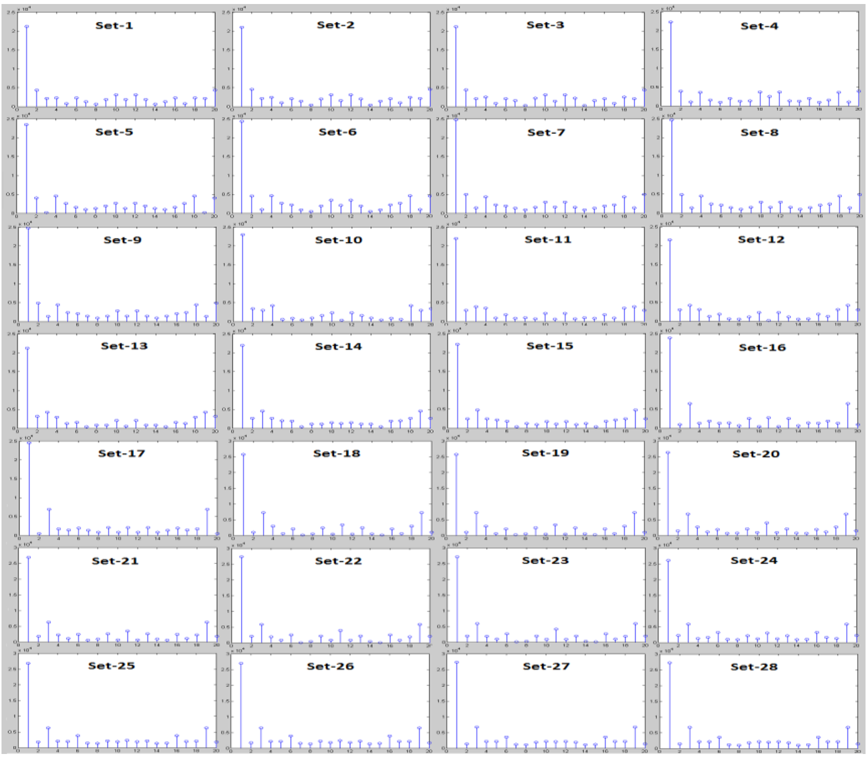


Figure 8. $f = A_1 * sin(B_1 * t - C_1) + A_2 * sin(B_2 * t - C_2)$ **Function**



It can clearly be observed from Figure 8 that the dominant sub function is $A_1 * sin(B_1 * t - C_1)$ in which case the dominant frequency is $f = 1/B_1$. In our case, the samples were transformed via FFT using Matlab and the outcome is shown in Figure 9.

Figure 9. FFT Outcomes of 28 Samples



It can readily be seen that the dominant frequency is the first one for all the sets, followed by third and the last. This means that the original data set can be explained by a mathematical model.

2.2.3. Mathematical Forecast

The aim is to model the total production volume mathematically, therefore we decided on a sinusoidal function of the generic type $Y = A*sin(B*t-C)$ which is similar to the fluctuations of the original data. The terms A, B and C represent amplitude, period and phase respectively. Emanating from the variability observed in the original production volume data, it is straightforward to conclude that all these three parameters vary dynamically. Hence, the main function is a summation of sub functions of the sinusoidal form. We represent this as;

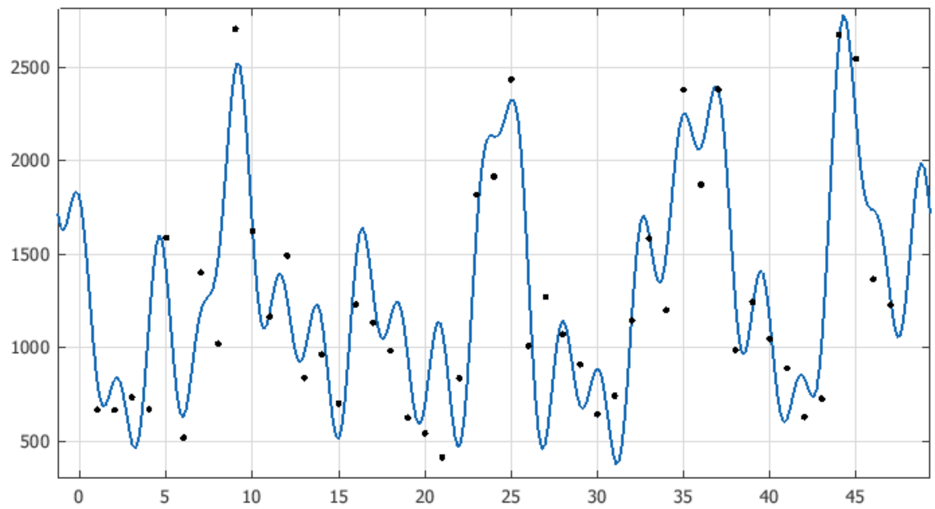
$$Y = \sum_{j} a_j * sin(b_j * t + c_j) \tag{4}$$

The Matlab solution for the parameters for $j = \{1,..,8\}$ is presented in Table 2 and the resulting function can be observed in Figure 10.

Table 2. Main Function Coefficients

a_1	8777,0	b_1	0,062	c_1	0,123
a_2	7615,0	b_2	0,069	c_2	3,111
a_3	434,3	b_3	0,501	c_3	2,538
a_4	400,7	b_4	0,682	c_4	2,793
a_5	261,2	b_5	0,931	c_5	-1,929
a_6	276,7	b_6	1,588	c_6	0,187
a_7	292,9	b_7	2,701	c_7	1,748
a_8	196,4	b_8	1,232	c_8	3,119

Figure 10. Total Production Volume Function over Time

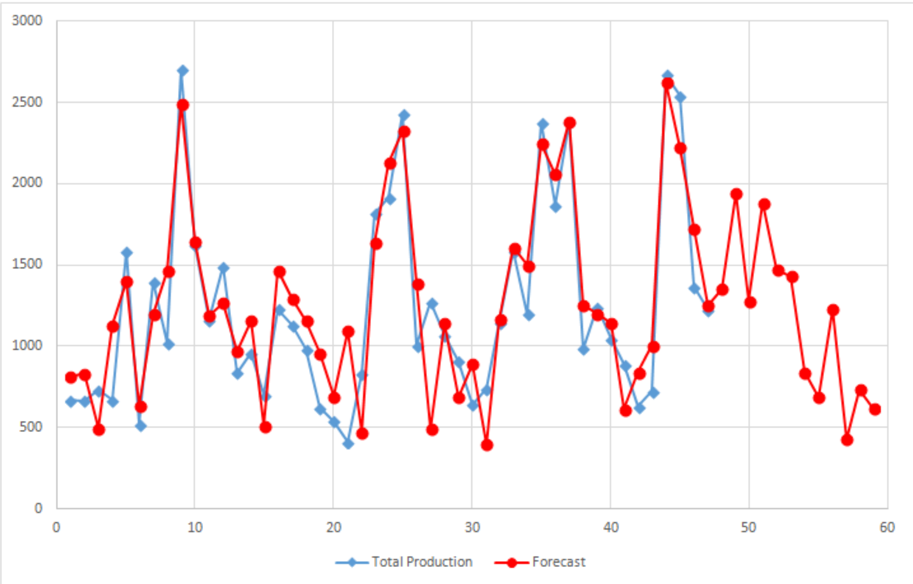


The MAPE for the resulting in-sample forecast was found as 24.11% which is somewhat large yet acceptable considering the erratic behaviour of the present data. It was considered as insightful in order to carry out the appropriate process enhancement activity planning. Under the expectation that the dominant sub functions will keep on effecting the change in production volumes, forecasts were made for the upcoming year. These are given in Table 3 and visualised along with past data and in-sample forecasts in Figure 11.

Table 3. Total Production Forecasts for 2016

Period	Forecasted Production	Period	Forecasted Production
Jan 16	1355	Jul 16	841
Feb 16	1942	Aug 16	689
Mar 16	1278	Sep 16	1229
Apr 16	1883	Oct 16	430
May 16	1471	Nov 16	735
Jun 16	1432	Dec 16	618

Figure 11. Total Past Production and Forecasts for 2016



CONCLUSION

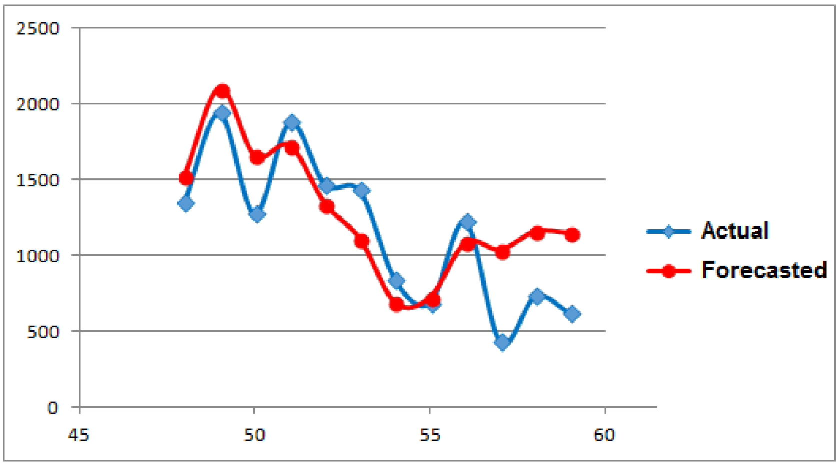
It is evident that modelling demand mathematically is a rather farfetched assumption unless the demand has an obvious reason to follow it. Causal models satisfy this assumption by definition, and function based models can only hope for a good future fit. There is no other way to verify this expectation other than observing the future data. We took this path, expecting actual data to match our forecasts, implying that sinusoidal functions stand for possible causal variables which were unknown.

Following actual production data was observed during year 2016 to analyze the efficiency of the forecasts. We present the values in Table 4 and the graph of actual and forecasted values for the in-house production amount which was the main concern of the study in Figure 12.

Table 4. Actual and Forecasted Production Volumes for 2016

Period	Forecasted Production	Actual Production	Absolute Error	Absolute Percentage Error
Jan 16	1355	1528	173	0.113
Feb 16	1942	2106	164	0.078
Mar 16	1278	1659	381	0.229
Apr 16	1883	1728	155	0.089
May 16	1471	1344	127	0.094
Jun 16	1432	1110	322	0.290
Jul 16	841	690	151	0.218
Aug 16	689	728	39	0.053
Sep 16	1229	1086	143	0.131
Oct 16	430	1040	610	0.586
Nov 16	735	1162	427	0.367
Dec 16	618	1149	531	0.462

Figure 12. Actual and Forecasted Production Volume for 2016



It can be observed from Figure 12 that DFT can actually estimate the changes even for such a volatile data structure. Forecast errors increase towards the end of the year, which can be accepted as a common phenomena based on previous studies applying different methods. This was the case for Zhang *et al.* (2010), Amjady *et al.*

(2010), Ichinose *et al*. (2012) and Kochak and Sharma (2015) who all used different methods for forecasting erratic patterns.

Apart from the last three months of 2016, the forecasts have a tendency to predict the actual values with a MAPE of 14.4 % which rises to a 22.6 % including the off-target values during the last quarter of 2016. Another possible advantage is that, even in the long run companies may benefit from the fact that FFT almost always is able to predict the direction change even when the discrepancy between the forecast and the actual value is relatively high. The correlations of two series are 0.885 for the first nine months and 0.782 for the 12-month period. We believe these outcomes are useful, and convincing in terms of MAPEs they reveal, for a data set which reveals no inner structural consistency and does not allow for any conventional estimation approaches. It is not surprising to obtain MAPEs well above 40 % (Fumi *et al*., 2013) which may still be deemed useful depending on the case.

The company benefited from the forecasts, planning their process enhancement activities during the June – August period where the production volume was expected to follow a local minimum and that was actually the case. Even though, later on October production turned out to be even lower than that of August's, a two month minimum would have been preferred since those activities took more than a month to complete. FFT performed well in defining the low demand periods without the aid of additional methodologies, yet improvements are deemed necessary if the forecasted demand is used directly as in an inventory planning problem. Researching a possible relation between FFT coefficients and cause variable coefficients of a multivariate regression, when they are visible and accessible, could justify the direct use of already well performing FFT for forecasting erratic demand.

REFERENCES

Amjady, N., F. Keynia, H. Zareipour (2010), "Short-Term Load Forecast of Micrograms by a New Bilevel Prediction Strategy", *IEEE Transactions on Smart Grid*, 1(3), 286-294.

Beiraghi, M., A.M. Ranjbar (2011), "Discrete Fourier Transform Based Approach to Forecast Monthly Peak Load", *Asia-Pacific Power and Energy Engineering Conference,* 1 – 5.

Bitaraf, H., S. Rahman, M. Pipattanasomporn (2015), "Sizing Energy Storage to Mitigate Wind Power Forecast Error Impacts by Signal Processing Techniques", *IEEE Transactions on Sustainable Energy*, 6(4), 1457-1465.

Bracewell, R. (1965), "The Fourier Transform and its Applications", New York: McGraw-Hill.

Brentan, B.M., Jr., E. Luvizotto, M. Herrera, J. Izquierdo, R. Pérez-García (2017), "Hybrid Regression Model For Near Real-Time Urban Water Demand Forecasting", *Journal of Computational and Applied Mathematics*, 309, 532-541.

Dong, S., L. Sun, T. Chang, H. Lu (2011), "Combined Short-Term Traffic Flow Forecast Model for Beijing Traffic Forecast System", *Proceedings of the 14th International IEEE Conference on Intelligent Transport Systems*, 638-643.

Ferbar Tratar, L., B. Mojškerc, A. Toman (2016), "Demand Forecasting with four-Parameter Exponential Smoothing", *International Journal of Production Economics*, 181, 162-173.

Ferbar Tratar, L., E. Strmčnik (2016), "The Comparison of Holt-Winters Method and Multiple Regression Method: A Case Study", *Energy*, 109, 266-276.

Fumi, A., A. Pepe, L. Scarabotti, M.M. Schiraldi (2013), "Fourier Analysis in Demand Forecasting in a Fashion Company", *International Journal of Engineering Business Management*, 5, 1-10.

González-Romera, E., M.A. Jaramillo-Morán, D. Carmona-Fernández (2008), "Monthly Electric Energy Demand Forecasting with Neural Networks and Fourier Series", *Energy Conversion and Management*, 49, 3135-3142.

Ichinose, T., S. Hirobayashi, T. Misawa, T. Yoshizawa (2012), "Forecast of Stock Market Based on Nonharmonic Analysis used on NASDAQ since 1985", *Applied Financial Economics*, 22, 197-208.

Kochak, A., S. Sharma (2015), "Demand Forecasting Using Neural Network for Supply Chain Management", *International Journal of Mechanical Engineering and Robotics Research*, 4(1), 96-104.

Malliavin, P., M.E. Mancino (2002), "Fourier Series Method for Measurement of Multivariate Volatilities", *Finance and Stochastics*, 6, 49-61.

Mollina, A., B. Ponte, J. Parreño, D. De la Fuente, J. Costas (2016), "Forecasting Erratic Demand of Medicines in a Public Hospital: A Comparison of Artificial Neural Networks and ARIMA Models", *Proceedings of the International Conference on Artificial Intelligence*, 401-406.

Onyeocha, C.E., J. Khoury, R.F. Harik (2014), "Evaluation of Forecasting and Inventory Control in Multi-Product Manufacturing Systems Operating under Erratic Demand: A Case Study in the Automotive Domain", *Computer Science and Applications*, 1(1), 31-47.

Shang, K.H. (2012), "Single-Stage Approximations for Optimal Policies in Serial Inventory Systems with Nonstationary Demand", *Manufacturing & Service Operations Management*, 14(3), 414 - 422.

Song, J.S., P. Zipkin (1993), "Inventory Control in a Fluctuating Demand Environment", *Operations Research*, 41(2), 351–370.

Treharne, J.T., C.R. Sox (2002), "Adaptive Inventory Control for Nonstationary Demand and Partial Information", *Management Science*, 48, 607-624.

Yu, X., X. Zhang, H. Qin (2018), "A Data-Driven Model Based on Fourier Transform and Support Vector Regression for Monthly Reservoir in Flow Forecasting", *Journal of Hydro-Environment Research*, 18, 12-24.

Yukseltan, E., A. Yucekaya, A.H. Bilge (2017), "Forecasting Electricity Demand for Turkey: Modeling Periodic Variations and Demand Segregation", *Applied Energy*, 193, 287-296.

Van Loan, C. (1992), "Computational Frameworks for the Fast Fourier Transform", Philadelphia: Society for Industrial and Applied Mathematics.

Zhang, R., Y. Bao, J. Zhang (2010), "Forecasting Erratic Demand by Support Vector Machines with Ensemble Empirical Mode Decomposition", *Proceedings of the 3rd International Conference on Information Sciences and Interaction Sciences*, 567-571.

Zong-chang, Y. (2013), "Fourier Analysis-Based air Temperature Movement Analysis and Forecast", *IET Signal Processing*, 7(1), 14-24.