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On Smarandache Curves in Affine 3-space

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Abstract — In this paper, we introduce Smarandache curves of an affine C^{∞}-curve in affine 3-space. Besides, we present the relationship between the Frenet frames of the curve couple and the Frenet apparatus of each obtained curve.

Keywords — *Affine curve, affine frame, Smarandache curves, affine 3-space*

Mathematics Subject Classification (2020) — 53A15, 53A55

1. Introduction

In the theory of curves in differential geometry, one of the interesting problems studied by many mathematicians is to characterize a regular curve and give information about its structure. Using the curvatures κ and τ of a regular curve, its shape and size can be determined, so the curvatures play an important role in the problem's solution. The relationship between the corresponding Frenet vectors of the two curves gives another approach to solving the problem. For example; involute-evolute curve couple, Bertrand mate curves and Mannheim mate curves result from this relationship. Another example is Smarandache curves, which are defined as regular curves with the location vector generated by the Frenet vectors of the regular curve. Smarandache curves have been widely studied in different ambient spaces ([1–17]).

While Euclidean differential geometry is the study of differential invariants regarding the group of rigid motions, affine differential geometry is the study of differential invariants regarding the group of affine transformations $x \longrightarrow Ax + b$, $A \in GL(n, \mathbb{R})$, $b \in \mathbb{R}^n$ acting on $x \in \mathbb{R}^n$, i.e., nonsingular linear transformations together with translations, denoted by the Lie group $A(n, \mathbb{R}) = GL(n, \mathbb{R}) \times \mathbb{R}^n$ with a semi-direct product structure, (see [18, 19]). Moreover, "affine geometry" is also called "equi-affine geometry", where we restrict to the subgroup $SA(n, \mathbb{R}) = SL(n, \mathbb{R}) \times \mathbb{R}^n$ of volume-preserving linear transformations together with translations.

In this paper, we introduce TN, TB, NB and $TNB-$Smarandache curves corresponding to a regular $C^{\infty}-$curve in affine $3-$space A_3. We also establish the relationship between the Frenet frames of the pair of curves and the Frenet apparatus of each curve.

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2. Basic Concepts

In this section, we give the information to understand the main subjects in this paper (see for details [20–22]).

A set of points whose elements correspond to a vector of the vector space V over a field is called the *affine space associated with* V. We refer to A_3 as an affine 3-space associated with $\mathbb{R}^3$.

An arbitrary curve $\alpha : I \subset \mathbb{R} \to A_3$ is called a *regular affine curve* if for all $r \in I$

$$\det\left(\frac{d\alpha}{dr}(r), \frac{d^2\alpha}{dr^2}(r), \frac{d^3\alpha}{dr^3}(r)\right) \neq 0$$

and the *arc-length* of α is defined as

$$s(r) := \int_{r_1}^{r_2} \left|\det\left(\frac{d\alpha}{dr}(r), \frac{d^2\alpha}{dr^2}(r), \frac{d^3\alpha}{dr^3}(r)\right)\right|^{1/6} dr$$

Here, s is called *the parameter of the affine arc-length* if

$$\det\left(\frac{d\alpha}{ds}(s), \frac{d^2\alpha}{ds^2}(s), \frac{d^3\alpha}{ds^3}(s)\right) = 1$$

Remark 2.1. In this paper, the prime denotes differentiation concerning the parameter s, i.e., $\alpha' = \frac{d\alpha}{ds}$ etc., while a dot is reserved for differentiation concerning any arbitrary parameter r, i.e., $\dot{\alpha} = \frac{d\alpha}{dr}$ etc..

For an affine $C^\infty-$curve α in A_3 parameterized by the parameter of the affine arc-length s, κ and τ are called the *affine curvature* and the *affine torsion* of α given by

$$\kappa(s) = \det\left[\alpha'(s), \alpha'''(s), \alpha^{(iv)}(s)\right] \tag{1}$$

and

$$\tau(s) = -\det\left[\alpha''(s), \alpha'''(s), \alpha^{(iv)}(s)\right] \tag{2}$$

From the definition of $\kappa(s)$ and $\tau(s)$, we get

$$\alpha^{(iv)}(s) + \kappa(s)\alpha''(s) + \tau(s)\alpha'(s) = 0$$

that is

$$\frac{d}{ds}\begin{bmatrix} \alpha'(s) \\ \alpha''(s) \\ \alpha'''(s) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\tau & -\kappa & 0 \end{bmatrix}\begin{bmatrix} \alpha'(s) \\ \alpha''(s) \\ \alpha'''(s) \end{bmatrix} \tag{3}$$

Let us set

$$T = \alpha', \quad N = \alpha'', \quad B = \alpha'''$$

Then, we can write the relation (3) as

$$\begin{bmatrix} T' \\ N' \\ B' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\tau & -\kappa & 0 \end{bmatrix}\begin{bmatrix} T \\ N \\ B \end{bmatrix} \tag{4}$$

Here, T, N and B are called the *tangent vector*, the *normal vector*, and the *binormal vector* of α, respectively. Also, $\{T, N, B\}$ is called an *affine Frenet frame* of α.

Example 2.2. Let α be an affine $C^\infty-$curve in A_3 with parametric equation

$$\alpha(s) = (\cos s, \sin s, s)$$

The affine Frenet frame of α reads

$$\begin{aligned} T(s) &= (-\sin s, \cos s, 1) \\ N(s) &= (-\cos s, -\sin s, 0) \\ B(s) &= (\sin s, -\cos s, 0) \end{aligned}$$

It follows that the curvatures of α have the form

$$\kappa(s) = \det\left[\alpha'(s), \alpha'''(s), \alpha^{(iv)}(s)\right] = 1$$

and

$$\tau(s) = -\det\left[\alpha''(s), \alpha'''(s), \alpha^{(iv)}(s)\right] = 0$$

Then, we can write

$$\begin{bmatrix} T' \\ N' \\ B' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix}$$

3. Smarandache Curves in Affine 3−space

In this section, we consider an affine $C^\infty-$curve α and define its affine Smarandache curves in affine 3−space A_3. Let $\alpha = \alpha(s)$ be a regular affine $C^\infty-$curve with affine Frenet frame $\{T, N, B\}$ in A_3. Denote by $\beta = \beta(u)$ arbitrary affine $C^\infty-$curve, where u is the parameter of the affine arc-length of β.

Definition 3.1. Let α be an affine $C^\infty-$curve in affine 3−space A_3. A curve β defined by

$$\beta(u(s)) = \frac{1}{\sqrt{2}}(T(s) + N(s)) \tag{5}$$

is called the $TN-$affine Smarandache curve of α.

Definition 3.2. Let α be an affine $C^\infty-$curve in affine 3−space A_3. A curve β defined by

$$\beta(u(s)) = \frac{1}{\sqrt{2}}(T(s) + B(s)) \tag{6}$$

is called the $TB-$affine Smarandache curve of α.

Definition 3.3. Let α be an affine $C^\infty-$curve in affine 3−space A_3. A curve β defined by

$$\beta(u(s)) = \frac{1}{\sqrt{2}}(N(s) + B(s)) \tag{7}$$

is called the $NB-$affine Smarandache curve of α.

Definition 3.4. Let α be an affine $C^\infty-$curve in affine 3−space A_3. A curve β defined by

$$\beta(u(s)) = \frac{1}{\sqrt{3}}(T(s) + N(s) + B(s)) \tag{8}$$

is called the $TNB-$affine Smarandache curve of α.

Next, we obtain the affine Frenet frame $\{T_\beta, N_\beta, B_\beta\}$, and the curvatures κ_β and τ_β of affine Smarandache curves of α.

3.1. $TN-$affine Smarandache curve

Taking the derivatives $\beta(u\,(s))$ concerning u, we obtain

$$\frac{d\beta}{du} = \frac{d\beta}{ds}\frac{ds}{du} \tag{9}$$

$$\frac{d^2\beta}{du^2} = \frac{d^2\beta}{ds^2}\left(\frac{ds}{du}\right)^2 + \frac{d\beta}{ds}\frac{d^2 s}{du^2} \tag{10}$$

$$\frac{d^3\beta}{du^3} = \frac{d^3\beta}{ds^3}\left(\frac{ds}{du}\right)^3 + 3\frac{d^2\beta}{ds^2}\frac{ds}{du}\frac{d^2 s}{du^2} + \frac{d\beta}{ds}\frac{d^3 s}{du^3} \tag{11}$$

and since u is the parameter of the affine arc-length of β, i.e., $\det(\frac{d\beta}{du}, \frac{d^2\beta}{du^2}, \frac{d^3\beta}{du^3}) = 1$, we can easily obtain

$$\left(\frac{du}{ds}\right)^6 = \det\left(\frac{d\beta}{ds}, \frac{d^2\beta}{ds^2}, \frac{d^3\beta}{ds^3}\right) \tag{12}$$

Using the relations (4) and (5) we get

$$\begin{aligned}
\frac{d\beta}{ds} &= \frac{1}{\sqrt{2}}\,(N+B) \\
\frac{d^2\beta}{ds^2} &= \frac{1}{\sqrt{2}}\,(-\tau T - \kappa N + B) \\
\frac{d^3\beta}{ds^3} &= \frac{1}{\sqrt{2}}\left[\left(-\tau' - \tau\right)T + \left(-\kappa' - \tau - \kappa\right)N - \kappa B\right]
\end{aligned}$$

and so, from the relation (12)

$$\frac{du}{ds} = \left(\frac{1}{2\sqrt{2}}\left[\left(\tau' + \tau\right)(\kappa+1) - \tau\left(\kappa' + \tau\right)\right]\right)^{1/6} \tag{13}$$

Without loss of generality, we assume that $du = ds$. Then the affine Frenet frame's vectors are given by

$$T_\beta = \frac{1}{\sqrt{2}}N + \frac{1}{\sqrt{2}}B \tag{14}$$

$$N_\beta = -\frac{\tau}{\sqrt{2}}T - \frac{\kappa}{\sqrt{2}}N + \frac{1}{\sqrt{2}}B \tag{15}$$

and

$$B_\beta = -\frac{\tau' + \tau}{\sqrt{2}}T - \frac{\kappa' + \tau + \kappa}{\sqrt{2}}N - \frac{\kappa}{\sqrt{2}}B \tag{16}$$

Differentiating the equation (16) concerning s and using the relations (4) we obtain

$$B'_\beta = -\frac{\tau'' + \tau' - \kappa\tau}{\sqrt{2}}T - \frac{\kappa'' + 2\tau' + \kappa' + \tau + \kappa^2}{\sqrt{2}}N - \frac{2\kappa' + \tau + \kappa}{\sqrt{2}}B \tag{17}$$

Since $B'_\beta = -\tau_\beta T_\beta - \kappa_\beta N_\beta$, from the relations (14)-(17) we have

$$\kappa_\beta = -\frac{\tau'' + \tau' - \kappa\tau}{\tau}$$

and

$$\tau_\beta = -\frac{\tau'' - 2\kappa'\tau - \tau' + \tau^2 + 2\kappa\tau}{\tau}$$

Theorem 3.5. Let $\alpha : I \subseteq R \mapsto A_3$ be an affine $C^{\infty}-$curve in affine $3-$space A_3 with the affine Frenet frame $\{T, N, B\}$ and the curvatures κ and τ. If $\beta : I \subseteq R \mapsto A_3$ is $TN-$affine Smarandache curve of α, then its frame $\{T_\beta, N_\beta, B_\beta\}$ is given by

$$\begin{bmatrix} T_\beta \\ N_\beta \\ B_\beta \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{\tau}{\sqrt{2}} & -\frac{\kappa}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{\tau'+\tau}{\sqrt{2}} & -\frac{\kappa'+\tau+\kappa}{\sqrt{2}} & -\frac{\kappa}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix} \tag{18}$$

and the corresponding curvature κ_β and τ_β read

$$\kappa_\beta = -\frac{\tau'' + \tau' - \kappa\tau}{\tau}, \quad \tau_\beta = -\frac{\tau'' - 2\kappa'\tau - \tau' + \tau^2 + 2\kappa\tau}{\tau} \tag{19}$$

3.2. $TB-$affine Smarandache curve

Using the relations (4) and (6) we get

$$\begin{aligned} \frac{d\beta}{ds} &= \frac{1}{\sqrt{2}} \left(-\tau T - (\kappa - 1) N\right) \\ \frac{d^2\beta}{ds^2} &= \frac{1}{\sqrt{2}} \Big(-\tau' T - \big(\kappa' + \tau\big) N - (\kappa - 1) B\Big) \\ \frac{d^3\beta}{ds^3} &= \frac{1}{\sqrt{2}} \Big[-\big(\tau'' - \kappa\tau + \tau\big) T - \big(\kappa'' + 2\tau' - \kappa^2 + \kappa\big) N - \big(2\kappa' + \tau\big) B\Big] \end{aligned}$$

and so, from the relation (12)

$$\frac{du}{ds} = \left(\frac{1}{2\sqrt{2}} \Big(-(\kappa - 1)^2 \big(\tau'' + \tau\big) + (\kappa - 1) \big(\tau\kappa'' + 3\tau\tau' + 2\kappa'\tau'\big) - \big(2\kappa' + \tau\big) \big(\kappa' + \tau\big) \tau\Big)\right)^{1/6} \tag{20}$$

Without loss of generality, we assume that $du = ds$. Then the affine Frenet frame's vectors are given by

$$T_\beta = -\frac{\tau}{\sqrt{2}} T - \frac{\kappa - 1}{\sqrt{2}} N \tag{21}$$

$$N_\beta = -\frac{\tau'}{\sqrt{2}} T - \frac{\kappa' + \tau}{\sqrt{2}} N - \frac{\kappa - 1}{\sqrt{2}} B \tag{22}$$

and

$$B_\beta = -\frac{\tau'' - \kappa\tau + \tau}{\sqrt{2}} T - \frac{\kappa'' + 2\tau' - \kappa^2 + \kappa}{\sqrt{2}} N - \frac{2\kappa' + \tau}{\sqrt{2}} B \tag{23}$$

Differentiating the equation (23) concerning s and using the relations (4) we obtain

$$B'_\beta = -\frac{\tau''' - 3\kappa'\tau - \kappa\tau' + \tau' - \tau^2}{\sqrt{2}} T - \frac{\kappa''' + 3\tau'' - 4\kappa\kappa' + \kappa' - 2\kappa\tau + \tau}{\sqrt{2}} N - \frac{3\kappa'' + 3\tau' - \kappa^2 + \kappa}{\sqrt{2}} B \tag{24}$$

Since $B'_\beta = -\tau_\beta T_\beta - \kappa_\beta N_\beta$, from the relations (21)-(24) we have

$$\kappa_\beta = -\frac{3\kappa'' + 3\tau' - \kappa^2 + \kappa}{\kappa - 1}$$

and

$$\tau_\beta = 3\kappa' + \tau - \frac{\tau''' - 2\kappa\tau' + \tau'}{\tau} - \frac{3\kappa''\tau' - 3(\tau')^2}{\kappa - 1}$$

where $\kappa(s) \neq 1$ for all s.

Theorem 3.6. Let $\alpha : I \subseteq R \mapsto A_3$ be an affine $C^\infty-$curve in affine 3$-$space A_3 with the affine Frenet frame $\{T, N, B\}$ and the curvatures κ and τ. If $\beta : I \subseteq R \mapsto A_3$ is $TB-$affine Smarandache curve of α, then its frame $\{T_\beta, N_\beta, B_\beta\}$ is given by

$$\begin{bmatrix} T_\beta \\ N_\beta \\ B_\beta \end{bmatrix} = \begin{bmatrix} -\frac{\tau}{\sqrt{2}} & -\frac{\kappa-1}{\sqrt{2}} & 0 \\ -\frac{\tau'}{\sqrt{2}} & -\frac{\kappa'+\tau}{\sqrt{2}} & -\frac{\kappa-1}{\sqrt{2}} \\ -\frac{\tau''-\kappa\tau+\tau}{\sqrt{2}} & -\frac{\kappa''+2\tau'-\kappa^2+\kappa}{\sqrt{2}} & -\frac{2\kappa'+\tau}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix} \tag{25}$$

and the corresponding curvature κ_β and τ_β read

$$\kappa_\beta = -\frac{3\kappa'' + 3\tau' - \kappa^2 + \kappa}{\kappa - 1}, \quad \tau_\beta = 3\kappa' + \tau - \frac{\tau''' - 2\kappa\tau' + \tau'}{\tau} - \frac{3\kappa''\tau' - 3(\tau')^2}{\kappa - 1} \tag{26}$$

everywhere $\kappa(s) \neq 1$.

3.3. $NB-$affine Smarandache curve

Using the relations (4) and (7) we get

$$\begin{aligned} \frac{d\beta}{ds} &= \frac{1}{\sqrt{2}}\left(-\tau T - \kappa N + B\right) \\ \frac{d^2\beta}{ds^2} &= \frac{1}{\sqrt{2}}\left(-\left(\tau' + \tau\right)T - \left(\kappa' + \tau + \kappa\right)N - \kappa B\right) \\ \frac{d^3\beta}{ds^3} &= \frac{1}{\sqrt{2}}\left[-\left(\tau'' + \tau' - \kappa\tau\right)T - \left(\kappa'' + 2\tau' + \kappa' + \tau - \kappa^2\right)N - \left(2\kappa' + \tau + \kappa\right)B\right] \end{aligned}$$

and so, from the relation (12)

$$\tfrac{du}{ds} = \begin{pmatrix} \frac{1}{2\sqrt{2}}(-2(\kappa')^2\tau - 3\kappa'\tau^2 - \tau^3 + \kappa''\kappa\tau + 3\kappa\tau'\tau + 2\tau'\kappa'\kappa - \kappa^2\tau'' + \kappa^3\tau + 2(\tau')^2 \\ +\kappa''\tau' - \kappa^2\tau' + \kappa''\tau + 2\tau'\tau + \kappa'\tau + \tau^2 - \kappa'\tau'' + \kappa'\kappa\tau - \tau''\tau + \kappa\tau^2 - \kappa\tau'' - \kappa\tau') \end{pmatrix}^{1/6} \tag{27}$$

Without loss of generality, we assume that $du = ds$. Then the affine Frenet frame's vectors are given by

$$T_\beta = -\frac{\tau}{\sqrt{2}}T - \frac{\kappa}{\sqrt{2}}N + \frac{1}{\sqrt{2}}B \tag{28}$$

$$N_\beta = -\frac{\tau' + \tau}{\sqrt{2}}T - \frac{\kappa' + \tau + \kappa}{\sqrt{2}}N - \frac{\kappa}{\sqrt{2}}B \tag{29}$$

and

$$B_\beta = -\frac{\tau'' + \tau' - \kappa\tau}{\sqrt{2}}T - \frac{\kappa'' + 2\tau' + \kappa' + \tau - \kappa^2}{\sqrt{2}}N - \frac{2\kappa' + \tau + \kappa}{\sqrt{2}}B \tag{30}$$

Differentiating the equation (30) concerning s and using the relations (4) we obtain

$$\begin{aligned} B'_\beta = &-\frac{\tau''' + \tau'' - 3\kappa'\tau - \kappa\tau' - \tau^2 - \kappa\tau}{\sqrt{2}}T - \frac{\kappa''' + \kappa'' + 3\tau'' + 2\tau' - 4\kappa'\kappa - 2\kappa\tau - \kappa^2}{\sqrt{2}}N \\ &-\frac{3\kappa'' + 2\kappa' + 3\tau' + \tau - \kappa^2}{\sqrt{2}}B \end{aligned} \tag{31}$$

Since $B'_\beta = -\tau_\beta T_\beta - \kappa_\beta N_\beta$, from the relations (27)-(31) we have

$$\kappa_\beta = -\frac{\tau''' + \tau'' - \kappa'\tau - \kappa\tau' - \kappa\tau + 3\kappa''\tau + 3\tau'\tau + \kappa^2\tau}{\tau' + \tau + \kappa\tau}$$

and

$$\tau_\beta = -\frac{\kappa\tau''' + \kappa\tau'' - 3\kappa''\tau' - 3(\tau')^2 - 2\kappa'\tau' - 3\kappa''\tau - 4\tau'\tau - 2\kappa'\tau - 3\kappa'\kappa\tau - \kappa\tau^2 - \tau^2}{\tau' + \tau + \kappa\tau}$$

Theorem 3.7. Let $\alpha : I \subseteq R \mapsto A_3$ be an affine $C^{\infty}-$curve in affine 3$-$space A_3 with the affine Frenet frame $\{T, N, B\}$ and the curvatures κ and τ. If $\beta : I \subseteq R \mapsto A_3$ is $NB-$affine Smarandache curve of α, then its frame $\{T_\beta, N_\beta, B_\beta\}$ is given by

$$\begin{bmatrix} T_\beta \\ N_\beta \\ B_\beta \end{bmatrix} = \begin{bmatrix} -\frac{\tau}{\sqrt{2}} & -\frac{\kappa}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{\tau'+\tau}{\sqrt{2}} & -\frac{\kappa'+\tau+\kappa}{\sqrt{2}} & -\frac{\kappa}{\sqrt{2}} \\ -\frac{\tau''+\tau'-\kappa\tau}{\sqrt{2}} & -\frac{\kappa''+2\tau'+\kappa'+\tau-\kappa^2}{\sqrt{2}} & -\frac{2\kappa'+\tau+\kappa}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix} \tag{32}$$

and the corresponding curvature κ_β and τ_β read

$$\begin{aligned} \kappa_\beta &= -\frac{\tau''' + \tau'' + 3\kappa''\tau + (3\tau - \kappa)\,\tau' - \kappa'\tau + \left(\kappa^2 - \kappa\right)\tau}{\tau' + \tau + \kappa\tau} \\ \tau_\beta &= -\frac{\kappa\tau''' + \kappa\tau'' - 3\,(\tau' + \tau)\,\kappa'' - 3(\tau')^2 - 2\kappa'\tau' - 4\tau'\tau - (2\tau + 3\kappa\tau)\,\kappa' - (1 + \kappa)\,\tau^2}{\tau' + \tau + \kappa\tau} \end{aligned} \tag{33}$$

3.4. $TNB-$affine Smarandache curve

The next theorem can be proved analogously as in the previous three cases.

Theorem 3.8. Let $\alpha : I \subseteq R \mapsto A_3$ be an affine $C^{\infty}-$curve in affine 3$-$space A_3 with the affine Frenet frame $\{T, N, B\}$ and the curvatures κ and τ. If $\beta : I \subseteq R \mapsto A_3$ is T$NB-$affine Smarandache curve of α, then its frame $\{T_\beta, N_\beta, B_\beta\}$ is given by

$$\begin{bmatrix} T_\beta \\ N_\beta \\ B_\beta \end{bmatrix} = \begin{bmatrix} -\frac{\tau}{\sqrt{3}} & -\frac{\kappa-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ -\frac{\tau'+\tau}{\sqrt{3}} & -\frac{\kappa'+\tau+\kappa}{\sqrt{3}} & -\frac{\kappa-1}{\sqrt{3}} \\ -\frac{\tau''+\tau'-(\kappa-1)\tau}{\sqrt{3}} & -\frac{\kappa''+2\tau'+\kappa'-\kappa^2+\tau+\kappa}{\sqrt{3}} & -\frac{2\kappa'+\tau+\kappa}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix} \tag{34}$$

and the corresponding curvature κ_β and τ_β read

$$\begin{aligned} \kappa_\beta &= -\frac{\tau''' + \tau'' + 3\kappa''\tau + (3\tau - \kappa + 1)\,\tau' - \kappa'\tau - \kappa^2\tau}{\tau' + \kappa\tau} \\ \tau_\beta &= -\frac{(\kappa-1)\tau''' + (\kappa-1)\tau'' - 3(\tau'+\tau)\kappa'' - 3(\tau')^2 + (\kappa-1)\tau' - (3\kappa-1)\,\kappa'\tau - (4\tau + 2\kappa')\,\tau' - \kappa\tau^2}{\tau' + \kappa\tau} \end{aligned}$$

4. Conclusion

Recently, many studies have been done on the curve theory in affine 3-space (see [23–26]). However, until now, Smarandache curves in affine 3-space have not been defined and their characteristics have not been examined. Therefore, in this paper, TN, TB, NB and TNB-Smarandache curves whose position vector are made by Frenet frame vectors on another regular affine C^{∞}-curve α with the affine Frenet frame $\{T, N, B\}$ in affine 3-space A_3 are introduced. The affine curvature κ_β, the affine torsion τ_β, and the expression of the affine frame vectors $\{T_\beta, N_\beta, B_\beta\}$ of Smarandache curves are obtained. Also, the relationship between the Frenet frames of the curve α and Smarandache curves is given.

Author Contributions

All authors contributed equally to this work. They all read and approved the last version of the manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

References

[1] H. Abdel-Aziz, M. K. Saad, *Computation of Smarandache Curves According to Darboux Frame in Minkowski 3-Space*, Journal of the Egyptian Mathematical Society 25(4) (2017) 382–390.

[2] A. T. Ali, *Special Smarandache Curves in the Euclidean space*, International Journal of Mathematical Combinatorics 2 (2010) 30–36.

[3] O. Bektaş, S. Yüce, *Special Smarandache Curves According to Darboux Frame in* E^3, Romanian Journal of Mathematics and Computer Science 3(1) (2013) 48–59.

[4] M. Çetin, Y. Tuncer, M. K. Karacan, *Smarandache Curves According to Bishop Frame in Euclidean 3-Space*, General Mathematics Notes 20(2) (2014) 50–66.

[5] M. Elzawy, S. Mosa, *Smarandache Curves in the Galilean 4-Space* G_4, Journal of the Egyptian Mathematical Society 25(1) (2017) 53–56.

[6] T. Kahraman, H. H. Uğurlu, *Dual Smarandache Curves and Smarandache Ruled Surfaces*, Mathematical Sciences and Applications E-Notes 2(1) (2014) 83–98.

[7] E. B. Koc Ozturk, U. Ozturk, K. İlarslan, E. Nešović, *On Pseudohyperbolical Smarandache Curves in Minkowski 3-Space*, International Journal of Mathematics and Mathematical Sciences Article ID 658670 (2013) 7 pages.

[8] E. B. Koc Ozturk, U. Ozturk, K. Ilarslan, E. Nešović, *On Pseudospherical Smarandache Curves in Minkowski 3-Space*, Journal of Applied Mathematics Article ID 404521 (2014) 14 pages.

[9] M. Mak, H. Altınbas, *Spacelike Smarandache Curves of Timelike Curves in Anti de Sitter 3-Space*, International Journal of Mathematical Combinatorics 3 (2016) 1–16.

[10] U. Ozturk, E. B. Koc Ozturk, *Smarandache Curves according to Curves on a Spacelike Surface in Minkowski 3-Space*, Journal of Discrete Mathematics Article ID 829581 (2014) 10 pages.

[11] U. Ozturk, E. B. Koc Ozturk, K. İlarslan, E. Neŝović, *On Smarandache Curves Lying in Lightcone in Minkowski 3-Space*, Journal of Dynamical Systems and Geometric Theories 12(1) (2014) 81–91.

[12] E. Solouma, *Special Equiform Smarandache Curves in Minkowski Space-time*, Journal of the Egyptian Mathematical Society 25(3) (2017) 319–325.

[13] S. Şenyurt, Y. Altun, C. Cevahir, *Smarandache Curves According to Sabban Frame Belonging to Mannheim Curves Pair*, Communications Faculty of Sciences University of Ankara Series A1 Mathematics and Statistics 68(1) (2019) 500–513.

[14] S. Şenyurt, C. Cevahir, Y. Altun, *On the Smarandache Curves of Spatial Quaternionic Involute Curve*, Proceedings of the National Academy of Sciences, India Section A: Physical Sciences 90(5) (2020) 827–837.

[15] S. Şenyurt, A. Çaliskan, U. Çelik, *Smarandache Curves of Bertrand Curves Pair According to Frenet Frame*, Boletim da Sociedade Paranaense de Matemática 39(5) (2021) 163–173.

[16] K. Taşköprü, M. Tosun, *Smarandache Curves on* S^2, Boletim da Sociedade Paranaense de Matemática 32(1) (2014) 51–59.

[17] M. Turgut, S. Yilmaz, *Smarandache Curves in Minkowski Space-Time* 3 (2008) 51–55.

[18] K. Nomizu, N. Katsumi, T. Sasaki, Affine Differential Geometry: Geometry of Affine Immersions, Cambridge University Press, 1994.

[19] U. Simon, *Affine Differential Geometry*, in: Handbook of Differential Geometry, Vol. 1, Elsevier, 2000, pp. 905–961.

[20] S. Buchin, Affine Differential Geometry, Gordon and Breach (China), 1983.

[21] N. Hu, *Affine Geometry of Space Curves and Homogeneous Surfaces*, PhD Dissertation, Department of Mathematics Graduate School of Science Hokkaaido University (2012) Japan.

[22] L. A. Santaló, *A Geometrical Characterization for the Affine Differential Invariants of a Space Curve*, Bulletin of the American Mathematical Society 52(8) (1946) 625–632.

[23] P. J. Olver, *Moving Frames and Differential Invariants in Centro-Affine Geometry*, Lobachevskii Journal of Mathematics 31(2) (2010) 77–89.

[24] Y. Tunçer, H. Kocayigit, M. K. Karacan, *Indicatrices of Curves in Affine 3-Space*, Palestine Journal of Mathematics 9(2) (2020) 858–865.

[25] Y. Tunçer, H. Kocayigit, M. K. Karacan, *Naturel Mates of Equiaffine Space Curves in Affine 3-Space*, Thermal Science 23(6) (2019) 2149–2157.

[26] Y. Tunçer, *Position Vectors of the Curves in Affine 3-Space According to Special Affine Frames*, International Journal of Mathematical Combinatorics 2 (2019) 43–59.