Alanya Akademik Bakış Dergisi
Yıl: 2021, C:5, S:2, ss.589-618

Alanya Academic Review Journal
Year: 2021, Vol:5, No:2, pp.589-618

Engle-Granger Cointegration Analysis Between GARCH-Type Volatilities of Gold and Silver Returns

(Research Article)

Altın ve Gümüş Getirilerinin GARCH Tipi Volatiliteleri Arasında Engle-Granger Koentegrasyon Analizi

Doi: 10.29023/alanyaakademik.838284

Yakup ARI
Department of Economics, Faculty of Economics and Administrative Sciences Alanya Alaaddin Keykubat University, Turkey
yakup.ari@alanya.edu.tr
Orcid:0000-0002-5666-5365

How to cite this article: *Arı, Y. (2021). "Engle-Granger Cointegration Analysis Between GARCH-Type Volatilities of Gold and Silver Returns." Alanya Academic Review, 5(2), pp.589-618.*

Keywords:
GARCH-type Volatility, Engle-Granger Cointegration, ECM, Granger Causality, Gold, Silver

Received: 10.12.2020
Accepted: 25.03.2021

ABSTRACT

The aim of this study is to reveal the cointegration relationship between the volatility of silver and gold returns. For this purpose, the volatility of silver and gold returns is modeled with GARCH-type models. The volatility data of the returns are obtained from 10 different GARCH-type models and using 8 different probability distributions in these models. The most adequate fit models for the volatility of silver and gold returns are found as ALLGARCH (1,1) and AVGARCH (1,1), respectively, where innovations are normal inverse Gaussian distributed. Volatility data are obtained from these models and it is determined that they are not stationary at the level. Therefore, the long-run and short-run relationships between the two volatilities are tested by the two-step Engle-Granger Cointegration method. Furthermore, the volatility spillover from silver returns to gold returns is shown by using the squared standardized residuals of the volatility model of silver returns as an exogenous variable in the nig-AVGARCH (1,1) volatility model of gold returns. It is concluded that the volatilities of silver and gold returns are cointegrated and the deviation from long-run equilibrium is rebalanced within 20 days.

1. INTRODUCTION

Although the use of gold as an investment tool in the financial system is as old as human history, it has been utilized to serve different financial purposes with the end of handling gold as money. However, in today's world, gold has taken its place in the financial sector as a tool of asset protection, hedging, and portfolio diversification. Along with all these, speculative use of gold is also possible for the sole purpose of generating returns. Moreover, gold, which is an important reserve asset today, is in the reserves of many central banks and international institutions around the world. Finding gold in the reserves of many Central Banks of the world is a tradition remaining from the gold standard era. Gold has been the focal point of the international financial system, along with the dollar, and has continued to exist as an important reserve tool. Nevertheless, as some European central banks started gold sales in the post-1980s, the importance of gold as a reserve instrument decreased relatively. As of 2020, a large part of the gold stocks is held by central banks of countries particularly the US (8133.53 tons), Germany (3362.47 tons), Italy (2451.86), France (2436.12 tons), Russian Federation (2298.67 tons), China (1948.32 tons), Switzerland (1040.01), Japan (765.22 tons), Turkey (691.01 tons), and India (668.25 tons) (Statista, 2020).

Another aspect that makes gold important is its physical properties. The most important feature of gold is its high resistance to external conditions. Being a good conductor of heat and electricity, gold is a very stable element that does not react easily, so it is not affected by many substances such as temperature, humidity and oxygen. Therefore, it never rusts, tarnishes, or becomes dull. One of the other features of gold is that it is an extremely soft and easily shaped material.

As it is mentioned before gold is considered an important investment asset among its other functions. It is one of the most preferred investment assets, especially in regions where financial markets are not sufficiently developed and therefore, financial product variety is limited. With the effect of gold prices, which have been on the rise in recent years, the use of gold as an investment tool has increased.

Silver has been the rival of gold for most of history. Along with gold, silver remains one of the precious metals in the commodity markets. In the 16th century, along with geographic discoveries, important silver resources were found in both Peru and Mexico, as well as new gold resources. For this reason, silver continued to be the main currency in Europe and America until the 19th century. Since gold coins were very valuable in many periods, it became compulsory to use two metals and the functioning of the money system was ensured. In this system, which is called the double metal mining standard, the value of the currencies is defined as gold and silver. However, towards the end of the 16th century, a serious competitor began to develop to compete with both precious metals. During this period, money in the form of paper issued by private institutions as debt instruments was put into use. Until the twentieth century, the use of gold as money continued, mostly in competition with silver coins (Menase, 2009).

In recent years, in addition to being the most conductive metal, it is considered among the metals that are highly demanded in the industry due to taking shape easily. Silver is among the most traded commodities in both the financial markets and the Forex market. Silver also provides the advantage of speculative transactions as it is considered among the reliable investment tools in uncertainty periods such as crisis and inflation. Today, gold and silver are subject to both spot and futures transactions in many different markets. Furthermore, gold and silver can also be preferred in order to protect against the risks brought by different variables. The main ones are exchange rates, inflation and interest rates. For instance, it is accepted that there is a positive relationship between inflation and gold prices and that the increase in gold prices is greater than the increase in inflation rates. Therefore, gold is used as an effective hedging tool against inflation. This situation, which is valid for prices formed in spot and physical markets, is also valid for future markets but does not show a similar feature for gold-based funds.

In this study, the relationship between the return volatility of these precious metals rather than the movement between gold and silver prices is examined. Volatility can be expressed as a measure of the high increase or decrease of any variable relative to a certain average value. Price volatility is expressed as a sudden variation in the price of any security. Volatility creates uncertainty and affects the decision-making processes of investors in financial markets. For this purpose, volatilities are modeled with the generalised version of Engle's (1982) autoregressive conditional variance model and and extensions of it. These models are called Generalized Autoregressive Conditional Heteroscedastic-type (GARCH-type) models. The cointegration tests of Engel and Granger (1987), Johansen (1991) and Paseran et al. (2001), which are used to reveal long-run and short-run relationships between variables, are widely used in the literature. The relationships between the volatility of variables are analyzed through multivariate GARCH models that one can see in the study Arı (2020a). However, in this study, after obtaining the volatilities of variables with GARCH-type models, the long and short-run relationship between volatilities are analyzed with the Engle-Granger cointegration approach. This method is similar to the Diebold-Yilmaz (2009, 2012) connectedness method and the Range-Based volatility spillover methods used in the study (Demiralay and Bayracı, 2015) to determine the transmission and propagation of volatility. Volatility spillover indicates the direction of information circulation and transmission structure between different industries and markets. Volatility spillover studies reveal the existence of the link between markets as well as measure the level of this link. In addition, in order to support the result obtained by the cointegration approach, the similar method in which Kanas (1998) determined volatility spillover using exponential GARCH models, is applied. The main contribution of this study is to include an alternative volatility spillover approach by using the volatility data obtained from 10 different GARCH-type models with 8 different probability distributions in the Engle-Granger cointegration test.

In light of all these, the volatilities of gold and silver returns are estimated using the GARCH model and its extensions. Afterward, the long-run relationship between the volatility of gold and silver returns is analyzed using the Engle-Granger cointegration method. For this purpose, there is a brief literature review in the second part of the study. Then, the volatility models and the cointegration method used in the methodology section are briefly explained. In the fourth section, the outputs of the volatility models and the results of the cointegration method are given. The last section includes a discussion of results and future work.

2. BRIEF LITERATURE REVIEW

There are many studies in the literature on gold and silver returns, and the volatility of these returns. In addition, the short and long-run relationships between the prices of these two metals have been studied extensively. One can see the survey study of Vigne et al 2017) that provides an extensive review of the literature on the financial economics of silver, platinum and palladium. The study covers the findings on a wide variety of topics related to the White Precious Metals and their relationships with other assets. Some studies on gold and silver prices in the literature can be listed as follows.

Christie-David et al. (2000) tried to determine whether macroeconomic news bulletins affect gold and silver prices. In their work; they followed the news bulletins on macroeconomic variables for 23 months between January 1992 and December 1995. As a result; they found that all precious metals market instruments were strongly influenced by the news on capacity utilization. It has been revealed that silver prices are affected by the bulletins regarding the unemployment rate.

The long-run relationship between the prices of the gold and silver futures contracts traded on the Tokyo Commodity Exchange was examined in the study of Ciner (2001). In a conclusion, he finds that the long-run relationship between the price of contracts is disappeared opposite of the frequently cited long-run stable relationship. Therefore, the changes in the gold to silver ratio should not be used to predict prices in the future is concluded in the study. Moreover, Ciner (2001) concluded that these two markets should not be regarded as substitutes to hedge against similar types of risks and these two commodities have different economic uses.

Chatrath et al. (2001) examined the structure of both gold and silver futures markets using daily prices of between 1975 and 1995 in their study. The authors tested for a correlation dimension as the first step then build on this correlation integral to test the price series for nonlinearity and deterministic chaos. Finally, a Kolmogorov entropy showed the measurements of the degrees at which the time series movements are predictable. As a conclusion, nonlinear dependencies were observed in the silver series, but those were not consistent with chaos, therefore allowing for a certain degree of predictability.

Gil-Alana et al. (2015) analyzed the long-run memory behavior of the price of silver using annual silver price data between 1792 and 2013. As a result of the study, it was found that real silver prices were mean-reverting. They also showed that there is no long-run memory behavior between silver and inflation rate. This reveals that external shocks affect real silver prices less than gold prices.

In another study, cointegration and causality analyzes were conducted in order to determine the effect of silver prices and the Dow Jones Index on gold prices. In the study in which daily data from January 1, 1973 - June 16, 2013, were used, it was investigated whether there was a long-run relationship between the series, and the existence of a long-run relationship was determined. A two-way causality has been determined between gold prices and silver prices (Elmas et al., 2015).

Göçer et al. (2019) examined the effects of gold, silver, oil, dollar, and natural gas prices on foreign trade between the years 1997 and 2018 for Turkey and they obtained the result that there was no long-run relationship between silver and gold prices using Maki cointegration test.

Cochran et al (2012) examined the returns and the long-memory properties of the return volatilities of copper, gold, platinum, and silver including the effect of the VIX and dummy variable 2008 Global Financial Crisis. They used the daily returns of the mentioned metals in the period January 4, 1999, to March 10, 2009. They concluded that the interaction effect of VIX and a financial crisis dummy variable was also found to be significant. Further, another result of the study was FIGARCH (1,d,1) was an appropriate model to describe the long-memory features of the returns.

Charles et al. (2015) test poor form efficiency for the presence of conditional variable variance in their studies using daily spot price data for silver and platinum between 1977 and 2013. They found that both markets meet the adaptive market hypothesis criterion and that the markets gradually become more efficient within the time frame considered. Nadarajah et al. (2015) tested which GARCH models performed better when modeling returns on different commodities, including gold and silver. Batten et al. (2016) examined possible silver price manipulation. Using 5-minute tick data between the 1st of January 2010 and the 30th of April 2015 the authors applied a cluster analysis procedure to try and detected price manipulation. Regarding silver, results showed a large concentration of returns around the derivative expiry date, suggesting possible manipulation. Furthermore, a three-component mixture model indicated abnormal market behaviour, which was also supported by a further method clustering the silver returns.

Küçükaksoy and Yalçın (2017) observed in their study that the shock in silver prices had no effect in the first period of gold prices. They observed that in the second period, the maximum response degree was realized as %0.34. They also stated that the effect of the shock disappeared after the third period. Among the precious metals, although silver is an important substitute for gold commodities, the effect of silver price shocks on the gold price is less than 1%. As a result of the study, they concluded that when there is a shock in silver commodities, investors turn to other investment instruments instead of gold.

The determinants of the gold price in Turkey were analyzed using cointegration tests using data between the period 2003.03- 2016.05. According to the test results, while the changes in the Istanbul Stock Exchange affect gold prices negatively, changes in the American Stock Exchange, oil prices and silver prices affect gold prices positively. However, it is found that changes in the consumer price index and real exchange rates have no effect on gold prices (Cicioğlu et al., 2018).

Dutta (2018) aims to investigate the implied volatility spillovers between gold and silver markets using two different forms of the bivariate VAR-GARCH mode. The findings of the study are that return and shocks significantly run from gold VIX (GVZ) to silver VIX (VXSLV), but not the other way around.

While a large number of studies estimate the cointegration and volatility spillover effects between gold and silver returns, the difference of this study can be said to apply GARCH-type models to find the best adequate volatility model.

3. METHODOLOGY

In this section, the GARCH model and its extensions and the Engle-Granger cointegration method are discussed respectively. The GARCH model and its extensions consist of 96 models in total according to their distributions of innovations. Therefore, only the model that best fits the volatility of gold returns according to the information criteria is considered in the GARCH model section.

3.1. GARCH Model and Extensions

One can state the return of the financial time series X_t as

$$X_t = \mu_t + a_t \tag{3.1}$$

where μ_t denotes conditional mean and a_t denotes residuals. The residuals of the conditional mean process can be expressed

$$a_t = \sigma_t \varepsilon_t \; and \; \varepsilon_t \sim f_v(0,1) \tag{3.2}$$

where σ_t and ε_t represent the volatility process and innovation process respectively. Further, innovations follow $f_v(0,1)$ that is the probability density function with zero mean and unit variance. If the distribution of the innovations is not normally distributed v represents additional distributional parameters for the scale and the shape of the distribution.

Bollerslev (1986) proposed the GARCH model in which conditional variance is dependent on the lag value of the squares of the error terms and its own lag values. GARCH (p,q) model can be expressed as

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i a_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 \tag{3.3}$$

where $p \geq 0,\ q > 0,\ \alpha_0 \geq 0,\ \alpha_i \geq 0$ for $i = 1,2,\dots,q$ and $\beta_i \geq 0$ for $i = 1,2,\dots,p$. GARCH(p,q) process is covariance stationary with E(a_t) = 0, var(a_t) = α_0 / ($1 - \sum_{i=1}^{q} \alpha_i - \sum_{i=1}^{p} \beta_i$) and cov($a_t$, a_s) = 0 for t ≠ s if and only if $\sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_i$ < 1.

Bollerslev (1986) utilized the Maximum Likelihood Estimation (MLE) Method for parameter estimation of the GARCH regression model. In the MLE method, the log-likelihood function that is maximized with respect to parameters is given below

$$L(\omega) = ln \prod_{t} f_v(a_t, E(a_t|I_{t-1}), \sigma_t) \tag{3.4}$$

where $E(a_t|I_{t-1})$ is the expected mean of residuals and $\omega = (v, \alpha_0, \alpha_1, \dots, \alpha_p,\ \beta_1, \dots, \beta_q)$.

In the literature, one can find various studies that show GARCH (1,1) is an adequate model to capture the volatility clustering, the study of Akgiray (1989) is one of the most important. Therefore, in this study, GARCH (1,1) model is used because of its ease of calculation, simplicity and success in capturing the volatility properties of time series. GARCH (1,1) model is as follows

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{3.5}$$

where $\alpha_0 \geq 0, \alpha_1 \geq 0, \beta_1 \geq 0$ and $\alpha_1 + \beta_1 < 1$.

Based on the GARCH (1,1) model, the features of the model can be listed as follows.

- The parameters α_1 and β_1 in Equation (3.5) are considered ARCH term and GARCH term respectively. The ARCH term α_1 is a measure that shows the effect of the existing volatility of today on the volatility of tomorrow. Moreover, it shows the short-run persistence of the shocks on the return variance.
- β_1 shows the effect of the old shocks on the long-run persistence of volatility. $(\alpha_1 + \beta_1)$ indicates volatility persistence and measures the rate of decay of the volatility feedback effect over time. The sum of $(\alpha_1 + \beta_1)$ close to 1 indicates high persistence, meaning that volatility shocks will be felt even less in the future. Volatility persistence makes it able to predict future economic variables and the changes in the risk-return trade-off over business cycles.
- Another inference from a GARCH model is the half-life of volatility shocks that is defined as half of the required time to reverse back to the expected variance value. For the mentioned model, it can be calculated using $h2l = -ln2/ln(\alpha_1 + \beta_1)$. The half-life calculation varies according to the extensions of the GARCH model.

In this study, the GARCH model and its extensions are not mentioned one by one. Only the best fitting model is briefly discussed. The models and the distributions of the innovations, which are used for volatility modeling, are given in the table below.

Table 1. GARCH-type Models and The Distributions of Innovations

Model	Distribution
1. Standard GARCH (Bollerslev, 1986) 2. Integrated GARCH (Engle & Bollerslev, 1986) 3. Exponential GARCH (Nelson, 1991) 4. GJR GARCH (Glosten et al.,1993) 5. Asymmetric power ARCH (Ding et al.,1993) 6. Absolute Value GARCH (Taylor, 1986)(Scwert, 1990) 7. Threshold GARCH (Zakoian, 1994) 8. All in the family Nesting symmetric and asymmetric GARCH – ALLGARCH (Hentschel, 1995) 9. Nonlinear GARCH (Higgins &Bera,1992) 10. Nonlinear Asymmetric GARCH (Engle & Ng, 1993) 11. Component sGARCH (Lee & Engle, 1999) 12. Multiplicative Component sGARCH (Engle & Sokalska, 2012)	1. Normal Distribution (Norm), 2. Skew-Normal Distribution (Snorm), 3. Student's T Distribution (Std), 4. Skew Student's Distribution (Sstd), 5. Generalized Error Distribution (Ged), 6. Skew Generalized Error Distribution (Sged), 7. Normal Inverse Gaussian Distribution (Nig) 8. Johnson's SU Distribution (Jsu)

One can see the studies of Ghalanos (2020b), Zang et al. (2017) and Chu et al. (2017) for details on on GARCH-type models. In this study, short definitions are made about the nig-ALLGARCH (1,1) and nig-AVGARCH (1,1) models, since they are found as the most adequate models for the volatility of gold and silver returns among 96 models. The estimation results of the mentioned models are given in the findings section.

The ALLGARCH (1, 1) model is

$$\sigma_t^{\delta} = \alpha_0 + \alpha_1 \sigma_{t-1}^{\delta}[|a_{t-1} - \eta_2| - \eta_1(a_{t-1} - \eta_2)]^{\delta} + \beta_1 \sigma_{t-1}^{\delta} \qquad (3.6)$$

with Normal Inverse Gaussian (NIG) distributed innovations. The parameters for this model are respectively; α_0 is variance intercept parameter, α_1 is ARCH parameter, β_1 is GARCH parameter, η_1 and η_2 that shows the rotation and shift on the news impact curve are the parameters of asymmetry, In this case, δ is asymmetry power parameter that is equal to conditional sigma power parameter.

The persistence of ALLGARCH (1,1) volatility is equal to $\beta_1 + \alpha_1\kappa_1$ where κ_1 is

$$\begin{aligned}\kappa_1 &= E[|a_{t-1} - \eta_2| - \eta_1(a_{t-1} - \eta_2)]^{\delta} \\ &= \int_{-\infty}^{\infty} [|a_{t-1} - \eta_2| - \eta_1(a_{t-1} - \eta_2)]^{\delta} f(a, 0, 1, \dots)\, da\end{aligned} \qquad (3.7)$$

The ALLARCH model allows the decomposition of the residuals in the conditional variance equation to be driven by different powers for a_t and σ_t and also allowing for both shifts and rotations in the news impact curve, where the shift is the main source of asymmetry for small shocks while rotation is the main source of asymmetry for large shocks (Ghalanos, 2020b).

ALLGARCH (1,1) model is reduced to a AVGARCH (1, 1) model when $\delta = 1$

$$\sigma_t = \alpha_0 + \alpha_1\sigma_{t-1}[|a_{t-1} - \eta_2| - \eta_1(a_{t-1} - \eta_2)] + \beta_1\sigma_{t-1} \tag{3.8}$$

with NIG distributed innovations where the log likelihood function of the NIG distribution is

$$L = ln(\underline{x}|\omega) = nln(\phi_1) - nln(\pi) - n\left(\tau_2\sqrt{\phi_1^2 - \phi_2^2} - \phi_2\tau_1\right) - \frac{1}{2}\sum_{t=p+q+1}^{T} lnf(a_t) + \sum_{t=p+q+1}^{T} \phi_2 a_t + \sum_{t=p+q+1}^{T} lnK_1\left(\phi_1\tau_1 lnf(a_t)^2\right) \tag{3.9}$$

where ϕ_1 and ϕ_2 are shape parameters, τ_1 and τ_2 are scale parameters, K_1 is the modified Bessel function of the third kind of the order r evaluated at a_t and the function $f(a_t)$ is defined by $f(a_t) = 1 + \left((a_t - \tau_1)/\tau_2\right)^2$. See (Kucharska, 2009).

3.2. Engle-Granger Cointegration

The concept of stationarity; it is defined as the time series data fluctuates around a zero mean and this fluctuation variance remains constant especially over time. Granger and Newbold (1974) discussed in their studies that non-stationary time series produce standard errors with deviation in the long run and therefore variables should have a stationary structure.

Engle and Granger (1987) cointegration test is used to reveal the long-run relationship between the two variables. According to the test; variables are assumed to be stationary at the same level. Both variables should have first-order stationarity. After creating a new regression with the variables whose stationarity is obtained, the stationarity of the residuals of this regression at the level value is tested. If it shows stationarity in level value, it is concluded that there is cointegration between variables.

In the Engle-Granger analysis based on mentioned idea, let Y_t and X_t be I(1) series which means that Y_t and X_t are not stationary at the level, but the first difference of the series are stationary. The regression model using Y_t and X_t series is as follows

$$Y_t = a_0 + a_1X_t + u_t \tag{3.10}$$

where $a_0, a_1 \in \mathbb{R}$ and u_t is error term. If u_t is I(0) or $\Delta\hat{u}_t = \rho_1\hat{u}_{t-1} + \sum_{i=0}^{p}\zeta_i\Delta\hat{u}_{t-i} + z_t$ where $|\rho_1| < 1$ then Y_t and X_t are cointegrated.

If the series are cointegrated, there is at least one causal relationship between the series. In order for the series to be cointegrated, they must be stationary. The differencing process is applied to ensure stability. However, applying the differencing process causes a loss of long-run information. Therefore, these imbalances are tried to be eliminated by using the error correction model. If there is a long-run relationship between series, an error correction model, which is used to determine the short-run relationship, shows a deviation period from a long-run relationship. The following error correction model (ECM) is used to determine the possible causality relationship between the cointegrated series and to determine the direction.

$$\Delta Y_t = \theta_0 + \sum_{i=0}^{p} \theta_{1i} \Delta Y_{t-i} + \sum_{i=0}^{q} \theta_{2i} \Delta X_{t-i} + \theta_3 \hat{u}_{t-1} + \varepsilon_t \tag{3.11}$$

where θ_0 is a constant parameter, θ_3 is error correction parameter or adjustment parameter and $-1 < \theta_3 < 0$ and $\hat{u}_{t-1}$ is equilibrium error term or error correction term where $\hat{u}_{t-1} = Y_{t-1} - a_0 - a_1 X_{t-1}$.

The special case of ECM is

$$\Delta Y_t = \theta_0 + \theta_1 \Delta Y_{t-1} + \theta_2 \Delta X_{t-1} + \theta_3 \hat{u}_{t-1} + \varepsilon_t \tag{3.12}$$

ECM describes how Y and X behave in the short run consistent with a long run cointegrating relationship.

4. DATA AND FINDINGS

The gold and silver prices data are downloaded from "finance.yahoo.com" via "quantmod" package (Ryan et al., 2020) in R. The time series plots of the price data and the returns of the gold and silver are given in Figure 1. The mentioned models are fitted the daily data between the period of 02-01-2018 and 09-30-2020. The descriptive statistics of data is given at Appendix-A in Table A1.

Looking at the time series plots below, it can be understood at first glance that the price data have a certain trend and are not stationary. Likewise, since it is seen that the return data have a mean-reverting structure that moves around zero, they can be said to be stationary. When all these cases are tested with Philip-Perron and Augmented Dickey-Fuller unit root tests, it is concluded that the price data are not stationary, besides, the return data are stationary. The outputs of the unit root tests are given at Appendix-A in Table A2.

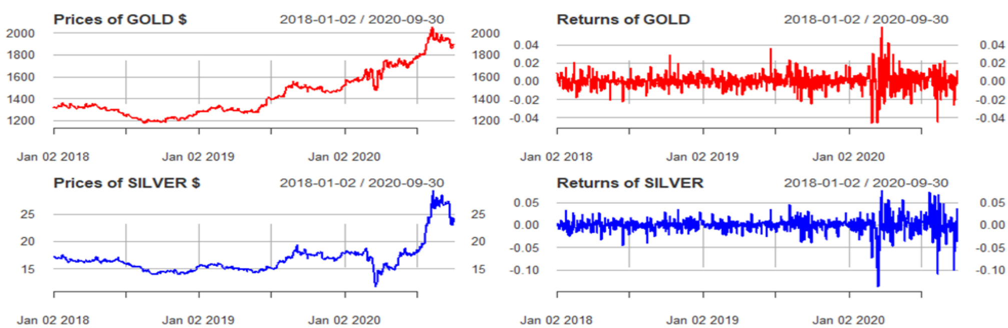

Figure 1. Time-series Plots of Gold and Silver Data

4.1. The Outputs of GARCH-type Volatility Models

The GARCH-type volatility models are ranked based on the information criterions Akaike (AIC), Bayesian (BIC), Shibata (SIC) and Hannan–Quinn (HQC). Moreover, the likelihood value is used for model evaluation and ranking. The volatility model evaluation results for the gold returns and silver returns are given in Table B1 and Table D1 at Appendix-B and Appendix-D, respectively. According to the evaluation results, it is concluded that the nig-AVGARCH (1,1) model for gold returns and the nig-ALLGARCH (1,1) model for silver returns are determined as best fit volatility models. The parameters of the mentioned models are estimated using MLE method via maximizing the log-likelihood function given in Equation 3.9 with respect to parameters of the volatility process. One can follow the studies of Ghalanos (2020a, 2020b) for the estimation and properties of GARCH-type models, and also see the study Arı (2020b) for the R codes used in this paper. The estimation results are given in Table 2 and Table 3.

Table 2. The Parameter Estimation of nig-AVGARCH (1,1) Model for Gold Volatility

Optimal Parameters				
parameter	estimate	Std.Error	t value	Pr(>|t|)
α_0	0.000091	0.000008	11.1101	0
α_1	0.088056	0.03482	2.5289	0.011443
β_1	0.912959	0.001133	805.7601	0
η_1	0.328551	0.191135	1.7189	0.085625
η_2	-1.032199	0.270611	-3.8143	0.000137
φ_1	-0.141552	0.064625	-2.1904	0.028498
φ_2	0.877951	0.245824	3.5715	0.000355

The persistence of nig-AVGARCH(1,1) volatility is equal to 0.9897062 that is calculated using to $\beta_1 + \alpha_1\kappa_1$ where κ_1 is given in Equation 3.7. The half-life is 66.98951 that is the number of days the volatility takes for half of the expected reversion back towards to the expected variance value. Except for the η_1 parameter, all the estimated parameters seem to be statistically significant at the 0.05 confidence level where φ_1 and φ_2 are skew and shape parameters. The η_1 parameter, which indicates that the source of asymmetry is rotation, is significant when the confidence level is accepted.

All parameters except the variance intercept parameter of the nig-ALLGARCH (1,1) process, which is the most suitable model for the volatility of silver returns, are statistically significant at the 0.05 confidence level. According to the model, the volatility persistence value is 0.9794035 and the half-life is 33.30586 days. In other words, the volatility that increases as a result of any shock is damped after a long period of time since the persistence effect is high and close to 1. Moreover, changes in returns increase volatility and because of the high volatility persistence, the volatility change decreases to an average level after approximately 33 days.

Table 3. The Parameter Estimation of nig-ALLGARCH(1,1) Model for Silver Volatility

Optimal Parameters				
parameter	estimate	Std.Error	t value	Pr(>|t|)
α_0	0.00003	0.000027	1.1091	0.267368
α_1	0.2037	0.014831	13.7353	0
β_1	0.82043	0.007346	111.68	0
η_1	0.72286	0.002664	271.3478	0
η_2	-2.87978	0.047257	-60.939	0
δ	1.55279	0.197598	7.8583	0
φ_1	-0.15468	0.05881	-2.6301	0.008536

φ_2	0.68752	0.149502	4.5987	0.000004

The diagnostics tests of the models are given in Appendix-C and Appendix-E. Looking at the diagnostics tests for the estimated models, it is seen that all the assumptions except one are satisfied. According to the results of Weighted Ljung-Box Test on Standardized Residuals and Weighted Ljung-Box and Weighted ARCH Langrange Multiplier tests on Standardized Squared Residuals, there is no autocorrelation between error terms. The Sign Bias Test results show that there is no model specification error and there is no leverage effect on the residuals. It is seen that the conditional probability distributions of the models are statistically fit and correct from the Adjusted Pearson Goodness-of-Fit Test result. This inference is also supported by QQ-plots given in Figure 2. From the individual and joint test statistics values of the Nyblom Stability Test, it is understood that there is a structural break in the time series and the stability of the parameters over time cannot be achieved.

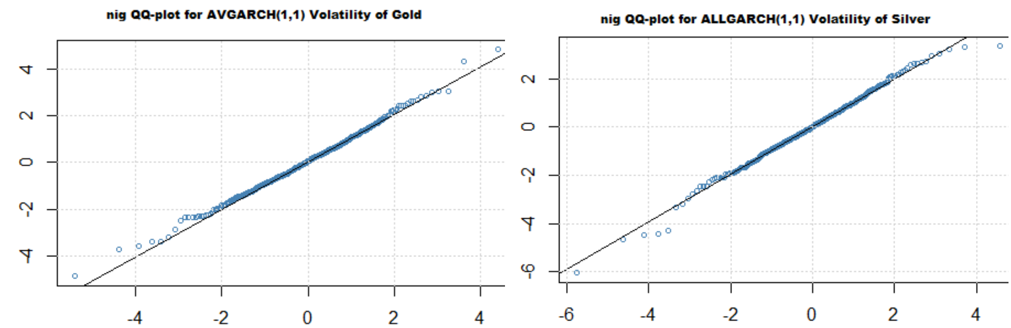

Figure 2. NIG QQ-Plots

4.2. The Outputs of Engle-Granger Cointegration Analysis with Error Correction Model

In this section, the long and short-run relationship between the volatility data obtained from AVGARCH (1,1) and ALLGARCH (1,1) processes estimated for the volatility of gold and silver returns are analyzed. Descriptive statistics values related to the volatility data are given at Appendix-A in Table A1 and the time-series graph of the data in Figure 3.

As can be understood from the time-series graph, variables do not have a mean-reverting structure that moves around zero. At the same time, it is seen that they do not have a constant variance. Therefore, it can be said that the variables have a unit root and are not stationary. This has been supported by Philips-Perron and Augmented Dickey-Fuller unit root tests that are given at Appendix-A in Table A2. As a result, the variables GoldVol (volatility of gold returns) and SilverVol (volatility of silver returns) are I (1).

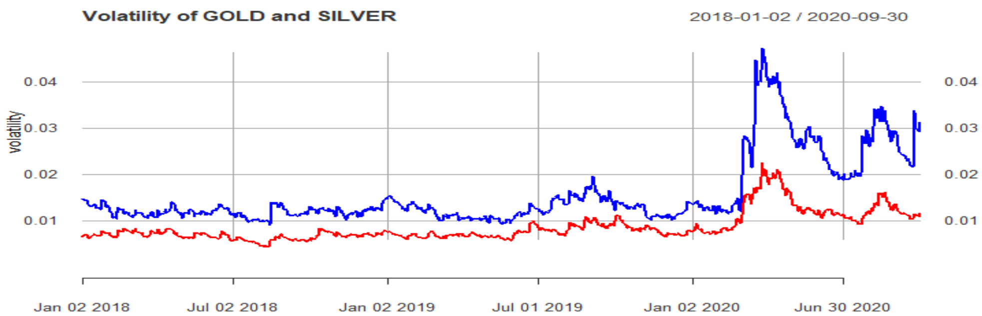

Figure 3. Time-series Plots of Gold Volatility and Silver Volatility

In the regression model that explains the long-run relationship in Engle-Granger cointegration analysis, GoldVol and SilverVol are accepted as the dependent variables, respectively. The coefficients of cointegration regressions are statistically significant. Standard errors and t-statistics of the coefficients are given in the below equations. The F-statistic value of the cointegration regression models is F (1,767) = 5889.958, which shows that the model is statistically significant. The R-square measure, which represents the percentage of the dependent variable variance that's explained by an independent variable in a regression model, is a high value such as 0.884782. The long-run equilibrium models are as follows.

$$GoldVol_t = 0.00257291 + 0.383857 SilverVol_t + u_{1t} \tag{4.1}$$

	(8.86181e-05)	(0.00500166)
t-stat	29.03	76.75

When the regression model, which shows the long-run relationship, given in Equation 4.1, is analyzed, it is seen that a unit change in the volatility of silver returns causes a change of approximately 0.38 units in the volatility of gold returns. In other words, one standard deviation change in silver returns causes a 0.38 standard deviation change in gold returns.

$$SilverVol_t = -0.00409028 + 2.30498 GoldVol_t + u_{2t} \tag{4.2}$$

	(0.000277795)	(0.0300338)
t-stat	−14.72	76.75

However, according to Equation 4.2, the effect of the volatility of gold returns on the volatility of silver returns, in the long run, is greater. A standard deviation change in gold returns causes a 2.3 standard deviation change in silver returns. When the stationarity of the error terms obtained from both equations is tested with the non-constant Augmented Dickey-Fuller equation, the asymptotic p values are found to be 0.0006947 for u_{1t} and 0.01056 for u_{2t}, respectively. This shows that the error terms are I(0) and the variables are cointegrated.

In order to compare the models given above, when the log-likelihood values, information criteria and forecasting performances are examined, it is revealed that the model in which the dependent variable is GoldVol is more appropriate. Therefore, after this point, only the first equation is used to explain the long-run relationship between variables. The comparisons of cointegration regression models are given in Table 4.

Table 4. The Comparison of cointegration regressions

Dependent Variable	**GoldVol**	**SilverVol**
Equation	**4.1**	**4.2**
Sum of squared residuals	0.000868	0.005212
S.E. of regression	0.001064	0.002607
Log-likelihood	4174.356	3485.119
Schwarz criterion	−8335.421	−6956.947
Akaike criterion	−8344.712	−6966.237
Hannan-Quinn	−8341.136	−6962.662
Root Mean Squared Error	0.0010624	0.0026034
Mean Absolute Error	0.00081517	0.001965
Mean Percentage Error	-1.4609	-1.7379
Mean Absolute Percentage Error	9.6165	12.889
Theil's U	2.4084	2.233

When examining the short-run relationship between variables, only the equilibrium equation obtained from the cointegration regression given in Equation 4.1, in other words, the error terms obtained from the first equation is used in the Error Correction Model (ECM). The ECM is

$$\underset{\text{t-stat}}{\Delta GoldVol_t} = \underset{\substack{(1.75\text{E-}05)\\ 0.3995}}{0.0000007} - \underset{\substack{(0.016528)\\ -0.5263}}{0.00869828}\Delta SilverVol_{t-1} - \underset{\substack{(0.0419265)\\ -0.4427}}{0.0185597}\Delta GoldVol_{t-1} - \underset{\substack{(0.0170618)\\ -3.690}}{0.0629555}\hat{u}_{t-1} + \varepsilon_t \quad (4.3)$$

where $\hat{u}_{t-1}$ is error correction term that is obtained by $\hat{u}_{t-1} = GoldVol_t - 0.0026 + 0.38SilverVol_t$. The error correction parameter or adjustment parameter that is statistically significant is equal to -0.063. It means that the long-run equilibrium between the volatilities of the gold returns and silver returns is re-balanced in (1/0.063) days which is approximately 16. Moreover, according to the Granger causality test, the volatility of gold returns is the Granger cause of the volatility of silver returns with F (12, 732) = 2.43 [p=0.0043]. At the same time, the volatility of silver returns is the Granger cause of the volatility of gold returns with F (12, 732) = 5.3827 [p=0.0000].

The results of Engle-Granger cointegration and Granger causality show that there is mutual volatility spillover between the gold and silver returns. To support this situation, the study of Kanas (1998) is followed. The squared standardized residuals of nig-ALLGARCH (1,1), which is the volatility model of silver returns, is defined as an exogenous variable in the nig-AVGARCH (1,1) volatility model of gold returns. Thus, the model to be estimated is as follows

$$\sigma_t = \alpha_0 + \alpha_1\sigma_{t-1}[|a_{t-1} - \eta_2| - \eta_1(a_{t-1} - \eta_2)] + \beta_1\sigma_{t-1} + \psi\Gamma_{t-1}^2 \quad (4.4)$$

where Γ_{t-1}^2 squared standardized residuals of nig-ALLGARCH (1,1) process. The estimation of the optimal parameters is given Table 5 and the diagnostics of the model can be found in Appendix-F.

Table 5. The Parameter Estimation of nig-AVGARCH(1,1) Volatility Spillover Model with Exogenous Variable

Optimal Parameters				
parameter	estimate	Std.Error	t value	Pr(>|t|)
α_0	0.004054	0.000301	13.45237	0
α_1	0.119143	0.043209	2.75734	0.005827
β_1	0.022156	0.034908	0.63469	0.525629
η_1	-0.38155	0.234167	-1.62938	0.103232
η_2	0.381517	0.096315	3.96115	0.000075
ψ	13.39991	1.610091	8.32245	0
φ_1	-0.13159	0.086568	-1.52012	0.128482
φ_2	4.272927	1.969696	2.16933	0.030057

The statistical significance of the volatility spillover coefficient ψ indicates that there is volatility spillover from silver returns to gold returns. This situation supports the findings obtained previously.

5. DISCUSSION AND CONCLUSION

In this study, the relationship between gold and silver returns is analyzed through the volatility of returns. The most appropriate volatility model is compared according to the information criteria and log-likelihood values of twelve different GARCH-type models in which innovations have eight different distributions. As a result, nig-AVGARCH model for gold returns and nig-ALLGARCH model for silver returns were found to be the best fitting volatility models. Later, by obtaining the volatility data that are predicted by the GARCH-type models, the long and short-run relationship between volatilities is analyzed using the Engle-Granger cointegration approach. In addition, short-run co-movements of these volatility data were analyzed by ECM. Subsequently, the Granger causality test was applied and mutual causality between variables was found. In this case, it shows that there are a volatility transition and spillover between gold and silver volatilities. To support this result, the volatility spillover was tested using a gold volatility model and volatility spillover from silver returns to gold returns was found. However, the volatility spillover from gold returns to silver returns was not made using both the cointegration analysis and the silver volatility model. It can be said as the deficiency of the study is not to compare with the volatility propagation methods using multivariate GARCH models. When comparing GARCH-type models, they can be compared according to their prediction performance as well as information criteria and log-likelihood values. At the same time, according to the results of diagnostics tests, the model that best satisfies the theory can be selected. Another shortcoming is that the cointegration regressions showing the long-run relationship and the ECM model do not provide the assumptions. This shows that other variables are needed in the analysis. Especially the presence of structural break reveals the need for dummy variables, and even the need to differentiate the methods used. The R codes, the results of the volatility spillover from gold returns to silver returns and the assumption tests of the cointegration regressions and the ECM model are available upon request. Finally, to mention the contribution of the study, it can be listed as obtaining the volatility data included in Engle-Granger cointegration analysis from 10 different GARCH-type models and using 8 different probability distributions in these models. At the same time, obtaining the volatility spillover similar to the Diebold-Yılmaz (2012) approach can be considered as a separate contribution. Based on this, modeling the volatility transmission and spillover between precious metal markets with Niebold-Yılmaz approach can be stated as a suggestion for future studies.

REFERENCES

AKGIRAY, V. (1989). "Conditional heteroscedasticity in time series of stock returns: Evidence and forecasts." *Journal of Business*, 62:55–80.

ARI, Y. (2020a). Volatility Transmission Model Using Dcc-Garch Representation. Book Chapter in: Studies at the Crossroads of Management & Economics, pp.237-250.ISBN: 978-1-912503-92-6 IJOPEC Publication.

ARI, Y. (2020b). From Discrete to Continuous GARCH Volatility Modelling With R. Available at https://math-stat.net/garch-and-cogarch-modelling.htm

BATTEN, J. A., LUCEY, B. M. & PEAT, M. (2016). "Gold and silver manipulation: What can be empirically verified?" *Economic Modelling*, 56:168–176

BOLLERSLEV, T. P. (1986). "Generalized autoregressive conditional heteroscedasticity", *Journal of Econometrics*, 31, pp. 307-327.

CHARLES, A., DARN_E, O. & KIM, J. H. (2015). "Will precious metals shine? A market efficiency perspective." *International Review of Financial Analysis*, 41:284-291.

CHATRATH, A., ADRANGI, B. & SHANK, T. (2001). "Nonlinear Dependence in Gold and Silver Futures: Is It Chaos?" *The American Economist*, 45(2):25–32.

CHRISTIE-DAVID, R. A., CHAUDHRY, M., AND KOCH, T. W. (2000). "Do macroeconomics news releases affect gold and silver prices?" *Journal of Economics and Business*, 52(5):405–421.

CICIOĞLU, Ş., ERASLAN, B., & TORUN, P. (2018). "Türkiye'de Altın Fiyatlarını Belirleyen Faktörler." *Journal of Human Sciences*, 15(3), 1551-1560. doi:10.14687/jhs.v15i3.47181553

CINER, C. (2001). "On the long run relationship between gold and silver prices: A note." *Global Finance Journal*, 12(2):299-303.

COCHRAN, S. J., MANSUR, I., & ODUSAMI, B. (2012). "Volatility persistence in metal returns: A FIGARCH approach." *Journal of Economics and Business*, 64(4):287-305.

DEMİRALAY, S. & BAYRACI, S. (2015). "Central and Eastern European Stock Exchanges under Stress: A Range-Based Volatility Spillover Framework." *Czech Journal of Economics and Finance (Finance a uver)*, vol. 65(5), pages 411-430, October.

DING Z., ENGLE R.F. & GRANGER C.W.J. (1993). "A long memory property of stock market return and a new model," *Journal of Empirical Finance* 1(1), 83-106.

DIEBOLD, F.X. & YILMAZ, K. (2009). “Measuring Financial Asset Return and Volatility Spillovers, With Application to Global Equity Markets.” *Economic Journal*, 119, 158-171.

DUTTA, A. (2018). “A note on the implied volatility spillovers between gold and silver markets.” *Resources Policy*, vol. 55(C), pages 192-195.

DIEBOLD, F.X. & YILMAZ, K. (2012), “Better to Give than to Receive: Forecast-Based Measurement of Volatility Spillovers.” *International Journal of Forecasting*, 28(1), 57-66.

ELMAS, B., & POLAT, M. (2015). “Gümüş Fiyatları ve Dow Jones Endeksi’nin Altın Fiyatlarına Etkisi Üzerine Eşbütünleşme ve Nedensellik Analizi.” *Bingöl Üniversitesi Sosyal Bilimler Enstitüsü Dergisi*, 3 (6), 33-48. Retrieved from http://busbed.bingol.edu.tr/tr/pub/issue/29510/402039

ENGLE, R. F. (1982). “Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation.” *Econometrica*, 50:4, pp. 987–1007

ENGLE, R. F., & BOLLERSLEV T., (1986). “Modelling the persistence of conditional variances.” *Econometric Reviews* 5: 1–50.

ENGLE, R. VE GRANGER, J. (1987). “Co-Integration and Error Correction: Representation, Estimation, and Testing”. *Econometrica*, Vol. 55, No. 2., pp. 251-276.

ENGLE, R. F., & SOKALSKA M. E. (2012). “Forecasting intraday volatility in the us equity market. multiplicative component garch.” *Journal of Financial Econometrics*, 10(1):54-83. doi: 10.1093/jj_nec/nbr005.

ENGLE, R. F., & VICTOR K. NG. 1993. “Measuring and testing the impact of news on volatility.” *Journal of Finance* 48: 1749–78.

GHALANOS, A. (2020a). rugarch: Univariate GARCH models. R package version 1.4-2. Available at: https://cran.r-project.org/web/packages/rugarch/rugarch.pdf

GHALANOS, A. (2020b). Introduction to the rugarch package. Technical Report Available at: https://cran.r-project.org/web/packages/rugarch/vignettes/ Introduction_to_the_rugarch_package. Pdf

GIL-ALANA, L. A., AYE, G. C. & GUPTA, R. (2015a). “Trends and cycles in historical gold and silver prices.” Journal of International Money and Finance, 58:98–109

GLOSTEN, L.R, JAGANNATHAN R. & RUNKLE, D.E. (1993). “Relationship between the expected value and the volatility of the nominal excess return on stocks.” *The Journal of Finance*,48(5), 1779-1801.

GRANGER, J. (1969). “Investigating Causal Relations by Econometric Models and Cross-spectral Methods”. Econometrica, Vol. 37, No. 3. pp. 424-438.

GÖÇER, Ş., BULUT DENİZ, M., & BURSAL, M. (2019). “Türkiye’de Dış Ticaret Hadlerinin Altın, Gümüş, Dolar, Petrol ve Doğalgaz Fiyatları ile İlişkisi: Eş Bütünleşme Analizi.” Bartın Üniversitesi İktisadi ve İdari Bilimler Fakültesi Dergisi, 10 (20), 191-203 . Retrieved from https://dergipark.org.tr/en/pub/bartiniibf/issue/50408/601636

HENTSCHEL, L. (1995). “All in the family nesting symmetric and asymmetric GARCH models.” *Journal of Financial Economics* 39: 71–104.

HIGGINS, M. L. & BERA A. K. (1992). “A class of nonlinear arch models.” *International Economic Review* 33: 137–58.

JOHANSEN, S. (1991). “Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models.” *Econometrica.* **59** (6): 1551–1580

KANAS, A. (1998). “Volatility Spillovers Across Equity Markets: European Evidence.” *Applied Financial Economics*, 8: 245-256

KUCHARSKA, M. & PIELASZKIEWICZ, J. (2009). NIG Distribution in Modelling Stock Returns with Assumption About Stochastic Volatility: Estimation of Parameters and Application to VaR and ETL. Retrieved from: https://www.diva-portal.org/smash/get/diva2:240092/FULLTEXT01.pdf. Master’s Thesis in Halmstad University.

KÜÇÜKAKSOY, İ. & YALÇIN, D. (2017). “Altın Fiyatlarını Etkileyebilecek Faktörlerin İncelenmesi.” *Ekonomik ve Sosyal Araştırmalar Dergisi*, 13 1-20. Retrieved from https://dergipark.org.tr/tr/pub/esad/issue/38974/456284

LEE, G.J. & ENGLE, R.F. (1999). A permanent and transitory component model of stock return volatility. In Cointegration Causality and Forecasting A Festschrift in Honor of Clive WJ Granger, pages 475-497. *Oxford University Pre*ss.

MENASE, M. (2009). Altın Piyasası ve Türkiye’de Altın Fiyatlarını Etkileyen Faktörlerin Analizi (Master Thesis). Retrieved from https://tez.yok.gov.tr/UlusalTezMerkezi/tezSorguSonucYeni.jsp

NADARAJAH, S., AFUECHETA, E., & CHAN, S. (2015). “GARCH modeling of five popular commodities.” *Empirical Economics*, 48(4):1691-1712.

NELSON, D. B. (1991). “Conditional heteroskedasticity in asset returns: A new approach.” *Econometrica* 59: 347-370.

PESARAN, M. H., SHIN, Y. & SMITH, R. J. (2001). “Bounds testing approaches to the analysis of level relationships.” *Journal of Applied Econometrics* 16(3), 289–326.

RYAN, J. A. & ULRICH, J.M. (2020). Quantmod: Quantitative Financial Modelling Framework. R package version 0.4.17. https://CRAN.R-project.org/package=quantmod

VIGNE, S. A., LUCEY, B.M., O’CONNOR, F.A. & YAROVAYA, L. (2017). “The financial economics of white precious metals — A survey.” *International Review of Financial Analysis*, Volume 52: 292-308, https://doi.org/10.1016/j.irfa.2017.04.006.

SCHWERT, G. W. (1990). “Stock volatility and the crash of ’87.” *Review of Financial Studies* 3: 103–6.

STATISTA. (2020, November 05). Gold reserves by country 2019. Retrieved November 09, 2020, from https://www.statista.com/statistics/267998/countries-with-the-largest-gold-reserves/

ZAKOIAN, J.-M. (1994). “Threshold heteroskedastic models.” *Journal of Economic Dynamics and Control*, 18: 931–55.

ZHANG, Y., DENG, Z., SHI, J., ZHANG, Y., & LIU, H. (2017). "Sea clutter modeling using an autoregressive generalized nonlinear-asymmetric GARCH model." *Digital Signal Processing, 62*, 52-64. doi:10.1016/j.dsp.2016.11.003

Appendix A. Philips-Perron and Augmented Dickey-Fuller Unit Root Tests

Table A1. Descriptive Statistics

	Gold Price	**Gold Returns**	**Gold Volatility**	**Silver Price**	**Silver Returns**	**Silver Volatility**
Mean	1451.4	0.0005257	0.0087038	16.894	0.00050022	0.015972
Median	1354.4	0.00085564	0.007545	16.362	0.0011108	0.012711
Range	875.3	0.105787	0.0181503	17.514	0.212947	0.0383522
Std. Dev.	214.73	0.0092819	0.0031319	2.9253	0.017901	0.0076746
C.V.	0.14794	17.656	0.35983	0.17316	35.785	0.48051
Skewness	0.90572	0.12003	1.8077	2.3195	-0.97267	1.875
Ex. kurtosis	-0.13016	6.2221	3.3139	5.6605	10.341	2.7937
5% perc.	1203.5	-0.014824	0.0057798	14.264	-0.024434	0.01022
95% perc.	1931.8	0.014213	0.015806	24.53	0.02782	0.032932
IQ range	277.75	0.0083464	0.0030928	2.487	0.013687	0.0041716

Table A2. Unit Root Test Table

PHILIPS-PERRON UNIT ROOT TEST TABLE							
		GOLD CLOSING PRICE	GOLD RETURNS	GOLD VOLATILITY	SILVER CLOSING PRICE	SILVER RETURNS	SILVER VOLATILITY
With Constant	t-Statistic	0.4913	-28.9299	-2.1218	-1.0789	-26.3424	-1.8448
	Prob.	***0.9864***	***0.0000***	***0.2362***	***0.7258***	***0.0000***	***0.3588***
With Constant & Trend	t-Statistic	-1.9779	-29.3955	-2.9806	-2.0106	-26.3879	-3.0279
	Prob.	***0.6119***	***0.0000***	***0.1384***	***0.5941***	***0.0000***	***0.1252***
Without Constant & Trend	t-Statistic	1.8510	-28.6166	-0.3535	0.4342	-26.3478	-0.3427
	Prob.	***0.9850***	***0.0000***	***0.5575***	***0.8073***	***0.0000***	***0.5616***
AUGMENTED DICKEY-FULLER UNIT ROOT TEST TABLE							
		GOLD CLOSING PRICE	GOLD RETURNS	GOLD VOLATILITY	SILVER CLOSING PRICE	SILVER RETURNS	SILVER VOLATILITY
With Constant	t-Statistic	0.4916	-6.9740	-2.6034	-2.0039	-6.2782	-2.0079
	Prob.	***0.9864***	***0.0000***	***0.0927***	***0.2853***	***0.0000***	***0.2835***
With Constant & Trend	t-Statistic	-2.0197	-9.0967	-3.9741	-2.8002	-6.3299	-3.2978
	Prob.	***0.5891***	***0.0000***	***0.0099***	***0.1976***	***0.0000***	***0.0674***
Without Constant & Trend	t-Statistic	1.7472	-6.7660	-0.5740	0.0795	-6.2576	-0.3231
	Prob.	***0.9809***	***0.0000***	***0.4686***	***0.7078***	***0.0000***	***0.5690***

Appendix B. GARCH-type Volatility Modeling of Gold
Table B1. GARCH Model Comparison for Gold Returns

No	Model	Akaike	Bayes	Shibata	Hannan-Quinn	Likelihood
1	nig avgarch11	-6.874684703	-6.832401691	-6.874848439	-6.858411235	2650.316268
2	jsu avgarch11	-6.874098385	-6.831815372	-6.87426212	-6.857824916	2650.090829
3	jsu nagarch11	-6.872174928	-6.835932346	-6.872295429	-6.858226241	2648.35126
4	nig nagarch11	-6.872100582	-6.835858	-6.872221083	-6.858151895	2648.322674
5	nig tgarch11	-6.871286291	-6.835043709	-6.871406792	-6.857337604	2648.009579
6	jsu tgarch11	-6.871195065	-6.834952483	-6.871315566	-6.857246378	2647.974502
7	jsu allgarch11	-6.869997352	-6.821673909	-6.870210846	-6.851399102	2649.513982
8	std allgarch11	-6.869466953	-6.827183941	-6.869630689	-6.853193485	2648.310044
9	sstd allgarch11	-6.869466953	-6.827183941	-6.869630689	-6.853193485	2648.310044
10	sged avgarch11	-6.869146182	-6.826863169	-6.869309917	-6.852872713	2648.186707
11	jsu egarch11	-6.869001909	-6.832759327	-6.86912241	-6.855053222	2647.131234
12	std avgarch11	-6.868835751	-6.832593169	-6.868956252	-6.854887064	2647.067346
13	sstd avgarch11	-6.868835751	-6.832593169	-6.868956252	-6.854887064	2647.067346
14	nig egarch11	-6.868789718	-6.832547135	-6.868910219	-6.85484103	2647.049646
15	nig aparch11	-6.868735551	-6.826452538	-6.868899286	-6.852462082	2648.028819
16	jsu aparch11	-6.868620538	-6.826337525	-6.868784273	-6.85234707	2647.984597
17	jsu igarch11	-6.86835344	-6.844191718	-6.86840718	-6.859054315	2644.881898
18	nig igarch11	-6.867406316	-6.843244594	-6.867460056	-6.858107191	2644.517729
19	std nagarch11	-6.867122434	-6.836920282	-6.867206258	-6.855498527	2645.408576
20	sstd nagarch11	-6.867122434	-6.836920282	-6.867206258	-6.855498527	2645.408576
21	std tgarch11	-6.866701984	-6.836499832	-6.866785808	-6.855078077	2645.246913
22	sstd tgarch11	-6.866701984	-6.836499832	-6.866785808	-6.855078077	2645.246913
23	jsu GARCH11	-6.866316347	-6.836114195	-6.866400172	-6.854692441	2645.098635
24	jsu mcsgarch11	-6.866315993	-6.836113841	-6.866399818	-6.854692087	2645.098499
25	nig mcsgarch11	-6.865972735	-6.835770583	-6.86605656	-6.854348829	2644.966517
26	nig GARCH11	-6.865969154	-6.835767002	-6.866052979	-6.854345248	2644.96514
27	sged nagarch11	-6.865969005	-6.829726422	-6.866089506	-6.852020317	2645.965082
28	jsu gjrgarch11	-6.865563865	-6.829321283	-6.865684366	-6.851615178	2645.809306
29	sged tgarch11	-6.865477862	-6.82923528	-6.865598364	-6.851529175	2645.776238
30	nig gjrgarch11	-6.865303304	-6.829060721	-6.865423805	-6.851354616	2645.70912
31	std egarch11	-6.864323581	-6.834121429	-6.864407406	-6.852699675	2644.332417
32	sstd egarch11	-6.864323581	-6.834121429	-6.864407406	-6.852699675	2644.332417
33	std aparch11	-6.864192332	-6.82794975	-6.864312833	-6.850243645	2645.281952
34	sstd aparch11	-6.864192332	-6.82794975	-6.864312833	-6.850243645	2645.281952
35	std igarch11	-6.863373475	-6.845252183	-6.863403755	-6.856399131	2641.967101
36	sstd igarch11	-6.863373475	-6.845252183	-6.863403755	-6.856399131	2641.967101
37	sged egarch11	-6.862945872	-6.82670329	-6.863066373	-6.848997185	2644.802688
38	sged allgarch11	-6.861613595	-6.813290152	-6.861827088	-6.843015345	2646.290427
39	std GARCH11	-6.861199513	-6.837037791	-6.861253253	-6.851900388	2642.131213
40	sstd GARCH11	-6.861199513	-6.837037791	-6.861253253	-6.851900388	2642.131213
41	std mcsgarch11	-6.861199232	-6.837037511	-6.861252972	-6.851900107	2642.131105
42	sstd mcsgarch11	-6.861199232	-6.837037511	-6.861252972	-6.851900107	2642.131105
43	jsu csgarch11	-6.86056285	-6.818279837	-6.860726585	-6.844289381	2644.886416

44	ged avgarch11	-6.860480833	-6.824238251	-6.860601334	-6.846532146	2643.85488
45	nig csgarch11	-6.86041568	-6.818132668	-6.860579416	-6.844142212	2644.829829
46	std gjrgarch11	-6.860279361	-6.830077209	-6.860363186	-6.848655455	2642.777414
47	sstd gjrgarch11	-6.860279361	-6.830077209	-6.860363186	-6.848655455	2642.777414
48	sged GARCH11	-6.860174174	-6.829972022	-6.860257999	-6.848550268	2642.73697
49	sged mcsgarch11	-6.860163901	-6.829961749	-6.860247725	-6.848539995	2642.73302
50	sged igarch11	-6.85980379	-6.835642068	-6.85985753	-6.850504665	2641.594557
51	sged gjrgarch11	-6.859447368	-6.823204785	-6.859567869	-6.84549868	2643.457513
52	jsu ngarch11	-6.858202734	-6.821960152	-6.858323235	-6.844254047	2642.978951
53	nig ngarch11	-6.85814762	-6.821905038	-6.858268121	-6.844198933	2642.95776
54	ged nagarch11	-6.856584165	-6.826382013	-6.85666799	-6.844960259	2641.356611
55	ged tgarch11	-6.85632304	-6.826120888	-6.856406865	-6.844699134	2641.256209
56	std csgarch11	-6.855517988	-6.819275405	-6.855638489	-6.8415693	2641.946666
57	sstd csgarch11	-6.855517988	-6.819275405	-6.855638489	-6.8415693	2641.946666
58	sged csgarch11	-6.854697899	-6.812414886	-6.854861634	-6.83842443	2642.631342
59	ged aparch11	-6.853784657	-6.817542074	-6.853905158	-6.839835969	2641.2802
60	std ngarch11	-6.852938241	-6.822736089	-6.853022065	-6.841314334	2639.954753
61	sstd ngarch11	-6.852938241	-6.822736089	-6.853022065	-6.841314334	2639.954753
62	ged egarch11	-6.852852972	-6.82265082	-6.852936797	-6.841229066	2639.921968
63	ged allgarch11	-6.852460588	-6.810177575	-6.852624323	-6.836187119	2641.771096
64	sged aparch11	-6.851788231	-6.809505219	-6.851951967	-6.835514763	2641.512575
65	sged ngarch11	-6.851750356	-6.815507773	-6.851870857	-6.837801668	2640.498012
66	ged GARCH11	-6.85038352	-6.826221798	-6.85043726	-6.841084395	2637.972463
67	ged mcsgarch11	-6.850383518	-6.826221796	-6.850437258	-6.841084393	2637.972463
68	ged gjrgarch11	-6.849620998	-6.819418846	-6.849704823	-6.837997092	2638.679274
69	ged igarch11	-6.848872752	-6.830751461	-6.848903033	-6.841898408	2636.391573
70	ged csgarch11	-6.844740232	-6.80849765	-6.844860733	-6.830791545	2637.802619
71	ged ngarch11	-6.84192018	-6.811718028	-6.842004004	-6.830296274	2635.718309
72	norm avgarch11	-6.774843914	-6.744641762	-6.774927739	-6.763220008	2609.927485
73	snorm avgarch11	-6.77428095	-6.738038367	-6.774401451	-6.760332262	2610.711025
74	norm nagarch11	-6.762393369	-6.738231648	-6.76244711	-6.753094245	2604.140251
75	snorm nagarch11	-6.762096138	-6.731893986	-6.762179962	-6.750472231	2605.025965
76	norm tgarch11	-6.759323354	-6.735161632	-6.759377094	-6.750024229	2602.95983
77	snorm allgarch11	-6.758568903	-6.71628589	-6.758732638	-6.742295434	2605.669743
78	snorm tgarch11	-6.758207955	-6.728005803	-6.75829178	-6.746584049	2603.530959
79	norm allgarch11	-6.757720837	-6.721478254	-6.757841338	-6.743772149	2604.343662
80	norm egarch11	-6.752145914	-6.727984192	-6.752199654	-6.742846789	2600.200104
81	norm mcsgarch11	-6.751839474	-6.733718183	-6.751869755	-6.74486513	2599.082278
82	norm GARCH11	-6.75183691	-6.733715619	-6.751867191	-6.744862567	2599.081292
83	snorm GARCH11	-6.751657301	-6.727495579	-6.751711041	-6.742358176	2600.012232
84	snorm mcsgarch11	-6.751657145	-6.727495423	-6.751710885	-6.74235802	2600.012172
85	snorm egarch11	-6.751170232	-6.72096808	-6.751254057	-6.739546326	2600.824954
86	norm gjrgarch11	-6.751111918	-6.726950196	-6.751165658	-6.741812793	2599.802532
87	snorm gjrgarch11	-6.750907587	-6.720705435	-6.750991412	-6.739283681	2600.723967
88	norm csgarch11	-6.742694395	-6.712492243	-6.742778219	-6.731070488	2597.565995
89	snorm csgarch11	-6.742459242	-6.70621666	-6.742579743	-6.728510555	2598.475579
90	snorm ngarch11	-6.740166911	-6.709964759	-6.740250736	-6.728543005	2596.594177

91	norm ngarch11	-6.739959419	-6.715797697	-6.740013159	-6.730660294	2595.514397
92	norm igarch11	-6.739295687	-6.727214826	-6.739309168	-6.734646124	2593.259192
93	snorm aparch11	-6.738759897	-6.702517314	-6.738880398	-6.724811209	2597.05318
94	norm aparch11	-6.738560431	-6.708358279	-6.738644255	-6.726936525	2595.976486
95	snorm igarch11	-6.738439641	-6.72031835	-6.738469922	-6.731465298	2593.930042
96	nig allgarch11	-4.79236525	-4.740755285	-4.792617528	-4.772424142	na

Appendix C. NIG-AVGARCH(1,1) Diagnostics for Gold Volatility

Table C1. Weighted Ljung-Box Test on Standardized Residuals

Weighted Ljung-Box Test on Standardized Residuals		
	statistic	p-value
Lag[1]	3.119	0.07737
Lag[2*(p+q)+(p+q)-1][2]	4.141	0.06911
Lag[4*(p+q)+(p+q)-1][5]	6.216	0.07996

Table C2. Weighted Ljung-Box Test on Standardized Squared Residuals

Weighted Ljung-Box Test on Standardized Squared Residuals			
	statistic	p-value	df
Lag[1]	0.2708	0.6028	2
Lag[2*(p+q)+(p+q)-1][5]	1.0737	0.8428	
Lag[4*(p+q)+(p+q)-1][9]	2.4919	0.8389	

Table C3. Weighted ARCH LM Tests

Weighted ARCH LM Tests					
		Statistic	Shape	Scale	P-Value
ARCH	Lag[3]	0.0954	0.5	2	0.7574
ARCH	Lag[5]	1.3588	1.44	1.667	0.6302
ARCH	Lag[7]	1.8599	2.315	1.543	0.747

Table C4. Sign Bias Test

Sign Bias Test		
	t-value	prob sig
Sign Bias	0.463036	0.6435
Negative Sign Bias	0.001231	0.999
Positive Sign Bias	0.313265	0.7542
Joint Effect	0.290307	0.9618

Table C5. Adjusted Pearson Goodness-of-Fit Test

Adjusted Pearson Goodness-of-Fit Test			
	group	statistic	p-value(g-1)
1	20	12.46	0.8652
2	30	25.86	0.6328
3	40	27.83	0.9088
4	50	47.06	0.5521

Table C6. Nyblom stability test

Nyblom stability test				
Parameters	Individual Stats:			
alpha0	0.04814			
alpha1	0.07889	Asymptotic Critical Values	10%	5%
beta1	0.05388	Joint Stat:	1.89	2.11
eta1	0.06703	Individual Stat:	0.35	0.47
eta2	0.04863			
skew	0.51887			
shape	0.24989			
Joint Stat:	2.2815			

Appendix D. GARCH-type Volatility Modeling of Silver

Table D1. GARCH Model Comparison for Silver Returns

No	Model	Akaike	Bayes	Shibata	Hannan-Quinn	Likelihood
1	nig allgarch11	-5.760820256	-5.712496813	-5.76103375	-5.742222006	2223.035389
2	nig nagarch11	-5.759395283	-5.7231527	-5.759515784	-5.745446595	2220.487486
3	jsu allgarch11	-5.759359476	-5.711036033	-5.759572969	-5.740761226	2222.473718
4	nig tgarch11	-5.759328331	-5.723085749	-5.759448832	-5.745379644	2220.461743
5	nig igarch11	-5.759079011	-5.73491729	-5.759132751	-5.749779886	2218.36588
6	jsu igarch11	-5.758579438	-5.734417717	-5.758633178	-5.749280313	2218.173794
7	jsu nagarch11	-5.758476919	-5.722234337	-5.758597421	-5.744528232	2220.134376
8	jsu tgarch11	-5.758451734	-5.722209152	-5.758572235	-5.744503046	2220.124692
9	nig gjrgarch11	-5.758076506	-5.721833923	-5.758197007	-5.744127818	2219.980416
10	nig ngarch11	-5.757464309	-5.721221726	-5.75758481	-5.743515621	2219.745027
11	jsu gjrgarch11	-5.757454363	-5.721211781	-5.757574864	-5.743505676	2219.741203
12	nig aparch11	-5.757097934	-5.714814921	-5.757261669	-5.740824465	2220.604156
13	nig csgarch11	-5.757012587	-5.714729574	-5.757176322	-5.740739118	2220.57134
14	nig GARCH11	-5.756951062	-5.72674891	-5.757034886	-5.745327155	2218.547683
15	nig mcsgarch11	-5.756951061	-5.726748909	-5.757034886	-5.745327155	2218.547683
16	jsu ngarch11	-5.756490981	-5.720248399	-5.756611483	-5.742542294	2219.370782
17	jsu aparch11	-5.756345785	-5.714062773	-5.756509521	-5.740072317	2220.314955
18	jsu mcsgarch11	-5.756095407	-5.725893255	-5.756179232	-5.744471501	2218.218684
19	jsu GARCH11	-5.756095371	-5.725893219	-5.756179196	-5.744471465	2218.21867
20	nig egarch11	-5.756071565	-5.719828982	-5.756192066	-5.742122877	2219.209517
21	nig avgarch11	-5.756043219	-5.713760207	-5.756206955	-5.739769751	2220.198618
22	jsu avgarch11	-5.755914602	-5.713631589	-5.756078337	-5.739641133	2220.149164
23	jsu egarch11	-5.755105822	-5.71886324	-5.755226323	-5.741157135	2218.838189
24	jsu csgarch11	-5.754364518	-5.712081506	-5.754528254	-5.73809105	2219.553157
25	std allgarch11	-5.752749965	-5.710466952	-5.7529137	-5.736476496	2218.932362
26	sstd allgarch11	-5.752749965	-5.710466952	-5.7529137	-5.736476496	2218.932362
27	std igarch11	-5.751254284	-5.733132993	-5.751284565	-5.74427994	2214.357272
28	sstd igarch11	-5.751254284	-5.733132993	-5.751284565	-5.74427994	2214.357272
29	std nagarch11	-5.751199941	-5.720997789	-5.751283766	-5.739576035	2216.336377
30	sstd nagarch11	-5.751199941	-5.720997789	-5.751283766	-5.739576035	2216.336377
31	std tgarch11	-5.750715296	-5.720513144	-5.750799121	-5.73909139	2216.150031
32	sstd tgarch11	-5.750715296	-5.720513144	-5.750799121	-5.73909139	2216.150031
33	std gjrgarch11	-5.749836706	-5.719634554	-5.74992053	-5.738212799	2215.812213
34	sstd gjrgarch11	-5.749836706	-5.719634554	-5.74992053	-5.738212799	2215.812213
35	std aparch11	-5.748747188	-5.712504605	-5.748867689	-5.7347985	2216.393294
36	sstd aparch11	-5.748747188	-5.712504605	-5.748867689	-5.7347985	2216.393294
37	std ngarch11	-5.748728531	-5.718526379	-5.748812356	-5.737104625	2215.38612
38	sstd ngarch11	-5.748728531	-5.718526379	-5.748812356	-5.737104625	2215.38612
39	std mcsgarch11	-5.748639819	-5.724478097	-5.748693559	-5.739340694	2214.35201
40	sstd mcsgarch11	-5.748639819	-5.724478097	-5.748693559	-5.739340694	2214.35201
41	std GARCH11	-5.74863981	-5.724478089	-5.74869355	-5.739340685	2214.352007
42	sstd GARCH11	-5.74863981	-5.724478089	-5.74869355	-5.739340685	2214.352007
43	std avgarch11	-5.747599336	-5.711356754	-5.747719837	-5.733650649	2215.951945
44	sstd avgarch11	-5.747599336	-5.711356754	-5.747719837	-5.733650649	2215.951945

45	sged tgarch11	-5.747464347	-5.711221764	-5.747584848	-5.733515659	2215.900041
46	std egarch11	-5.747416651	-5.717214499	-5.747500475	-5.735792744	2214.881702
47	sstd egarch11	-5.747416651	-5.717214499	-5.747500475	-5.735792744	2214.881702
48	sged csgarch11	-5.747307441	-5.705024428	-5.747471176	-5.731033972	2216.839711
49	sged nagarch11	-5.746899407	-5.710656825	-5.747019908	-5.73295072	2215.682822
50	sged igarch11	-5.746439543	-5.722277822	-5.746493283	-5.737140418	2213.506004
51	sged ngarch11	-5.746295749	-5.710053167	-5.74641625	-5.732347062	2215.450715
52	std csgarch11	-5.745642999	-5.709400417	-5.7457635	-5.731694312	2215.199733
53	sstd csgarch11	-5.745642999	-5.709400417	-5.7457635	-5.731694312	2215.199733
54	sged mcsgarch11	-5.745066565	-5.714864413	-5.74515039	-5.733442659	2213.978094
55	sged GARCH11	-5.745066049	-5.714863897	-5.745149874	-5.733442143	2213.977896
56	sged gjrgarch11	-5.745013786	-5.708771204	-5.745134288	-5.731065099	2214.957801
57	sged aparch11	-5.744925108	-5.702642095	-5.745088843	-5.728651639	2215.923704
58	sged allgarch11	-5.744583894	-5.696260451	-5.744797388	-5.725985645	2216.792507
59	sged avgarch11	-5.744554576	-5.702271563	-5.744718311	-5.728281107	2215.781234
60	sged egarch11	-5.743926649	-5.707684067	-5.74404715	-5.729977962	2214.539797
61	ged csgarch11	-5.740147601	-5.703905018	-5.740268102	-5.726198913	2213.086752
62	ged tgarch11	-5.737919422	-5.70771727	-5.738003247	-5.726295516	2211.230018
63	ged nagarch11	-5.737902322	-5.70770017	-5.737986147	-5.726278416	2211.223443
64	ged igarch11	-5.737563131	-5.71944184	-5.737593412	-5.730588787	2209.093024
65	ged ngarch11	-5.73710719	-5.706905038	-5.737191015	-5.725483284	2210.917715
66	ged allgarch11	-5.736782204	-5.694499191	-5.736945939	-5.720508735	2212.792757
67	ged GARCH11	-5.736665014	-5.712503293	-5.736718755	-5.72736589	2209.747698
68	ged mcsgarch11	-5.736664881	-5.712503159	-5.736718621	-5.727365756	2209.747647
69	ged gjrgarch11	-5.735844141	-5.705641989	-5.735927966	-5.724220235	2210.432072
70	ged avgarch11	-5.735796371	-5.699553789	-5.735916872	-5.721847684	2211.413705
71	ged aparch11	-5.735441811	-5.699199228	-5.735562312	-5.721493123	2211.277376
72	ged egarch11	-5.734535335	-5.704333183	-5.734619159	-5.722911429	2209.928836
73	snorm csgarch11	-5.619562214	-5.583319632	-5.619682715	-5.605613526	2166.721671
74	norm csgarch11	-5.616243846	-5.586041694	-5.616327671	-5.60461994	2164.445759
75	snorm allgarch11	-5.610811812	-5.5685288	-5.610975547	-5.594538344	2164.357142
76	snorm nagarch11	-5.608300691	-5.578098539	-5.608384515	-5.596676784	2161.391616
77	snorm avgarch11	-5.607483281	-5.571240699	-5.607603782	-5.593534594	2162.077322
78	snorm tgarch11	-5.607427773	-5.577225621	-5.607511598	-5.595803867	2161.055979
79	snorm ngarch11	-5.606873997	-5.576671845	-5.606957822	-5.595250091	2160.843052
80	norm allgarch11	-5.605682209	-5.569439627	-5.60580271	-5.591733522	2161.384809
81	snorm igarch11	-5.605050336	-5.586929045	-5.605080617	-5.598075993	2158.141854
82	snorm aparch11	-5.604835241	-5.568592659	-5.604955742	-5.590886554	2161.05915
83	norm avgarch11	-5.604777569	-5.574575417	-5.604861394	-5.593153663	2160.036975
84	snorm GARCH11	-5.604439513	-5.580277792	-5.604493253	-5.595140388	2158.906993
85	snorm mcsgarch11	-5.604439329	-5.580277608	-5.604493069	-5.595140204	2158.906922
86	snorm egarch11	-5.604082174	-5.573880022	-5.604165999	-5.592458268	2159.769596
87	snorm gjrgarch11	-5.603435766	-5.573233614	-5.603519591	-5.59181186	2159.521052
88	norm tgarch11	-5.602303598	-5.578141876	-5.602357338	-5.593004473	2158.085733
89	norm ngarch11	-5.602214302	-5.57805258	-5.602268042	-5.592915177	2158.051399
90	norm nagarch11	-5.601742345	-5.577580623	-5.601796085	-5.59244322	2157.869932
91	norm igarch11	-5.601124934	-5.589044073	-5.601138415	-5.596475372	2155.632537

92	norm mcsgarch11	-5.600124129	-5.582002838	-5.60015441	-5.593149786	2156.247728
93	norm GARCH11	-5.600124071	-5.582002779	-5.600154352	-5.593149727	2156.247705
94	norm aparch11	-5.599721498	-5.569519346	-5.599805323	-5.588097592	2158.092916
95	norm egarch11	-5.598207815	-5.574046094	-5.598261555	-5.58890869	2156.510905
96	norm gjrgarch11	-5.598139716	-5.573977994	-5.598193456	-5.588840591	2156.484721

Appendix E. NIG-ALLGARCH(1,1) Diagnostics for Silver Volatility

Table E1. Weighted Ljung-Box Test on Standardized Residuals

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	1.614	0.20392
Lag[2*(p+q)+(p+q)-1][2]	4.072	0.07213
Lag[4*(p+q)+(p+q)-1][5]	6.352	0.07423

Table E2. Weighted Ljung-Box Test on Standardized Squared Residuals

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value	df
Lag[1]	5.737	0.01661	2
Lag[2*(p+q)+(p+q)-1][5]	8.331	0.02438	
Lag[4*(p+q)+(p+q)-1][9]	9.868	0.05353	

Table E3. Weighted ARCH LM Tests

Weighted ARCH LM Tests

		Statistic	Shape	Scale	P-Value
ARCH	Lag[3]	0.4448	0.5	2	0.5048
ARCH	Lag[5]	2.3257	1.44	1.667	0.4036
ARCH	Lag[7]	2.8661	2.315	1.543	0.5392

Table E4. Sign Bias Test

Sign Bias Test

	t-value	prob sig
Sign Bias	0.1979	0.84319
Negative Sign Bias	2.5816	0.01002
Positive Sign Bias	0.9322	0.35151
Joint Effect	8.8015	0.03205

Table E5. Adjusted Pearson Goodness-of-Fit Test

Adjusted Pearson Goodness-of-Fit Test

	group	statistic	p-value(g-1)
1	20	15.73	0.675
2	30	19.62	0.9043
3	40	30.64	0.8284
4	50	39.65	0.8274

Table E6. Nyblom stability test

Nyblom stability test

Parameters	Individual Stats:			
alpha0	0.2031	Asymptotic Critical Values	10%	5%
alpha1	0.1597	Joint Stat:	1.89	2.11
beta1	0.152	Individual Stat:	0.35	0.47
eta1	0.1606			
eta2	0.1585			
lambda	0.2012			
skew	0.6292			
shape	0.1497			
Joint Stat:	2.8601			

Appendix F. NIG-AVGARCH(1,1) Diagnostics for Volatility Spillover Model

Table F1. Weighted Ljung-Box Test on Standardized Residuals

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	6.585	0.01028
Lag[2*(p+q)+(p+q)-1][2]	6.597	0.01525
Lag[4*(p+q)+(p+q)-1][5]	8.869	0.01786

Table F2. Weighted Ljung-Box Test on Standardized Squared Residuals

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value	df
Lag[1]	0.7078	0.4002	2
Lag[2*(p+q)+(p+q)-1][5]	4.1496	0.236	
Lag[4*(p+q)+(p+q)-1][9]	5.7249	0.3311	

Table F3. Weighted ARCH LM Tests

Weighted ARCH LM Tests

		Statistic	Shape	Scale	P-Value
ARCH	Lag[3]	0.09536	0.5	2	0.7575
ARCH	Lag[5]	0.33494	1.44	1.667	0.9308
ARCH	Lag[7]	1.18573	2.315	1.543	0.8816

Table F4. Sign Bias Test

Sign Bias Test

	t-value	prob sig
Sign Bias	0.1759	0.8604
Negative Sign Bias	0.1526	0.8788
Positive Sign Bias	0.4723	0.6368
Joint Effect	0.2709	0.9654

Table F5. Adjusted Pearson Goodness-of-Fit Test

Adjusted Pearson Goodness-of-Fit Test			
	group	statistic	p-value(g-1)
1	20	24.06	0.19402
2	30	41.16	0.06674
3	40	40.21	0.41658
4	50	58.24	0.17179

Table F6. Nyblom stability test

Nyblom stability test				
Parameters	Individual Stats:			
alpha0	2.07904	Asymptotic Critical Values	10%	5%
alpha1	1.66905	Joint Stat:	1.89	2.11
beta1	1.52041	Individual Stat:	0.35	0.47
eta1	0.33366			
eta2	0.08266			
psi	1.12546			
skew	0.241			
shape	0.09025			
Joint Stat:	7.5582			