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RESEARCH ARTICLE

Investigation of Circular, Elliptical and Obround Shaped Vessels by Finite Element Method(FEM) Analysis under Internal Pressure Loading

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HIGHLIGHTS

- *FEA shows that deformed shape elliptical /obround under internal pressure is not uniformly enlarge due to non-uniform bending. But,circle deforms uniformly due to uniform curvature.*
- *It is found that stress in elliptical/obround shape much higher than circular.*
- *This is useful in selection of shape according to functional requirement.*

Keywords:

- Pressure vessel shape
- Circular
- Elliptical
- Obround
- FEM

GRAPHICAL ABSTRACT

The forms of pressure vessels applicable withinside the aerospace enterprise are starting from essential load-carrying imperative tank systems to small auxiliary tanks and pressurized cabins. Elliptical and obround stress vessels are used in which there may be a space-constrained. In the present work nonlinear (geometric and material within tensile strength) finite element evaluation is performed for the determination of its stress state in open-ended pressure vessels of various shapes (round, elliptical, and obround). A parametric examine has been completed and outcomes offered for different inside areas of vessel shapes and thicknesses of the cross-section. As the inside areas of the pressure vessel increases, the effective stress increases for a particular thickness. The deformed shape elliptical/obround under internal pressure might no longer be uniformly enlarged because of non-uniform bending. Effective stress in the elliptical/obround shapes are found higher than circular shape for a particular thickness and inside area of vessel (Table A).

Configuration	Location in figure A1, A2 & A3 A	B	C	D
Circular (Fig A1)	4.101	3.950	4.101	3.950
Elliptical (Fig.A2)	310.07	315.32	430.26	358.65
Obround (Fig.A3)	387.38	391.56	482.60	462.50

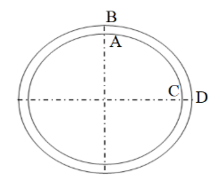

Fig.A1

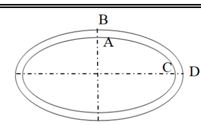

Fig.A2

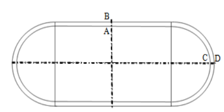

Fig.A3

Table A. Effective Stress(MPa) at various location for 2.5 mm thick for different shape pressure vessel(circular(Fig.A1), elliptical(Fig.A2) and obround(Fig.A3)) with 10000π inside area.

Aim of Article: *Evaluation of stress state in open ended pressure vessels of different shape (circular, elliptical and obround). Parametric study carried out by varying inside areas of vessel and thickness of cross sections.*

Theory and Methodology: *The numerical FEM method is used to find the results.*

Findings and Results: *The effective stress is very much higher in elliptical and obround pressure vessel as compared to circular. As the thickness of the cross section increases the stress value decreases.*

Conclusion: *Nonlinear (geometric and material within tensile strength) FEA is performed to capture more realistic behavior of structure. It has been shown based on FEA that, a pressure vessel with a non-circular cross-section will produce significantly higher wall stresses than that of a circular due to its change in shape. The future work will be aimed at determining failure pressure (instability pressure) of these shapes to known their capacity for a given inside area and thickness.*

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HIGHLIGHTS

- *A parametric examine has been carried out and outcomes presented for different inside areas of specific pressure vessel shapes and thickness of the cross-sections.*
- *FEA shows that the deformed shape elliptical /obround under internal pressure could not uniformly enlarge due to non-uniform bending. However, circle deforms uniformly due to its uniform curvature.It is found that obround/elliptical shape give higher stress than circular vessel.*
- *This analysis is useful in selection of shape of pressure vesslel in design according to functional requirement.*

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ABSTRACT

The forms of pressure vessels applicable withinside the aerospace enterprise are starting from essential load-carrying imperative tank systems to small auxiliary tanks and pressurized cabins. Elliptical and obround stress vessels are used in which there may be a space-constrained. In the present work nonlinear(geometric and material within tensile strength) finite element evaluation is performed for the determination of its stress state in open-ended pressure vessels of various shapes(round, elliptical, and obround). A parametric examine has been completed and outcomes offered for different inside areas of vessel shapes and thickness of the cross-sections. As the inside area of the shape of pressure vessel increases, the effective stress increases for a particular thickness. The deformed shape elliptical/obround under internal pressure might no longer be uniformly enlarged because of non-uniform bending. Deformation and effective stress in the elliptical/obround shapes are found higher than circular shape for a particular thickness and inside area of vessel.

Keywords: *Pressure vessel shape, Circular, Elliptical, Obround, FEM*

I. INTRODUCTION

A pressure vessel is a vessel designed to hold a pressure that is significantly different from the ambient pressure, either internally or externally. The class of pressure vessels associated with the aerospace industry is propellant tanks ranging from main load-carrying integral tank structures to small auxiliary tanks, storage tanks, motor cases, high-pressure gas bottles, and pressurized cabins. Commonly used shapes for pressure vessels are spheres, cylinders, and cones. The advantage of a spherical vessel over a cylindrical vessel is that for a given pressure and diameter, the spherical vessel requires a thinner wall than an equivalent cylinder. The design of the pressure vessel for a given ultimate pressure is mainly based on the diameter and thickness of the cylinder section. Optimal design means maximum enclosed volume for the storage with minimum structural mass. The most important point is to minimize the discontinuous stress at the junction. It is important to perform non-linear analysis of geometry and materials to capture structural behavior [1,2,3]. Elastic stress of pressure vessels with a mismatch in circumferential weld seam [4], presence of weld sinkage [5], and the

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estimation of longitudinal seam mismatch to compare the test strain [6] have been studied. Finite element analysis (FEA) with modeling the measured profile of cylindrical shell compared strain close to measured strain in the circumferential direction [7].

Spheres and cylinders are two basic shapes generally used for internally pressurized vessels. The sphere will impart the lowest value of membrane stress on its walls as compared to any other shape. That's why, for a given enclosed volume, a sphere represents the most weight-efficient design. In many situations, a straight cylinder is the preferred shape, as it is much more easily fabricated than a sphere while still providing a reasonably weight-efficient design. However, there appear circumstances where neither a cylindrical or spherical vessel may be appropriate for a particular purpose. Situations encountered in the design of aircraft propulsion systems where the engine is incorporated into the airframe structure. In such situations, elliptical pressure vessels are used where there is a space-constrained such as propulsion systems with respect to the engine cavity [8]. In some situations, due to the limited space available, exit pipes are made of elliptical or obround shape [9-10]. Shells of circular and elliptical cross-sections [11] with equivalent volume comparison show that cargo tanks with elliptical cross-sections possess a lower center of gravity than those of circular cross-sections. Consequently, at the lower center of gravity, the chance of topple over is decreased. Having the same height for the center of gravity of two tanks, the elliptical cross-section tank will be able to carry more shipping material than a circular cross-section tank due to its larger lateral dimension.

Ansys finite element software was used to calculate the stress concentration factor of functionally graded round square cross-section and flattened tube cross-section (obround) [12].The ASME code [13] specifies two non-circular shapes, a rectangle, and a obround. The difference in behavioral between circular and non-circular pressure vessels is different. Limited studies have been reported in the literature on calculating stress in non-circular pressure vessels using numerical/analytical methods.

The linear FEM simplifies many things. For example, the material does not yield, creating unrealistic high stresses in the model. Also, because the non-linear shape is not taken into account, the state of the membrane is unpredictable (or very inadequate). Non-linear FEA, if properly defined, handles all these problems. When using linear analysis, solver assumes that a "small deformation". This actually means that two assumptions are made: (i) deformation does not affect the behavior of the structure (considering go in for membrane state), (ii) there is no impaired stability. This means that if a thickset solid actually needs to be analyzed, it will not be in a membrane state. Non-linear geometric analysis is not required in such cases. Therefore, to take care different shape of structure whose behavior is not known apriori (i.e. whether it will behave linearly or nonlinearly), nonlinear analysis is appropriate. But it doesn't mistreat either. What works linearly, using nonlinear geometry analysis gives the same results as linear analysis. In complicated vessel shapes the simple membrane-stress concepts do not enough to give adequate information of the true stress state. So, in the present article, nonlinear FEA using Ansys is carried out for the determination of stresses and deformations in open-ended pressure vessels of different shapes (circular, elliptical, and obround), which seems not carried out earlier[8,9,10]. Due to change in shape in elliptical or obround vessel, at the change in profile location high stress is expected. Therefore, to carry out more realistic and qualitatively analysis, both geometric and material analysis is performed. Further, a parametric study is carried out, by varying inside areas of various shapes of vessel and thickness of cross-section of the pressure vessel.

II. CONFIGURATION

The open ended pressure vessels of various geometries (Figure.1.) such as circular, elliptical and obround (circular half sections with flat plate walls) are used for the stress analysis.

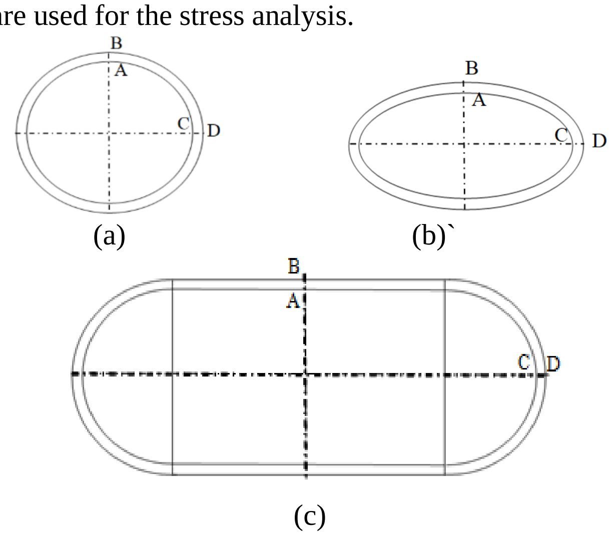

Figure. 1. Pressure vessels (a) circular (b) elliptical (c) obround

III. FINITE ELEMENT ANALYSIS

Finite element software ANSYS [14], is used for the modeling of circular shape pressure vessel and geometric and material non linear [2,3,4] analysis performed. The plane 82 element has 8-nodes and having two degrees of freedom per node. A quarter models considered and symmetric boundary condition applied in the symmetric plane. Internal pressure (P) of 0.1MPa is applied and analysis is carried out for 5000п inside area of vessel and thickness (t) of 2.5 mm. Material is HSLA steel (YS=834 MPa, UTS=981 MPa., E=206010 MPa and ν=0.33). The material card supplied to Ansys as follows.

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MPTEMP,1,0
MPDATA,EX,1,,206010
MPDATA,PRXY,1,,0.33
TB,Miso,     1,   1, 100
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A. Mesh Convergence Study

Convergence study is carried out to arrive at the appropriate finite element size.

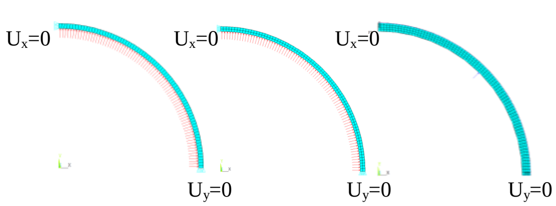

Figure.2. Finite element model of circular pressure vessel (a) one element across thickness (b) two elements across thickness (c) 4 elements across thickness

Table I. Mesh convergence study.

Mesh size	1x80	2x80	4x80
Hoop Stress in MPa	2.878	2.879	2.879

Based on above convergence study, it is seen that a mesh of 2 elements and 4 elements across thickness (Figure 2) do not change the values of hoop stress. Therefore, the mesh size of 2 elements across thickness is considered for present analysis.

B. Validation of Finite Element Analysis

The finite element model for a circular vessel (R=100mm, t=2.5mm) is shown in Figure 3. Hoop stress is responsible for change in diameter or radius. Longitudinal stress is responsible for change in length of cylinder. When the pressure vessel is open and free of end restraints, longitudinal stress will not be prodded into. It will develop only if the ends of vessel are closed. Radial stress is very small as compared to longitudinal and hoop stress. It can be neglected. Hence the radial stress is not shown in the paper.

The displacement and hoop stress are determined through analytical solution for circular pressure vessels.

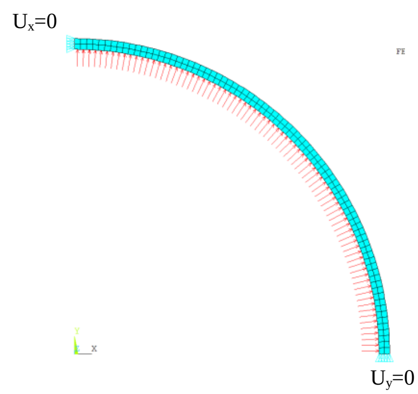

Figure. 3. Finite element model of circular pressure vessel

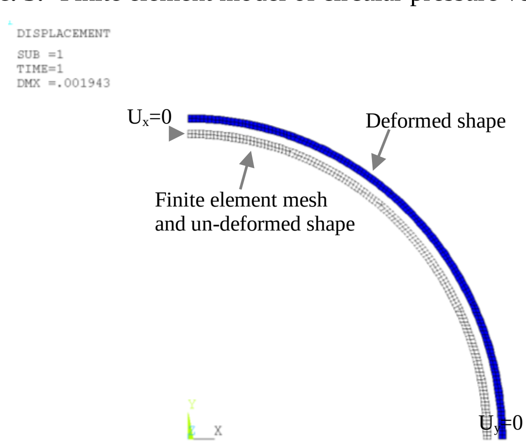

Figure.4. Deformed and un-deformed shape of circular pressure vessel with 10000π mm^2 inside area

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The radial displacement is calculated using formulae

$$\delta = \frac{PR^2}{2tE}(2-\nu) = 0.0018\text{mm}$$

Figure 4 indicates the deformed shape along with the un-deformed shape of the circular pressure vessel.

Hoop Stress = (P R)/t = (0.1 x 100)/2.5 = 4 MPa,

This is compared well to current finite element solutions (Figure 5, first principal stress) and with literature value[8-10].This validates finite element modeling and mesh size.

IV. RESULTS AND DISCUSSION

Circular pressure vessel: Effective stress (MPa) at various locations for the inside area of circular shape vessel of 5000 π mm^2 with different thicknesses is shown in Table II. Effective Stress (MPa) at the various locations for 2.5 mm thick circular pressure vessels with the different inside areas is shown in Table III. As inside area of vessel increases the effective stress increases. As thickness increases, the effective stress reduces. The deformed and un-deformed shape of the circular pressure vessel with 10000 π mm^2 inside area is shown in Figure 4.

The first principal stress (MPa) and effective stress of circular pressure vessels for 10000 π mm^2 inside area are shown in Figures 5 and 6 respectively. The effective stress at location A and C, B and D (Figure.1.) are close due to uniform curvature (Table II & Table III).

The effective stress for 5000 π mm^2, 10000 π mm^2 and 15000 π mm^2 inside areas are 2.930 MPa, 4.1 MPa and 4.999 MPa respectively (Table III).

Table II. Effective Stress (MPa)at various locations for circular shape with inside area of 5000 π mm^2 with different thickness.

Thickness in mm	Location in Figure 1			
	A	B	C	D
2.5	2.930	2.779	2.930	2.779
3.0	2.460	2.310	2.460	2.310
3.5	2.123	1.971	2.123	1.971
4.0	1.870	1.720	1.870	1.720

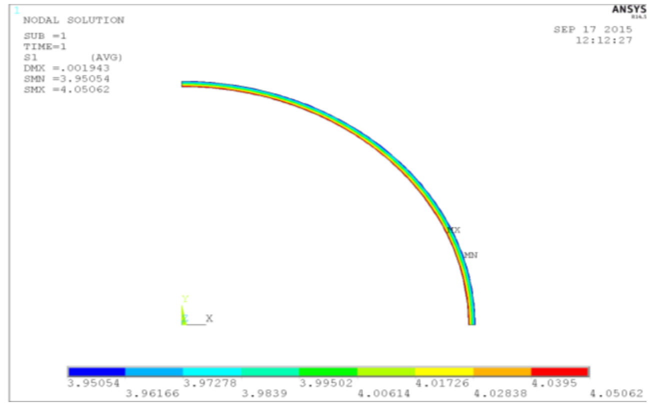

Figure. 5. First principal stress (MPa) in circular pressure vessel with 10000π mm^2 inside area.

Table III. Effective Stress (MPa) at various location with 2.5 mm cross sectional thickness for different inside area of circular shape pressure vessel

Inside area of vessel	Location in Figure 1			
	A	B	C	D
5000 mm^2	2.930	2.779	2.930	2.779
10000 π mm^2	4.101	3.950	4.101	3.950
15000π mm^2	4.999	4.849	4.999	4.849

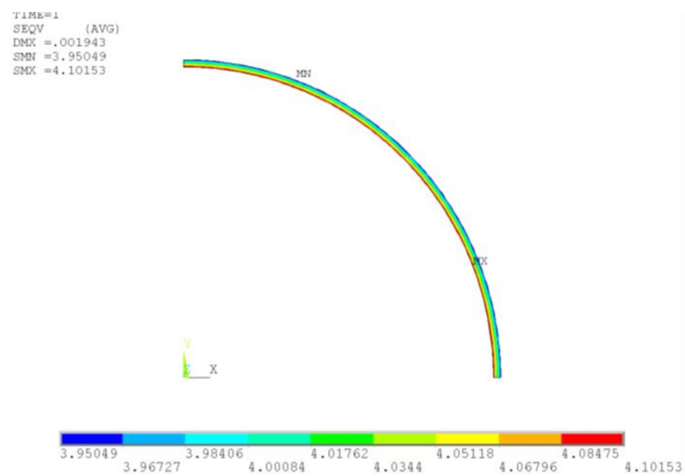

Figure. 6. Effective stress (MPa) in circular pressure vessel with 10000π mm^2 inside area

Elliptical Pressure Vessel: The finite element mesh with deformed and un-deformed shape of elliptical pressure vessel subjected to internal pressure (P= 0.1MPa) is shown in Figure 7. A quarter model considered and symmetric boundary condition applied in the symmetric plane. Effective Stress (MPa) at the various locations for 2.5 mm thick with the different inside areas of vessel is shown in Table IV. As inside area of vessel increases the effective stress increases. Effective Stress (MPa) at various locations of inside areas of vessel of 5000 π mm^2 with different thicknesses is shown in Table V. As thickness increases, the effective stress reduces. The effective stress (MPa) of a vessel is shown in Figure 8. Comparison of elliptical vessel results with circular vessel show that effective stresses are very much higher than that of the circular pressure vessel. The maximum deformation is higher than that of the circular pressure vessels.

The maximum effective stress is found for 10000 π mm^2 inside area of 430.26 MPa at location C (Table IV). The maximum stress is increased to 589.81 MPa for 15000 π mm^2 inside area (Table IV and Figure 8).

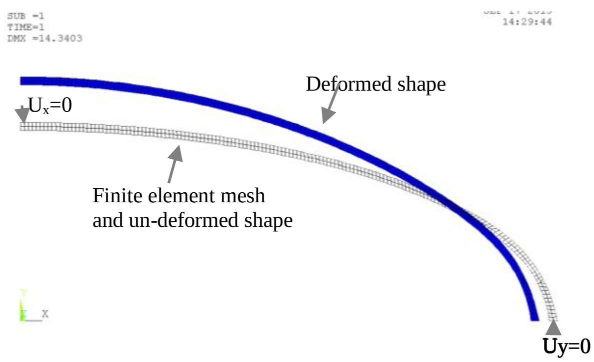

Figure.7. Deformed and un-deformed (FE mesh 2 elements across thickness) shape of elliptical pressure vessel with 15000π mm^2 inside area.

Table. IV. Effective Stress (MPa) at various locations with 2.5 mm cross sectional thickness for different inside areas of elliptical shape pressure vessel

Inside area of vessel	Location in Figure 1 A	B	C	D
5000π mm^2	219.11	222.22	327.19	167.25
10000π mm^2	310.07	315.32	430.26	358.65
15000π mm^2	414.95	421.73	589.81	361.32

Table. V. Effective Stress (MPa) at various locations for elliptical pressure vessel with different cross sectional thickness with inside area of 5000π mm^2

Thickness in mm	Location in Figure 1 A	B	C	D
2.5	219.11	222.22	327.18	167.25
3.0	165.80	168.34	281.25	98.31
3.5	115.54	124.82	182.71	68.31
4.0	97.32	99.33	181.67	42.27

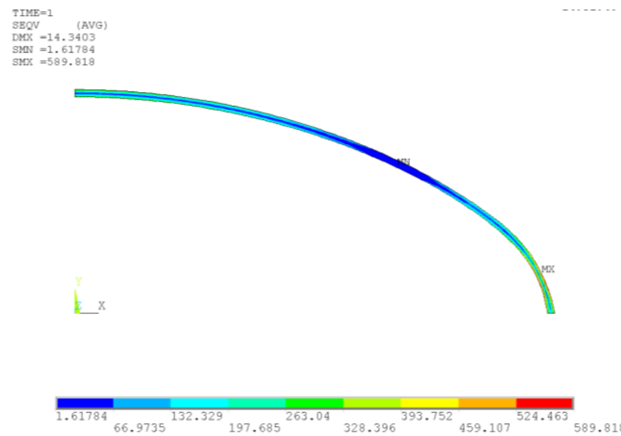

Figure. 8. Effective stress (MPa) of elliptical pressure vessel with 15000π mm^2 inside area.

Obround Pressure Vessel: The finite element model of an obround pressure vessel is shown in Figure 9. A quarter model considered and symmetric boundary condition applied in the symmetric plane. Internal pressure (P) of 0.1MPa is applied. The deformed and un-deformed shape is shown in Figure 10. Effective stress (MPa) at the various locations for 2.5 mm thick with the different inside areas of vessel is shown in Table VI. As inside area increases the effective stress increases. Effective Stress (MPa) at various locations with inside areas of vessel of 5000 π mm^2 and different thicknesses is shown in Table VII. As thickness increases, the effective stress reduces. The effective stress (MPa) of a vessel with 10000π mm^2 inside area of vessel is shown in Figure 11. From the above study, it is found that values of deformation and effective stresses are very much higher than that of the circular shape pressure vessel.

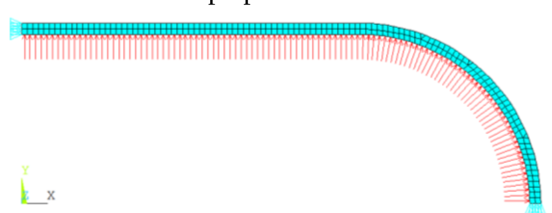

Figure. 9. Finite element model of obround pressure vessel

Table. VI. Effective Stress (MPa) at various locations with 2.5 mm cross sectional thickness for different inside areas of obround shape pressure vessel

Inside area of vessel	Location in Figure 1 A	B	C	D
5000π mm^2	196.24	199.30	242.64	228.67
10000π mm^2	387.38	391.56	482.60	462.50
15000π mm^2	588.96	592.95	696.71	679.80

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Table.VII. Effective Stress (MPa) at various locations for obround pressure vessel with different cross sectional thickness with inside area of 5000π mm^2

Thickness in mm	Location in Figure 1			
	A	B	C	D
2.5	196.24	199.30	242.63	228.67
3.0	148.68	151.21	185.37	172.06
3.5	109.50	111.67	137.36	125.75
4.0	87.47	89.36	110.40	99.70

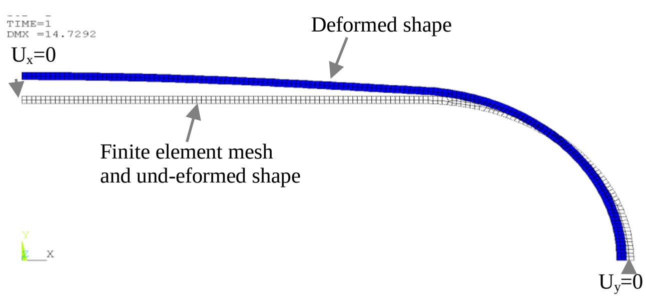

Figure. 10. Deformed and un-deformed shape of obround pressure vessel

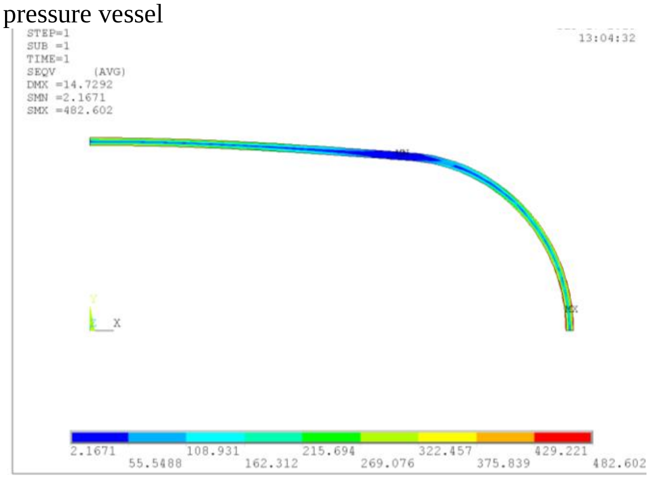

Figure.11. Effective stress (MPa) in obround pressure vessel with 10000π mm^2 inside area.

The maximum effective stress is found of 482.6 MPa for 10000 π mm^2 inside area of vessel at location C (Table VI, Figure. 11). The maximum stress is increased to 696.7 MPa for 15000 π mm^2 inside area (Table VI).

V. CONCLUSION

In the present article FEA detail is presented in open-ended specific shapes (circular, elliptical, and obround) pressure vessels for the evaluation of its stress state. Based on nonlinear FEA, it is determined that a pressure vessel designed with a particular thickness and inside area, non-circular cross-section will produce substantially higher wall stresses and deformation than that of a round or circular vessel because of the alternate in curvature. As the thickness of the cross-section will increase for a given inside area of the vessel, the stress value decreases. As the inside area of the pressure vessel increases the effective stress increases for a particular thickness.

In an elliptical and obround vessel, the internal pressure induces a bending moment at the wall. So, the elliptical / obround pressure vessel the deformed shape will no longer be deformed uniformly because of non-uniform bending. The center portion would be undergone a larger deflection than that of the corner. Therefore the total stress for a noncircular pressure vessel is the sum of the stress because of bending and the membrane stress. However, in the case of circle bending impact is absent because of its uniform curvature.

The future work will be aimed at determining of failure pressure (instability pressure) of these shapes to known their capacity for a given area of cross section and thickness. To carry out failure analysis of these structural shapes, the non-linear analysis is must to find out its capacity. This non-linear evaluation is to find out its pressure at instability. However, in the present article the stability point is not addressed (i.e. structural analysis is within tensile strength of material) and only non-linear analysis is performed to capture more realistic behavior of structure.

CONFLICTS OF INTEREST

There was no conflict of interest.

RESEARCH AND PUBLICATION ETHICS

In the studies carried out within the scope of this article, the rules of research and publication ethics were followed.

REFERENCES

[1] J. F. Harvey, “Theory and design of pressure vessels,” ISBN No. 0-442-23248-9, New York, USA: Van Nostrand Reinhold Company Inc,1985.
[2] C. Pany, M.K.Sundaresan, B.N. Rao et al., “On the bursting of an HSLA steel rocket motor case during proof pressure testing,” *Steel Grips (J. of Steel and Related Mat.) Application*, vol.10, 2012, pp.434-438. https://www.researchgate.net/publication/319128256.
[3] J.T.Merlin, and C. Pany, “Design and analysis of pressure vessel with different end domes,” *Int. J. of*

Sci. Engg. and Tech. Res., vol.6, no.8, 2017,pp.1225-1233.
[4] M.G. Sreelakshmi, and C. Pany, "Stress analysis of metallic pressure vessels with circumferential mismatch using finite element method," *Int. J. of Sci. and Engg. Res.*, vol .7, no.4, 2016,pp.479-484.
[5] C.Pany, "Structural analysis of metallic pressure vessels with weld sinkage in the circumferential joint," *J. of Sci. Tech. and Engg. Res.*, vol.2, no.1, 2021, pp.4-10. DOI:10.5281/zenodo.4586360.
[6] C.Pany, "Estimation of correct long-seam mismatch using FEA to compare the measured strain in a non-destructive testing of a pressurant tank - a reverse problem," *Int. J. of Smart Veh. and Smart Transportation*,vol.4,no.2,2021,pp.16-28. DOI:10.4018/IJSVST.2021010102.
[7] C.Pany, "Cylindrical shell pressure vessel profile variation footprint in strain comparison of test data with numerical analysis," *Liquid and Gaseous Energy Resources (JVE journals)*, vol.1, no.2, pp.91-101, 2021. https://doi.org/10.21595/lger.2021.22163.
[8] J.C.Wang, "Stress analysis of an elliptical pressure vessel under internal pressure, "*MS Report*, Rensselaer at Hartford, Hartford, CT, December 8, 2005.
[9] M.M.Utagikar and S.B. Naik, "Finite element analysis of elliptical pressure vessels," *American J. of Engg. Res.*, Vol. 2, no.12, 2013, pp.343-349.
[10] M.M.Utagikar and S.B. Naik, "Finite element analysis of obround pressure vessels," *Int. J. of Modern Engg. Res.*, Vol. 3, no. 5, 2013, pp.2717-2725.
[11] Q.S. Zheng, "Analysis of a shell of elliptical cross-section under internal pressure and body force," PhD Dissertation, Mechanical Engineering Texas Tech University, December, 1997.
[12] Z. Ebrahimi, and S. Negahban, "Investigation of stress concentration factors for functionally graded hollow tubes with curved edges under torsion," *Iranian J. of Mat. Forming.*, Vol. 8, no.2, 2021. pp.22-34. https://doi.org/10.22099/ijmf.2021.39443.1175
[13] American Society of Mechanical Engineers (ASME), Mandatory Appendix 13: Vessels of noncircular cross section, from the *ASME boiler & pressure vessel code*, ASME International, 2004.
[14] ANSYS Software Version 14.0 Documentation, (Copyright – SAS IP, Inc.).