Özkan and Ulucan, International Advanced Researches and Engineering Journal 06(02): 132-141, 2022

Available online at www.dergipark.org.tr/en

INTERNATIONAL ADVANCED RESEARCHES
and
ENGINEERING JOURNAL

Journal homepage: www.dergipark.org.tr/en/pub/iarej

e-ISSN: 2618-575X

International Open Access

Volume 06
Issue 02

August, 2022

Research Article

A Mixed-Integer Linear Programming approach for university timetabling problem with the multi-section courses: an application for Hacettepe University Department of Business Administration

Akın Özkan [a,*] ***and Aydın Ulucan*** [b]

[a]*Mus Alparslan University, Faculty of Economics and Administrative Sciences Department of Business Administration, Mus, Turkey*
[b]*Hacettepe University, Faculty of Economics and Administrative Sciences Department of Business Administration, Ankara, Turkey*

ARTICLE INFO

Article history:
Received 18 May 2022
Accepted 04 August 2022
Published 15 August 2022

Keywords:
Integer programming
Multi-section courses
Timetabling
University course timetabling

ABSTRACT

University course timetabling is an NP-Complete problem type which becomes even more difficult due to the specific requirements of each university. In this study, it was aimed to solve a university course timetabling problem by using integer programming and to develop assignment models that can be easily adapted to similar problems. The models that we developed for the solution are based on the integer programming model of Daskalaki et al. [1]. In addition, the models were developed taking into account the fact that there was an availability of multi-section courses, the minimum overlap of elective courses, and the ability to divide courses into sessions in terms of effective use of the capacity. In this framework, two different models (model 1 and model 2) were developed. Whereas model 1 assumes that all courses are processed as a single session (If a course has 3 time periods per week, then it is taught as a single session), model 2 assumes courses can be assigned by divided into multiple sessions (If a course has 3 time periods per week, then it can be divided into 1+1+1 or 2+1 sessions.). In model 2, a structure in which the model itself could determine how to split the courses in the framework of predetermined options was developed. Both models were formulated in such a way as to maximize the satisfaction of the lecturers. Finally, a larger scale problem was derived from the first problem and the performance of these two models were compared for both problems. The results showed that the optimal solution was obtained within the specified constraints, and the solution time significantly increased with an increase in the size of the problem.

1. Introduction

Many timetabling problems such as nurse rostering, sports timetabling, transportation planning, university course schedules planning and university exam schedules planning have been the subject of operations research for more than 40 years [2]. The timetabling problem which is the most difficult to solve belongs to the problem class called NP-Complete. It is difficult to solve such problems optimally and efficiently. The course scheduling problem also falls under the class of NP-Complete problem due to the rapid increase in the number of decision variables and the rapid growth of the solution space [3-5]. What makes it difficult to solve the problem is that the solution time dramatically increases as the problem size grows [1]. The increase in solution time also depends on the structure of the course timetabling problem because this problem varies depending on each institution that has its own specific requirements. The fact that courses can be mandatory, elective, or in multi-section status can be shown as an example of these requirements. To accomplish most of the wishes and requirements of the instructors and students is also an important and difficult task. In the literature, there are many studies in which different methods have been used and different models have been developed for the timetabling problem by taking into account these and similar reasons.

Educational timetabling problems are usually divided into two main categories: high school timetabling problems and university timetabling problems. These problems in themselves are also divided into two categories: course timetabling and exam timetabling [6, 7]. The university examination timetabling problem can be defined as assigning courses to a specific number of

* Corresponding author. Tel.: *+90-436-249-4949.*
E-mail addresses: a.ozkan@alparslan.edu.tr *(A. Özkan)*, aulucan@hacettepe.edu.tr *(A. Ulucan)*
ORCID: 0000-0003-2862-2496 *(A. Özkan)*, 0000-0002-0439-2211 *(A. Ulucan)*
DOI: 10.35860/iarej.1118213

classrooms suitable for the requirement of each course and usually to specific time periods for five days a week or for ten days two weeks. The focus of this study is university course timetabling problems. The university course timetabling problem can be defined as assigning a course to specific time periods only for five days a week and to specific classrooms suitable for student capacity and the requirement of the course. In both problem types mentioned above, for those called “hard constraints”, there should never be an overlap; and for the remaining constraints called “soft constraints”, the aim should be to minimize the number of overlaps. It is seen that the constraints are divided into two groups as hard and soft constraints in many of the timetabling studies [6-10]. Hard constraints must be fulfilled fully so that the generated solution would be a feasible solution and there would be no overlap. The possibility of assigning a lecturer or student to maximum one course in the same time slot can be shown as an example of such constraints. Soft constraints are not indispensable but desirable constraints for increasing the quality of a timetabling. In practice, it is impossible to fully satisfy all soft constraints. For this reason, in most timetabling studies, these constraints are included in the objective function of mathematical programming models. For example, in this paper, the level of satisfaction of lecturers is optimized in the objective function.

Since the university course timetabling problem differs depending on the structure of each institution, there is no specific problem structure. When looking at past studies, it seems that different models and different solution methods have been used to solve this problem both because of unspecific problem structure and because of NP-Complete problem structure. As Babaei et al. [9] and Feizi-Derakhshi et al. [11] mentioned in their studies, these methods and approaches which solved the timetabling problem can be classified under four categories. The first category includes techniques based on Operations Research. Graph Coloring [3, 12, 13], Linear Programming (LP)/ Integer Linear Programming (IP) [1, 14-18], Goal Programming [19], and Constraint Satisfaction Programming [4, 20, 21] can be given as examples of these techniques. The second category includes approaches based on the metaheuristic strategies. As examples of these, many heuristic approaches including primarily the tabu search [22-24] and genetic algorithm [25, 26] methods can be shown. The third category includes methods based on Multi-criteria and multi-purpose approaches. The final category of these approaches includes Modern Intelligence Novel Methods. Fuzzy theory [27, 28], hybrid [29-31] and artificial intelligence-based approaches can be shown as examples of this category.

In our study, primarily, it was aimed to solve a university course timetabling problem by using integer programming and to create assignment models that can easily be adapted to many universities that have similar structures. In this framework, two models were developed. These models that we developed for the solution are based on the integer programming model of Daskalaki et al. [1]. In addition, the models were developed taking into account the availability of multi-section courses, the minimum overlap of elective courses, and the ability to split courses in terms of effective use of the capacity. Model 1 solves the timetabling problem by meeting all the requirements of the course program of Hacettepe University Business Administration (HUBA). Since all courses in HUBA are given as a single session in the morning or afternoon, it is assumed in model 1 that all courses are presented as only a single session. Unlike Model 1, Model 2 contains the assumption that course hours can be assigned by dividing instead of the single session.

The contributions of this study are: (1) The fact that the courses can become mandatory, elective, and in multi-section status was taken into account in the both models; (2) In the Model 2, a structure in which the model itself can determine how to split the courses in the framework of predetermined options (e.g., assigning a 3-hour course as 2 + 1 or as a single session) was developed. Also, in this paper, a larger scale problem (problem 2) was derived from the HUBA problem (problem 1) and then the performances of both models were compared. This last analysis is important in that it shows how dramatically the solution time for the NP-Complete problem class changes. Both models have an objective function. These objective functions maximize the satisfaction of the teaching staff and minimize the overlap of elective courses.

2. Structure of University Course Timetabling Problem Section

University course problems differ depending on the structure of each institution, its possibilities, and its own specific requirements. For this reason, although the basic structure of the models developed to solve the university course problem is generally similar, the specific requirements render the models more complex and different.

The structure of the university course timetabling mainly consists of day, time period, student group, lecturer, course, and classroom dimensions. We can explain these dimensions through the example of HUBA modeled in our study as follows:

The time period refers to a one-hour interval, and there are eight time periods per day between 09:00 am and 17.45 pm, from Monday to Friday.

The student groups are the student cluster taking common courses. Therefore, in HUBA, which has a four-year training program, every training year refers to a student group. The courses in the curriculum are taught by lecturers who are experts in their field. In HUBA, which course will be taught by which lecturer is determined in advance by the decisions of the board of directors.

The courses in the curriculum are divided into mandatory and elective. Mandatory courses are courses that students must take in order to graduate from a program. In terms of elective courses, students have the option of taking or not taking these courses according to their interests or claim. However, they have to take a certain number of these courses to complete the total amount of credit required for graduation. In HUBA, all courses in the first two years are in the mandatory course category, and most of the courses in the last two years are in the elective course category. Because of the lack of staff and physical space in the HUBA, the course timetabling problem should be resolved in such a way that the overlap of the elective courses given to student groups is minimized. In addition, due to the same reasons, some mandatory courses are divided into sections. For this reason, the course timetabling problem should be modeled in such a way that the students can take any section of the mandatory course according to their own program.

All courses in HUBA are processed as a single session in a way that they will be three hours per week. The courses that require special equipment are taught in the laboratory, and the others are taught in normal classrooms. Some courses outside the department are taught by the faculty. For this reason, the department's courses should not be assigned to the classrooms and time periods in which these courses are held.

3. Modeling the University Course Timetabling Problem

Two different models are developed for solving the course timetabling problem. While Model 1 completely meets HUBA's requirements, Model 2 meets the need for programs that the courses can be divided into parts according to the time period. The notation used in these models is mostly similar to the notation used in the study of Daskalaki et al. [1]. The sets, decision variables, and parameters used in both models respectively defined below:

3.1 Sets

- The set of days that are appropriate for assigning in a week is indicated by the letter I, e.g.I= $\{1,2,3,4,5\}$.
- The set of the time periods that are appropriate for assigning in a day is indicated by the letter J. The time period usually refers to a time of one-hour, which is received 45-minutes lesson and 15-minutes break. For this reason, in Model 2, the set of the time period in a day that starts at 09:00 and ends at 17.45 is as follows: $J = \{1,2,3,4,5,6,7,8\}$. In Model 1, on the other hand, as all courses in HUBA are given as a single session in the morning or after lunch, the set of the time period (a time period is three hours) in a day is as follows: $J = \{1,2\}$.
- The set of the student groups is indicated by the letter$K$, *e.g.* $K = \{1,2,3,4\}$.
- The set of the lecturers is indicated by letter L, *e.g.* $L = \{Lec\#1, lec\#2, \dots, Lec\#L\}$.
- The set of the courses is indicated by letter M, *e.g.* $M = \{course\#1, course\#2, \dots, course\#M\}$.
- The set of the classrooms is indicated by the letter N, *e.g.* $N = \{class\#1, class\#2, \dots, class\#N\}$.

3.2 Decision Variables

In this study, two decision variables were included in both models. The first (*i.e.*, basic) decision variable which is adopted as the set of binary variables is denoted as follows:

$$x_{i,j,k,l,m,n} = \begin{cases} 1, & when\ course\ m\ taught\ by\ lecturer\ l\ to\ the\ student\ group\ k\ is\ assigned\ for\ the\ jth\ time\ period\ of\ day\ i\ in\ classroom\ n.\ where\ i \in I,\ j \in J,\ k \in K, l \in L,\ m \in M,\ n \in N \\ 0, & otherwise \end{cases}$$

The second (*i.e.*, auxiliary) decision variable which is adopted as the set of integer variables is denoted as follows:

$$y_{i,j,k} = \begin{cases} Z^{+}, & when\ the\ number\ of\ conflicts\ in\ the\ jth\ time\ period\ of day\ i\ of\ student\ group\ k\ is.\ where\ i \in I,\ j \in J,\ k \in K \\ 0, & when\ there\ is\ no\ conflict. \end{cases}$$

3.3 Parameters

The parameter sets are defined in the form of subsets according to the basic indices I, J, K, L, M, and N. Thus, the number of variables is further reduced and modeling of constraints becomes much easier. These parameters are the followings:

K^1={$k \in K$: $k =$ the student groups of lower-grade years, usually from the first two years in a four-year department.}.

K^2 ={ $k \in K$: $k =$ the student groups of higher-grade years, usually from the last two years in a four-year department, and there are also elective courses in this period.}.

K_l ={ $k \in K$, the set of the student group taught by lecturer l. }.

L_i = { $l \in L$, the set of the lecturer available on day i. }.

L_k = { $l \in L$, the set of the lecturer who teaches the student group k. }.

$L_{ki} = L_k \cap L_i$

M_k = { $m \in M$, the set of the course designed for the student group k.}.

M_k^C = { the set of mandatory course designed for the student group k, $k \in K^2$. }.

M_n = { $m \in M$, the set of the course taught in the classroom n.}.

$M_{kl} = M_k \cap M_l$, $M_{kln} = M_k \cap M_l \cap M_n$

M_k^{Sec} = { $m \in M$, the set of the multi-section course designed for the student group k. }.

M_{ks}^{Sec} = { $m \in M$, $(k,s) \in KxS$, k the set of the s^{th} section of the multi-section course designed for the student group k. The set of S shows the total number of sections in one multi-section course, $s \in \{1,2 \dots, Sec\#S\}$.}

N_{mk}= { $n \in N$, the set of the classroom that fits student group k for the course m }.

I_n = { $i \in I$, the set of the day on which classroom n is suitable for use}.

I_l = { $i \in I$, the set of the day on which lecturer l is suitable for lecture}.

$I_{ln} = I_l \cap I_n$

J_{in} = { $j \in J$, the set of the time period of the day i on which classroom n is suitable for use.}.

J_{iln} = { $j \in J$, the set of the time period of the day i on which lecturer l and classroom n are suitable for assigning.}.

PRA = { $(i,j,k,l,m,n) \in IxJxKxLxMxN$, PRA, course m taught by lecturer l to student group k is pre-assigned for j^{th} time period of day i in classroom n, that is, $x_{i,j,k,l,m,n} = 1$.}

$C_{i,j,m}$ = { cost coefficient of course m for j^{th} time period of day i, $i \in I$, $j \in J$, $m \in M$ }

a_k^1 = total number of courses for student group k (only Model 1).

a_k^2 = total number of time periods for student group k (only Model 2).

b_m = total length of course m (only Model 2)

hm = total number of time periods for course m (only Model 2)

3.3 Model 1 (with single session)

Model 1 is modelled to meet all the requirements of course timetabling of HUBA. The constraints of this model are as follows:

$$\forall i \in I, \quad \forall j \in J, \quad \forall l \in L_i \qquad (1)$$

$$\sum_{k \in K_l} \sum_{m \in M_{kl}} \sum_{n \in N_{mk}} x_{i,j,k,l,m,n} \le 1,$$

$$\forall k \in K^1, \quad \forall i \in I, \quad \forall j \in J, \quad \forall s \in S$$

$$\sum_{l \in L_{ki}} \sum_{m \in M_{kl} - M_k^{Sec}} \sum_{n \in N_{mk}} x_{i,j,k,l,m,n} + \sum_{l \in L_{ki}} \sum_{m \in M_{ks}^{Sec}} \sum_{n \in N_{mk}} x_{i,j,k,l,m,n} \le 1 \qquad (2)$$

$$\forall k \in K^2, \quad \forall i \in I, \quad \forall j \in J, \quad \forall s \in S$$

$$\sum_{l \in L_{ki}} \sum_{m \in M_{kl} - M_k^{Sec}} \sum_{n \in N_{mk}} x_{i,j,k,l,m,n} + \sum_{l \in L_{ki}} \sum_{m \in M_{ks}^{Sec}} \sum_{n \in N_{mk}} x_{i,j,k,l,m,n} \le 1 + y_{i,j,k} \qquad (3)$$

$$\forall k \in K^2, \quad \forall i \in I, \quad \forall j \in J, \quad \forall s \in S$$

$$\sum_{l \in L_{ki}} \sum_{m \in M_k^C - M_k^{Sec}} \sum_{n \in N_{mk}} x_{i,j,k,l,m,n} + \sum_{l \in L_{ki}} \sum_{m \in M_{ks}^{Sec}} \sum_{n \in N_{mk}} x_{i,j,k,l,m,n} \le 1 \qquad (4)$$

$$\forall n \in N, \quad \forall i \in I_n, \quad \forall j \in J_{in}$$

$$\sum_{k \in K} \sum_{l \in L_{ki}} \sum_{m \in M_{kln}} x_{i,j,k,l,m,n} \le 1, \qquad (5)$$

$$\forall k \in K$$

$$\sum_{l \in L_k} \sum_{m \in M_{kl}} \sum_{n \in N_{mk}} \sum_{i \in I_{ln}} \sum_{j \in J_{iln}} x_{i,j,k,l,m,n} = a_k^1, \qquad (6)$$

$$\forall k \in K, \forall l \in L_l, \forall m \in M_{kl}$$

$$\sum_{n \in N_{mk}} \sum_{i \in I_{ln}} \sum_{j \in J_{iln}} x_{i,j,k,l,m,n} = 1 \qquad (7)$$

$$\forall (i,j,k,l,m,n) \in PRA$$

$$x_{i,j,k,l,m,n} = 1 \qquad (8)$$

Equation (1) ensures that every lecturer can be assigned at most one student group, one classroom, and one course in every time period of the week. Equation (2) ensures that every student group can be assigned at most one course, one lecturer, and one classroom in every time period of the week. However, this constraint provides that only mandatory courses of students in the same lower-grade student groups do not overlap. In addition, this constraint allows the sections of the multi-section mandatory courses to overlap in accordance with the structure of this problem. Equation (3) and Equation (4) are designed for higher-grade student groups that have mostly elective courses. Equation (3) is a soft constraint that allows the least overlap of elective courses of higher-level student groups. For this reason, it is used for the decision variable "y" that indicates the number of overlapping elective courses in this constraint. At least the conflict is ensured by minimizing this decision variable in the objective function. Equation (4) is almost the same as Equation (2). The only difference, Equation (4) ensures that mandatory courses of students in the same higher-grade student groups do not overlap. Equation (5) ensures that every classroom in every time period of the week can be assigned at most one student group, one lecturer, and one course. Equation (6) is to ensure that all courses of every student group take place in the course timetabling. Equation (7) is to ensure that every course takes place only once in the course timetabling. Equation (8) ensures that course m taught by lecturer l to student group k is pre-assigned for j^{th} time period of day i in classroom n.

3.5 Objective Function

In general, the main purpose of the course timetabling is to assign the course appropriately under certain constraints. These assignments can be done with constraints (*i.e.*, with constraint satisfaction programming) without objective function, or different objective functions can be used depending on the structure of the problem. For example, Daskalaki et al. [1] used a model that minimizes the objective function by determining one cost coefficient for each course according to every time period of every day in a week. Thus, in this way, courses were able to be assigned to the desired time period and day. Similarly, the purpose of Bakir andAksop [32]'s study was to minimize the objective function in terms of student and lecturer satisfaction.

$$\text{Min Z}= \sum_{i\in I}\sum_{j\in J}\sum_{k\in K^2} y_{i,j,k} + \sum_{i\in I}\sum_{j\in J}\sum_{m\in M} C_{i,j,m} * (\sum_{k\in K}\sum_{l\in L_k}\sum_{n\in N_{mk}} x_{i,j,k,l,m,n}) \tag{9}$$

The objective function of this study consists of two parts. In the first part, the overlap of elective courses is minimized. For this, we use the decision variable $y_{i,j,k}$, $i \in I,\ j \in J,\ k \in K$, which minimizes the number of overlaps of elective courses of student groups k for j^{th} time period of day i. In the second part, the satisfaction level of the lecturer is maximized. For this, we use the cost coefficient $C_{i,j,m}$, $i \in I,\ j \in J,\ m \in M$, which is formed by giving the minimum value for the most desired time period of day, and the maximum value for the minimum desired time period of day. In these models, these cost coefficients are determined using values between 1 and 5. In addition, these two parts can be multiplied by constant coefficients of appropriate magnitude to form a priority order in the objective function.

3.6 Model 2 (with multi-session)

In Model 1, all courses in HUBA can be assigned to only one of two different time periods (morning and afternoon) due to the fact that they are held once a week. For this reason, each course entered as a parameter into the model is considered as a time period in Model 1; thus, there is no need for additional constraints that ensure a course to have consecutive time periods in the day. The opposite is true in Model 2 because the course hours can be assigned in this model by dividing instead of the single session. As a result, additional constraints are needed to ensure that the courses have consecutive time periods throughout the day. Also, a new decision variable $w_{i,k,h,m,n}$ $i \in I,\ k \in K,\ h \in H_m,\ m \in M,\ n \in N$ is used in model 2. This decision variable ensures that the model itself can determine how to partition the courses in the framework of predetermined options (*e.g.*, to assign a 3-hour course as 2 + 1 or as a single session). The fact that the partitioning is left to the model itself is important in terms of solution quality. A different approach can be seen in Daskalaki et al. [1]. In their study, how the courses would be divided was entered as a parameter into the model.

The decision variable that indicates which part of any course will be given on any day is as follows:

$$w_{i,k,h,m,n} = \begin{cases} 1, & \begin{array}{l} when\ the\ hth\ section\ of\ course\ m \\ given\ to\ the\ student\ group\ k\ is \\ assigned\ on\ day\ i\ in\ classroom\ n. \\ where\ \ i\ \in I, k\ \in K, h\ \in H_m, m\ \in M, \\ \qquad n\ \in N \end{array} \\ 0, & otherwise \end{cases}$$

where h refers to the number of time periods of part or a whole of a course. For example, if $h \in H_m = \{1,2\}$ for a 2-hour course m, this course can be assigned as 1+1 or a single session. Similarly, if $h \in H_m = \{1,2,3\}$ for a 3-hour course m, this course can be assigned as 1+1+1, 2+1 or a single session.

The above constraints (1), (2), (3), (4), (5), and (8) were taken in the same way and included in Model 2. In addition, the objective function can also be used in Model 2. The other constraints of Model 2 are as follows:

$$\forall k \in K \qquad \sum_{l\in L_k}\sum_{m\in M_{kl}}\sum_{n\in N_{mk}}\sum_{i\in I_{ln}}\sum_{j\in J_{iln}} x_{i,j,k,l,m,n} = a_k^2, \tag{10}$$

$$\forall k \in K, \forall l \in L_l, \forall m \in M_{kl} \qquad \sum_{n\in N_{mk}}\sum_{i\in I_{ln}}\sum_{j\in J_{iln}} x_{i,j,k,l,m,n} = b_m \tag{11}$$

$$\forall i \in I,\quad \forall k \in K,\quad \forall l \in L_{ki},\quad \forall m \in M_{kl},\quad \forall n \in N_{mk} \qquad \sum_{j\in J_{iln}} x_{i,j,k,l,m,n} - \sum_{h\in H_m}(w_{i,k,h,m,n} * h) = 0 \tag{12}$$

$$\forall k \in K,\quad \forall m \in M_k,\quad \forall h \in H_m - hm \qquad \sum_{i\in I}\sum_{n\in N_{mk}}(w_{i,k,h,m,n} + w_{i,k,hm,m,n)} = 1 \tag{13}$$

$$\forall k \in K, \quad \forall m \in M_k, \quad \forall i \in I$$
$$\sum_{h \in H_m - hm} \sum_{n \in N_{mk}} w_{i,k,h,m,n} \leq 1 \tag{14}$$

$$\forall i \in I,\ \forall k \in K,\ \forall l \in L_{ki},\ \forall m \in M_{kl},\ \forall n \in N_{mk},\ \forall h \in H_m \wedge h > 1,\ \forall t \in \{1, \dots, h-1\},$$
$$x_{i,1,k,l,m,n} - x_{i,t+1,k,l,m,n} \leq (1 - w_{i,k,h,m,n}) \tag{15}$$

$$\forall i \in I,\ \forall k \in K,\ \forall l \in L_{ki},\ \forall m \in M_{kl},\ \forall n \in N_{mk},\ \forall h \in H_m \wedge h > 1,\ \forall t \in \{1, \dots, h-1\},$$
$$x_{i,8,k,l,m,n} - x_{i,8-t,k,l,m,n} \leq (1 - w_{i,k,h,m,n}) \tag{16}$$

$$\forall i \in I,\ \forall k \in K,\ \forall l \in L_{ki},\ \forall m \in M_{kl},\ \forall n \in N_{mk},\ \forall h \in H_m \wedge h > 1,\ \forall j \in J, \forall t \in \{2, \dots, h-1\} \wedge j + t \leq 8,$$
$$- x_{i,j,k,l,m,n} + x_{i,j+1,k,l,m,n} - x_{i,j+t,k,l,m,n} \leq (1 - w_{i,k,h,m,n}) \tag{17}$$

Equation (10) ensures that the number of all courses of every student group k in the model is equal to the total number of their time periods. Equation (11) ensures that the number of course m in the course timetabling is equal to the number of its time periods. Equation (12) ensures that the number of course m on day i is equal to h, $h \in H_m$. Equation (13) ensures that all time periods for every course during a week are in the course timetabling. hm in these constraints represents the total number of time periods for course m. If the value of the decision variable is $w_{i,k,hm,m,n}$ is 1, it means that the relevant course is assigned as a single session. If the course m is not a single session, Equation (14) ensures each part of the course m to be assigned to a different day. For example, if a 3-hour course is divided into 2 and 1 (*i.e.*, 2+1), 2-hours part and 1-hour part must be assigned to different days. Eqs. (15, 16, 17) ensure that time periods follow each other if the course is assigned to more than one-time period on a day. Equation (15) ensures a consecutive sequence when a course is assigned to the first time period of the day. Equation (16) ensures a consecutive sequence when a course is assigned to the last time period of the day. Finally, Equation (16) ensures a consecutive sequence when a course is assigned to the other time periods of the day.

4. Optimal Solution of Course Timetabling Problem

Model 1 and Model 2 developed in the scope of the study were applied to data of Hacettepe University Department of Business Administration (HUBA).

In the fall semester curriculum of HUBA, 33 courses are offered in total. All of these courses are given as a 3-hour single session. For this reason, the total number of time periods are taken as 33. However, the total number of time periods is 99 because the time period is considered as 1-hour in Model 2.

In Table 1, the total number of courses in HUBA is given taking into account the mandatory and elective course categories. For these courses, there is available seven regular classrooms and one specialized classroom (lab). Also, these courses are assigned to 23 lecturers.

The problem of HUBA (problem 1) and a larger scale problem (problem 2) were modeled with the Python programming language and solved using solver Gurobi6.2.5-64bit. The solution found is a proposed timetable for both problems.

Table 1. Number of mandatory/elective courses offered for each student groups

	Student groups				Total
Courses	First	Second	Third	Fourth	
Mandatory	5	7	2	2	14
Elective	0	0	9	8	19
Total	5	7	11	10	33

The students of HUBA have 14 mandatory and 19 elective courses for the fall semester. These courses were planned as follows: 5 mandatory courses for the first year, 7 mandatory courses for the second year, 2 mandatory and 9 elective courses for the third year, and 2 mandatory and 8 elective courses for the fourth year. In Table 2, the solution of problem 1 in Model 1 is shown using the code numbers of the courses and classrooms. Looking at this table, it seems that there is no overlap among all mandatory courses, but there is a few overlaps among elective and multi-section mandatory courses in accordance with the structure of the problem. Also, it appears that all the courses required by the curriculum exist in the course timetabling and are assigned to suitable classrooms without overlap.

In Model 2, as mentioned above, one day is divided into eight time periods to be between 09: 00 am and 17: 45 pm. Thus, the courses can be assigned by splitting according to the hours of the course. In these problems, since all courses are 3-hour courses, these courses can be split in the framework of predetermined options (*e.g.*, a 3-hour course can be assigned as 2 + 1 or as a single session). The solution of problem 1 obtained using model 2 is shown in Table 3. Similarly, as also shown in this table, firstly, there appears no overlap among all mandatory courses. Secondly, there seem to be a few overlaps among elective and multi-section mandatory courses. Thirdly, it appears that all the courses required by the curriculum exist in the course timetabling and are assigned to suitable classrooms without overlap. Finally, it is seen that the time periods of the course assigned to more than one time period on a day are in a consecutive sequence, and this course is assigned to the same classroom.

Table 2. Course timetabling for the fall semester – Model 1

		Time periods				Time Periods	
Group	Days	Morning	Afternoon	Group	Days	Morning	Afternoon
Student group 1	Monday	M101-01, D5		Student group 3	Monday	M311, D6	M317, D5
		M101-02, D4					
	Tuesday				Tuesday	M301-01, A1	M321, D 5
						M301-02, D4	
	Wednesday		M107, D6		Wednesday	M313, D4	M305, D3
	Thursday	M105, D5			Thursday	M303, D3	M319, D7
	Friday		M103, D4		Friday	M307, LAB-C	M309, D5
Student group 2	Monday	M203, D3	M205-02, LAB-C	Student group 4	Monday	M403, A1	M409, D8
	Tuesday				Tuesday	M425, D7	M417, D7
	Wednesday	M201, D3			Wednesday	MAN401-01, D6	M407, D7
	Thursday	M209, A1	M205-02, LAB-C		Thursday	M415, D7	M405, D3
			M207-01, D4				
	Friday		M207-02, D6		Friday	MAN419, D7	M401-02, D7

The code numbers indicate which course belongs to which student group and which department, and the digit after "-" indicates the section of the course. The dark grey cells represent mandatory courses, while the light grey cells represent elective courses.

5. The Performance of the Model 1 and Model 2

The solution of the course timetabling problem becomes increasingly difficult due to the special needs of each educational institution, increasing the number of courses, lecturers, and classrooms. Depending on the size of the problem, the solution of the models developed for the solution of the timetabling problems sometimes takes a very long time and sometimes the solution becomes impossible. For this reason, the solution performance of the models developed for the university course timetabling problem in terms of problem 1 and problem 2 are compared in Table 4. For these problems of different sizes, the number of courses varied from 33 to 66, the number of lecturers from 23 to 46, and the number of time periods from 33 to 198. When both problems are solved by Model 1, the solution times are 1.6 and 32.7 seconds. On the other hand, when both problems are solved by Model 2, the solution times are 13.1 and 163.5 seconds.

The solution times are also significantly affected by both the number of binary variables and the number of constraints. Therefore, within the scope of problem 1 and problem 2, some significant results of both models are shown in Table 5. For problems 1 and 2, Model 1 contained 283-686 equations, 2150-4280 integer variables, and 2130-4260 binary variables, respectively, while the non-zero values varied from 9450 to 17440. On the other hand, Model 2 contained 27144-77648 equations, 11795-70770 integer variables, and 11715-70290 binary variables, respectively, while the non-zero values varied from 152930 to 930360.

6. Conclusions

In this study, firstly, using integer programming, the course timetabling problem of the Hacettepe University Business Administration Department was solved optimally considering the level of satisfaction of the lecturers. Secondly, this problem was also solved under the assumption that the courses can be assigned by splitting. Two models were developed to reach these solutions by taking into account that the courses can be mandatory, elective, and multi-section status. Also, these models allow a minimum overlap of the elective and multi-mandatory courses provided that students should have the chance to choose at least one of the multi-section mandatory courses. Within the scope of these models, new constraints were developed to ensure that at least one of the multi-section mandatory courses can be chosen and that the courses can be split by the model itself in the framework of predetermined options. These new constraints that we developed extend the research in this area of timetabling, and they provide that these models can be easily adapted to the problems in similar structures.

The solution of the university course timetabling problem, which is about assigning the courses in the curriculum in the way that there is no problem in terms of student groups and lecturers, becomes more difficult as the size and special requirements of the problem increase. The results obtained in this study support this situation. Looking at the solution tables, it can be seen that the optimal solutions are obtained within the specified constraints and that the solution times significantly increase depending on the problem size.

Table 3. Course timetabling for the fall semester – Model 2

		Time periods							
Group	Days	PER1	PER2	PER3	PER4	PER5	PER6	PER7	PER8
Student group 1	Monday	M101-01,D8		M105,D4					M107,D6
		M101-02,A1							
	Monday			M103,A1				M107,D3	
	Wednesday	M105,D8							
	Thursday				M103,D3				
	Friday	M101-02,A1				M101-01,D5			
Student group 2	Monday			M205-01,LAB-C			M207-01,D3		
	Tuesday			M209,D6			M207-02,D5		
	Wednesday				M209,D7	M207-01,D4			
	Thursday			M205-01,LAB-C		M207-02,D8		M201,D6	
	Friday		M203,D4			M205-02,LABC			M201,D7
Student group 3	Monday						M311,D6		
	Tuesday	M309,D4		M303,D3	M307,LAB-C		M319,D6		M321,D5
	Wednesday	M301-01,A1		M311,D8	M313,D6	M317,D6		M321,D5	
		M301-02,D5							
	Thursday		M301-01,D7	M313,D7		M305,D4		M301-02,D3	
	Friday	M305,D4	M319,D6	M303,D3		M317,D6	M309,D8		M307,LAB-C
Student group 4	Monday	M403,D3				M401-02,A1		M417,D3	
							M401-01,D4		
	Tuesday	M405,D3		M407,D8	M425,D3		M415,D8		
	Wednesday		M419,D4		M405,A1	M409,D5	M417,D7	M407,D6	
	Thursday		M419,D3	M403,D8		M401-01,D3			
							M401-02,D7		
	Friday				M409,A1				M425,D5

The code numbers indicate which course belongs to which student group and which department, and the digit after "-" indicates the section of the course. The dark grey cells represent mandatory courses, while the light grey cells represent elective courses.

Table 4. The solution times

	Problem sizes					Solution times	
	Student groups	Courses	Lecturers	**Classrooms/Labs**	**Time Periods For of M 1/ M 2**	**Model 1 (Second)**	**Model 2 (Second)**
Problem 1	4	33	23	8/1	33/99	1,6	13,1
Problem 2	8	66	46	16/2	66/198	32,7	163.5

M 1: Model 1, M 2: Model 2.

Table 5. The solution sizes

	Model 1			**Model 2**		
	No. of rows	**No. of columns**	**No. of non-zeros**	**No. of rows**	**No. of columns**	**No. of non-zeros**
Problem 1	283	2150 integer (2130 binary)	9450	27144	11795 integer (11715 binary)	152930
Problem 2	686	4280 integer (4260 binary)	17440	77648	70770 integer (70290 binary)	930360

Declaration

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article. The authors also declared that this article is original, was prepared in accordance with international publication and research ethics, and ethical committee permission or any special permission is not required.

Author Contributions

A. Özkan developed the methodology, analyzed and wrote the manuscript. A. Ulucan supervised, improved and proofread the study. This study was derived from the PhD thesis of A. Özkan.

References

1. Daskalaki, S., T. Birbas, and E. Housos, *An integer programming formulation for a case study in university timetabling.* European Journal of Operational Research, 2004. **153**(1): p. 117-135.
2. Burke, E.K., et al., *A graph-based hyper-heuristic for educational timetabling problems.* European Journal of Operational Research, 2007. **176**(1): p. 177-192.
3. D. de Werra, D., *The combinatorics of timetabling.* European Journal of Operational Research, 1997. **96**(3): p. 504-513.
4. Deris, S., S. Omatu, and H. Ohta, *Timetable planning using the constraint-based reasoning.* Computers & Operations Research, 2000. **27**(9): p. 819-840.
5. Parker, R.G. and R.L. Rardin, *Discrete optimization*. 2014: Elsevier.
6. Burke, E.K. and S. Petrovic, *Recent research directions in automated timetabling.* European Journal of Operational Research, 2002. **140**(2): p. 266-280.
7. MirHassani, S. and F. Habibi, *Solution approaches to the course timetabling problem.* Artificial Intelligence Review, 2013. **39**(2): p. 133-149.
8. Vermuyten, H., et al., *Developing compact course timetables with optimized student flows.* European Journal of Operational Research, 2016. **251**(2): p. 651-661.
9. Babaei, H., J. Karimpour, and A. Hadidi, *A survey of approaches for university course timetabling problem.* Computers & Industrial Engineering, 2015. **86**: p. 43-59.
10. Birbas, T., S. Daskalaki, and E. Housos, *School timetabling for quality student and teacher schedules.* Journal of Scheduling, 2009. **12**(2): p. 177-197.
11. Feizi-Derakhshi, M.-R., H. Babaei, and J. Heidarzadeh. *A survey of approaches for university course timetabling problem*. in *Proceedings of 8th international symposium on intelligent and manufacturing systems (IMS 2012)*. 2012.
12. Dandashi, A. and M. Al-Mouhamed. *Graph Coloring for class scheduling*. in *ACS/IEEE International Conference on Computer Systems and Applications-AICCSA 2010*. 2010. IEEE.
13. Welsh, D.J. and M.B. Powell, *An upper bound for the chromatic number of a graph and its application to timetabling problems.* The Computer Journal, 1967. **10**(1): p. 85-86.
14. Dimopoulou, M. and P. Miliotis, *Implementation of a university course and examination timetabling system.* European Journal of Operational Research, 2001. **130**(1): p. 202-213.
15. Dimopoulou, M. and P. Miliotis, *An automated university course timetabling system developed in a distributed environment: A case study.* European Journal of Operational Research, 2004. **153**(1): p. 136-147.
16. Daskalaki, S. and T. Birbas, *Efficient solutions for a university timetabling problem through integer programming.* European Journal of Operational Research, 2005. **160**(1): p. 106-120.
17. Al-Yakoob, S.M. and H.D. Sherali, *Mathematical programming models and algorithms for a class–faculty assignment problem.* European Journal of Operational Research, 2006. **173**(2): p. 488-507.
18. Al-Yakoob, S.M. and H.D. Sherali, *Mixed-integer programming models for an employee scheduling problem with multiple shifts and work locations.* Annals of Operations Research, 2007. **155**(1): p. 119-142.
19. Badri, M.A., et al., *A multi-objective course scheduling model: Combining faculty preferences for courses and times.* Computers & operations research, 1998. **25**(4): p. 303-316.
20. Kang, L. and G.M. White, *A logic approach to the resolution of constraints in timetabling.* European Journal of Operational Research, 1992. **61**(3): p. 306-317.
21. Zhang, L. and S. Lau. *Constructing university timetable using constraint satisfaction programming approach*. in *International Conference on Computational Intelligence for Modelling, Control and Automation and International Conference on Intelligent Agents, Web Technologies and Internet Commerce (CIMCA-IAWTIC'06).* 2005. IEEE.
22. Alvarez-Valdes, R., E. Crespo, and J.M. Tamarit, *Design and implementation of a course scheduling system using Tabu Search.* European Journal of Operational Research, 2002. **137**(3): p. 512-523.
23. Aladag, C.H., G. Hocaoglu, and M.A. Basaran, *The effect of neighborhood structures on tabu search algorithm in solving course timetabling problem.* Expert Systems with Applications, 2009. **36**(10): p. 12349-12356.
24. Hao, J.-K. and U. Benlic, *Lower bounds for the ITC-2007 curriculum-based course timetabling problem.* European Journal of Operational Research, 2011. **212**(3): p. 464-472.
25. Deris, S., et al., *Incorporating constraint propagation in genetic algorithm for university timetable planning.* Engineering applications of artificial intelligence, 1999. **12**(3): p. 241-253.
26. Wang, Y.-Z., *An application of genetic algorithm methods for teacher assignment problems.* Expert Systems with Applications, 2002. **22**(4): p. 295-302.
27. Asmuni, H., E.K. Burke, and J.M. Garibaldi. *Fuzzy multiple heuristic ordering for course timetabling*. in *Proceedings of the 5th United Kingdom workshop on computational intelligence (UKCI 2005)*. 2005. Citeseer.
28. Chaudhuri, A. and K. De, *Fuzzy genetic heuristic for university course timetable problem.* Int. J. Advance. Soft Comput. Appl, 2010. **2**(1): p. 100-123.
29. Abdullah, S., E.K. Burke, and B. McCollum. *A hybrid evolutionary approach to the university course timetabling problem*. in *2007 IEEE congress on evolutionary computation*. 2007. IEEE.
30. Kohshori, M.S. and M.S. Abadeh, *Hybrid genetic algorithms for university course timetabling.* International Journal of Computer Science Issues(IJCSI), 2012. **9**(2).

31. Badoni, R.P., D. Gupta, and P. Mishra, *A new hybrid algorithm for university course timetabling problem using events based on groupings of students.* Computers & Industrial Engineering, 2014. **78**: p. 12-25.
32. Bakir, M.A. and C. Aksop, *A 0-1 integer programming approach to a university timetabling problem.* Hacettepe Journal of Mathematics and Statistics, 2008. **37**(1): p. 41-55.