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DYNAMIC ANALYSIS OF A QUADCOPTER USING PID, ADAPTIVE AND LQR CONTROL METHODS

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Abstract

Original scientific paper

In this study, different control methods for controlling the position and angle values for the quadcopter (UAV) that take off and land vertically are simulated with Matlab/Simulink. First of all, a mathematical model was created with the Newton-Euler method, taking into account the dynamics of the quadcopter system. The focus of the study is to investigate the appropriate control method for the position control of the quadcopter. For the linear model of the quadcopter system, PID, LQR and Adaptive Control methods, and for the nonlinear model of the quadcopter system. PD Control simulation has been done. The mentioned control methods have been applied to the system and the control of the movement of the system in each axis has been examined. The obtained results are compared with each other to see the performance of the controllers and the most appropriate control method for the quadcopter was determined with comparisons and tracking scenarios.

Keywords: *Adaptive, LQR, PID, Quadcopter, UAV*

1 Introduction

Unmanned Aerial Vehicles (UAV) are systems that do not contain human elements, can adjust their speed and direction through their sensors and software methods, go from their location to the target point and perform the defined tasks [1, 2]. A quadcopter is an under-actuated UAV with six degrees of freedom and four symmetrically positioned rotors. The main forces and moments acting on a quadcopter are produced by its rotors. Under-actuated system; means that the six degree of freedom (θ, ϕ, ψ, X, Y, Z) quadcopter system is controlled by only four inputs. In another words, a quadcopter can be controlled simply by variation in motor speed and do not require any complicated actuators. Due to their small size and maneuverability, quadcopters are one of the most used UAVs both in open and closed areas. Quadcopters do not need mechanical connections used to change the propeller angle. So it is simple to design and maintain. The second advantage is that its kinetic energy is less during flight as four motors with small diameters are used [3]. Good maneuverability, not needing large areas for landing and take-off, easy control and installation, their ability to hover in the air, are the reasons why they are preferred more than helicopters using a main rotor and tail rotor.

The motion of the quadcopter is driven by the thrust generated by its rotors. As shown in Figure 1.a, while one pair of opposed rotor pairs rotates clockwise, the other pair rotates counterclockwise. As a result of the simulations in the study, the most appropriate control method for the quadcopter was determined with comparisons and tracking scenarios [4].

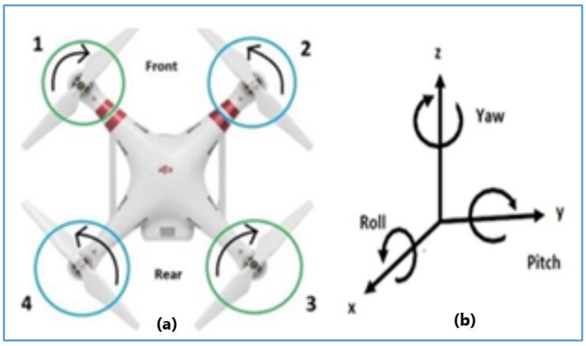

Figure 1: a) Quadcopter with four rotors
b) Roll, pitch and yaw angles that define the motion of the quadcopter.

2 Quadcopter Motion Definitions and Equations

Quadcopters have two pairs of rotors rotating mutually in the same direction to prevent torque imbalance [5,6]. As seen in Figure 2, the first and third rotors rotate in the same direction (clockwise), the second and fourth rotors in the same direction but in the opposite direction (counterclockwise) to the other pair of rotors.

Quadrotors have three basic flight angles as shown in Figure 1.b. These are called roll, pitch and yaw angles. For the movement of the quadcopter in the x, y, z axes, the the roll (ϕ), pitch (θ) and yaw (ψ) angles must be changed by controlling the speed of the rotors [7].

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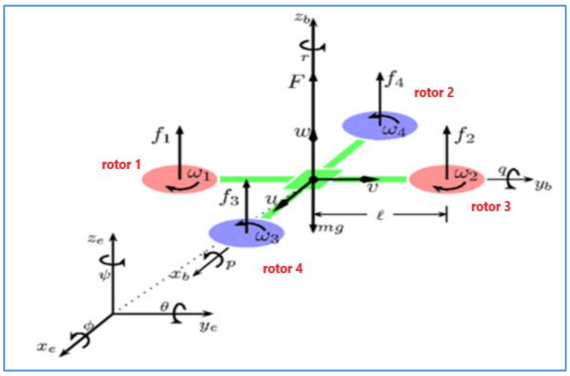


Figure 2: Rotation directions of quadcopter rotors

The quadcopter moves in the direction of the propeller with less angular velocity. If the total thrust generated by the four rotors is equal to the total weight of the quadcopter, the quadcopter will be suspended in the air. For movement on the vertical axis, the angular velocities of the two opposing rotor pairs rotating in opposite directions must be changed at the same rate. Take off by increasing the angular velocities of the rotor pairs at the same rate and landing by decreasing them. The roll angle is the angle the quadcopter makes about the x-axis. The quadcopter moves on the y-axis with the opposite speed change of the rotors 2 and 4, provided that the speeds of the rotors 1 and 3 remain constant. As it is known, the quadcopter moves in the direction of the propeller with less angular speed. Pitch angle is the angle the quadcopter makes about the y-axis. Similar to the roll angle, the quadcopter moves forward and backward with the opposite speed change of the rotors 1 and 3, provided that the speeds of the rotors numbered 2 and 4 remain constant. The angle of yaw refers to the rotation of the quadcopter around the z-axis. It is caused by the balance mismatch caused by the change of speed of one of the mutually placed rotor pairs. In other words, the angle of deviation in the z-axis occurs from the speed difference between the number 1 and 3 rotors and the number 2 and 4 rotors [8]. Movement directions of a quadcopter according to rotor speeds are shown in Figure 3. Green and white lines mean slower speed, and red lines mean faster speed.

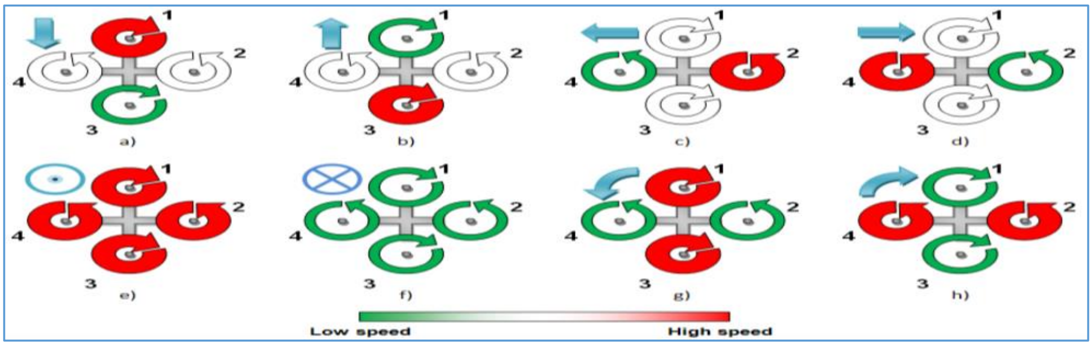


Figure 3. Various directions of movement of the quadcopter [9]

2.1 Equations of Motion of the Quadcopter

$$\ddot{\phi}=\dot{\psi}\dot{\theta}\left(\frac{I_y-I_z}{I_x}\right)-\frac{J}{I_x}\dot{\theta}(-\Omega_1+\Omega_2-\Omega_3+\Omega_4)+\frac{l}{I_x}U_2 \quad (1)$$

$$\ddot{\theta}=\dot{\psi}\dot{\phi}\left(\frac{I_x-I_z}{I_y}\right)-\frac{J}{I_y}\dot{\phi}(-\Omega_1+\Omega_2-\Omega_3+\Omega_4)+\frac{l}{I_y}U_3 \quad (2)$$

$$\ddot{\psi}=\dot{\phi}\left(\frac{I_x-I_y}{I_z}\right)+\frac{1}{I_y}U_4 \quad (3)$$

$$\ddot{x}=(C\psi S\theta S\phi+S\psi C\phi)\frac{1}{m}U_1 \quad (4)$$

$$\ddot{y}=(S\psi S\theta C\phi-C\psi S\phi)\frac{1}{m}U_1 \quad (5)$$

$$\ddot{z}=-g+(C\theta C\phi)\frac{1}{m}U_1 \quad (6)$$

Motion equations of a quadcopter are given in equation number 1 to 6. These nonlinear equations obtained for the quadcopter are linearized around a chosen operating point. In the study, the hover point is considered as the operating point and is given by equation 7.

$$[\phi\ \theta\ \psi]=[0\ 0\ 0] \quad U_1=g \quad (7)$$

Due to small orientations, the angles are approximately become; $\sin\theta\approx\theta$, $\sin\phi\approx\phi$, $\sin\psi\approx\psi$, $\cos\theta\approx1$, $\cos\phi\approx1$, $\cos\psi\approx1$.

So, the motion equations of the quadcopter can be written as seen in equations 8-13.

$$\ddot{\phi}=\frac{U3}{Ix} \quad (8)$$

$$\ddot{\theta}=\frac{U2}{Iy} \quad (9)$$

$$\ddot{\psi}=\frac{U4}{Iz} \quad (10)$$

$$\ddot{x}=\frac{U1}{m}\sin\theta \quad (11)$$

$$\ddot{y}=-\frac{U1}{m}\sin\emptyset \quad (12)$$

$$\ddot{z}=\frac{U1}{m}-g \quad (13)$$

3 Application of PID, LQR and Adaptive Control Methods for Quadcopter

In this section, simulations and results of DJI-F450 model quadcopter with PID, LQR and Adaptive Control methods created in Matlab/Simulink software are given [17].

3.1 Quadcopter Control with Proportional Integral Derivative (PID) Controller

In order to control the movement of the quadcopter in the x and y axes, roll and pitch angles must be controlled. With the formation of the pitch angle, the quadcopter moves in the y-axis, with the formation of the roll angle in the x-axis. After controlling the Roll (θ) and Pitch (ϕ) angles with PID, the quadcopter's x and y axes were controlled [10-16]. The simulation models and results for height, x-axis, y-axis and deviation are shown in Figure 4., Figure 5., Figure 6. and Figure 7., respectively.

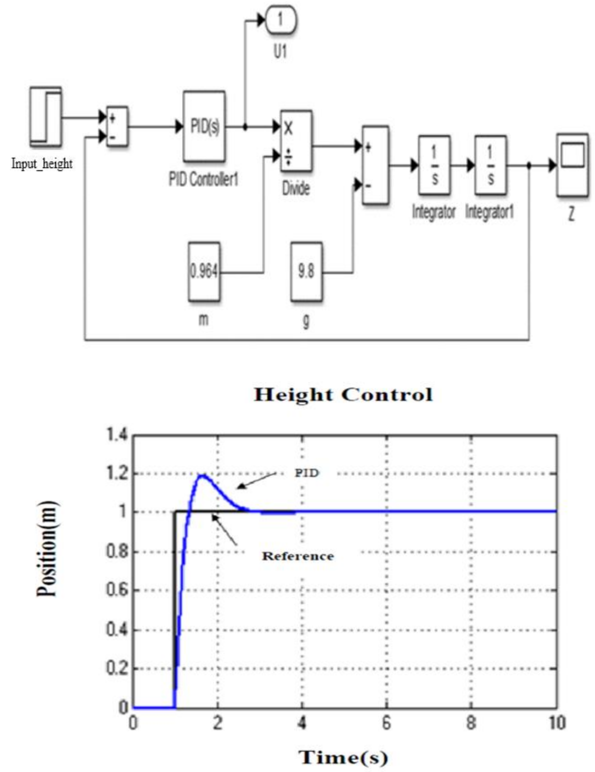


Figure 4: PID simulation model and simulation result for height (m=0.964 kg, g=9.8 m/s²)

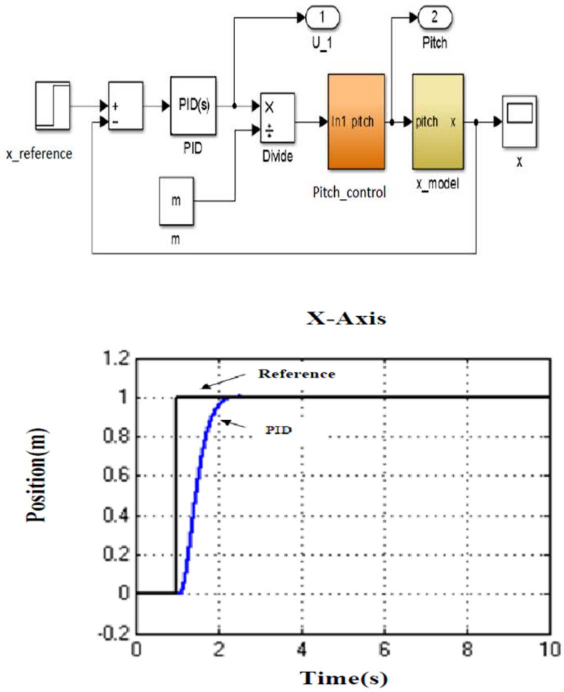
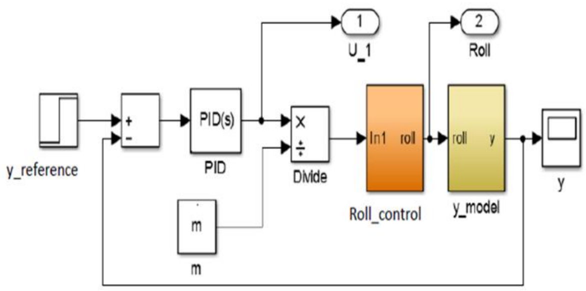


Figure 5: PID simulation model and simulation result for x-axis (m=0.964 kg, I_{xx} = 8.5532x10^{-3}kgm^2, x_{ref}= 1m)



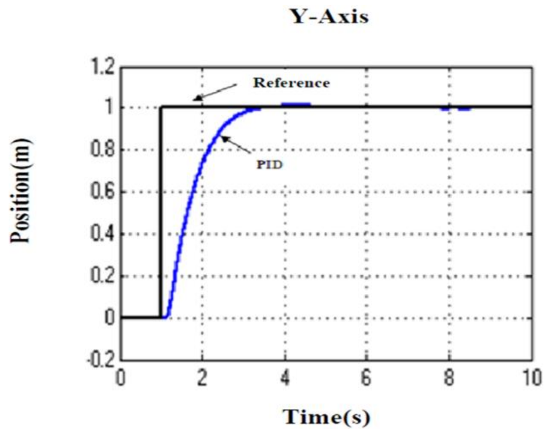


Figure 6: PID simulation model and simulation result for y-axis (m=0.964 kg, I_{yy} = 8.5532x10^{-3} kgm^2, y_{ref} = 1 m)

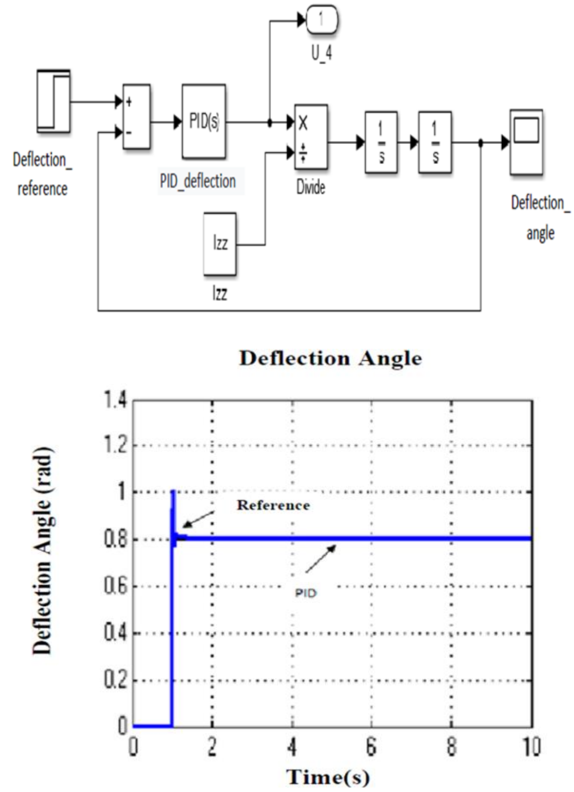


Figure 7: PID simulation model and simulation result for Roll angle (I_{zz} = 8.5532x10^{-3} kgm^2, ψ_{ref} = 0.8 rad.)

The purpose of the position control for the quadcopter is to enable it to follow the desired travelers in the x, y, z cartesian space. According to the graph obtained for height control, maximum overshoot is observed within reasonable limits, and the steady-state error is almost zero. It was observed that the vehicle reached the desired height in the z-axis in a short time. When we look at the x and y positions, the x value reached the desired target in the 2nd second and the y value in the 4th second. Looking at the graph in Figure 7 for the angle of deviation, it is at stable position, at zero.

3.2 Quadcopter Control with Linear Quadratic Regulator (LQR) Controller

LQR is the optimal control method commonly used to find state feedback gain for closed loop systems [16, 18]. In this method, the closed loop response is found by obtaining the system's feedback coefficients. Performance index function (J) is used for this. State space representation is written in equations 14 and 15;

$$\dot{x} = Ax+Bu \tag{14}$$
$$y = Cx+Du \tag{15}$$

and the general form of the performance function, including the state feedback controller in the form of; u = -K_{lqr}x is seen in equation 16;

$$J(x,u) = \frac{1}{2}\int_0^T [x^T(t)Qx(t) + u^T(t)Ru(t)]dt. \tag{16}$$

The value of K is based on establishing the balance between transient response and control success. This balance is achieved by the choice of Q and R matrices which are positively defined symmetric matrices. The matrix K_lqr is obtained from the defined equation. The matrix K_{lqr}; K_{lqr} A+$A^T K_{lqr}$ −$K_{lqr}\, BR^{-1}B^T K_{lqr} + Q = 0$

The LQR control of the system was performed with the A and B state matrices obtained from the state space form representation of the quadcopter and the K_{lqr} gain matrix calculated from the Q and R matrices selected in accordance with the system.

For the quadcopter;

x state vector $x = [x\ \dot{x}\ y\ \dot{y}\ z\ \dot{z}\ \theta\ \dot{\theta}\ \phi\ \dot{\phi}\ \ \psi\ \dot{\psi}\ ]$T, positions x, y, z and ψ angle is output. $y = [x\ y\ z\ \psi]$T

LQR simulation model for x, y, z axes and deflection angle are seen in Figure 8 and 9. LQR Control simulation results are shown in Figure 10.

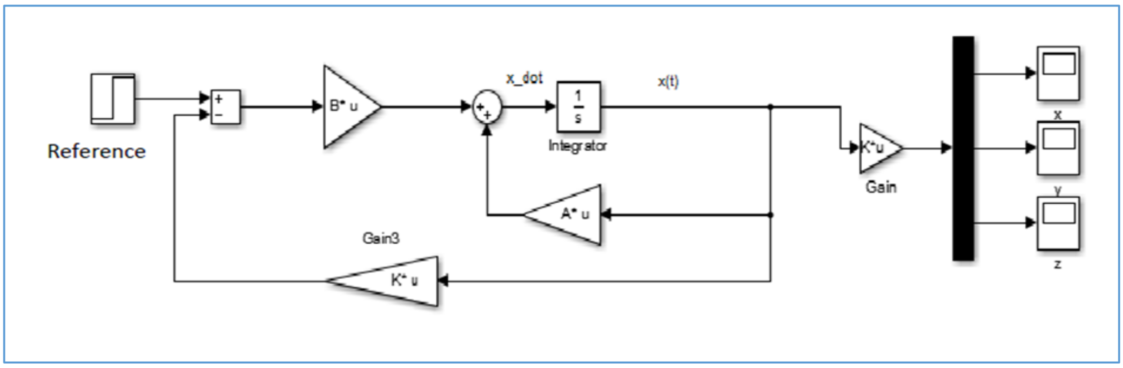


Figure 8: LQR simulation model for x, y, z axes

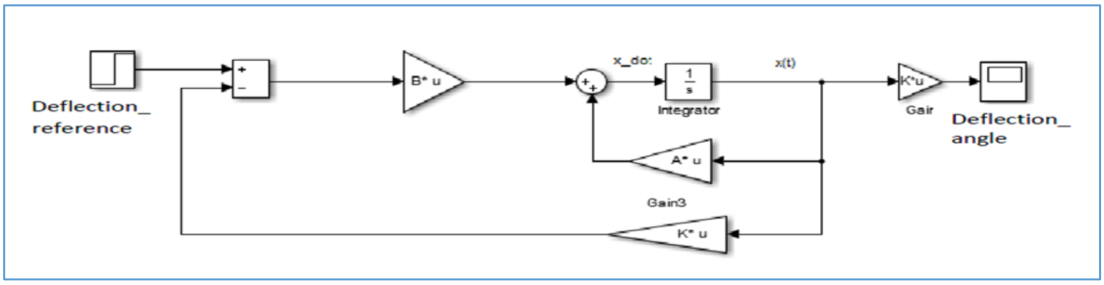


Figure 9: LQR simulation model for deflection angle.

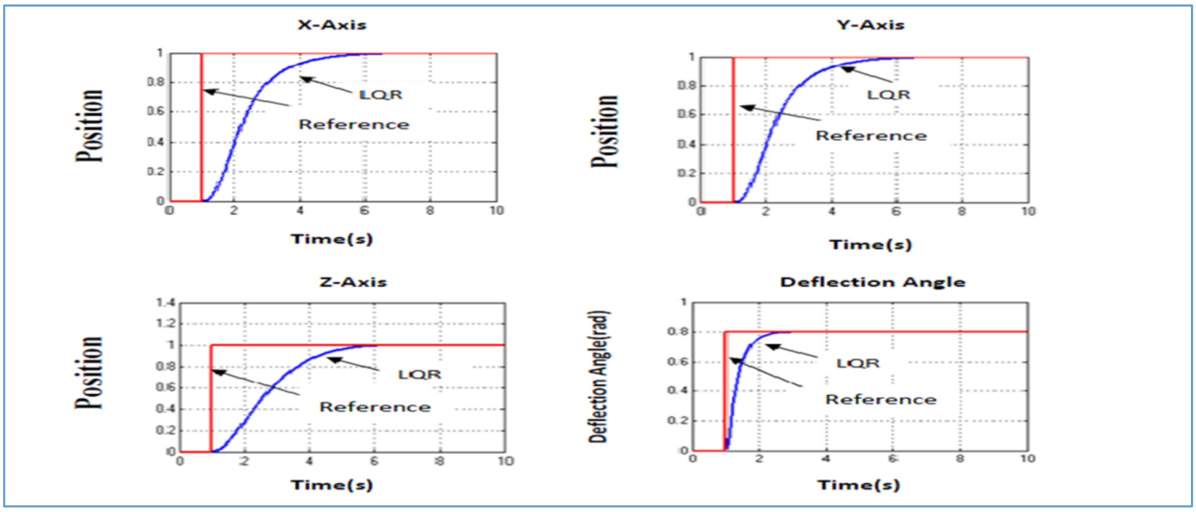


Figure 10: LQR Control simulation results

3.3 Quadcopter Control with Adaptive Controller

Adaptive control is one of the most important areas of modern control and deals with the control of systems in structural disorders and environmental changes where uncertainties exist. The adaptive control concept is based on the principle of automatic fulfillment of the controller settings in order to provide the ability to measure the system behavior at any time and to provide the most appropriate system response. Adaptive controllers differ from conventional controllers in that they do not need prior knowledge of limits on indefinite or time-varying parameters. There are two methods in adaptive controller design. The first one is "Model Reference Adaptive Control" (MRAC) and the other is "Self-Tuning Regulator Control" (STR) [19-21]. In the study, MRAC, one of the adaptive control methods, was used for the quadcopter. The reference model characterizes the desired system behaviors such as overshoot, rise time, settling time, and stability. When the linearized model is considered, transfer functions and references are modeled for x, y, z and ψ system outputs that are desired to be controlled. Transfer functions for z and ψ outputs 2nd order and one low order system order reference model is selected with 2nd order transfer functions for x and y outputs.

The transfer function for 1st order systems is characterized by; G(s) = $\frac{a}{(s+a)}$ and for 2nd order systems; G(s) = $\frac{b}{s^2+as+b}$. For the vertical axis movement and deflection angle, the reference model from the 1st degree shown with the transfer function at 15 with a set-up time of 0.066 seconds and a rise time of 0.036 was chosen. $G_Z(s) = \frac{60}{s+60}$. For x, y, z axes and angle ψ, low damped, critically damped and extremely damped reference models were chosen. $G_{X-Y}(s) = \frac{60}{s^2+15s+60}$ Critically damped reference model was chosen with the transfer function; $G_{X-Y}(s) = \frac{225}{s^2+30s+225}$. Reference model with over damping; $G_{X-Y}(s) = \frac{9}{s^2+9s+9}$.

Adaptive Control simulation models used for the motion control of the quadcopter in x-axis and y-axis are shown in Figure 11. Adaptive Control simulation model for the control of vertical movement and the control of deflection angle are seen in Figure 12. Figure 13 shows the Adaptive Control simulation model.

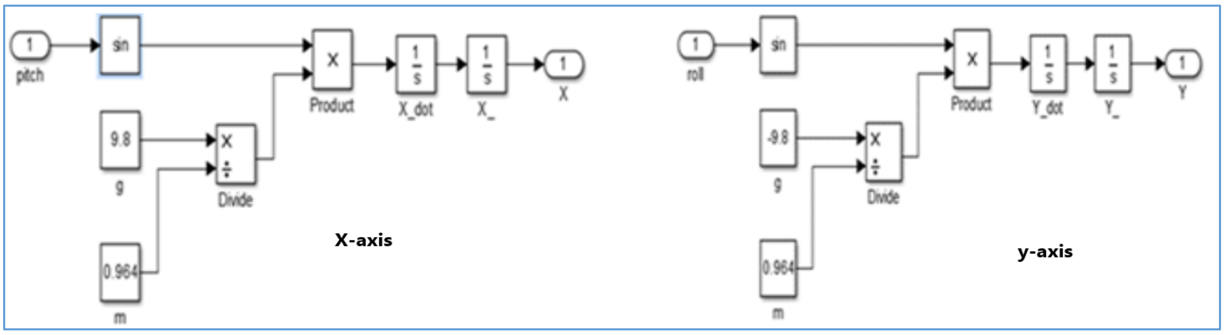


Figure 11: Adaptive Control simulation model on the x and y axes

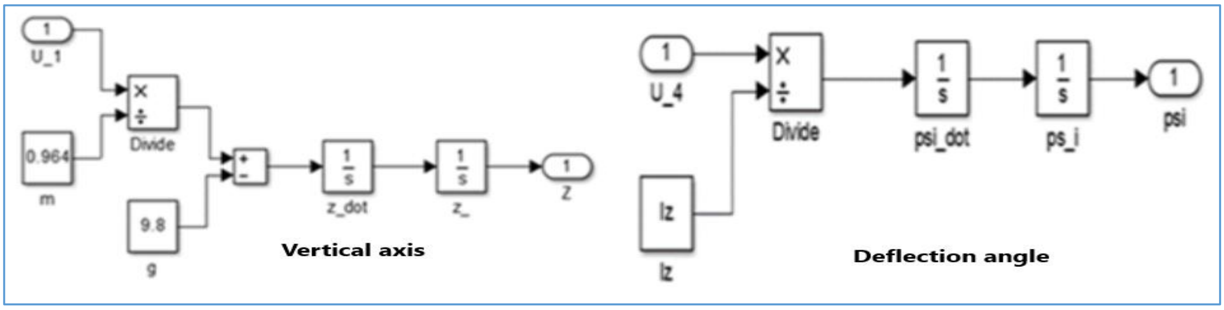


Figure 12: Adaptive Control simulation model for Vertical axis and Deflection angle

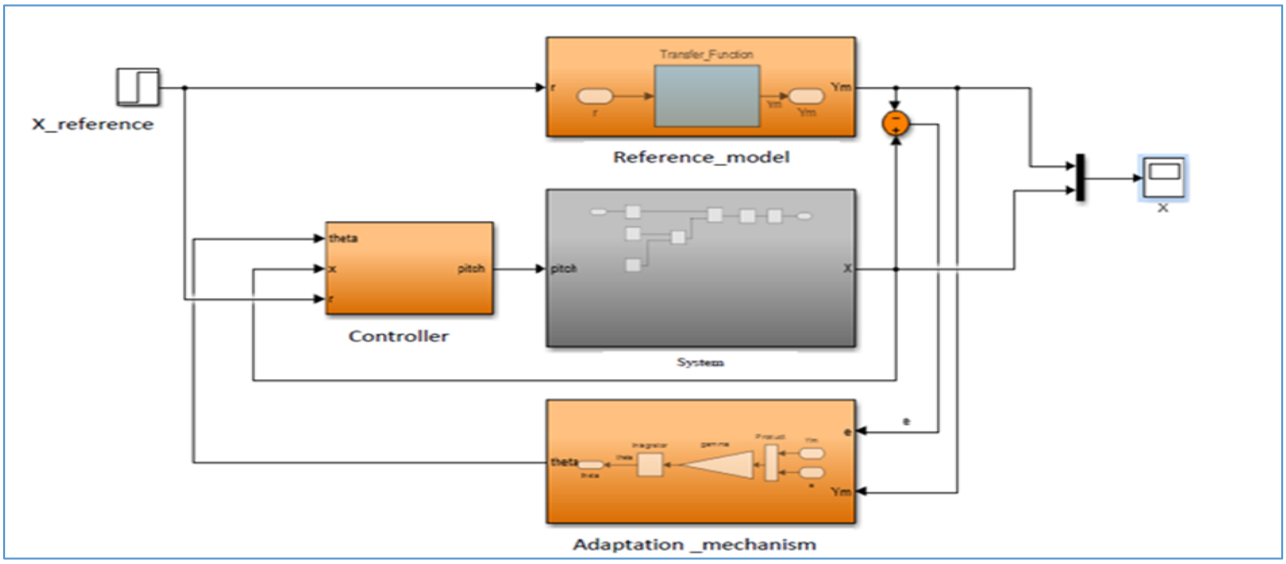


Figure 13: Adaptive Control Simulation Model

The simulation results obtained with different adaptation rates for different reference models using the MRAC method for the quadcopter are shown in Figure 14 to 21.

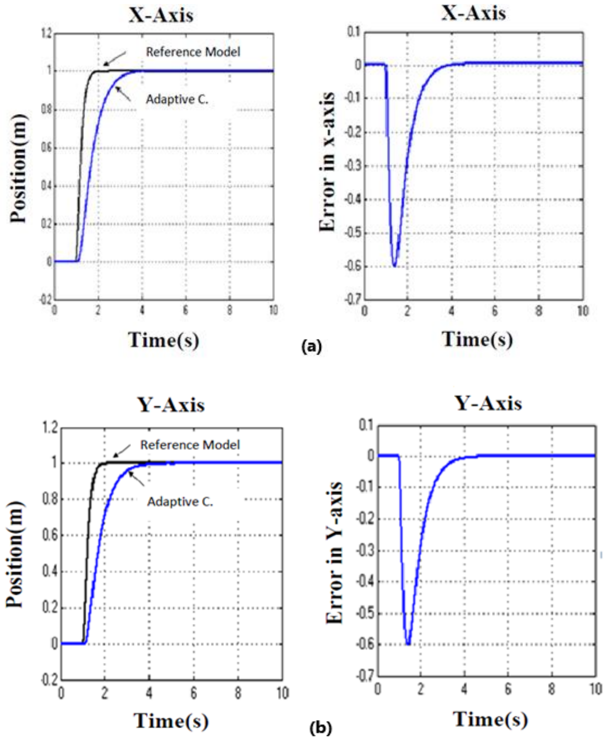


Figure 14: For; $G_{X-Y}(s) = \frac{60}{s^2+15s+60}$ reference model and $\gamma = 0.3$ (a) x-axis, (b) y-axis simulation results

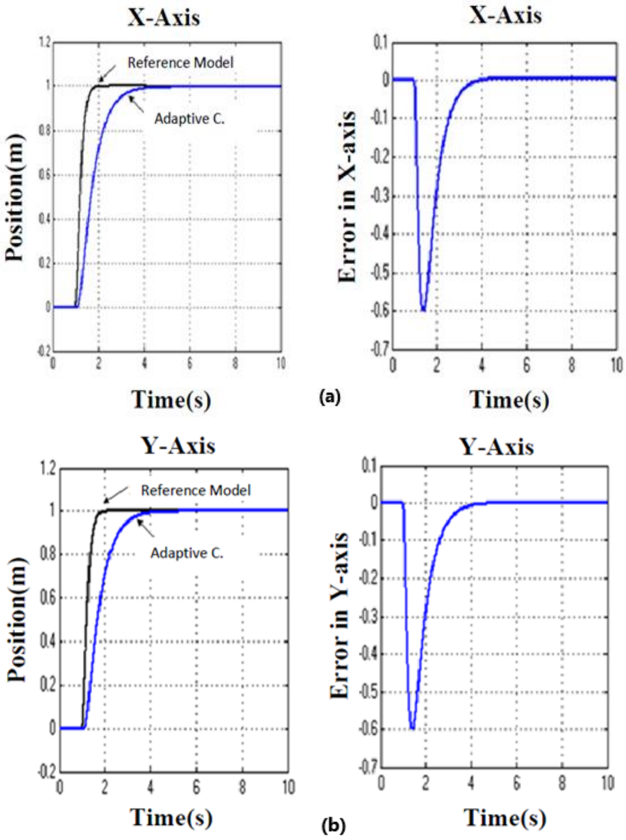


Figure 15: For; $G_{X-Y}(s) = \frac{60}{s^2+15s+60}$ reference model and $\gamma = 0.1$ (a) x-axis, (b) y-axis simulation results

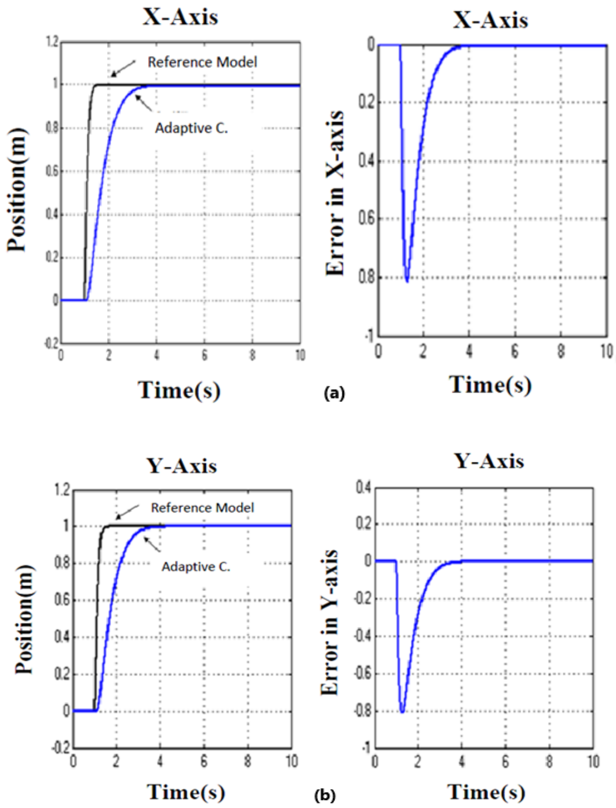


Figure 16: For; $G_{X-Y}(s) = \frac{225}{s^2+30s+225}$ reference model and $\gamma = 0.5$ (a) x-axis, (b) y-axis simulation results

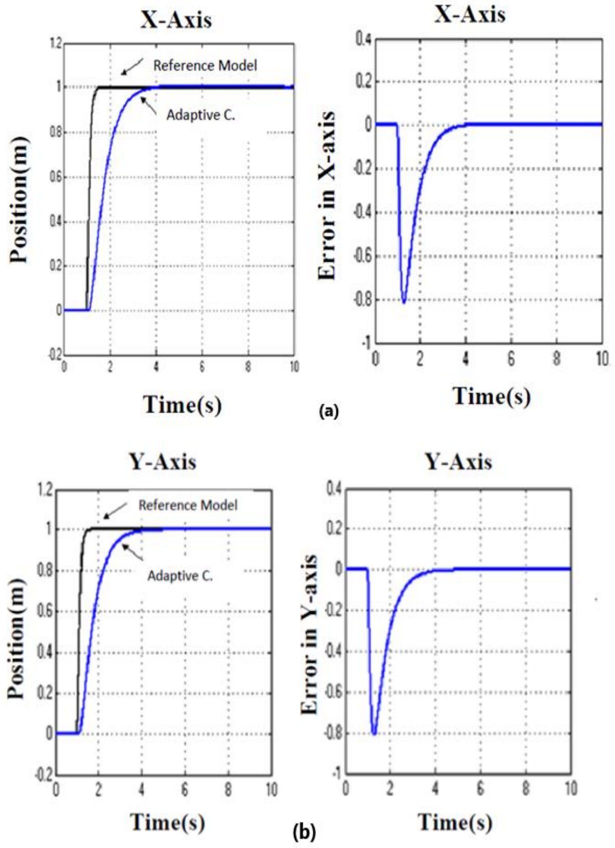


Figure 17: For; $G_{X-Y}(s) = \frac{225}{s^2+30s+225}$ reference model and $\gamma = 0.1$ (a) x-axis, (b) y-axis simulation results

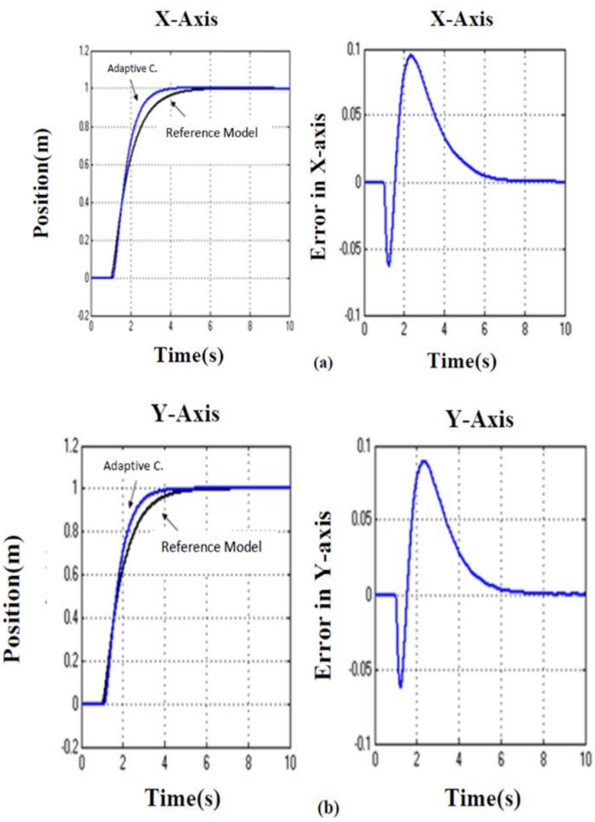


Figure 18: For; $G_{X-Y}(s) = \frac{9}{s^2+9s+9}$ reference model and $\gamma = 0.5$
(a) x-axis, (b) y-axis simulation results

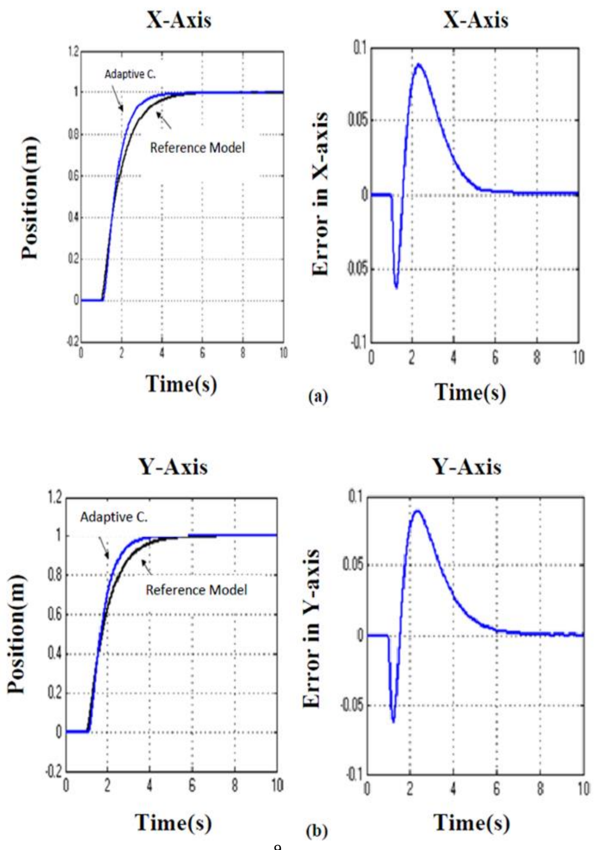


Figure 19: For; $G_{X-Y}(s) = \frac{9}{s^2+9s+9}$ reference model and $\gamma = 0.1$
(a) x-axis, (b) y-axis simulation results

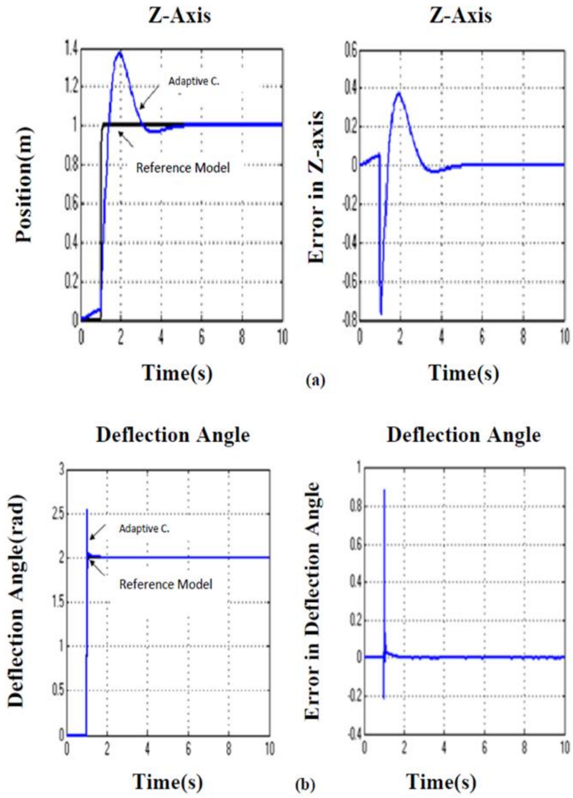


Figure 20: For; $G_{Z-\psi}(s) = \frac{60}{s+60}$ reference model and $\gamma = 0.5$
(a) z-axis, (b) deflection angle simulation results

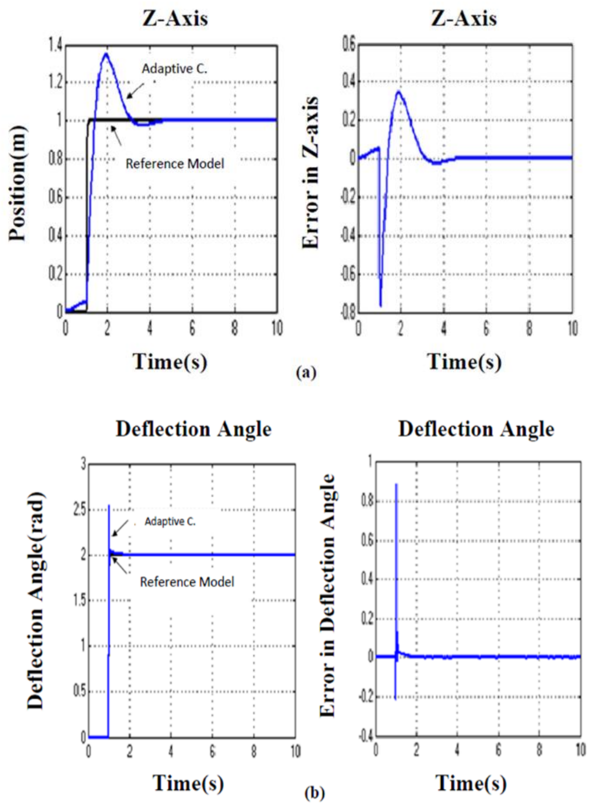


Figure 21: For; $G_{Z-\psi}(s) = \frac{60}{s+60}$ reference model and $\gamma = 0.5$
(a) z-axis, (b) deflection angle simulation results

The simulation results show that the choice of adaptation rate is very important. According to the simulation results in the studies, the system gave the best answer when the adaptation rate was 0.1.

3.4 PD Control for Quadcopter Nonlinear Model

In this section, a PD controller has been created for the quadcopter nonlinear model. Rotor voltage values U_1, U_2, U_3, U_4 are multiplied by the appropriate PD coefficients in the PD control block, and the rotor voltage values are checked for the case where the propellers are mounted.

In the quadcopter system block, the general angular velocity value of the quadcopter and the angular velocities of the propellers, ϕ (phi), θ (theta), ψ (psi) were calculated and the desired x, y, z values were obtained according to this angle data. The quadcopter PD simulation model is shown in Figure 22 and the simulation model results are shown in Figure 23.

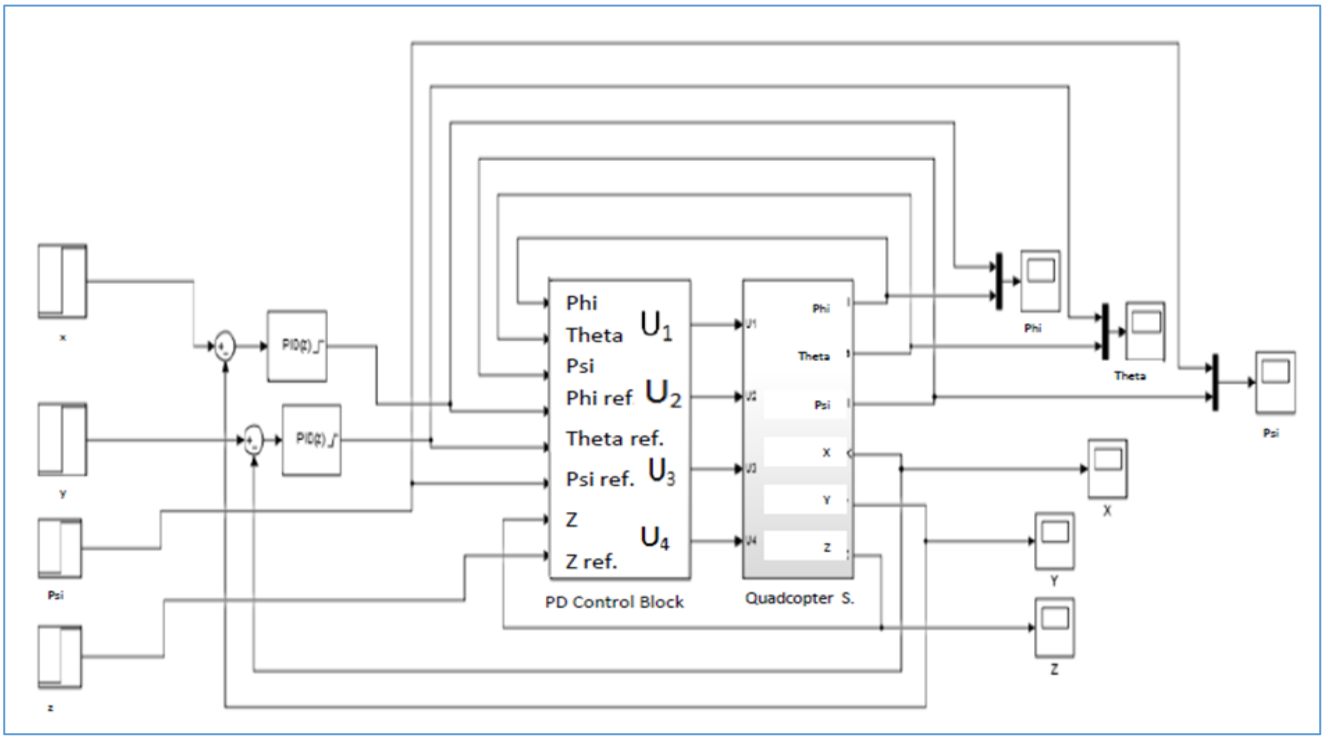


Figure 22: Quadcopter PD control simulation model

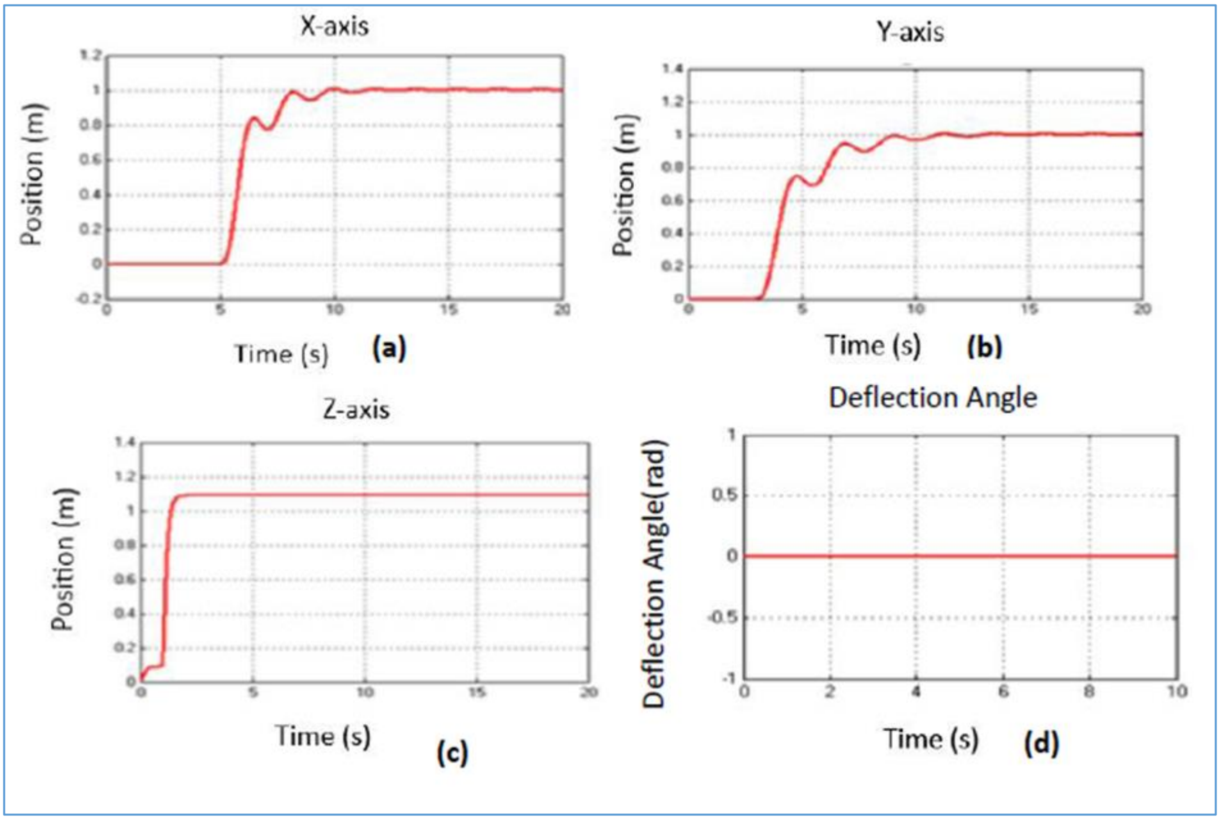


Figure 23: Quadcopter PD control simulation results a) x-axis b) y-axis c) z-axis d) deflection angle

When the PD control results for the quadcopter nonlinear model in Figure 23 are analyzed, it is seen that they pass the desired position values of x, y and z with a short settling time and without not much oscillation. A steady state error has been observed in the z-axis. The declination angle is at the stable position, ie at zero, throughout the simulation.

3.5 Tracking Scenarios

Route tracking scenarios were applied for PID, LQR and Adaptive Control methods for the quadcopter and the results obtained are shown in Figure 24.

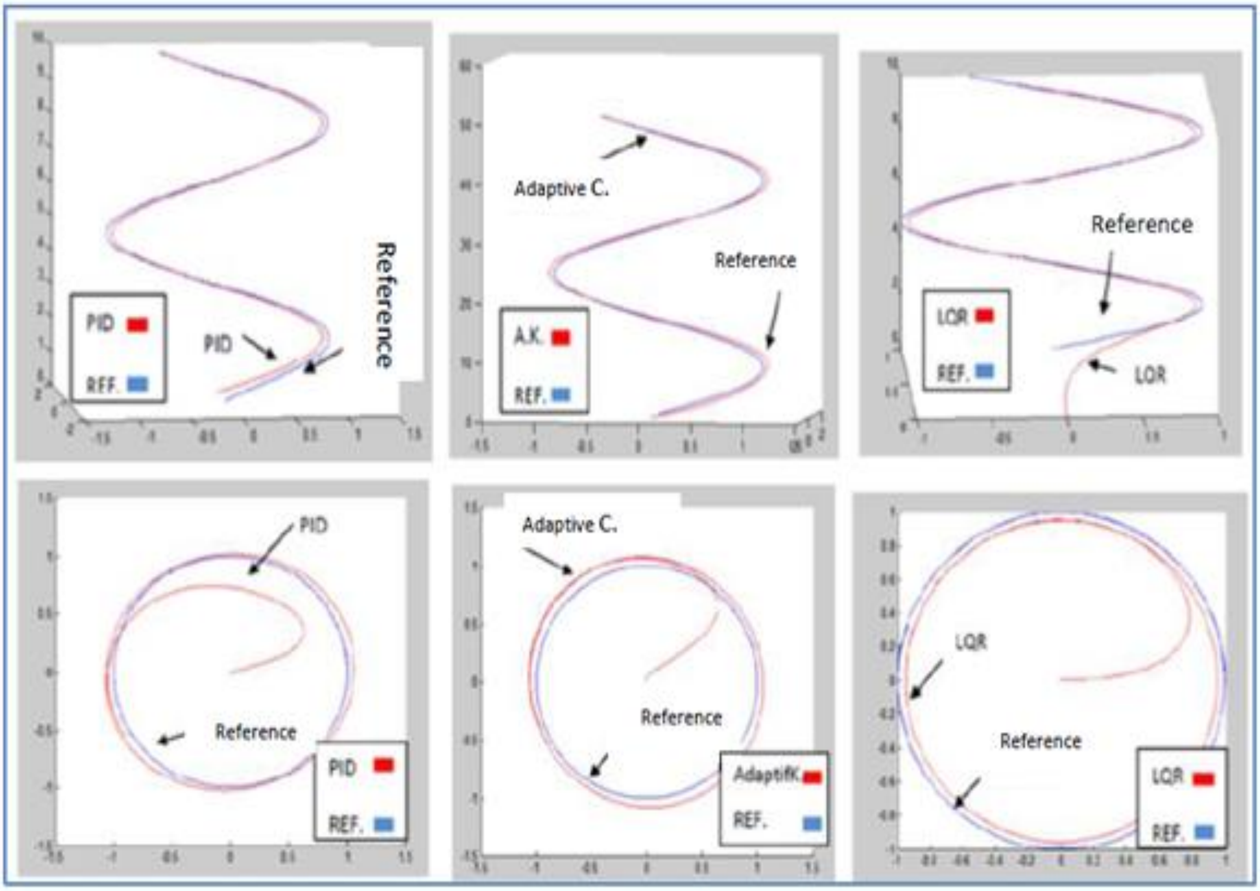


Figure 24: Results of route tracking scenarios

4 Conclusion

In this study, the dynamic model of the quadcopter (UAV), which has become widely used recently, was examined and simulation studies were carried out with PID, Adaptive Control and LQR methods for the quadcopter linear model, and PD Control methods for the nonlinear model.

In the PID structure, pitch and roll angles were first checked and then a control was applied for the x and y axes. Linear model is used for LQR design. In the simulation phase, the desired performances were achieved with the linear model. Looking at the simulation results, it was seen that the settling time was shorter in the PID Control method. Steady state error is almost nonexistent in both control methods.

When the PD control results for the quadcopter nonlinear model are examined, it is seen that they pass the desired position values of x, y and z with a short settling time and not much oscillation. A steady state error has been observed in the z-axis. The roll angle is at the stable position, ie at zero, throughout the simulation.

In the simulations, it has been observed that the Adaptive Control method gives more successful results in following the desired reference positions for x, y, z-axes and angle ψ. In MRAC method, selection of reference model and adaptation rate greatly affects system performance. In the study, desired results were obtained with control studies performed with different reference model and adaptation ratio selections for x, y, z axes. It was obtained from the adaptation rate $(\gamma) = 0.1$. According to the simulation studies, the best results for all reference models were obtained when the adaptation ratio was 0.1.

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