Hacettepe Journal of Mathematics & Statistics

Hacet. J. Math. Stat.
Volume 51 (1) (2022), 235 – 238
DOI : 10.15672/hujms.933537

Corrigendum

Corrigendum to "On generalized open sets"

Serap Erdem[1], Murad Özkoç*[2], Faical Yacine Issaka[1]

[1] *Muğla Sıtkı Koçman University, Graduate School of Natural and Applied Sciences, Mathematics, 48000 Menteşe-Muğla, Turkey*

[2] *Muğla Sıtkı Koçman University, Faculty of Science, Department of Mathematics, 48000 Menteşe-Muğla, Turkey*

Abstract

In this corrigendum, we show that Theorem 3.9, Theorem 2.9, Theorem 2.10(1) and Theorem 2.21(9)(10) given in the paper entitled "On generalized open sets" [Hacet. J. Math. Stat. 47(6) (2018), 1438-1446] are incorrect.

Mathematics Subject Classification (2020). 54A05, 54A10, 54D99

Keywords. γ-open set, γ-semiinterior, γ-semiclosure, γ-semiclosed set, γ-b-closure, γ-b-closed set, γ-b-closed function

1. The results

We refute the following Theorems stated by Hussain in [1] through six examples.

Theorem 1.1 ([1, Theorem 3.9]). *Let X and Y are topological spaces and function $f : X \to Y$ be a functions. Then the following statements are equivalent:*

(1) *f is γ-b-open.*

(2) *For each set B of Y and for each γ-open set A in X such that $f^{-1}(B) \subseteq A$, there is a γ-b-open set U of Y such that $B \subseteq U$ and $f^{-1}(U) \subseteq A$.*

The conclusion mentioned in Theorem 1.1 is incorrect. For example, let $X := \{a, b, c\}$ and let $\tau := 2^X$ be discrete topology on X. For $b \in X$, define an operation $\gamma : \tau \to 2^X$ by

$$\gamma(A) = A^\gamma := \begin{cases} \{a\} & , \quad A = \{a\} \\ A \cup \{b\} & , \quad A \neq \{a\} \end{cases}.$$

Clearly, γ-open sets in X are $\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}$. Calculations show that $BO_\gamma(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$. Also, let $Y := \{1, 2, 3\}$ and $\sigma := \{\emptyset, Y, \{3\}, \{1, 3\}\}$. For $2 \in Y$, define an operation $\gamma : \sigma \to 2^Y$ by

$$\gamma(A) = A^\gamma := \begin{cases} \{3\} & , \quad A = \{3\} \\ A \cup \{2\} & , \quad A \neq \{3\} \end{cases}.$$

*Corresponding Author.
Email addresses: seraperdem123@gmail.com (S. Erdem), murad.ozkoc@mu.edu.tr (M. Özkoç), faicalyacine@gmail.com (F.Y. Issaka)
Received: 05.05.2021; Accepted: 17.09.2021

Clearly, γ-open sets in Y are $\emptyset, Y, \{3\}$. Calculations show that

$$BO_\gamma(Y) = \{\emptyset, Y, \{3\}, \{1,3\}, \{2,3\}\}.$$

Define the function $f : X \to Y$ by $f := \{(a,3),(b,3),(c,2)\}$. Simple calculations show that f is γ-b-open function, but the proposition in (2) is not provided.

Theorem 1.2 ([1, Theorem 2.9]). *Let X be a space and $A \subseteq X$. Then*

(1) $scl_\gamma(A) = A \cup int_\gamma(cl_\gamma(A))$,

(2) $sint_\gamma(A) = A \cap cl_\gamma(int_\gamma(A))$.

The conclusion mentioned in Theorem 1.2 is incorrect. For example, let $X := \{1,2,3\}$ and $\tau := \{\emptyset, X, \{1\}, \{2\}, \{1,2\}\}$. Let us consider the operation $\gamma : \tau \to 2^X$ by $\gamma(T) := cl(T)$. Simple calculations show that $\gamma O(X) = \gamma C(X) = SO_\gamma(X) = SC_\gamma(X) = \{\emptyset, X\}$. For the subset $A := \{1\} \subseteq X$, we have

$$\begin{aligned} scl_\gamma(A) &= scl_\gamma(\{1\}) \\ &= \bigcap\{F | (\{1\} \subseteq F)(F \in SC_\gamma(X))\} \\ &= \bigcap\{X\} \\ &= X. \end{aligned}$$

On the other hand,

$$\begin{aligned} A \cup int_\gamma(cl_\gamma(A)) &= \{1\} \cup int_\gamma(cl_\gamma(\{1\})) \\ &= \{1\} \cup int_\gamma(\{1,3\}) \\ &= \{1\} \cup \{1\} \\ &= \{1\}. \end{aligned}$$

These calculations show that $scl_\gamma(A) = X \neq \{1\} = A \cup int_\gamma(cl_\gamma(A))$.

Similarly, for the subset $B := \{2,3\} \subseteq X$, we have

$$\begin{aligned} sint_\gamma(B) &= sint_\gamma(\{2,3\}) \\ &= \bigcup\{E | (E \subseteq \{2,3\})(E \in SO_\gamma(X))\} \\ &= \bigcup\{\emptyset\} \\ &= \emptyset. \end{aligned}$$

On the other hand,

$$\begin{aligned} B \cap cl_\gamma(int_\gamma(B)) &= \{2,3\} \cap cl_\gamma(int_\gamma(\{2,3\})) \\ &= \{2,3\} \cap cl_\gamma(\{2\}) \\ &= \{2,3\} \cap \{2,3\} \\ &= \{2,3\}. \end{aligned}$$

These calculations show that $sint_\gamma(B) = \emptyset \neq \{2,3\} = B \cap cl_\gamma(int_\gamma(B))$.

Theorem 1.3 ([1, Theorem 2.10(1)]). *Let X be a space and $A \subseteq X$. Then*

$$scl_\gamma(sint_\gamma(A)) = sint_\gamma(A) \cup int_\gamma(cl_\gamma(int_\gamma(A))).$$

The conclusion mentioned in Theorem 1.3 is incorrect. For example, let $X := \{1,2,3\}$ and $\tau := \{\emptyset, X, \{1\}, \{2\}, \{1,2\}\}$. Let us consider the operation $\gamma : \tau \to 2^X$ by $\gamma(B) :=$

$cl(B)$. Simple calculations show that $\gamma O(X) = \gamma C(X) = SO_\gamma(X) = SC_\gamma(X) = \{\emptyset, X\}$. For the subset $A := \{2,3\} \subseteq X$, we have

$$\begin{aligned} scl_\gamma(sint_\gamma(A)) &= scl_\gamma(sint_\gamma(\{2,3\})) \\ &= scl_\gamma(\cup\{E|(E \subseteq \{2,3\})(E \in SO_\gamma(X))\}) \\ &= scl_\gamma(\cup\{\emptyset\}) \\ &= scl_\gamma(\emptyset) \\ &= \bigcap\{F|(\emptyset \subseteq F)(F \in SC_\gamma(X))\} \\ &= \bigcap\{\emptyset, X\} \\ &= \emptyset. \end{aligned}$$

On the other hand,

$$\begin{aligned} sint_\gamma(A) \cup int_\gamma(cl_\gamma(int_\gamma(A))) &= sint_\gamma(\{2,3\}) \cup int_\gamma(cl_\gamma(int_\gamma(\{2,3\}))) \\ &= \emptyset \cup int_\gamma(cl_\gamma(\{2\})) \\ &= int_\gamma(\{2,3\}) \\ &= \{2\}. \end{aligned}$$

These calculations show that $scl_\gamma(sint_\gamma(A)) = \emptyset \neq \{2\} = sint_\gamma(A) \cup int_\gamma(cl_\gamma(int_\gamma(A)))$.

Theorem 1.4 ([1, Theorem 2.21(9)(10)]). *Let X be a space and $A \subseteq X$. Then*

(1) $sint_\gamma(bcl_\gamma(A)) = scl_\gamma(A) \cap cl_\gamma(int_\gamma(A))$,

(2) $scl_\gamma(bint_\gamma(A)) = sint_\gamma(A) \cup int_\gamma(cl_\gamma(A))$.

The conclusion mentioned in Theorem 1.4 is incorrect. For example, let $X := \{1,2,3\}$ and $\tau := \{\emptyset, X, \{1\}, \{2\}, \{1,2\}\}$. Let us consider the operation $\gamma : \tau \to 2^X$ by $\gamma(T) := cl(T)$. Simple calculations show that $\gamma O(X) = \gamma C(X) = SO_\gamma(X) = SC_\gamma(X) = \{\emptyset, X\}$ and $BO_\gamma(X) = BC_\gamma(X) = 2^X$. For the subset $A := \{2,3\} \subseteq X$, we have

$$\begin{aligned} sint_\gamma(bcl_\gamma(A)) &= sint_\gamma(bcl_\gamma(\{2,3\})) \\ &= sint_\gamma(\bigcap\{F|(\{2,3\} \subseteq F)(F \in BC_\gamma(X))\}) \\ &= sint_\gamma(\bigcap\{\{2,3\}, X\}) \\ &= sint_\gamma(\{2,3\}) \\ &= \bigcup\{E|(E \subseteq \{2,3\})(E \in SO_\gamma(X))\} \\ &= \bigcup\{\emptyset\} \\ &= \emptyset. \end{aligned}$$

On the other hand,

$$\begin{aligned} scl_\gamma(A) \cap cl_\gamma(int_\gamma(A)) &= scl_\gamma(\{2,3\}) \cap cl_\gamma(int_\gamma(\{2,3\})) \\ &= (\bigcap\{F|(\{2,3\} \subseteq F)(F \in SC_\gamma(X))\}) \cap cl_\gamma(\{2\}) \\ &= (\bigcap\{X\}) \cap \{2,3\} \\ &= X \cap \{2,3\} \\ &= \{2,3\}. \end{aligned}$$

These calculations show that $sint_\gamma(bcl_\gamma(A)) = \emptyset \neq \{2,3\} = scl_\gamma(A) \cap cl_\gamma(int_\gamma(A))$.

Similarly, for the subset $B := \{1\} \subseteq X$, we have

$$\begin{aligned} scl_\gamma(bint_\gamma(B)) &= scl_\gamma(bint_\gamma(\{1\})) \\ &= scl_\gamma(\bigcup\{E|(E \subseteq \{1\})(E \in BO_\gamma(X))\}) \\ &= scl_\gamma(\bigcup\{\emptyset, \{1\}\}) \\ &= scl_\gamma(\{1\}) \\ &= \bigcap\{F|(\{1\} \subseteq F)(F \in SC_\gamma(X))\} \\ &= \bigcap\{X\} \\ &= X. \end{aligned}$$

On the other hand,

$$\begin{aligned} sint_\gamma(B) \cap int_\gamma(cl_\gamma(B)) &= sint_\gamma(\{1\}) \cap int_\gamma(cl_\gamma(\{1\})) \\ &= (\bigcup\{E|(E \subseteq \{1\})(E \in SO_\gamma(X))\}) \cap int_\gamma(\{1,3\}) \\ &= (\bigcup\{\emptyset\}) \cap \{1\} \\ &= \emptyset \cap \{1\} \\ &= \emptyset. \end{aligned}$$

These calculations show that $scl_\gamma(bint_\gamma(B)) = X \neq \emptyset = sint_\gamma(B) \cap int_\gamma(cl_\gamma(B))$.

References

[1] S. Hussain, *On generalized open sets*, Hacet. J. Math. Stat. **47** (6), 1438–1446, 2018.