Gazi University Journal of Science
GU J Sci
29(1):159-165 (2016)

An Adroit Randomized Response New Additive Scrambling Model

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Received: 18/08/2014 Revised:20/08/2015 Accepted: 01/09/2015

ABSTRACT

In this paper, an improved new additive model has been proposed. The proposed model is found to be more efficient than the randomized response models studied by Gjestvang and Singh (2009) and Singh (2010). The relative efficiency of the proposed model has been studied with respect to the Gjestvang and Singh (2009) and Singh (2010) model. It is found that the envisaged model is superior to those additive models earlier considered by Gjestvang and Singh (2009) and Singh (2010).Numerical illustrations are also given in support of the present study.
Key Words: Randomized response sampling, Estimation of proportion, sensitive quantitative variable.

1. INTRODUCTION

Warner (1965) was first to introduce a randomized response (RR) model to estimate proportion for sensitive attributes including sexual orientation, criminal activity, child abuse, suicidal tendency in teenagers, all cases of AIDS, abortion or drug addiction, such that the respondent's privacy should be protected. Some recent contribution to randomized response sampling is given by Fox and Tracy (1986), Singh and Mathur (2004, 2005), Gjestvang and Singh (2006, 2009), Gupta et al. (2010,2012) and Singh and Tarray (2013, 2014, 2015).We below give the description of the models due toGjestvang and Singh (2009) and Singh (2010) additive models:

1.1 Gjestvang and Singh (2009) additive model:

Let α and β be two known positive real numbers. Then Gjestvang and Singh (2009) proposed an additive model in which each respondent in the sample is requested to

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draw a card secretly from a well – shuffled deck of cards. In the deck, let P be the proportion of cards bearing the statement, "Multiply scrambling variable S with α and add to the real value of the sensitive variable Y_i", and (1-P) be the proportion of cards bearing the statement, "Multiply scrambling variable S with β and subtract it from the real value of the sensitive variable Y_i". Mathematically, each respondent is requested to report the scrambled response Z_i as:

$$Z_i = \begin{cases} Y_i + \alpha S & \text{with probability } P = \beta/(\alpha+\beta) \\ Y_i - \beta S & \text{with probability } (1-P) = \alpha/(\alpha+\beta) \end{cases} \quad (1.1)$$

Gjestvang and Singh (2009) defined an unbiased estimator of the population mean μ_Y as

$$\hat{\mu}_{GS} = \frac{1}{n}\sum_{i=1}^{n} Z_i \quad (1.2)$$

and the variance of $\hat{\mu}_Y$ is given by

$$V(\hat{\mu}_{GS}) = \frac{1}{n}\left[\sigma_y^2 + \alpha\beta(\theta^2 + \gamma^2)\right] \quad (1.3)$$

1.2 Singh (2010) additive model:

Suppose there are k scrambling variables denoted by S_j, j = 1,2,…,k whose mean θ_j (i.e. $E(S_j) = \theta_j$) and variance γ_j^2 (i.e. $V(S_j) = \gamma_j^2$) are known. In Singh (2010) proposed optimal new orthogonal additive model named as (POONAM), each respondent selected in the sample is requested to rotate a spinner, as shown in Fig. 1, in which the proportion of the k shaded areas, say P_1, P_2, … P_k are orthogonal to the means of the k scrambling variables, say $\theta_1, \theta_2, \ldots, \theta_k$ such that:

$$\sum_{j=1}^{k} P_j \theta_j = 0 \quad (1.4)$$

and

$$\sum_{j=1}^{k} P_j = 1 \quad (1.5)$$

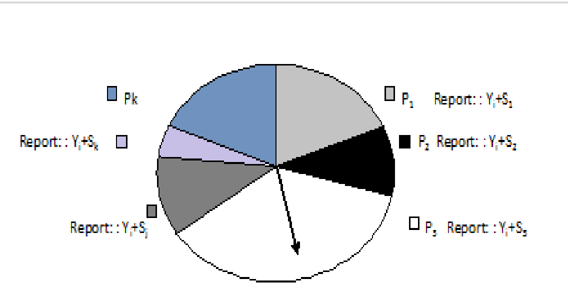

Fig. 1 Spinner for POONAM (Singh (2010))

Now if the pointer stops in the j^{th} shaded area, then the i^{th} respondent with real value of the sensitive variable, say Y_i, is requested report the scrambled response Z_i as:

$$Z_i = Y_i + S_i \quad (1.6)$$

Assuming that the sample of size n is drawn from the population using simple random sampling with replacement (SRSWR).Singh (2010) suggested an unbiased estimator of the population mean μ_Y as

$$\hat{\mu}_Y = \frac{1}{n}\sum_{j=1}^{k} Z_j \quad (1.7)$$

The variance of $\hat{\mu}_Y$ is given by

$$V(\hat{\mu}_Y) = \frac{1}{n}\left[\sigma_y^2 + \sum_{j=1}^{k} P_j(\theta_j^2 + \gamma_j^2)\right] \quad (1.8)$$

2. The proposed procedure

It is to be noted that the mean θ_j and variance γ_j^2 of the j^{th} scrambling variable S_j (j=1,2,…,k) are known. But these information's have not utilized by the previous authors in building up the randomization models. It is possibility that the use of the prior information regarding the parameters of the scrambling variable S_j may improve the efficiency of the randomized response model. This led authors to propose a new additive model based on standardized scramblingvariable

$S_j^* = \left(\frac{S_j - \theta_j}{\gamma_j}\right)$, j = 1,2,…,k whose mean is "zero" (i.e. $E(S_j^*) = 0$) and the variance is "unity" (i.e. $V(S_j^*) = 1$).

Then in the proposed additive model, each respondent selected in the sample is requested to rotate a spinner, as demonstrated in Fig. 2, in which the proportion of the k shaded areas, say P_1, P_2, … P_k such that:

$$\sum_{j=1}^{k} P_j = 1 \quad (2.1)$$

Now if the pointer stops in the j^{th} shaded area, then the i^{th} respondent with real value of the sensitive variable, say Y_i, is requested report the scrambled response Z_i^* as:

$$Z_i^* = Y_i + S_j^* \quad (2.2)$$

Let a sample of size n be drawn from the population using the simple random sampling with replacement (SRSWR). Then we prove the following theorems.

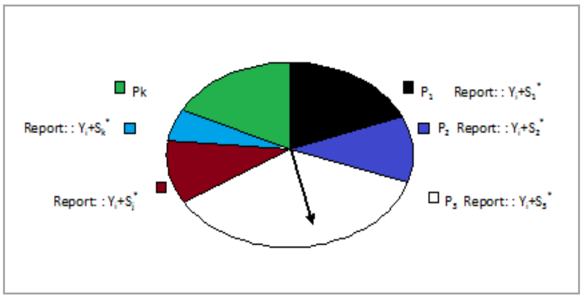

Fig. 2 Spinner for proposed procedure.

Theorem 2.1 An unbiased estimator of the population mean μ_Y is given by

$$\hat{\mu}_{ST} = \frac{1}{n}\sum_{i=1}^{n} Z_i^* \tag{2.3}$$

Proof. Let E_1 and E_2 denote the expectation over the sampling design and the randomization device respectively, we have

$$E(\hat{\mu}_{ST}) = E_1 E_2\left[\frac{1}{n}\sum_{i=1}^{n} Z_i^*\right]$$

$$= E_1\left[\frac{1}{n}\sum_{i=1}^{n} E_2(Z_i^*)\right]$$

$$= E_1\left[\frac{1}{n}\sum_{i=1}^{n}\left\{Y_i\sum_{j=1}^{k}P_j + \sum_{j=1}^{k}P_j E_2(S_j^*)\right\}\right]$$

$$= E_1\left[\frac{1}{n}\sum_{j=1}^{n} Y_i\right] = \mu_Y \quad ,$$ since

$\sum_{j=1}^{k} P_j = 1$ and $E_2(S_j^*) = 0$,

Which proves the theorem.

The variance of the proposed estimator $\hat{\mu}_Y$ is given in the following theorem.

Theorem 2.2 The variance of the proposed estimator $\hat{\mu}_Y$ is given by

$$V(\hat{\mu}_{ST}) = \frac{1}{n}\left[\sigma_y^2 + 1\right] \tag{2.4}$$

Proof. Let V_1 and V_2 denote the variance over the sampling design and over the proposed randomization device, respectively, then we have

$$V(\hat{\mu}_Y) = E_1 V_2(\hat{\mu}_Y) + V_1 E_2(\hat{\mu}_Y)$$

$$= E_1\left[V_2\left(\frac{1}{n}\sum_{i=1}^{n} Z_i^*\right)\right] + V_1\left[E_2\left(\frac{1}{n}\sum_{i=1}^{n} Z_i^*\right)\right] \tag{2.5}$$

$$= E_1\left[\frac{1}{n^2}\sum_{i=1}^{n} V_2(Z_i^*)\right] + V_1\left[\frac{1}{n}\sum_{i=1}^{n} E_2(Z_i^*)\right]$$

$$= E_1\left[\frac{1}{n^2}\sum_{i=1}^{n}(1)\right] + V_1\left[\frac{1}{n}\sum_{i=1}^{n} Y_i\right]$$

$$= \frac{1}{n}\left(\sigma_y^2 + 1\right)$$

Note that:

$$V_2(Z_i^*) = E_2\left(Z_i^{*2}\right) - \left(E_2\left(Z_i^*\right)\right)^2$$

$$E_2(Z_i^{*2}) = E_2\left(Y_i + S_j^*\right)^2 = E_2\left[Y_i^2 + S_j^{*2} + 2Y_i S_j^*\right]$$

$$= Y_i^2\sum_{j=1}^{k}P_j + \sum_{j=1}^{k}P_j \quad ,$$ since

$E_2(S_j^{*2}) = 1$ and $E_2(S_j^*) = 0$,

$$= (Y_i^2 + 1),$$

and

$$E_2(Z_i^*) = E_2\left(Y_i + S_j^*\right) = Y_i\sum_{j=1}^{k}P_j = Y_i,$$ since

$E_2(S_j^*) = 0$,

Thus

$$V_2(Z_i^*) = Y_i^2 + 1 - Y_i^2 = 1.$$

which proves the theorem.

3. Efficiency Comparison

From (1.3) and (2.4), we have

$V(\hat{\mu}_{ST}) < V(\hat{\mu}_{GS})$ if

$$\frac{1}{n}\left[\sigma_y^2 + 1\right] < \frac{1}{n}\left[\sigma_y^2 + \alpha\beta(\theta^2 + \gamma^2)\right]$$

i.e. if $1 < \alpha\beta(\theta^2 + \gamma^2)$

i.e. if $$\alpha\beta < \frac{1}{(\theta^2 + \gamma^2)} \tag{3.1}$$

Thus the proposed estimator $\hat{\mu}_{ST}$ is better than Gjestvang and Singh (2009) estimator $\hat{\mu}_{GS}$ as long as condition (3.1) is satisfied.

Further from (1.8) and (2.4), we have

$V(\hat{\mu}_{ST}) < V(\hat{\mu}_Y)$

if

i.e.if $$\frac{1}{n}\left[\sigma_y^2 + 1\right] < \frac{1}{n}\left[\sigma_y^2 + \sum_{j=1}^{k}P_j\left\{(\theta_j^2 + \gamma_j^2)\right\}\right]$$

i.e if $1 < \sum_{j=1}^{k} P_j(\theta_j^2 + \gamma_j^2)$

i.e if $\sum_{j=1}^{k} P_j(\theta_j^2 + \gamma_j^2 - 1) > 0, \quad \text{since} \quad \sum_{j=1}^{k} P_j = 1,$

i.e. if $\left(\theta_j^2 + \gamma_j^2\right) > 1$, $\forall$ j (3.2)

The condition (3.2) clearly indicates that if one chooses (θ_j, γ_j) such that $\left\{\left(\theta_j^2 + \gamma_j^2\right) > 1\right\}$ then the proposed model is always better than the Singh's (2010) model.

The percent relative efficiency (PRE) of the proposed estimator $\hat{\mu}_{ST}$ with respect toSingh's (2010) estimator $\hat{\mu}_Y$ and Gjestvang and Singh's (2009) estimator $\hat{\mu}_{GS}$ are respectively given by

$$\mathrm{PRE}(\hat{\mu}_{ST}, \hat{\mu}_Y) = \frac{\left[\sigma_y^2 + \sum_{j=1}^{k} P_j\left\{\left(\theta_j^2 + \gamma_j^2\right)\right\}\right]}{\left[\sigma_y^2 + 1\right]} \times 100 \qquad (3.2)$$

and

$$\mathrm{PRE}(\hat{\mu}_{ST}, \hat{\mu}_{GS}) = \frac{\left[\sigma_y^2 + \alpha\beta(\theta^2 + \gamma^2)\right]}{\left[\sigma_y^2 + 1\right]} \times 100 \qquad (3.3)$$

By keeping the respondents cooperation in mind, we decided to chooseα = 0.4, β =0.6 (similarly to Gjestvang and Singh (2009)), γ=40, γ_1= 30, γ_2= 40, γ_3 = 20, γ_4= 10, P_1=0.02,P_2=0.05,P_3=0.06,P_4=0.87 with k = 4(similarly to Singh (2010)). In addition we choose different values $\sigma_y^2, \theta, \theta_1, \theta_2, \theta_3$ and θ_4 as listed in Tables 1 and 2 respectively.

We have computed the percent relative efficiencies $\mathrm{PRE}(\hat{\mu}_{ST}, \hat{\mu}_Y)$ and $\mathrm{PRE}(\hat{\mu}_{ST}, \hat{\mu}_{GS})$ and findings are displayed in Tables 1 and 2 respectively.

Table 1. The PRE $(\hat{\mu}_{ST}, \hat{\mu}_Y)$

σ_Y^2	θ_1	θ_2	θ_3	θ_4	PRE
	300	200	100	-25.20	19948.02
	800	700	600	-100.00	260900.00
	1300	1200	1100	-174.70	789178.76
25	1800	1700	1600	-249.40	1604801.20
	300	200	100	-25.20	4195.62
	800	700	600	-100.00	53915.87
	1300	1200	1100	-174.70	162925.78
125	1800	1700	1600	-249.40	331228.82
	300	200	100	-25.20	2383.40
	800	700	600	-100.00	30103.54
	1300	1200	1100	-174.70	90878.97
225	1800	1700	1600	-249.40	184711.64
	300	200	100	-25.20	1682.97
	800	700	600	-100.00	20900.00
	1300	1200	1100	-174.70	63032.66
325	1800	1700	1600	-249.40	128082.30
	300	200	100	-25.20	1311.38
	800	700	600	-100.00	16017.37
	1300	1200	1100	-174.70	48259.74
425	1800	1700	1600	-249.40	98039.51
	300	200	100	-25.20	1081.08
	800	700	600	-100.00	12991.25
	1300	1200	1100	-174.70	39103.89
525	1800	1700	1600	-249.40	79419.83
	300	200	100	-25.20	924.36
	800	700	600	-100.00	10931.95
	1300	1200	1100	-174.70	32873.24
625	1800	1700	1600	-249.40	66748.93
	300	200	100	-25.20	810.81
	800	700	600	-100.00	9439.94
	1300	1200	1100	-174.70	28359.02
725	1800	1700	1600	-249.40	57568.64
	300	200	100	-25.20	724.76
	800	700	600	-100.00	8309.20
	1300	1200	1100	-174.70	24937.83
825	1800	1700	1600	-249.40	50611.18

Table 2. The PRE $(\hat{\mu}_{ST}, \hat{\mu}_{GS})$

σ_Y^2	θ_1	θ_2	θ_3	θ_4	θ	PRE
	300	200	100	-25.20	200.00	38496.154
	800	700	600	-100.00	700.00	453880.77
	1300	1200	1100	-174.70	1200.00	1330803.8
25	1800	1700	1600	-249.40	1700.00	2669265.4
	300	200	100	-25.20	200.00	8023.0159
	800	700	600	-100.00	700.00	93737.302
	1300	1200	1100	-174.70	1200.00	274689.68
125	1800	1700	1600	-249.40	1700.00	550880.16
	300	200	100	-25.20	200.00	4517.2566
	800	700	600	-100.00	700.00	52304.867
	1300	1200	1100	-174.70	1200.00	153189.82
225	1800	1700	1600	-249.40	1700.00	307172.12
	300	200	100	-25.20	200.00	3162.2699
	800	700	600	-100.00	700.00	36291.104
	1300	1200	1100	-174.70	1200.00	106229.75
325	1800	1700	1600	-249.40	1700.00	212978.22
	300	200	100	-25.20	200.00	2443.4272
	800	700	600	-100.00	700.00	27795.54
	1300	1200	1100	-174.70	1200.00	81316.667
425	1800	1700	1600	-249.40	1700.00	163006.81
	300	200	100	-25.20	200.00	1997.9087
	800	700	600	-100.00	700.00	22530.228
	1300	1200	1100	-174.70	1200.00	65876.236
525	1800	1700	1600	-249.40	1700.00	132035.93
	300	200	100	-25.20	200.00	1694.7284
	800	700	600	-100.00	700.00	18947.125
	1300	1200	1100	-174.70	1200.00	55368.85
625	1800	1700	1600	-249.40	1700.00	110959.9
	300	200	100	-25.20	200.00	1475.0689
	800	700	600	-100.00	700.00	16351.102
	1300	1200	1100	-174.70	1200.00	47756.061
725	1800	1700	1600	-249.40	1700.00	95689.945
	300	200	100	-25.20	200.00	1308.5956
	800	700	600	-100.00	700.00	14383.656
	1300	1200	1100	-174.70	1200.00	41986.562
825	1800	1700	1600	-249.40	1700.00	84117.312

It is observed from Tables 1 and 2 that the value of $PRE(\hat{\mu}_{ST}, \hat{\mu}_Y)$ and $PRE(\hat{\mu}_{ST}, \hat{\mu}_{GS})$ are greater than 100. It follows that the proposed estimator is more efficient than the estimator $\hat{\mu}_Y$ due to Singh (2010) and $\hat{\mu}_{GS}$ due to Gjestvang and Singh (2009) with substantial gain in efficiency. Thus, based on our simulation results, the use of the proposed estimator $\hat{\mu}_{ST}$ over Singh (2010) estimator $\hat{\mu}_Y$ and Gjestvang and Singh (2009) estimator $\hat{\mu}_{GS}$ is recommended for all situations close to Tables 1and 2 respectively. It should be mentioned here that the experience is must in real surveys while making a choice of randomization device to be used in practice.

Further we consider a situation where θ =0 as well as θ_j = 0 for j= 1,2,3,4, and rest of the parameters are kept same as in Tables 1 and 2. The percent relative efficiency of the proposed estimator over Gjestvang and Singh (2009) and Singh (2010) estimators has been shown in Tables 3 and 4 respectively.

Table 3. Percent relative efficiencies of the proposed estimator $\hat{\mu}_{ST}$ over the Singh (2010) estimator $\hat{\mu}_Y$.

σ_y^2	25	125	225	325	425	525	625	725	825
PRE	900.00	256.08	192.04	163.80	148.83	139.54	133.23	128.65	125.18

Table 4. Percent relative efficiencies of the proposed estimator $\hat{\mu}_{ST}$ over the Gjestvang and Singh (2009) estimator $\hat{\mu}_{GS}$.

σ_y^2	25	125	225	325	425	525	625	7 25	825
PRE	1573.07	403.96	269.46	217.48	189.90	172.81	161.18	152.75	146.36

The minimum values of the percent relative efficiencies in Tables 3 and 4 is observed as 125.18 and 146.36 and maximum 900.00 and 1573.07 with a median of 148.83 and 189.90 based on 9 situations investigated in Tables 3 and 4 for different choices of parameters respectively. It is observed from tables 3 and 4 that the percent relative efficiency remains higher if the value of σ_y^2 is small. In order to see as the maximum gain we also investigate lower values of σ_y^2 given that in practice, for example, the number of abortions by a woman could vary from 0 to 3 or 4, because it may not be practical for a woman to go for more than 3 or 4 abortions. In that case the value of σ_y^2 will be around 0.5 to 5.0 [see Singh (2010), p.79]. We observed that the percent relative efficiency value decreases from 13966.67 to 3566.76 (in case of Singh (2010)) and 25633.33 to 6483.34 (in case of Gjestvang and Singh (2009)) as the value of σ_y^2 increases from 0.5 to 5.0 when all the means of the scrambling variables are zero level.

We have given the various choices of parameters for k =2 in Tables 5 and 6 such that the suggested estimator $\hat{\mu}_{ST}$ remains better than the Singh's (2010) estimator $\hat{\mu}_Y$ and Gjestvang and Singh (2009) estimator $\hat{\mu}_{GS}$. Thus, based on our findings, the use of the envisaged estimator $\hat{\mu}_{ST}$ over Singh's (2010) estimator $\hat{\mu}_Y$ and Gjestvang and Singh (2009) estimator $\hat{\mu}_{GS}$ is recommended for all situations close to Tables 1 to 6 in real practice.

Table 5. Percent relative efficiencies of the proposed estimator $\hat{\mu}_{ST}$ over the Singh (2010) estimator $\hat{\mu}_{Y}$ with k =2.

P_1	θ	θ_1	θ_2	σ_Y^2	PRE
				25	1630711.54
				125	336575.40
				225	187692.48
				325	130148.77
				425	99620.89
				525	80700.57
				625	67825.08
				725	58496.56
0.2	1700	1300	-325.0	825	51426.76
				25	235942.31
				125	48765.87
				225	27232.30
				325	18909.51
				425	14494.13
				525	11757.60
				625	9895.37
				725	8546.14
0.4	700	300	-200.0	825	7523.61
				25	1646116.67
				125	339754.23
				225	189464.75
				325	131377.40
				425	100561.11
				525	81462.04
				625	68464.91
				725	59048.26
0.4	1700	800	-533.3	825	51911.66
				25	1388711.54
				125	286638.89
				225	159851.77
				325	110848.16
				425	84850.94
				525	68738.59
				625	57773.96
				725	49829.89
0.8	1700	300	-1200.0	825	43809.32

Table 6. Percent relative efficiencies of the proposed estimator $\hat{\mu}_{ST}$ over the Gjestvang and Singh (2009) estimator $\hat{\mu}_{GS}$ with k =2.

P_1	θ	θ_1	θ_2	σ_Y^2	PRE
				25	2669265.4
				125	550880.16
				225	307172.12
				325	212978.22
				425	163006.81
				525	132035.93
				625	110959.9
				725	95689.945
0.2	1700	1300	-325.0	825	84117.312
				25	453880.77
				125	93737.302
				225	52304.867
				325	36291.104
				425	27795.54
				525	22530.228
				625	18947.125
				725	16351.102
0.4	700	300	-200.0	825	14383.656
				25	2669265.4
				125	550880.16
				225	307172.12
				325	212978.22
				425	163006.81
				525	132035.93
				625	110959.9
				725	95689.945
0.4	1700	800	-533.3	825	84117.312
				25	2669265.4
				125	550880.16
				225	307172.12
				325	212978.22
				425	163006.81
				525	132035.93
				625	110959.9
				725	95689.945
0.8	1700	300	-1200.0	825	84117.312

4. CONCLUSIONS AND RECOMMENDATIONS

This paper illustrates enrichment over the Gjestvang and Singh's (2009) randomized response model and Singh (2010). We have suggested the new additive randomized response model which is to be more efficient both theoretically as well as numerically than the additive randomized response model studied by Gjestvang and Singh (2009) and the additive model due to Singh (2010). Thus the proposed randomized response procedure is therefore recommended for its use in practice as an alternative to Gjestvang and Singh's (2009) and Singh (2010) model.

ACKNOWLEDGEMENTS

The authors are heartily thankful to the learned referees and the Associate - Editor for their fruitful and constructive comments regarding the improvement of the paper.

CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

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