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Research Article

Mathematical Discourse of a Middle School and a Senior Prospective Mathematics Teacher

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Abstract

The purpose of this study was to investigate and analyze mathematical discourse of a senior prospective teacher who is educated about mathematical communication and of an experienced middle school mathematics teacher. Besides, the aim was to shed light on future research studies about the effects of education about mathematical communication and teaching experience on mathematical discourse and discourse analysis. The study was designed as a phenomenological study. In the process of data collection, a teaching scenario was developed by the researchers. Afterwards, clinical interviews and lesson observations were conducted with the participants. The participants of this study were a middle school mathematics teacher who has ten years of experience in teaching and a senior prospective teacher. Data were analyzed qualitatively by using focal analysis technique and data collected from clinical interviews were analyzed by using content analysis. The findings of the study

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revealed that the participants' mathematical discourse and their discourse analysis were quite different from each other.

Keywords: *Discourse analysis; focal analysis; mathematical discourse*

Introduction

Developing mathematical communication skill both orally and in writing has become an important outcome of mathematics learning. For this reason, communicational perspective constitutes an integral part of mathematical research. Mathematical discourse has received considerable attention due to its importance for developing mathematical communication skill that requires students to explain their thoughts clearly and to make sense of other people's thoughts. In other words, communicating mathematically requires becoming an active participant of mathematical discourse (Nakamura, 2009). Participating in mathematical discourse can be conveyed through talking and acting in the ways that mathematically competent people talk and act (Moschkovich, 2003; Sfard, 2008). Therefore, discourse becomes a key component of the development of mathematical communication skill. In mathematics education literature, there are different definitions of the concept of discourse. For instance, Gee and Green (1998) state that discourse offers a way to characterize how to construct and share knowledge. According to Sfard (2001), discourse can be defined as an activity of communication with others or with oneself. Therefore, discourse can be used to denote any specific instance of personal or interpersonal communication in a verbal or nonverbal way. Besides a discourse counts as mathematical if it features mathematical words (Sfard, 2008). According to National Council of Teachers of Mathematics [NCTM] (2014), discourse is defined as participating in classroom discussions and communicating mathematically by using various written or visual representations.

The importance of developing students' mathematical communication skill and making them active participants of mathematical discourse is highlighted by mathematics educators (Thompson, 2007; Schleppegrell, 2010). Lynch and Bolyard (2012) state that engaging in meaningful, structured discourse gives students opportunities to explain and evaluate their thinking. Discourse enables students to build shared understanding of mathematical ideas and enables teachers to assess their students' mathematical thinking (Walshaw & Anthony, 2008). According to Hamm and Perry (2002), becoming participants of mathematical discourse provides students with opportunities to develop their own understandings. Similarly, in learning environments intended to develop mathematical discourse, students can take an active and participatory role in their own learning processes (NCTM, 2014). In addition,

development of mathematical discourse is effective in improving students' communication skills and achievements (Truxaw & DeFranco, 2007).

It is a commonly accepted fact that as long as students engage in mathematical discourse they can develop their ability to use language of mathematics (Nakamura, 2009). Since it includes vocabulary, syntax, semantics and linguistic features of the language of mathematics, mathematical discourse facilitates learners' active use of mathematics language (Kersaint, 2015). Besides, students need the teachers' guidance in learning the language of mathematics and, by this way; knowledge of subject matter develops simultaneously (Pimm, 1987). Therefore, type and quality of teachers' discourse has an important impact on conceptual learning (Chapin, O'Connor & Anderson, 2003). Therefore, teachers' discourse emphasizing mathematical meanings of concepts gains importance. Indeed, Stein (2007) indicates that students need both 'motivational discourse' to encourage participation in mathematical discourse and 'cognitive discourse' to promote conceptual understanding.

Moschkovich (2003) emphasizes the critical role of orchestrating student discourse in conceptual understanding. Kersaint (2015) highlights the need for various teaching strategies to facilitate students' participation in mathematical discourse. According to Adler (1997), teachers need to listen to students carefully and analyze their conceptions in order to turn informal expressions of their mathematical thinking into formal expressions. In other words, teachers should manage students' discourse to guide them toward learning objectives (Funahashi & Hino, 2014) and build a bridge between students' informal discourse and mathematical discourse (Lee, 2006). Hiebert, Morris and Glass (2003) indicate that most teachers did not participate in mathematical discourse during their training process and for this reason; they have difficulty in creating learning environments for developing mathematical discourse. Razfar (2012) points out that language and discourse contents are absent in most mathematics teaching courses for preservice teachers because language is typically treated as a subject matter in teacher education and they are separated from the content subjects. Similarly, Esty (2004) signifies the importance of preservice mathematics teachers' awareness of the language of mathematics, how to teach it and the importance of mathematics teaching courses intended for developing mathematical communication.

From the perspective of conceptualizing learning mathematics as change in discourse (Sfard, 2008), teachers' mathematical discourse comes into prominence. Therefore, there is a need to investigate teachers' mathematical discourse. In literature of mathematics education, most of the studies are about investigating teacher discourse in mathematics classrooms and the effects of mathematical discourse on instructional practices, mathematical understanding and achievement. For instance, Peressini and Knuth (1998) state that mathematics teachers' discourse is primarily univocal. In this respect, scholars indicate the importance of providing experience about creating learning environments intended to improve mathematical discourse. On the other hand, there are also various studies focusing on developing mathematical discourse of mathematics teachers, who participated in professional development programs. Within the domain of the current study, teaching strategies like "asking qualified questions", "using real world scenarios" were developed by the researchers. It was seen that there were few studies conducted with preservice teachers compared to the ones conducted with mathematics teachers. These studies aimed to investigate preservice mathematics teachers' discourse during their teaching practice processes or to develop their ability to orchestrate mathematical discourse (Blanton, Berenson, Norwood, 2001; Mosvold, 2015). For instance, Spangler and Hallman-Thrasher (2014) asked pre-service teachers to write task dialogues between a child and teacher about a problem-solving task in which they practice responding to correct, partially correct and incorrect student responses. Then, the researchers asked them to analyze these task dialogues, to anticipate students' responses, to redirect partially correct or incorrect responses, and to match follow-up questions to the student's thinking. According to the findings, researchers indicate that teacher-training process has an important effect on pre-service mathematics teachers' ability to create learning environments aiming to improve mathematical discourse. Similarly, Wille (2016) used imaginary dialogues to raise pre-service mathematics teachers' awareness of their own mathematical speech. Then, she found that their language use revealed sentences that may cause misconceptions or incorrect sentences. She also indicated that these kind of mathematical discourse may lead to problems for students' mathematics learning.

As a result, most studies related to mathematical discourse analysis were conducted with mathematics teachers or pre-service mathematics teachers. However, studies conducted with both mathematics teachers and pre-service mathematics teachers are missing. In this regard, the purpose of this study is to investigate mathematical discourse and discourse analysis of a

senior prospective teacher who got training about mathematical communication and an experienced middle school mathematics teacher. Besides, the current study aims to shed a light on future research studies about the effects of mathematical communication education and teaching experience on mathematical discourse and discourse analysis. Therefore, it is believed that this study is believed to fill a strategic gap in the existing mathematics education literature. Accordingly, the research questions addressed within the scope of this study are:

1. How do the in-service mathematics teacher and the prospective mathematics teacher analyze mathematical discourse?
2. How is mathematical discourse of the mathematics teacher and the prospective teacher shaped in classroom?
3. What is the relationship between mathematical discourse and discourse analysis of the mathematics teacher and the prospective teacher?

Research Methodology

The purpose of this study is to investigate mathematical discourse and discourse analysis of a senior prospective teacher and an experienced middle school mathematics teacher. In this regard, this study is designed as a phenomenological study. The fundamental goal of phenomenological research is to arrive at a description of the nature of a particular phenomenon (Creswell, 2013). In other words, phenomenological research is searching for the meaning of a true-life experience from the point of view of a person having experienced it (McMillan, 2004). For this purpose, the results can be presented comparatively (Yıldırım & Şimşek, 2011).

Participants

The participants of this study are a middle school mathematics teacher, who has 10 years of experience in teaching and a senior prospective teacher (see Table 1). As can be seen from the Table 1, Algebraic Concepts and Teaching Approaches were courses taken by the prospective mathematics teacher. The course is instructed by one of the researchers as an elective course and includes algebraic concepts in middle school mathematics, algebraic reasoning and development of algebraic language.

Table 1

Demographic Information

Status	Teaching Experience	Course	Graduate Education
In-service mathematics teacher	10 years	-	Master's degree
Prospective mathematics teacher	-	The Language of Mathematics, Algebraic Concepts and Teaching Approaches	-

Participation in the study was completely voluntary and purposeful sampling technique was used. Purposeful sampling involves selecting cases that meet some predetermined criterion of importance (Yıldırım and Şimşek, 2011). In this study, it was aimed to investigate the effects of having education about mathematical communication and teaching experience on participants' mathematical discourse and discourse analysis. In accordance with the purpose of the study, criteria for selecting participants were determined as follows:

- Having training in faculty of education
- Getting training about mathematical communication and to receive high level of achievement (for the prospective mathematics teacher)
- Having teaching experience that can be characterized as long (for mathematics teacher)
- To access participants easily

Data Collection and Analysis

In the process of data collection, a scenario about teaching the concept of variable was developed by the researchers. Accordingly, a 6^{th} grade-learning objective as *"to write algebraic expressions for verbal expressions"* was chosen. The scenario includes possible dialogues related to the learning objective. In this context, teacher discourse was written in a way to include inappropriate/insufficient mathematical sentences. In this way, inappropriate/insufficient teacher discourse was characterized as discourse that requires improvement in terms of supporting teaching the concept of variable. In addition, a directive

was prepared by the researchers to facilitate participants' analysis of inappropriate/insufficient teacher discourse. Besides, the directive provided an example of how to do discourse analysis (see Table 2). Then, the participants analyzed the scenario within mathematical language use in mathematical discourse. The steps of discourse analysis were defined as i) to determine what the inappropriate/inadequate discourse is, ii) to determine what the aim of discourse is, iii) to determine what the reason of inappropriateness/inadequateness is, iv) to edit and rewrite discourse by using mathematical language, so that it is grammatically correct. Within the domain of the aforementioned steps, it is aimed to investigate how participants analyze the scenario within mathematical language use in mathematical discourse.

Table 2

An Example of Discourse Analysis

Teacher's discourse	Aim of discourse	The reason of inappropriateness /inadequateness	Revised discourse
Let's count from Ali's house toward the bank. 10 meters, 20 meters, 30 meters, 40 meters and 50 meters.	The teacher aims to express the distance between Ali's house and the bank in terms of unit of measurement.	The unit of measurement is meter. Therefore, the expression of 'count' is inappropriate.	Let's look at the distance between Ali's house and the bank. According to the model in the problem, there are five buildings between Ali's house and the bank. In addition, the distance between the buildings is 10 meters. Therefore, the distance between Ali's house and the bank is 50 meters.

Afterwards, clinical interviews were conducted with the participants. Clinical interviews are used to collect and analyze data on mental processes of the participants and to expose hidden structures and processes in the subject's thinking (Clement, 2000). During the interviews, firstly, the participants' mathematical content knowledge was questioned. Secondly, their mathematical discourse analysis was questioned in depth by addressing questions like 'how' and 'why'. In this respect, it was aimed not only to shed a light on the participants' discourse analysis but also to provide data for triangulation. An interview form was developed and sub-questions were shaped according to the participants' answers during the interview process.

Then, a mathematics educator was asked whether the questions were in accordance with research problems or not. Accordingly, the questions were reviewed and put into final form (see Table 3).

Table 3

Content of Interview Form and Sample Questions

Part	Content of Part	Sample Questions
1	Questions about mathematical content knowledge	Can you explain mathematical meaning of the concept of variable? Which teacher discourse should be emphasized when teaching the concept of variable?
2	Questions about discourse analysis	Why do you think that this discourse is inappropriate? What is the reason of rewriting the discourse in a way like that?

Clinical interviews were nearly 45 minutes long and audiotaped by a voice recorder. Lastly, lesson observations were conducted to investigate participants' mathematical discourse in the classroom. In this regard, prospective mathematics teacher was observed in an 8^{th} grade classroom within the Teaching Practice Course. The mathematics teacher was observed in a 5^{th} grade classroom because of teaching only 5^{th} graders. These lessons were videotaped by a camera.

Data Analysis

Participants' discourse analysis was analyzed to find all inappropriate/insufficient discourse, to explain the reason of inappropriateness/inadequateness correctly and to rewrite discourse in order to make it grammatically correct. In addition, data obtained from clinical interviews were also taken into consideration. Participants' discourse in classroom was analyzed based on the data obtained from lesson observations. For this purpose, data related to discourse analysis and classroom discourse was analyzed qualitatively by using focal analysis technique (Sfard, 2001) and data collected from clinical interviews was analyzed by content

analysis. The main goal of content analysis is to arrive at concepts and relations that can explain data clearly (Yıldırım & Şimşek, 2011).

Theoretical framework for discourse analysis

Sfard (2008) developed communicational approach to cognition based on the view that there is a strong relationship between mathematical communication and mathematical thinking. In this regard, she considers learning as moving towards a more sophisticated mathematical discourse through participation (Sfard, 2008). In other words, learning mathematics can be interpreted as a process and change in students' engagement in mathematical discourse (Güçler, 2016). From this point of view, the purpose of teaching mathematics is talking and acting in the ways that mathematically competent people talk and act. Talking and acting in the ways that mathematically competent people talk and act requires learners' effective communication with each other's and with their teachers. In this regard, communication will not be regarded as effective unless all the participants feel confident that all parties involved refer to the same things when using the same words. According to Sfard (2000), effectiveness of communication is dependent on the degree of clarity of discursive focus. That is, the consistence between participants' word use is irreplaceable for effectiveness of communication. Sfard (2001) uses two types of analyses named 'focal analysis' and 'pre-occupational analysis' to investigate the effectiveness of communication between participants of mathematical discourse.

Focal analysis gives information about students' communication with each other or a teacher's communication with his/her students. The effectiveness of communication is determined by the degree of clarity of discursive focus presented in the communication. Based on this assumption, three components of discursive focus as i) pronounced focus, ii) attended focus and iii) intended focus were distinguished. The pronounced focus is the component of "the word used by an interlocutor to identify the object of her attention" (p.304). The attended focus is the component of "what and how we are attending when speaking" (p. 304). The intended focus is the component of "interlocutor's interpretation of the pronounced and attended foci" (p. 304). More than one pronounced and attended foci are important discursive clues for presence of the intended focus. In some cases, although the participants' pronounced and attended foci are different, their intended foci may be the same.

For this reason, effectiveness of communication primarily depends on the consistency between participants' intended foci.

Analysis of participants' discourse within theoretical framework

Participants' mathematical discourse they produced during their analyzing process was analyzed by using focal analysis technique. In this regard, discourse written in "editing and rewriting the discourse" column in analysis form was determined as "pronounced focus". Columns of "intention of the discourse" and "the reason of inappropriateness" were determined as "intended focus" (see Table 4).

Table 4

An Example of Focal Analysis

Inappropriate /Inadequate discourse	Intension	Reason of inappropriateness	Revised discourse
The starting point for Emre's money is the number of the week.	Number of the week changes all the time. It means we can find money by looking number of the week.	The expression of "starting point"	Amount of money changes depending on number of the week. Therefore, algebraic expression is written depending on number of the week.

Intended focus

Pronounced focus

Focal analysis didn't produce data about the attended foci. Because participants' mathematical discourse analysis was investigated by using their analysis forms. Indeed, Sfard (2000) states that attended foci are determined according to what and how participants are attending the speech. As it depends on the researcher's interpretation, according to Sfard (2000), intended focus of a mathematical discourse can be changeable. For this reason, the most important thing for the researcher is to provide a convincing interpretation of the observed phenomena. Because of not gathering data about attended foci of participants' mathematical discourse and the results' dependency to researcher's interpretations, clinical

interviews were conducted to strengthen the trustworthiness of the results. On the other hand, participants' discourse in the classroom was recorded by a video camera. Therefore, all three ingredients of focal analysis can be investigated.

Findings

Findings Related to the Participants' Discourse Analysis

Although it seems like there is no difference between the participants about determining the inappropriate/inadequate mathematical discourse, there is a significant difference about determining the intension of editing and rewriting discourse. It was seen that during discourse analysis process, the mathematics teacher ignored mathematical meaning of variable. However, the prospective mathematics teacher made sentences constructing the concept and its mathematical meaning. Participants' explanations about intention of a discourse and their revision are presented as follows.

Table 5

An Example Discourse Analysis Related to an Inappropriate Mathematical Discourse

Participant	Inappropriate Discourse	Intention	Revised Discourse
Mathematics teacher		Let's think some numerical values as Sevgi's weight and then write them on the table.	Let's write some numbers for Sevgi's weight and create a table.
Prospective mathematics teacher	Ok then, let's create a table with some values.	To try constructing the concept of variable and state that variables can take any of weight values	Let's create a table by taking into consideration the relationship between Pınar's and Sevgi's weight.

Additionally, when participants' discourse analysis is investigated in detail, it is seen that if the source of inappropriateness/inadequateness is quantity of weight and using kilogram as a unit of weight, the mathematics teacher could determine inappropriate discourse. However, similar inappropriate discourse was seen when explaining the intention or revising and rewriting the discourse. Although the mathematics teacher used the word "weight" in some inappropriate discourse, she did not use this word in the revised version. Besides, she preferred using the word "kilo" in her explanations related the intension of discourse.

Table 6

An Inappropriate Discourse and Teacher's Analysis

Inappropriate/Inadequate discourse	Intension	Reason of inappropriateness	Revised discourse
For instance, what if Sevgi is 50 kilos, how many is Pınar?	It is questioned how many Pınar's kilos are.	If there is a unit, we use unit.	How many is Pınar's kilos?

When the revised discourse is considered within the concept of variable, it is seen that mathematics teacher did not use any expressions that reflect "varying quantity" meaning of variable. She just emphasized the 'unknown' meaning of variable (see Table 7).

Table 7

A Revised Discourse Focused On Unknown Meaning of Variable

Inappropriate/Inadequate discourse	Intension	Reason of inappropriateness	Revised discourse
We can change Sevgi's weight arbitrarily. So if we represent Sevgi's weight with 's' then Pınar's weight should be 's+5'.	To represent variable with the letter 's'	Students don't know what 's' is	We don't know Sevgi's weight and we represent unknown ones with letters. Let's use the letter 's' for Sevgi's weight.

Indeed, when her mathematical knowledge of variable was questioned in interview process, she tried to explain the concept of variable as "letter in the algebraic expression",

"expressions that change according to some things" and "unknown expression in a problem situation". In this way she showed evidence of her cognitive conflict. An excerpt from interview related to her cognitive conflict is as follows:

> ***Mathematics teacher****: "I conceptualize the concept of variable as letters in an algebraic expression. Changeable situation in the problem is our variable."*

When the participant was questioned about teaching the concept of variable, she stated that focus concepts of learning objectives should take part in teacher's discourse clearly. She also stated that "variable" and "unknown" should be frequently emphasized. The teacher's view reflects her belief that constructing concepts requires repetition of these terms. Her point of view is as follows:

> ***Mathematics teacher:*** "*As long as we use new terms, mathematical concepts will develop more deeply. If we don't use new terms, learners don't know how to use them."*

When discourse analysis and explanations during interviews are considered, it was seen that prospective teacher's discourse was as it should be. She explained the intensions of inappropriate discourse by taking into consideration the construction process of variable and its mathematical meaning. Besides, she was able to make well-constructed and meaningful sentences to revise mathematical discourse and used the language of mathematics effectively.

Table 8

An Example of Prospective Mathematics Teacher's Discourse Analysis

Inappropriate/Inadequate discourse	Intention	Reason of inappropriateness	Revised discourse
Ok, then the starting point for decreasing value is Pınar's weight. So if we represent Pınar's weight with 'p', Sevgi's weight is 'p-5'.	To reach a mathematical expression by considering the expression that changes in its domain.	The expression of 'starting point' is inadequate.	The expression that changes in its domain is Pınar's weight. So, if we represent Pınar's weight with letter 'p' then Sevgi's weight is 'p-5'.

On the other hand, the prospective teacher's mathematical discourse was supportive to develop the quantitative reasoning within the problem situation in the scenario (see Table 9).

Table 9

An Example of Revised Discourse That Supports Quantitative Reasoning

Inappropriate/Inadequate discourse	Intension	Reason of inappropriateness	Revised discourse
Let's think some values arbitrarily and show in a table.	Thinking arbitrary values to get students understand the relationship between Emre's money and number of the week.	The expression of "think some values arbitrarily"	Let's create a table by considering the values that number of the week can take.

Prospective mathematics teacher used conjunctions like "depending on" and "according to" properly. In this way, she tried to reflect "varying quantity" meaning of variable in revised version of inappropriate discourse. In addition, supporting development of concept of variable with her mathematical discourse, prospective teacher's mathematical expressions like "account of money", "number of week" and "Pınar's weight" were supportive for quantitative reasoning.

Prospective mathematics teacher supported these teaching approaches with expressions related to variable and teaching of it during the interview process. The participant described the meaning of variable as "expressions that vary in the set we are working on and used for the unknown in a problem". Therefore, she emphasized the set we are working on and taking different values from this set. Although this definition doesn't reflect the formal description of the concept, she often emphasized the importance of relationship between quantities and different values. In this regard, her explanations such as "it takes varying values", "the important thing is the relationship between quantities" draw attention. Her views about teaching variable are as follows:

> ***Prospective teacher**: "... I thought how to concentrate on the concept of variable. That is, I tried to construct this concept."*

Findings Related to Participants' Mathematical Discourse in the Classroom

The findings of the current study revealed that the participants' mathematical discourse in classroom were quite different from each other. The in-service mathematics teacher ignored the mathematical discourse that should be emphasized to gain learning objectives. However, the prospective mathematics teacher focused on discourse emphasizing mathematical meaning.

The mathematics teacher was observed in a lesson in which the 5th grade-learning objectives are: "to know units of length measurement, to converse units of measurement (meter-kilometer, meter-centimeter-millimeter) and to solve problems". In this regard, it is necessary that a teacher's mathematical discourse should focus on the quantity of length and measurement of it. However, it was seen that the teacher's discourse included some inappropriate/inadequate usage of quantity of length and measurement of it. For instance, she asked students to measure a specific distance by foot and then to compare their measurement results. In this way, she aimed at getting learners to understand the necessity of units of length measurement. This activity requires a teaching process supporting quantitative reasoning. In spite of this, the teacher's discourse requiring emphasis on quantity of length included expressions that may cause misconceptions. An example of this kind of discourse is as follows:

Table 10

An Example of Inappropriate Teacher Discourse

Teacher discourse	Intension	An example of discourse that reflect intension
Let's measure through the door. What is the length of the door from here?	The length of distance between the door and location of the teacher.	What is the length of the distance between the door and location of the teacher?

After making sense of the necessity of standard measurement of length, the teacher started using length conversation table to convert units of length measurement. In this way, she missed the part of learning objective as "to know units of length measurement". While using length conversation table, the teacher frequently used expressions like "coming down",

"moving up" and "coming/going". However, these expressions make conceptual learning difficult and don't emphasize concepts of quantity and unit of measurement. Afterwards, the teacher wrote some questions near the conversation table on the board in order to make a conversation between units of measurement. She solved them by using the conversation table directly. An example of prospective teacher discourse related to solving a question is as follows:

Table 11

An Example of Mathematical Discourse That Doesn't Support Conceptual Learning

Teacher discourse	Intension	An example of discourse that reflect intension
I'm coming down from kilometer to meter. We said that one zero should be added when coming down.	To converse kilometer to meter	Here, to write the length in terms of meter, we multiply it by 1000. We said that to multiply by 1000, we should add three zeros.

On the other hand, it was seen that the teacher's inappropriate/inadequate mathematical discourse affected the students' discourse. As can be seen from the excerpt below, types of students' discourse related to mathematical concepts were quite similar to the teacher's discourse.

***Teacher**: How many meters are there in 134 kilometers?*

***Student**: Coming from kilometer to meter... Two zeros.*

***Teacher**: Was that two zeros in coming from kilometer to meter?*

***Student**: Three zeros*

***Teacher**: That's right. Because, I am coming down three times. Hectometer, decameter and meter.*

The teacher's another behavior that doesn't support students' conceptual learning is the lack of feedback to their inappropriate/inadequate discourse.

***Student**: Measuring length of a line*

***Teacher**: Ok*

***Student**: Both of you are right, because, Melek's foot is shorter than yours.*

***Teacher**: Ok then, what will I say if the manager asks me how long the distance of the door and the table is?*

The prospective mathematics teacher was observed in a lesson in which the 8th grade-learning objective was the aim: *"to determine and construct basic elements of a sphere"*. She aimed at getting students to understand basic elements of a sphere by relating them to the model of earth. In this process, she supposed that shape of earth is a sphere. She also paid attention to suitability of her own discourse with mathematical terminology and she could use mathematical terms correctly. In addition, her discourse emphasized the concepts of "surface", "center", "big circle" and "radius", which are focal components of the learning objective.

Table 12

An Example of Discourse Reflecting Its Intention

Prospective teacher's discourse	Intention
We'll call surface of sphere for earth's surface; center for core; big circle for equator and radius for distance between core and equator.	To determine basic elements of a sphere

After introducing basic elements of a sphere by using model of earth, the prospective teacher aimed at introducing dependency relationship of these elements. Therefore, she used two different sized balls and questioned similar and different features of the balls. During this process, she tried to guide students to concentrate on the fact that the reason of difference between sizes of the balls is having different radius length. Besides, as can be seen from the excerpt below, her discourse was supportive for conceptual learning.

***Teacher:** Are these balls the same size?*
***Student:** No.*

***Teacher:** They are not the same size. What is the reason of having different sizes?*

***Student:** The distance between core and equator is different.*

***Teacher:** Yes, that's right. What did we call for the distance between core and equator, which is center and big circle?*

***Student:** Radius*

***Teacher:** Ok then, the reason of having different sizes is having different length of radius.*

When students' discourse is considered, it was seen that the prospective teacher's discourse and her students' discourse were parallel to each other. Mathematics teacher's and her students' discourse were parallel too. However, prospective teacher's discourse supports conceptual learning and was appropriate to mathematical terminology. Therefore, her explanations lead to the intended student discourse like "the distance between core and equator" and "length of radius".

While the teacher gave no feedback to her students, the prospective mathematics teacher improved her students' inappropriate/inadequate mathematical discourse. It was seen that prospective teacher improved students' discourse so that it emphasizes the mathematical meaning.

> ***Teacher****: Now, I'll give all of you identical spheres. What I mean by identical sphere?*
>
> ***Student****: All the same.*
>
> ***Teacher****: Which property is the same?*

Discussion and Conclusion

The findings of the present study revealed that the participants' mathematical discourse and their discourse analysis were quite different from each other. That is, while the middle school mathematics teacher undervalued the mathematical meaning both in her discourse analysis and classroom discourse, the prospective teacher focused on discourse emphasizing mathematical meaning. This result shows that prospective mathematics teacher's discourse emphasizing mathematical meaning and support constructing the concept can be characterized as cognitive discourse (Stein, 2007), however, mathematics teacher's discourse can not be regarded in this way. However, it is frequently emphasized that types of discourse emphasizing mathematical meaning of the concepts have an important role for conceptual understanding (Trocki, Taylor, Starling, Sztajn & Heck, 2014). Nathan and Knuth (2003) stated that teacher discourse have an important role in conceptual understanding and promoting learners' mathematical thinking.

When participants' discourse analysis, treatment of the variable concept in revised discourse and constructing the concept is considered, it was seen that mathematics teacher emphasized only 'unknown' meaning of variable. In spite of this fact, prospective teacher's discourse was supportive for quantitative reasoning related to teaching the concept of variable. This result shows that mathematics teacher restricted teaching variable with "unknown" meaning of it. However, prospective teacher focused on the relationship between quantities and constructing "varying quantity" meaning of the concept. It is clear that thinking variable just as an unknown lead to some misconceptions as the concept has multiple meanings and fundamental for high-level mathematical concepts. In mathematics education literature, it was mostly seen that after solving an equation and finding the value of an unknown (x), students don't think 'x' as an unknown matter anymore. Similarly, they don't think the quantities, which they know their values (e.g. body height, body weight) as an unknown. Aforementioned misconceptions stem from thinking variable and unknown as the same (Mac Gregor & Stacey, 1997).

Another result of this study is that, participants' mathematical discourse in classroom was quite different from each other. It was seen that during discourse analysis process, the mathematics teacher undervalued mathematical meaning of variable. However, the prospective mathematics teacher made sentences constructing the concept and its mathematical meaning. In this regard, participants' discourse analysis and their discourse in classroom were parallel. Additionally, it was seen that students' discourse is affected by the participants' discourse. However, mathematics teacher's discourse leads to inappropriate/inadequate student discourse but prospective teacher's discourse enable students to make sentences appropriate to the mathematical terminology. This result conforms the findings of Gillies (2004) in that teachers who get educated about mathematical communication affect quality of student discourse positively.

Finally, the mathematics teacher provided no feedback to inaccuracies in students' discourse but prospective teacher reconstructed students' inappropriate discourse. Walshaw and Anthony (2008) indicated that supporting the development of students' discourse requires teacher feedback encouraging students to think deeply. In this regard, the emphasis on the effects of teacher feedback on students' achievement, interest and motivation (Hattie and Timperley, 2007) makes this result more crucial. On the other hand, giving productive

feedback depends on interpreting intensions of students' discourse. Therefore, it was seen that prospective teacher listen to her students carefully and analyzed their thinking in depth. Besides, the necessity and importance of listening students' thinking carefully is emphasized in literature of mathematics education. Martino and Maher (1999) stated that teachers' ability of listening learners is as important as questioning their mathematical thinking. Similarly, NCTM (1991) points out that, learning environments supportive for development of discourse require listening students, analyze their thinking and observe their communication in classroom.

To sum up, the participants' discourse and their discourse analysis was quite different from each other. The results of this study made researchers believe that teaching experience can be ineffective for developing mathematical communication and training about this topic is a fundamental necessity. It is known that mathematics teacher hasn't been provided courses about mathematical communication but prospective mathematics teacher has. Indeed, Gillies and Khan (2008) conducted a survey with two mathematics teachers. One of the teachers is educated about development of mathematical communication skill and the other is not. In this regard, student discourse in well-educated teacher's class is more coherent with teacher discourse. Besides, the students' problem solving and mathematical reasoning is improved. Similarly, Esty and Teppo (1994) indicated that graduate or undergraduate courses about the language of mathematics and mathematical communication have positive effects on teaching language of mathematics both orally and in writing. These emphases related having education strengthened our belief that prospective mathematics teacher's education affected her mathematical discourse and discourse analysis. Therefore, the need for in-service training about mathematical communication for teachers, who have not any training during undergraduate level, arises clearly. Development of teachers' mathematical communication skill, mathematical discourse and discourse analysis can be supported by in-service training seminars organized by Ministry of Education.

When it is considered that characteristics of participants may be effective on the results, it is suggested that this study should be conducted with more participants. Additionally, lack of studies about effects of teaching experience on mathematical discourse makes researchers believe that results of upcoming studies conducted with more participants will contribute to the literature of mathematics education.

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