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A Novel, Nelder-Mead Optimization Approach, based on Neuro-regression modeling for the Energy Efficiency Parameters of End Milling Process

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Abstract

Global crises are increasing day by day due to the rapid depletion of energy supplies around the planet. One of the goals of engineering is to prevent this situation by developing innovative solutions to this rapid energy consumption that has disappeared in the world. A solution could be to reduce the energy consumption of the machines that are used during production. In this study, a new design technique based on the neuro-regression approach and non-linear regression modeling was offered as an alternative to Taguchi design to reduce energy consumption. Thus, a cutting parameter optimization model was created to examine the effects of the constraint conditions on energy consumption. The cutting power, the surface roughness of the part, and tool life were handled as objective functions(constraint conditions). First of all, the multiple non-linear regression modeling was created using design variables in end milling . These design variables were determined as spindle rotational speed, feed rate power, radial cut depth, axial cut depth, and cutting speed. Then, objective functions were brought to the proper minimum optimal levels due to this optimization modeling. As a result of the optimization model built with design variables, accurate modeling was achieved in this work by studying several optimization models utilized to optimize the minimum objective functions, which play a significant role in reducing energy consumption in end milling. After the optimization, the maximum value was found as 110.791. At the end of the study, some options of direct search method to maximize and minimize results were applied.

Keywords: End milling; energy efficiency; optimization.

1. Introduction

The industrial sector has been pushed to cut energy consumption and enhance energy efficiency since natural energy resources have been depleted due to rising global energy crises and the resulting climate change. Manufacturing is one of the most energy-consuming industries, so there is much potential for energy-saving options to think about, analyze, and test. Energy resources must be used correctly in the manufacturing industry, and energy efficiency must be continuously improved. For example, a suitable energy-oriented machine tool component design or better machine usage, both in terms of machining strategy and process parameter selection, can save energy in machine tools [1].

Machining is a significant part of the manufacturing industry, which is one of the most critical industries. Machining is generally defined as removing material in the form of chips from a workpiece or part. A mechanical part can be machined using different techniques without significant differences in final results. However, machining methods such as end milling play a significant role in producing and shaping parts. End milling can be used to produce slots, shoulders, die cavities, contours, profiles, and other milling parts. It is widely used to create auto parts, aircraft parts, etc. Machine tools are the primary electricity-consuming devices in milling processes, and they are also the source of carbon dioxide emissions [2].

For this reason, end milling machines consume so much energy. Therefore, studying energy parameters is important since cutting parameters in end milling can enhance energy efficiency. Several studies on optimizing cutting parameters have been published in this field; several of them used surface roughness, cutting force, cutting power, tool life, and material removal rate as optimization criteria [3].

There are many combinations of parameters, such as feed rate, spindle speed, axial or radial depth of cut, to achieve varied results in terms of machined surface quality and tool wear, depending on the machining goal and the choice of cutting tool [4]. In addition, each combination of cutting parameters will provide a varied surface roughness and lifetime of the tool [5].

One way to increase energy efficiency is to increase the machine tool's lifetime and achieve minimum surface roughness and cutting power consumption. Many different studies have been conducted to reach these results. Different optimization methods were used in these studies.

Velchev et al. [6] presented an approach to optimize cutting parameters to minimize direct energy consumption during turning. Negrete-Compesto [7] optimized the cutting parameters (cutting speed, cutting depth, and feed rate) for minimizing electrical energy consumption in turning of AISI 6061 T6 by the Taguchi method.

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Mativenga and Rajemi [8] established an energy consumption model for a single pass turning process, including energy during machine setup, machining, tool change, and tool production. The energy model could derive the optimal tool for economic life and cutting speed under a minimum energy criterion. Their model can be improved if machine stand-by power was not used to replace power consumption during their study's tool-changing operations. The minimum energy criterion introduced is exploited to improve and apply a methodology for optimum cutting conditions' selection.

The paper by Rajemi et al. [9] aims to improve a novel model and technique for optimizing the energy footprint for a machined product. They introduced the essential parameters in minimizing energy use and therefore reducing the energy cost and environmental footprint.

Asilturk et al. [10] applied response surface methodology to optimize cutting parameters (spindle speed, feed, depth of cut, and tool radius) and developed a surface roughness model of medical alloy machined on a CNC lathe.

In work presented by Bhushan [11], the optimization of turning cutting parameters to minimize electrical energy consumption and maximize tool life was presented. The response surface methodology (RSM) was applied to establish the cutting parameters' electrical energy consumption and tool life models. Results of the research work showed that electrical energy consumption could be reduced by 13.55%, and tool life can be increased by 22.12% with the optimized cutting parameters. Li et al. [12] integrated Taguchi, the response surface methodology and multi-objective particle swarm optimization to optimize energy saving parameters and selected specific energy consumption to evaluate energy efficiency. The results showed that feed rate is the most significant factor for minimizing electrical energy consumption. A higher feed rate provides minimum electrical energy consumption.

The main objective of this paper is to minimize the surface roughness and cutting power of the tool and maximize the lifetime of the machine tool by using a new neuro-regression analysis for improving energy efficiency in end milling. Furthermore, the direct search approach, modified versions of the Nelder-Mead algorithm have been thoroughly tested and shown.

2. Materials and Methods

2.1. Optimization

The notion of optimization, which is the inherent attribute of achieving the best or most beneficial (minimum or maximum) outcome from a given situation, has enormous significance in human affairs and natural laws. Since the beginning of civilization, the human species has faced countless technological obstacles, including determining the best answer to various issues such as control technologies, power sources constructions, economic applications, mechanical engineering, and energy distribution, amongst others. Optimization problems are ubiquitous in science, and even in our daily lives, we optimize how we get to work every morning or how we should navigate to a new place. There is an objective you want to either maximize or minimize in an optimization problem, and there may be constraints within which you need to operate. This shows how design optimization may aid not just in the human activity of producing optimal designs for products, processes, and systems but also in the understanding and analysis of mathematical and physical phenomena and the solution of mathematical problems. [13]

2.1.1. Regression analysis

Regression analysis is one of the reliable tools frequently used in economics, science, and engineering. Regression analysis allows us to determine which variables have an effect on the subject we are studying. It helps us determine whether we should improve or ignore these variables. The purpose of regression analysis is to express the response variable as a function of the predictive variables. The duality of the fit and the accuracy of the result depend on the data used [14].

Regression analysis can be examined in different categories according to the number and linearity of the predictive variables: Simple and multiple regression; linear and non-linear regression. The most common form of regression analysis is linear regression, it is a model that assesses the relationship between a dependent variable and an independent variable. Multiple linear regression analysis is essentially similar to the simple linear model, except that multiple independent variables are used in the model. When the model function is not linear in the parameters, an iterative procedure must minimize the sum of squares. This introduces many complications, which are summarized in differences between linear and non-linear least squares [15].

2.1.2. Neuro-regression approach

To increase the accuracy of the predictions, a technique that combines the strengths of regression analysis and Artificial Neural Network (ANN) is utilized in the modeling step. With this approach, all the data in our table are divided into training, testing, and validation, approximately 80%, 15%, and 5%. However, it is used by choosing an appropriate regression model. If we have only one variable, the simple regression model is used here. If there is more than one variable, a multiple regression model is used. If the variables contain non-linear terms, the non-linear regression model should be used. First, in the training part, the coefficients in the regression model are adjusted to minimize the error between the experimental and predicted values. Then, the inconsistencies of the regression model in the testing part and the validation part are minimized, and estimated results are tried to be reached. After obtaining the appropriate values, each variable's maximum and minimum values in the given ranges are calculated [14, 15].

In the Mathematica code, some terms are used. "Length" will give you the number of data sets in the experiment. The sum of squares total, denoted SST, is the squared differences between the observed dependent variable and its mean. The sum of squares error or SSE is the summation of each element's testing and prediction response data. "Ybar" is the mean of the training data [15]

2.1.3. Regression models

In this study, different models were used to reach the appropriate values. Some of them are multiple linear regression, and some are non-linear regression models. The table below contains these models.

Table 1. *Regression models name with nomenclature – formula.* [14, 15]

Model Name	Nomenclature	Formula
Multiple Linear	L	a[1] + a[2] x1 + a[3] x2 + a[4] x3 + a[5] x4
Multiple Linear Rational	LR	(a[1] + a[2] x1 + a[3] x2 + a[4] x3 + a[5] x4)/(b[1] + b[2] x1 + b[3] x2 + b[4] x3 + b[5] x4)
Second Order Multiple Nonlinear	SON	a[1] + 2 x1 a[2] + x1^2 a[3] + 2 x2 a[4] + 2 x1 x2 a[5] + x2^2 a[6] + 2 x3 a[7] + 2 x1 x3 a[8] + 2 x2 x3 a[9] + x3^2 a[10] + 2 x4 a[11] + 2 x1 x4 a[12] + 2 x2 x4 a[13] + 2 x3 x4 a[14] + x4^2 a[15]
Second-Order Multiple Nonlinear Rational	SONR	(a[1] + 2 x1 a[2] + x1^2 a[3] + 2 x2 a[4] + 2 x1 x2 a[5] + x2^2 a[6] + 2 x3 a[7] + 2 x1 x3 a[8] + 2 x2 x3 a[9] + x3^2 a[10] + 2 x4 a[11] + 2 x1 x4 a[12] + 2 x2 x4 a[13] + 2 x3 x4 a[14] + x4^2 a[15])/(b[1] + 2 x1 b[2] + x1^2 b[3] + 2 x2 b[4] + 2 x1 x2 b[5] + x2^2 b[6] + 2 x3 b[7] + 2 x1 x3 b[8] + 2 x2 x3 b[9] + x3^2 b[10] + 2 x4 b[11] + 2 x1 x4 b[12] + 2 x2 x4 b[13] + 2 x3 x4 b[14] + x4^2 b[15])
Third Order Multiple Nonlinear	TON	a[1] + a[2] 3 x1 + a[3] 3 x1^2 + a[4] x1^3 + a[5] 3 x2 + a[6] 6 x1 x2 + a[7] 3 x1^2 x2 + a[8] 3 x2^2 + a[9] 3 x1 x2^2 + a[10] x2^3 + a[11] 3 x3 + a[12] 6 x1 x3 + a[13] 3 x1^2 x3 + a[14] 6 x2 x3 + a[15] 6 x1 x2 x3 + a[16] 3 x2^2 x3 + a[17] 3 x3^2 + a[18] 3 x1 x3^2 + a[19] 3 x2 x3^2 + a[20] x3^3 + a[21] 3 x4 + a[22] 6 x1 x4 + a[23] 3 x1^2 x4 + a[24] 6 x2 x4 + a[25] 6 x1 x2 x4 + a[26] 3 x2^2 x4 + a[27] 6 x3 x4 + a[28] 6 x1 x3 x4 + a[29] 6 x2 x3 x4 + a[30] 3 x3^2 x4 + a[31] 3 x4^2 + a[32] 3 x1 x4^2 + a[33] 3 x2 x4^2 + a[34] 3 x3 x4^2 + a[35] x4^3
Fourth Order Multiple Nonlinear	FON	a[1] 1 + a[2] 4 x1 + a[3] 6 x1^2 + a[4] 4 x1^3 + a[5] x1^4 + a[6] 4 x2 + a[7] 12 x1 x2 + a[8] 12 x1^2 x2 + a[9] 4 x1^3 x2 + a[10] 6 x2^2 + a[11] 12 x1 x2^2 + a[12] 6 x1^2 x2^2 + a[13] 4 x2^3 + a[14] 4 x1 x2^3 + a[15] x2^4 + a[16] 4 x3 + a[17] 12 x1 x3 + a[18] 12 x1^2 x3 + a[19] 4 x1^3 x3 + a[20] 12 x2 x3 + a[21] 24 x1 x2 x3 + a[22] 12 x1^2 x2 x3 + a[23] 12 x2^2 x3 + a[24] 12 x1 x2^2 x3 + a[25] 4 x2^3 x3 + a[26] 6 x3^2 + a[27] 12 x1 x3^2 + a[28] 6 x1^2 x3^2 + a[29] 12 x2 x3^2 + a[30] 12 x1 x2 x3^2 + a[31] 6 x2^2 x3^2 + a[32] 4 x3^3 + a[33] 4 x1 x3^3 + a[34] 4 x2 x3^3 + x3^4 + a[35] 4 x4 + a[36] 12 x1 x4 + a[37] 12 x1^2 x4 + a[38] 4 x1^3 x4 + a[39] 12 x2 x4 + a[40] 24 x1 x2 x4 + a[41] 12 x1^2 x2 x4 + a[42] 12 x2^2 x4 + a[43] 12 x1 x2^2 x4 + a[44] 4 x2^3 x4 + a[45] 12 x3 x4 + a[46] 24 x1 x3 x4 + a[47] 12 x1^2 x3 x4 + a[48] 24 x2 x3 x4 + a[49] 24 x1 x2 x3 x4 + a[50] 12 x2^2 x3 x4 + a[51] 12 x3^2 x4 + a[52] 12 x1 x3^2 x4 + a[53] 12 x2 x3^2 x4 + a[54] 4 x3^3 x4 + a[55] 6 x4^2 + a[56] 12 x1 x4^2 + a[57] 6 x1^2 x4^2 + a[58] 12 x2 x4^2 + a[59] 12 x1 x2 x4^2 + a[60] 6 x2^2 x4^2 + a[61] 12 x3 x4^2 + a[62] 12 x1 x3 x4^2 + a[63] 12 x2 x3 x4^2 + a[64] 6 x3^2 x4^2 + a[65] 4 x4^3 + a[66] 4 x1 x4^3 + a[67] 4 x2 x4^3 + a[67] 4 x3 x4^3 + a[68] x4^4

First Order Trigonometric Multiple Nonlinear	FOTN	a[1] + a[2] Cos[x1] + a[3] Cos[x2] + a[4] Cos[x3] + a[5] Cos[x4] + a[6] Sin[x1] + a[7] Sin[x2] + a[8] Sin[x3] + a[9] Sin[x4]
First Order Trigonometric Multiple Nonlinear Rational	FOTNR	a[1] + a[2] Cos[x1] + a[3] Cos[x2] + a[4] Cos[x3] + a[5] Cos[x4] + a[6] Sin[x1] + a[7] Sin[x2] + a[8] Sin[x3] + a[9] Sin[x4])/(b[1] + b[2] Cos[x1] + b[3] Cos[x2] + b[4] Cos[x3] + b[5] Cos[x4] + b[6] Sin[x1] + b[7] Sin[x2] + b[8] Sin[x3] + b[9] Sin[x4]
Second Order Trigonometric Multiple Nonlinear	SOTN	a[1] + 2 a[2] Cos[x1] + a[3] Cos[x1]^2 + 2 a[4] Cos[x2] + 2 a[5] Cos[x1] Cos[x2] + a[6] Cos[x2]^2 + 2 a[7] Cos[x3] + 2 a[8] Cos[x1] Cos[x3] + 2 a[9] Cos[x2] Cos[x3] + a[10] Cos[x3]^2 + 2 a[11] Cos[x4] + 2 a[12] Cos[x1] Cos[x4] + 2 a[13] Cos[x2] Cos[x4] + 2 a[14] Cos[x3] Cos[x4] + a[15] Cos[x4]^2 + 2 a[16] Cos[x5] + 2 a[17] Cos[x1] Cos[x5] + 2 a[18] Cos[x2] Cos[x5] + 2 a[19] Cos[x3] Cos[x5] + 2 a[20] Cos[x4] Cos[x5] + a[21] Cos[x5]^2 + 2 a[22] Sin[x1] + 2 a[23] Cos[x1] Sin[x1] + 2 a[24] Cos[x2] Sin[x1] + 2 a[25] Cos[x3] Sin[x1] + 2 a[26] Cos[x4] Sin[x1] + 2 a[27] Cos[x5] Sin[x1] + a[28] Sin[x1]^2 + 2 a[29] Sin[x2] + 2 a[30] Cos[x1] Sin[x2] + 2 a[31] Cos[x2] Sin[x2] + 2 a[32] Cos[x3] Sin[x2] + 2 a[33] Cos[x4] Sin[x2] + 2 a[34] Cos[x5] Sin[x2] + 2 a[35] Sin[x1] Sin[x2] + a[36] Sin[x2]^2 + 2 a[37] Sin[x3] + 2 a[38] Cos[x1] Sin[x3] + 2 a[39] Cos[x2] Sin[x3] + 2 a[40] Cos[x3] Sin[x3] + 2 a[41] Cos[x4] Sin[x3] + 2 a[42] Cos[x5] Sin[x3] + 2 a[43] Sin[x1] Sin[x3] + 2 a[44] Sin[x2] Sin[x3] + a[45] Sin[x3]^2 + 2 a[46] Sin[x4] + 2 a[47] Cos[x1] Sin[x4] + 2 a[48] Cos[x2] Sin[x4] + 2 a[49] Cos[x3] Sin[x4] + 2 a[50] Cos[x4] Sin[x4] + 2 a[51] Cos[x5] Sin[x4] + 2 a[52] Sin[x1] Sin[x4] + 2 a[53] Sin[x2] Sin[x4] + 2 a[54] Sin[x3] Sin[x4] + a[55] Sin[x4]^2 + 2 a[56] Sin[x5] + 2 a[57] Cos[x1] Sin[x5] + 2 a[58] Cos[x2] Sin[x5] + 2 a[59] Cos[x3] Sin[x5] + 2 a[60] Cos[x4] Sin[x5] + 2 a[61] Cos[x5] Sin[x5] + 2 a[62] Sin[x1] Sin[x5] + 2 a[63] Sin[x2] Sin[x5] + 2 a[64] Sin[x3] Sin[x5] +2 a[65] Sin[x4] Sin[x5] + a[66] Sin[x5]^2
First Order Logarithmic Multiple Nonlinear	FOLN	a[1] + a[2] Log[x1] + a[3] Log[x2] + a[4] Log[x3] + a[5] Log[x4] + a[6] Log[x5]
Second Order Logarithmic Multiple Nonlinear	SOLN	a[1] + 2 a[2] Log[x1] + a[3] Log[x1]^2 + 2 a[4] Log[x2] + 2 a[5] Log[x1] Log[x2] + a[6] Log[x2]^2 + 2 a[7] Log[x3] + 2 a[8] Log[x1] Log[x3] + 2 a[9] Log[x2] Log[x3] + a[10] Log[x3]^2 + 2 a[11] Log[x4] + 2 a[12] Log[x1] Log[x4] + 2 a[13] Log[x2] Log[x4] + 2 a[14] Log[x3] Log[x4] + a[15] Log[x4]^2 + 2 a[16] Log[x5] + 2 a[17] Log[x1] Log[x5] + 2 a[18] Log[x2] Log[x5] + 2 a[19] Log[x3] Log[x5] + 2 a[20] Log[x4] Log[x5] + a[21] Log[x5]^2

2.1.4. Problem definition

The main problem in this study is to obtain suitable results due to optimizing the objective functions used to increase energy efficiency. Tables 2-4 give the information about the results of surface roughness and cutting power of the machine, the lifetime of the machine, and constraints of design variables of objective functions, respectively. Surface roughness and cutting power should be minimized, and the lifetime of the machine should be increased. These outputs depend on some predictive variables. As a result of controlling these predictive variables, maximum or minimum values of outputs can be reached. These constraint (predictive) variables are feed rate, spindle speed, axial and radial depth of cut, and cutting speed. Output 1 (surface roughness) and output 2 (cutting power) have the same constraint variables. These variables are feed rate, spindle speed, axial and radial depth of cut. The constraint variables of output 3 (lifetime of the machine) are feed rate, spindle speed, axial and radial depth of cut, and cutting speed. Output 1 and output 2 have 25 different data sets; Output 3 has nine different data sets. Therefore, appropriate data must be captured for these output values. For this purpose, at the end of the neuro-regression approach, the $R^2_{training}$ value must be greater than 0.90, the $R^2_{testing}$ value must be greater than 0.85, and finally, the $R^2_{validation}$ value must be greater than 0.85 to fit the accurate model. If we obtain these values, the appropriate model is considered to have been reached.

Table 2. *Data table of output 1 (surface roughness) and output 2 (cutting power).* [2]

No.	n (r/min)	v_f (mm/min)	a_p (mm)	a_e (mm)	R_a (µm)	Power (kW)
1	800	240	0.5	2	2.141	0.020
2	800	290	1	3	2.793	0.060
3	800	340	1.5	4	2.837	0.100
4	800	380	2	5	3.512	0.200
5	800	420	2.5	6	4.013	0.300
6	1200	240	1	4	2.171	0.069
7	1200	290	1.5	5	2.331	0.134
8	1200	340	2	6	2.424	0.244
9	1200	380	2.5	2	1.924	0.119
10	1200	420	0.5	3	2.187	0.047
11	1600	240	1.5	6	2.024	0.140
12	1600	290	2	2	1.897	0.086
13	1600	340	2.5	3	1.799	0.155
14	1600	380	0.5	4	2.17	0.060
15	1600	420	1	5	2.373	0.150
16	2000	240	2	3	1.533	0.143
17	2000	290	2.5	4	1.565	0.253
18	2000	340	0.5	5	1.684	0.068
19	2000	380	1	6	1.76	0.178
20	2000	420	1.5	2	1.741	0.113
21	2400	240	2.5	5	1.497	0.290
22	2400	290	0.5	6	1.728	0.115
23	2400	340	1	2	1.415	0.085
24	2400	380	1.5	3	1.321	0.185
25	800	240	0.5	2	2.141	0.020

Table 3. *Output 3 (lifetime of machine tool)T_{life}.*

No.	n (r/min)	v_f (mm/min)	a_p (mm)	a_e (mm)	V_c (m/s)	T_{life} (min)
1	1300	120	2.5	3	0.816	80
2	1300	140	3	6	0.816	62
3	1300	180	3.5	9	0.816	53
4	1700	120	3	9	1.068	49
5	1700	140	3.5	3	1.068	55
6	1700	180	2.5	6	1.068	50
7	2100	120	3.5	6	1.319	20
8	2100	140	2.5	9	1.319	30
9	2100	180	3	3	1.319	25

Table 4. *Constraints of design variables.*

Constraints of Design Variables For Output 1 (Surface Roughness) and Output 3 (Cutting Power)	Constraints of Design Variables For Output 3 (Lifetime of Machine)
800 < Spindle Rotation Speed (n) < 2400	1300 < Spindle Rotation Speed (n) < 2100
240 < Feed Rate (v_f) < 420	120 < Feed Rate (v_f) < 180
0.5 < Radial Cut Depth (a_p) < 2.5	2.5 < Radial Cut Depth (a_p) < 3.5
2 < Axial Cut Depth (a_e) < 6	3 < Axial Cut Depth (a_e) < 9
	0.816 < Cutting Speed (V_c) < 1.319

2.1.4.1. Optimization process of problem

The neuro-regression approach should be used as a first step. The data sets of the outputs should be divided into 80%, 15%, and 5% as training, testing, and validation randomly. Then the appropriate regression models should be used respectively. In this study, non-linear regression modeling was used for three different outputs. The suitable models for outputs 1-3 are the second-order non-linear, third-order non-linear, and second-order trigonometric non-linear regression functions. Then, the regression coefficients were calculated in line with the regression analysis. Mathematica's *NonlinearModelFit* solver was applied to each model. Some values were obtained as a result of the applied neuro-regression approach. Then, the maximum or minimum desired values of the objective functions were calculated. The modified versions of the Nelder-Mead algorithm have been thoroughly tested on output 1 and output 2 to minimize their results and output 3 to maximize its result more and shown in different scenarios.

3. Result and Discussion

In this study, the neuro-regression approach was used. Different trials have been made to achieve appropriate results. Moreover, as a result of these trials, suitable models have been reached in order to reach the minimum and maximum values of the objective functions. Inputs and outputs are taken from tables as experimental data sets. It is known that constraint variables directly affect the results of the objective function. In addition to this, appropriate values were tried to be reached as a result of each model. Our first output, the second order non-linear regression model, which gives the most relevant results among the different models, was found suitable and applied for the surface roughness value. As a result of different models applied for our second output, cutting power, the third-order non-linear regression model gave appropriate results. Minimum values have been found for these two objective function values considering different scenarios.

Table 5. *Results of output 1.*

Model of Output 1	$R^2_{training}$	$R^2_{testing}$	$R^2_{validation}$	Maximum	Minimum
SON	0.997314	0.927675	0.883081	4.83422	-2.97059

Table 6. *Results of output 2.*

Model of Output 2	$R^2_{training}$	$R^2_{testing}$	$R^2_{validation}$	Maximum	Minimum
TON	1	0.90423	0.942348	0.488696	0.0275759

Table 7. *Results of output 3.*

Model of Output 3	$R^2_{training}$	$R^2_{testing}$	$R^2_{validation}$	Maximum	Minimum
L	1	0.1505	0.646551	80.2801	15.1206
SON	1	0.7819	-0.08065	80.6516	15.1206
TON	1	0.2675	-1.09194	92.2776	3.67565
FOLN	1	-0.3127	-0.02565	85.6958	0.225738
SOLN	1	0.6429	0.619779	88.2654	8.67709
FOTN	1	0.0351	0.708655	87.8171	-0.536101
SOTN	1	-1.4135	0.768006	102.188	9.56213
HYBRID	1	0.4996	0.996241	110.791	2.43927

Eight different models have been implemented for the third output, the lifetime of the machine. Due to the limited data set of the objective function, relevant results could not be reached. For this reason, the second-order trigonometric non-linear regression model, which is the modeling that gives the closest results we want to obtain, was used. The following tables show the appropriate models for objective functions and their resulting values, $R^2_{training}$, $R^2_{testing}$, $R^2_{validation}$.Results are maximized and minimized for the best models. As a result of these processes, suitable design variables were determined to bring each output to appropriate values. For each model, the design variables required to achieve optimum results are given in Table 8-10.

Table 8. *Design variables values for output 1.*

Model for Output 1	Design Variables Values for Nminimize			
SON	$x1 \to 800.$	$x2 \to 240.$	$x3 \to 2.5$	$x4 \to 2.$

Table 9. *Design variables values for output 2.*

Model for Output 2	Design Variables Values for Nminimize			
TON	$x1 \to 800.$	$x2 \to 240.$	$x3 \to 2.5$	$x4 \to 2.$

Table 10. *Design variables values for output 3.*

Model for Output 3	Design Variables Values for Nminimize				
SOTLN	$x1 \to 1697$	$x2 \to 180.$	$x3 \to 3.5$	$x4 \to 9.$	$x5 \to 1.32$

Table 11. *Results of optimization based on Modified Nelder-Mead algorithm for output 1.*

Model for Output 1	Scenario Number	Algorithm options	Outputs	Design Variables
SON	1 $800 < x1 < 2400$ $240 < x2 < 420$ $0.5 < x3 < 2.5$ $2 < x4 < 6$	Default	-2.97059	$x1 \to 800.\ x2 \to 240. x3 \to 2.5, x4 \to 2.$
		Random Seed → 200	-2.97059	$x1 \to 800.\ x2 \to 240.\ x3 \to 2.5 x4 \to 2.$
		Contract Ratio → 0.10	-2.97059	$x1 \to 800 x2 \to 240 x3 \to 2.5 x4 \to 2.$
		Reflect Ratio → 2	-2.97059	$x1 \to 800 x2 \to 240 x3 \to 2.5 x4 \to 2.$
	2 $800 < x1 < 2400$ $240 < x2 < 420$ $0.5 < x3 < 2.5$ $2 < x4 < 6$ $\{x1, x2, x3, x4\} \in$ Integers	Default	-1.04013	$x1 \to 801\ x2 \to 241\ x3 \to 2\ x4 \to 3$
		Random Seed → 200	-1.04013	$x1 \to 801 x2 \to 241 x3 \to 2 x4 \to 3$
		Contract Ratio → 0.10	-1.04013	$x1 \to 801 x2 \to 241 x3 \to 2 x4 \to 3$
		Reflect Ratio → 2	-1.04013	$x1 \to 801, x2 \to 241, x3 \to 2, x4 \to 3$
	3 $x1 \geq 800 \vert\vert x1 \leq 2400,$ $x2 \geq 240 \vert\vert x2 \leq 420,$ $x3 \geq 0.5 \vert\vert x3 \leq 2.5,$ $x4 \geq 2 \vert\vert x4 \leq 6$	Default	-1.33004×10^{245}	$x1 \to -5.285 \times 10^{119}, x2 \to 240.$ $x3 \to 4.709 \times 10^{122}, x4 \to 2.121 \times 10^{121}$
		Random Seed → 50	$-9.988998 \times 10^{235}$	$x1 \to -1.44 \times 10^{115}, x2 \to -1.3 \times 10^{116},$ $x3 \to 1.290 \times 10^{118}, x4 \to 5.81 \times 10^{116}$
		Contract Ratio → 0.9	$-9.988998 \times 10^{235}$	$x1 \to -1.5 \times 10^{115}, x2 \to -1.38 \times 10^{116},$ $x3 \to 1.290 \times 10^{118}, x4 \to 5.81 \times 10^{116}$
		Reflect Ratio → 2	$-9.812743 \times 10^{276}$	$x1 \to -4.605 \times 10^{135}, x2 \to 240.$ $x3 \to 4.053 \times 10^{138}, x4 \to 6.$

Table 12. *Results of optimization based on Modified Nelder-Mead algorithm for output 2.*

Model for Output 2	Scenario Number	Algorithm options	Outputs	Design Variables
TON	1 $800 < x1 < 2400$ $240 < x2 < 420$ $0.5 < x3 < 2.5$ $2 < x4 < 6$	Default	0.027576	x1 → 800. x2 → 240. x3 → 2.5 x4 → 2.
		Random Seed → 111	0.004081	x1 → 2400. x2 → 420. x3 → 0.5 x4 → 2.
		Contract Ratio → 0.9	0.004081	x1 → 2400. x2 → 420. x3 → 0.5x4 → 2.
		Reflect Ratio → 2	0.004081	x1 → 2400. x2 → 420. x3 → 0.5x4 → 2.
	2 $800 < x1 < 2400$ $240 < x2 < 420$ $0.5 < x3 < 2.5$ $2 < x4 < 6$ $\{x1, x2, x3, x4\} \in$ Integers	Default	0.060015	x1 → 801 x2 → 284 x3 → 1 x4 → 3
		Random Seed → 111	0.056212	x1 → 801x2 → 241 x3 → 2 x4 → 3
		Contract Ratio → 0.7	0.056212	x1 → 801x2 → 241 x3 → 2 x4 → 3
		Reflect Ratio → 2	0.056212	x1 → 801x2 → 241 x3 → 2 x4 → 3
	3 $x1 \geq 800 \| x1 \leq 2400$, $x2 \geq 240 \| x2 \leq 420$, $x3 \geq 0.5 \| x3 \leq 2.5$, $x4 \geq 2 \| x4 \leq 6$	Default	-1.22984×10^{620}	x1 → 2400 x2 → 3.3142×10^{206} x3 → 0.5 x4 → 4.72×10^{207}
		Random Seed → 50	-1.22984×10^{620}	x1 → 2400 x2 → 3.3142×10^{206} x3 → 0.5 x4 → 4.72×10^{207}
		Contract Ratio → 0.5	-1.22984×10^{620}	x1 → 2400 x2 → 3.3142×10^{206} x3 → 0.5 x4 → 4.72×10^{207}
		Reflect Ratio → 1	-1.22984×10^{620}	x1 → 2400 x2 → 3.3142×10^{206} x3 → 0.5 x4 → 4.72×10^{207}

Table 13. *Results of optimization based on Modified Nelder-Mead algorithm for output 3.*

Model for Output 3	Scenario Number	Algorithm options	Outputs	Design Variables
	1 $1300 < x1 < 2100$ $120 < x2 < 180$ $2.5 < x3 < 3.5$ $3 < x4 < 9$ $0.816 < x5 < 1.319$	Default	110.791	x1 → 1709.03x2 → 120. x3 → 3.5x4 → 3.x5 → 1.319.
		Random Seed → 111	110.791	x1 → 1709.03x2 → 120. x3 → 3.5x4 → 3.x5 → 1.319.
		Shrink Ratio → 1	110.791	x1 → 1709.03x2 → 120. x3 → 3.5x4 → 3.x5 → 1.319.
		Expand Ratio → 3	110.791	x1 → 1709.03x2 → 120. x3 → 3.5x4 → 3.x5 → 1.319.
	2 $1300 < x1 < 2100$ $120 < x2 < 180$ $2.5 < x3 < 3.5$ $3 < x4 < 9$ $0.816 < x5 < 1.319$ $\{x1, x2, x3, x4, x5\} \in$ Integers	Default	98.3408	x1 → 1332x2 → 139. x3 → 3x4 → 5x5 → 1
		Random Seed → 300	98.3534	x1 → 1464x2 → 140 x3 → 3x4 → 5x5 → 1
		Shrink Ratio → 3	98.3534	x1 → 1464x2 → 140 x3 → 3x4 → 5x5 → 1
		Expand Ratio → 3	98.3534	x1 → 1464x2 → 140 x3 → 3x4 → 5x5 → 1

4. Conclusion

This study aimed to increase the machine's life span while minimizing the surface roughness and cutting power. There were four different variables and 25 data sets for surface roughness and cutting power. For the life span, there were five different variables and 9 data sets. The aim here was to reach a suitable regression model that can give us the necessary optimum output values depending on the variables. The neuro-regression approach obtained an appropriate regression model so that the $R^2_{training}$ value must be greater than 0.90, the $R^2_{testing}$ value must be greater than 0.85, and finally, the $R^2_{validation}$ value must be greater than 0.85. For output 1 (surface roughness), these values were obtained conveniently as 0.997314, 0.927675, and 0.883081 for $R^2_{training}$, $R^2_{testing}$, and $R^2_{validation}$, respevtively. As the desired values were found to be suitable, the second-order non-linear regression model was used. After the optimization, the minimum value was found -2.97059.

For output 2 (cutting power), these values were obtained as $R^2_{training}$ is equal to 1, $R^2_{testing}$ equals 0.90423, and $R^2_{validation}$ equals 0.94235. As the desired values were found to be suitable, the third-order non-linear regression model was used. After the optimization process , the minimum value was found as 0.0275759. For output 3 (lifetime of the machine), these values were obtained as $R^2_{training}$ is equal to 1, $R^2_{testing}$ is equal to 0.4996, and $R^2_{validation}$ is equal to 0.996241. An accurate model could not be reached due to the scarcity of a number of data sets for output 3. Instead, a model was used in which the values closer to the desired values are provided. As the desired values were found to be closer, the second-order trigonometric non-linear regression model was used. After the optimization, the maximum value was found as 110.791. At the end of the study, some options of the direct search method to maximize and minimize results were applied. After applied some scenarios, it has been seen that the third scenario is more suitable for output 1 and output 2, and it has been seen that the first scenario is more suitable for output 3.

Declaration of Interest

The authors declare that there is no conflict of interest.

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APPENDIX

Model Name For Output 1	Model
SON	$-6.509885663403495 + 0.005618299339813021x1 - 3.177186608513434 \times 10^{-7}x1^{2} + 0.044188550823988065x2 - 0.000019259068529200848x1x2 - 0.000045373242358123256x2^{2} - 4.828733842049232x3 + 0.0013505663904117497x1x3 + 0.012871712805669044x2x3 - 0.5974390681670687x3^{2} + 0.3084440279573145x4 - 0.0001638361563178854x1x4 + 0.0005907347821132539x2x4 - 0.053084237534413624x3x4 - 0.009306210170207618x4^{2}$

Model Name For Output 2	Model
TON	$0.0873885736338913 + 0.000023034134840759 54x1 - 2.472633303621079 \times 10^{-8}x1^{2} + 1.901217799089097 \times 10^{-11}x1^{3} - 0.00011773726012124803x2 + 9.271486175818838 \times 10^{-9}x1x2 - 1.709038139214161 \times 10^{-10}x1^{2}x2 - 4.579728866634524 \times 10^{-7}x2^{2} + 9.684495546735244 \times 10^{-11}x1x2^{2} + 1.930501597745149 \times 10^{-9}x2^{3} - 0.020880825356986286x3 + 0.000021022527585904126x1x3 - 3.431749926300749 \times 10^{-9}x1^{2}x3 - 0.000068809817757313 41x2x3 + 1.2381322305199 \times 10^{-7}x1x2x3 + 2.531771100982355 \times 10^{-7}x2^{2}x3 - 0.01704700883302848x3^{2} - 0.000006308008574305507x1x3^{2} + 0.000011876273887436986x2x3^{2} + 0.0037771244976612574x3^{3} + 0.000079532418213709 5x4 + 0.0000023390536628784 61x1x4 + 3.547220566752366 \times 10^{-9}x1^{2}x4 - 0.000042854986446547 08x2x4 + 3.68911641138247 \times 10^{-8}x1x2x4 - 3.11930870044312 \times 10^{-7}x2^{2}x4 - 0.003852659541721482x3x4 + 0.0000014614401340257 31x1x3x4 + 0.000009925109721374419x2x3x4 - 0.0011541084133987402x3^{2}x4 + 0.0015191113584700662x4^{2} - 0.0000024485578207915 96x1x4^{2} + 0.000036044890566014 4x2x4^{2} + 0.0024467728436591216x3x4^{2} - 0.001172013876941x4^{3}$

Model Name For Output 3	Model
L	$198.69752228249897 - 0.021872444963457895x1 - 0.2098362545789043x2 - 14.689949495039151x3 + 0.046687461628923115x4 - 34.924242558607936x5$
SON	$64.64181129905197 + 0.014496276936404161x1 - 0.000006176621808466241x1^{2} + 0.12513812861414014x2 - 0.000045885621883926725x1x2 - 0.0004157216592296709x2^{2} + 10.266290320622979x3 - 0.0014366289458181746x1x3 - 0.0194434340032 2525x2x3 - 0.34088885496069x3^{2} + 3.1801123453702664x4 + 0.000744889105442118x1x4 - 0.004855967937663978x2x4 - 0.5082296220692951x3x4 - 0.4210065345862815x4^{2} + 23.04638571573809x5 - 0.009850257650718444x1x5 - 0.07317520758202213x2x5 - 2.2979404074579386x3x5 + 1.1871412946005615x4x5 - 15.708776136010252x5^{2}$
TON	$29.893416052350194 + 0.009998210925432327x1 + 9.376243055860865 \times 10^{-7}x1^{2} - 1.618567835027597 \times 10^{-9}x1^{3} + 0.08706345078529117x2 + 0.000012266608043995348x1x2 - 1.382244525610618 \times 10^{-8}x1^{2}x2 + 0.0000989361937438921 3x2^{2} - 1.169847073579434 \times 10^{-7}x1x2^{2} - 0.000001049713657160243x2^{3} + 6.749036088653819x3 + 0.0016134269352232533x1x3 - 5.231257594690665 \times 10^{-7}x1^{2}x3 + 0.011705349758856239x2x3 - 0.0000046474769753224 77x1x2x3 - 0.000048904101344540 04x2^{2}x3 + 1.0986152072047146x3^{2} - 0.000040686408621468 88x1x3^{2} - 0.0013582576962779232x2x3^{2} + 0.03364286339330275 3x3^{3} + 1.9292715836575525x4 + 0.0007827780599396093x1x4 + 6.157699555843764 \times 10^{-8}x1^{2}x4 + 0.002720884172060242 6x2x4 - 4.687682771145666 \times 10^{-7}x1x2x4 - 0.0000163091690383647 94x2^{2}x4 + 0.178630893902604 06x3x4 - 0.0000163660691536345 42x1x3x4 - 0.0010496122326251554x2x3x4 - 0.05254895658354053x3^{2}x4 - 0.0493071567430364x4^{2} - 0.0000392236769705405x1x4^{2} - 0.0006867757799132519x2x4^{2} - 0.04433749007359964x3x4^{2} - 0.02334700715594493 4x4^{3} + 15.904266395356187x5 + 0.0014855044925990678x1x5 - 0.0000025795886726125 22x1^{2}x5 + 0.01947007270661679x2x5 - 0.0000220318147377862 93x1x2x5 - 0.0001864554149830 69x2^{2}x5 + 2.564600271016065x3x5 - 0.000835040027238563 7x1x3x5 - 0.007417199206766824x2x3x5 - 0.06604840572904243x3^{2}x5 + 1.2459646316622992x4x5 + 0.0000976375020726504 6x1x4x5 - 0.000746984439577417x2x4x5 - 0.02618375437935298 3x3x4x5 - 0.06237891065962948 6x4^{2}x5 + 2.3534753013457177x5^{2} - 0.00411120514182041 9x1x5^{2} - 0.0351167764444061 7x2x5^{2} - 1.332922694384214x3x5^{2} + 0.1548134057316448x4x5^{2} - 6.552197319709215x5^{3}$
FOLN	$45.90174309425046 + 6.166940354629804Log[x1] + 6.75342737350235Log[x2] - 58.46700292964906Log[x3] - 9.162245880230312Log[x4] - 104.1950039454285Log[x5]$
SOLN	$13.521477710151082 + 1.8019277442891868Log[x1] + 0.23921348829604344Log[x1]^{2} + 2.272941128627296Log[x2] + 0.30268754648928725Log[x1]Log[x2] + 0.37330579473965075Log[x2]^{2} + 3.2634283187566187Log[x3] + 0.4323933826400694Log[x1]Log[x3] + 0.2944387382572189Log[x2]Log[x3] - 5.581917444517096Log[x3]^{2} + 0.36856740788861364Log[x4] + 0.07472954482071655Log[x1]Log[x4] - 0.10582033908667163Log[x2]Log[x4] - 5.31571810653157Log[x3]Log[x4] - 3.6916571182915106Log[x4]^{2} - 50.98377870921349Log[x5] - 6.966087885114664Log[x1]Log[x5] - 8.0553545871115Log[x2]Log[x5] - 15.365463543423846Log[x3]Log[x5] + 37.66337831228693Log[x4]Log[x5] - 107.59435628174604Log[x5]^{2}$
FOTN	$10.64081282241565 + 5.034370478891869Cos[x1] + 5.257553787711318Cos[x2] - 10.820874858752916Cos[x3] - 2.6575799620321767Cos[x4] + 23.40478067680203Cos[x5] - 17.964586796717672Sin[x1] - 0.11024584068657649Sin[x2] + 23.65088730257444Sin[x3] + 5.42737821990518Sin[x4] + 11.4753228Sin[x5]$
SOTN	$2.1268778951890837 + 1.0817233708633547Cos[x1] + 3.6445800900623873Cos[x1]^{2} + 0.3880832315407555Cos[x2] + 3.304927252Cos[x1]Cos[x2] + 5.2056Cos[x2]^{2} - 2.261725936207216Cos[x3] - 1.325791Cos[x1]Cos[x3] - 0.2787001Cos[x2]Cos[x3] + 2.335767Cos[x3]^{2} - 0.9418334606330854Cos[x4] - 3.33052Cos[x1]Cos[x4] - 2.220531592Cos[x2]Cos[x4] + 1.0549779Cos[x3]Cos[x4] + 2.3010Cos[x4]^{2} + 4.175128Cos[x5] + 2.51310Cos[x1]Cos[x5] + 1.486520Cos[x2]Cos[x5] - 4.646382Cos[x3]Cos[x5] - 2.399166459Cos[x4]Cos[x5] + 6.50245Cos[x5]^{2} - 2.1200571Sin[x1] - 2.8746Cos[x1]Sin[x1] - 1.4208Cos[x2]Sin[x1] + 1.982694Cos[x3]Sin[x1] + 1.0817273Cos[x4]Sin[x1] - 6.309403629Cos[x5]Sin[x1] + 2.2206202Sin[x1]^{2} - 0.5198319Sin[x2] + 2.2207702Cos[x1]Sin[x2] + 4.4370Cos[x2]Sin[x2] + 0.64428Cos[x3]Sin[x2] - 1.334958Cos[x4]Sin[x2] - 0.402099036Cos[x5]Sin[x2] - 0.2044859Sin[x1]Sin[x2] + 3.1245758Sin[x2]^{2} + 3.9929145Sin[x3] + 0.3146275Cos[x1]Sin[x3] + 3.3513020Cos[x2]Sin[x3] - 4.5746521Cos[x3]Sin[x3] - 0.7033Cos[x4]Sin[x3] + 6.52852Cos[x5]Sin[x3] - 6.86444229Sin[x1]Sin[x3] + 1.415727Sin[x2]Sin[x3] + 9.3460Sin[x3]^{2} + 2.032636Sin[x4] + 8.4765371Cos[x1]Sin[x4] + 4.2485003Cos[x2]Sin[x4] - 2.4465862Cos[x3]Sin[x4] - 7.4238472Cos[x4]Sin[x4] + 4.7515586Cos[x5]Sin[x4] - 4.00293Sin[x1]Sin[x4] + 1.83231Sin[x2]Sin[x4] - 5.021096Sin[x3]Sin[x4] + 15.25544Sin[x4]^{2} + 2.407062Sin[x5] + 1.02498Cos[x1]Sin[x5] + 0.18563Cos[x2]Sin[x5] - 2.50865Cos[x3]Sin[x5] - 0.88696Cos[x4]Sin[x5] + 5.3772Cos[x5]Sin[x5] - 2.033612Sin[x1]Sin[x5] - 0.760593Sin[x2]Sin[x5] + 4.974744Sin[x3]Sin[x5] + 1.88647Sin[x4]Sin[x5] + 2.5913Sin[x5]^{2}$
HYBRID	$16.59202757244946 + 0.03173224x2 - 0.0001105735x2^{2} + 2.6612967x3 - 0.004938x2x3 - 0.0779593x3^{2} + 0.50618x4 - 0.0030x2x4 - 0.2226835641837039x3x4 - 0.163622x4^{2} + 6.661x5 - 0.0145x2x5 - 0.2780144954732378x3x5 + 0.051825641727912805x4x5 - 2.668534322994205x5^{2} + 15.996592242917572Cos[x1] - 0.008617296752240735x2Cos[x1] + 2.774913789008801x3Cos[x1] - 2.18612688614873x4Cos[x1] + 7.627534289932837x5Cos[x1] + 60.149392252627514Cos[x1]^{2}$