EVALUATION OF LEARNING MANAGEMENT SYSTEMS USING INTERVAL VALUED INTUITIONISTIC FUZZY-Z NUMBERS

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Received: 02/09/2022 **Accepted:** 23/01/2023

ABSTRACT

The use of online education tools has increased rapidly with the transition to distance education caused by the pandemic. The obligation to carry out all activities of face-to-face education online made it very important for the tools used in distance education to meet the increasing needs. In line with these needs, radical changes have occurred in the learning management systems used in distance education. Therefore, in this study, it is aimed to determine the features that the systems used in distance education should have and to compare the existing systems according to these features. For this purpose, a novel fuzzy extension, interval valued intuitionistic fuzzy Z-numbers, is defined for modeling uncertainty, and AHP and WASPAS methods using proposed fuzzy numbers are developed to determine the importance of decision criteria and compare alternatives.

Keywords: E-Learning, Interval Valued Intuitionistic Fuzzy Z-Numbers, Fuzzy AHP, Fuzzy WASPAS.

INTRODUCTION

In today's Information Age, Information and Communication Technologies (ICTs) can be easily accessed from every corner of the world and become widespread day by day. ICTs have entered the lives of people with the decrease of costs and have significantly affected their lifestyles. In this sense, developing technologies and widespread use of the Internet have brought advantages such as adapting to rapid changes, differentiation, fast access to the information needed, problem solving and creative thinking. Developments in ICTs directly affect the field of education as well as every area of our lives and the concept of E-learning that emerged as a result of these developments has also started to gain importance today. The ability to obtain, absorb and apply proper information effectively has become one of the key skills today. With the concept of learning, which is the key to achieving one's full potential, the survival of individuals, organizations and nations in the 21st century will depend on their learning capacity and their application of learned things in daily life.

With the development of ICTs, accessing information from different sources in a shorter time has enabled the development and diversification of distance education/distance learning environments. In this way, in the 21st century, individuals can easily access the information they need anywhere, anytime, by any means. Therefore, with the development of each new communication technology, e-learning and individualized distance education opportunities are gradually increasing. As a result of this situation, distance education environments are now designed as processes that are more flexible, easily accessible and include daily life as much as possible. Also, achieving success and quality in distance education services has become the focus of both educators and researchers.

Distance education is an education system model that brings together learners, instructors and teaching activities with different communication technologies infrastructure. Distance education is the fastest developing and spreading type of education service in Turkiye and the world. This system, which has been offered to people in different infrastructures with its rapid development from the past to the present, currently serves as a web-based system in Turkiye.

E-learning / online learning stands for electronic learning or in other terms web-based education that involves learning and information management activities carried out through internet technologies. This concept allows users to efficiently gather information and content with both simultaneous and asynchronous methodologies and effectively meet the need to gain up-to-date knowledge. E-learning technologies find more usage areas as open and/or distance education applications become widespread in the world and in parallel with this, they undergo a rapid development process. It is aimed to personalize learning by using technology, that is, teaching by taking into account the average learning needs and styles of an audience in the group, to develop one's own learning skills and to enable him to learn by determining his own needs. In line with this goal, subjects such as e-learning methods, e-learning tools, and evaluation of e-learning have come to the fore in order to enable learning using technology.

In recent years, the use of web-based learning in the higher education system has been increasing. While the effect of using the Internet in education has gradually increased, the inclusion of new technologies in education has brought the inevitable result. While this situation improves the learning of students, distance education has become a crucial part of education. With the development of technology in education, the need for distance education tools has increased and therefore most universities have started to use web-based distance education systems and e-learning tools. The learning management system (LMS) is one of the e-learning tools that has become a critical tool for almost all higher education institutions and a propellent force in online learning. Some of these tools are open source while others are for commercial purposes.

The main purpose of this paper is to determine a suitable learning management system platform to meet the requirements of universities in Turkiye. For this purpose, interval valued intuitionistic fuzzy Z-numbers (IVIF-Z), a new fuzzy extension for modeling uncertainty in linguistic expressions, is developed for the first time in this study. Then, the IVIF-Z AHP method, which will be used in weighting the criteria to be used in the selection of distance education systems, and the IVIF-Z WASPAS method, which will be used to compare the existing LMS platforms, have been developed.

The organization of the paper is as follows: First literature review on learning management systems and proposed methodology are given. Then, the basics of e-learning and e-learning tools are determined. After giving the preliminaries of the proposed fuzzy extensions, interval valued intuitionistic fuzzy Z-AHP (IVIF Z-AHP) and interval valued intuitionistic fuzzy Z-WASPAS (IVIF Z-WASPAS) are proposed. Later, the proposed method is applied to the learning management systems used in Turkiye and the results are discussed. Finally, the article ends with the discussions and conclusions.

LITERATURE REVIEW ON LEARNING MANAGEMENT SYSTEMS

Distance education and e-learning / online learning, which come to the fore with the development of technology, are two distinguishable teaching/learning methods that emerged at that time (Micha, 2019). This study has been focused on e-learning tools and more specifically on LMSs. In the literature, some studies show learning management systems have positive effects on the teaching and learning process (Han and Shin, 2016; Ramirez-Correa et al., 2017). Also, some studies introduce learning management systems whose use with distance education is increasing day by day, comparing them in terms of their features and usage, and investigating their effect on the learning and teaching process. Machado and Tao (2007) created two study groups, a faculty group and a student group, and compared the user experience between the proprietary solution Blackboard and the open-source solution Moodle. In the study, the user experience of each system's basic functions as communication tools, student-student interaction tools, student-instructor interaction tools were compared using online questionnaires. Miyazoe (2008) examines whether different LMS affects students' participation in online interaction and their evaluation of the course. To answer these, it was planned to use a semi-identical course design and an LMS to compare two different LMS, Blackboard and Moodle. A questionnaire consisting of 20 five-point Likert-scale questions and five open questions that consist of basic demographics, specific purposes of computer usage, usage of mobile phones etc. was applied in four classes, and correlation analysis was performed between variables. Cheung (2007), WebCT, Blackboard and Moodle functionally examined three web-based learning management systems commonly used in higher education. The study presents a comparison of the functional framework of LMS systems in terms of curriculum design, communication and discussion, performance evaluation and course management, focusing on their use in teaching and learning of continuing education courses.

Payette and Gupta (2009) examine the transition from one type of commercial software, Blackboard to another open-source software, Moodle. 34 faculty members and 390 students were surveyed to gain insight into the transition from one LMS system to another. The aim is to identify issues that can be addressed with targeted training and insights that will improve the transition process. Al-Ajlan (2012) conducted a comparative study between Moodle and other LMS systems to meet the requirements of Qassim University. In this study, in which three comparisons were made by dividing the features into technical tools, support tools and learner tools, the features and capabilities and technical aspects of 10 LMS tools, Moodle, ATutor, Scholar360, Sakai, Blackboard etc. were examined. As a result of the study, it was determined that the best platforms were Moodle and Sakai, which have missed only two of the forty features, while extensive and in-depth analyses proved Moodle should be selected as the most suitable platform for Qassim University. Cavus and Zabadi (2014) focused on the file exchange/internal mail, whiteboard/video services, discussion forums, live chat and online journal mail features of each of the six open-source learning management systems such as ATutor, Claroline, Dokeos, Ilias, Moodle ve Sakai. This article aims to make it easier for educators who want to make the best choice when choosing a learning management system by revealing which learning management system has the best communication tools. The comparison result showed that Moodle and ATutor have the best communication tools with a user-friendly interface. Orfanou et al. (2015) stated that the perceived ease of use of learning management systems had an effect on students' learning effectiveness and learning experience. In the study which 769 students participated, they examined the perceived ease of two learning management systems, eClass and Moodle, using the System Usability Scale.

Cigdem and Ozturk (2016) aimed to examine the factors that determine the behaviours of 155 students in using learning management systems through a questionnaire in the study. As a result of their study, it was revealed that multimedia features and interaction affected students' perceived satisfaction. The study conducted by Kasim and Khalid (2016) is discussed several potential Learning Management Systems that Higher education institutions such as Moodle, Sakai, ATutor, Blackboard and SuccessFactors can be used for teaching and learning processes. In the study, a comparison is made among selected LMS providers based on various features such as flexibility, ease of use, accessibility, user-friendliness, and the ability to integrate with other systems, and results are presented which is about the preference of platform to be used. Juarez Santiago et al. (2020) conducted a study to evaluate a model in which architectural design, configuration, metadata and statistical coefficients were obtained using four LMSs as Edmodo, Schoology, Classroom, Moodle. This model enabled the determination of reliability, accuracy and correlation by using and integrating factors previously used in many studies such as Anxiety - Innovation (AI), Utility and Use (UU), Tools Learning (TL), System Factors (SF), Access Strategies (AS), Virtual Library (VL), and Mobile Use (MU).

The lack of recent studies on learning management systems in the literature and the insufficiency of studies comparing between LMSs systems led to the emergence of this study. In this study, it is aimed to examine the platforms used by universities in the distance education process, especially during the COVID-19 pandemic, in various dimensions (instructional, formal, educational program and program compatibility) and evaluate the platforms through decision makers' views. In this respect, it is considered an original study.

DISTANCE EDUCATION

Distance education offers learners and lecturers a learning environment where the lessons are taught live, visually and audibly in a virtual environment, without time and space limitations, and where the participant can watch them again whenever they want. In today's conditions, distance education is an innovative education system in which education and training are rapidly passed to the computer environment. E-learning, which is a component of distance education, and e-learning tools that enable distance education are tools that are part of this digital transformation in education and training. In this section, e-learning and e-learning tools are explained in detail.

E-Learning

The technologies that have developed over time within the scope of the needs and requirements of the age, the widespread use of the internet and the computers, which have become essential for education have changed the scope of education, and the concept of e-learning has started to come to the fore in education. Internet and online communication tools, which enable cheap, global, interactive and intensive computer communication, have created a learning environment independent of time and place, unlike traditional education (Collins and Halverson, 2018). When it comes to learning independent of time and space, the first thing that comes to mind is the concept of distance education (Bicer and Korucu, 2020). In this sense, the Internet has also transformed the concept of distance education and has become an accelerating factor in this transformation process. As a result of this, the concept of E-learning has emerged, which is a new learning environment that provides the learner with many flexibilities such as being able to learn anytime and anywhere, parallel to the purpose of distance education, and even considered by most researchers as a sub-topic of distance education. Although E-learning and distance education are sometimes confused with each other, E-learning is just a form of distance education (Rosenberg and Foshay, 2002). Although there are different definitions, E-learning is most simply defined as conducting educational activities in electronic environments or transferring knowledge and skills through electronic technologies (Gulbahar, 2017).

In the early 90s, after the use of radio and television channels in education, with the use of Flash-based multimedia contents and through CD-ROMs and DVD-ROMs, distance learning activities began, and these activities evolved into e-learning with the spread of the Internet (Ulker and Yilmaz, 2016). E-learning can be seen as the most effective and significant technological solution, together with the technological facilities provided to meet the needs of both individuals and society, to complete the development by providing life-long learning and rapid learning in the context of using technology, in the economic context and line with personal needs (Bicer and Korucu, 2020). As technological innovations continue on their way without slowing down, especially the use of e-learning technologies for education and training is becoming more widespread day by day, and the transfer of knowledge with technology has started to be the focus on the attention of universities. Because while these technologies provide a wide area for learning courses, seminars, discussion forums and other approaches, offer innovative approaches to instructor-learner interaction (Singh et al., 2011). Therefore, e-learning technologies and developments in this field have made educational design a major skill for organizations that manage with open and distance education, especially during the COVID-19 outbreak. In today's conditions, the number of universities providing education with e-learning continues to increase day by day, and practices such as universities' orientation towards distance education, and open education programs at some universities have left many learners confronted with e-learning systems (Bahadir, 2020). Therefore, technology-supported systems, in other words, e-learning environments and technologies, are used to better meet the learning needs of learners in different ways in the education performed inside or outside the classroom.

With the individualization of education by e-learning, multiple-learning environments have gained prominence. The fact that the curriculum and course contents are constantly available in the virtual environment and the course can be repeated continuously can be considered as some of the contributions of e-learning. Factors such as supporting the contents with visual materials, and thus simplifying comprehension are another positive contribution of e-learning to the teaching-learning process. Nowadays, with e-learning, it is possible to reach any information from anywhere for not only the registered student group but also every segment of society. These possibilities are becoming more and more intense in parallel with the development of information technologies. On the other hand, individuals who receive education within the scope of e-learning are also allowed to manage their own time. While e-learning has positively affected the motivation of the individual towards learning by supporting individual teaching, it has largely eliminated the psychological pressure of group learning. These opportunities have been significant in terms of revealing the individual's own originality. E-learning has become an important alternative in enabling different segments to participate more in the learning process by making learning more interesting and attractive. At this point, with e-learning, individuals and/or groups can get or share information/data by finding the opportunity to reach different individuals and groups that they cannot reach in traditional learning. Within the scope of e-learning, the individual is not only dependent on a single resource but also gets the opportunity to benefit from many different object-based and visual web environments to understand more easily the same subject. The opportunity of interaction offered by e-learning allows the learner to benefit from the internet environment in accordance with his/her level of knowledge. Therefore, many possibilities that traditional education cannot offer can be offered with e-learning.

Today, overcrowded classrooms in educational institutions in Turkiye has always been a problem, so the instructor-learner interaction has remained very limited. Therefore, e-learning has become a necessity to eliminate the limitations in the instructor-learner interaction level and to bring this interaction level to an equal level for all students.

Although e-learning provides many benefits for teachers and students, it also brings some problems. The factors that make e-learning difficult are the fact that individuals do not have self-discipline in working, the possibility of preventing the socialization process of individuals, the process of creating content is comprehensive, time-consuming and costly, the inability to give up traditional learning habits easily, and the need of having sufficient knowledge and technological infrastructure. In addition to the limitations of e-learning, also it can be costly for students to own a computer. While technical problems on the computer or the internet can hinder teachers and students, they also may not have sufficient knowledge about computers and the Internet. While teaching with e-learning can be costly at the beginning, those who take lessons with e-learning may be new in this field and there may not be knowledgeable and experienced people in their environment who can help them.

Making preparations by knowing all these disadvantages and taking into account the benefit to be gained can provide e-learning more effective and beneficial. A simple comparison between traditional education and e-learning is given in the table below (Gowda and Suma, 2017).

Table 1. Comparison of Traditional education and E-learning (Gowda and Suma, 2017)

Factor	Traditional Education	E-Learning
Time	Dependent, periodic	Independent, lifelong
Place	Dependent, restricted	Independent, theoretically unlimited
Transfer	Not dependent on technology	Technology dependent
Learning Process	Slow	Fast
Learning Environment	Under control, regular, face-to-face, limited time	Uncontrolled, no rules, learner away from the instructor, unlimited time
Material	Depends on books	Depends on LMS
Flexibility	Inflexible, not reconfigurable	Flexible, can be reconfigured depending on the individual, time and purpose
Utilization / Access	Limited, a certain number of learners	Unlimited theoretically
Setup Cost	Low	High
Operating cost	Relatively expensive	Cheap

E-learning basically includes concepts such as web-based learning, computer-based learning, virtual classrooms and digital collaboration. In this context, hardware and software tools are required for the development and implementation of E-learning. These tools, which are indispensable parts of e-learning, can be classified into two groups as creation tools and learning tools as seen in Table 2:

Table 2. Classification of e-learning tools (Kisla and Karaoglan, 2011)

Productivity Tools	Learning Tools
• Authoring Tools	• Learning Management Systems
• Content Management Systems	• Learning Content Management Systems
• Video Editing Tools	• ePortfolio
• Audio Editing Tools	• Assessment Tools
• Chart Drawing Tools	• Online Interview Tools
• Animation Tools	• Virtual Classrooms
• Simulation Tools	• Other
• Other	

While creation tools are used in the design and development of e-learning environments, learning tools are used in sub-processes such as transferring the information to the learner, repeating it, evaluating the learner and so on.

E-learning Tool: LMSs

In this period of the Information Age, rapid developments in communication technologies affect the structure and form of education and force educators to develop new educational programs and learning-teaching models (Altiparmak et al., 2011). One of these models is distance education and the application of distance education has started to become widespread in the form of e-learning. In this context, how to realize the most effective distance education and training has led experts and organizations that develop education programs to think about Learning Management Systems (LMSs). LMSs are software that manage learning activities (Bezovski and Poorani, 2016). LMSs, which have come to a very significant point among e-learning tools, are defined by Ellis et al. (2009) as web-based software that enables the management of educational material, control of documents, monitoring of learners and instructors and reporting operations, as well as online classroom activities to be held. Besides, these integrated systems provide functions such as presenting learning material, sharing and discussing the presented learning material, managing course catalogues, taking assignments, taking exams, providing feedback on these assignments and exams, organizing learning materials, and keeping system records (Sezer and Yilmaz, 2019). The main purpose of these systems is to facilitate e-learning activities and to realize them in a more systematic and planned way. Although there are many different LMSs, the common usage purposes of LMSs are to support teaching, to allow the student to structure the information herself/himself, to increase the quality of education and to increase permanence (Bahceci and Yildiz, 2016). Since learning activities can be evaluated through these systems, the learning style is continuously improved at this point.

The most important criteria for success in such applications is to be able to access extensive information quickly, easily and regularly. The high level of interaction between the user and the system, the ability to answer the user's questions, to provide a more effective learning service by taking advantage of the multimedia support and the opportunities provided by the internet constitute substantial advantages. Despite the fact that instructors and students are far from each other, it is ensured that are close them to each other with the tools in the application and at the same time, it is also possible to bring together learning materials from a wide variety of sources. The main reasons for the widespread use of LMS are that learners can access 24/7 learning materials, that the management of large user groups and learning materials at the same time saves time and cost, and the advanced reporting system allows data analytics (Poyraz and Ozkul, 2019). Also, the features such as the ability to instantly respond to students who want to ask questions through the live-chat environment, and the ability to send students' documents to the system with the "Upload" play an important role in choosing this software.

Nowadays, especially with the COVID-19 pandemic, schools and universities have switched to 100% online education mode, which has forced education to transform (Dwivedi et al., 2020) and LMSs have started to be used more actively by many universities. There are many LMSs produced for commercial purposes and open-source. Among the commercial LMSs, globally the most used are Blackboard/WebCT, Desire2Learn (D2L), eCollege, it's learning, eLeaP. In addition, the main open-source learning management systems can be listed as Moodle, Chamilo, Totara Learn, Canvas, ILIAS, Opigno, ATutor, OLAT, Sakai, Claroline, eFront, Dokeos, Bodington, Drupal, LAMS, Docebo, DotLRN, eLedge, Openelms.

METHODOLOGY

This section consists of three subheadings in which IVIF-Z numbers are proposed and the steps of IVIF-Z AHP and IVIF-Z WASPAS methods are developed.

Preliminaries on Interval Valued Intuitionistic Fuzzy Z-Numbers

In this subsection, firstly, the preliminaries of fuzzy Z numbers and the interval valued intuitionistic fuzzy numbers that form the basis for the proposed IVIF-Z numbers are given. Then the definitions of the proposed IVIF-Z numbers are determined.

Fuzzy Z-Numbers

Z -numbers, an ordered pair of fuzzy sets $Z(\tilde{A},\tilde{B})$, are introduced by Zadeh in 2011 (Zadeh, 2011). The first component $(\tilde{A})$ of a Z-number $Z(\tilde{A},\tilde{B})$ is a fuzzy restriction of the values of X variable, and the second component $(\tilde{B})$ is referred to as certainty of the fuzzy restriction.

The restriction $R(X)$: X *is* A is referred to a possibilistic restriction shown in Eq. (1) where $\mu_{\tilde{A}}(x)$ is the membership function of $\tilde{A}$:

$$R(X): X \text{ is } A \rightarrow Poss(X = x) = \mu_{\tilde{A}}(x) \tag{1}$$

Figure 1 represents a simple fuzzy Z-number; $Z = (\tilde{A},\tilde{B})$ which has a trapezoidal membership function for fuzzy restriction and a triangular membership function for fuzzy reliability.

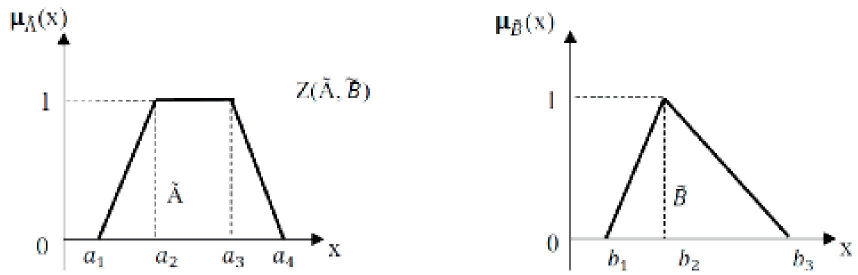

Figure. 1. A simple fuzzy Z-number

Let $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x)\rangle | \mu(x) \in [0,1]\}$ and $\tilde{B} = \{\langle x, \mu_{\tilde{B}}(x)\rangle | \mu(x) \in [0,1]\}$ where $\mu_{\tilde{A}}(x)$ is a trapezoidal membership function and $\mu_{\tilde{B}}(x)$ is a triangular membership function as shown in Figure 1. To convert a Z-number into a regular fuzzy number, Eqs. (2-4) could be used [95]. First to convert the reliability into a crisp number Eq. 2 can be used:

$$\alpha = \frac{b_1 + 2b_2 + b_3}{4} \tag{2}$$

Then, the weighted Z-number can be denoted as $\tilde{Z}^{\alpha}$ by adding the weight of the reliability to the restriction:

$$\tilde{Z}^{\alpha} = \{\langle x, \mu_{\tilde{A}^{\alpha}}(x)\rangle | \mu_{\tilde{A}^{\alpha}}(x) = \alpha\mu_{\tilde{A}}(x), \mu(x) \in [0,1]\} \tag{3}$$

The weighted Z-number, in other words weighted restriction, can be converted to an ordinary trapezoidal fuzzy number $\tilde{Z}'$ which is shown in Figure 2 using Eq. (4) and Eq. (5):

$$\tilde{Z}' = \left\{ \langle x, \mu_{Z'}(x) \rangle \middle| \mu_{Z'}(x) = \mu_{\tilde{A}}\left(\frac{x}{\sqrt{\alpha}}\right), \mu(x) \in [0,1] \right\} \quad (4)$$

$$Z' = \left(\sqrt{\alpha}a_1, \sqrt{\alpha}a_2, \sqrt{\alpha}a_3, \sqrt{\alpha}a_4\right) \quad (5)$$

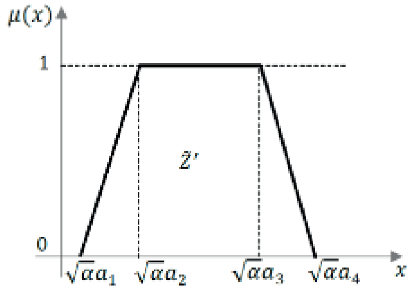

Figure 2. Ordinary fuzzy number transformed from Z-number

Interval Valued Intuitionistic Fuzzy Sets

While $D \subseteq [0,1]$ is the set of all closed sub-intervals of the interval and X is a discourse universe, the interval-valued intuitionistic fuzzy set $\tilde{A}$ is defined over X as follows (Atanassov, 1986):

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) \rangle | x \in X\} \quad (6)$$

where $\mu_{\tilde{A}} \rightarrow D \subseteq [0,1]$, $\nu_{\tilde{A}}(x) \rightarrow D \subseteq [0,1]$ by satisfying the condition $0 \leq sup\, \mu_{\tilde{A}}(x) + sup\, \nu_{\tilde{A}}(x) \leq 1$, $\forall x \in X$.

The membership and non-membership functions of the element x to the set A are represented as intervals $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ respectively. Thus, $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ are closed intervals for each $x \in X$ and $\mu_{\tilde{A}}^-(x), \mu_{\tilde{A}}^+(x), \nu_{\tilde{A}}^-(x)$ and $\nu_{\tilde{A}}^+(x)$ are indicated as starting and ending points of closed intervals. Interval-valued intuitionistic fuzzy set (IVIF) $\tilde{A}$ is indicated by

$$\tilde{A} = \{\langle x, [\mu_{\tilde{A}}^-(x), \mu_{\tilde{A}}^+(x)], [\nu_{\tilde{A}}^-(x), \nu_{\tilde{A}}^+(x)] \rangle | x \in X\} \quad (7)$$

where $0 \leq \mu_{\tilde{A}}^+(x) + \nu_{\tilde{A}}^+(x) \leq 1$, $\mu_{\tilde{A}}^-(x) \geq 0$, $\nu_{\tilde{A}}^-(x) \geq 0$.

For each element x, we compute the hesitancy degree (unknown degree) of an IVIF of $x \in X$ in $\tilde{A}$ defined as follows:

$$\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x) = ([1 - \mu_{\tilde{A}}^+(x) - \nu_{\tilde{A}}^+(x)], 1 - \mu_{\tilde{A}}^-(x) - \nu_{\tilde{A}}^-(x)) \quad (8)$$

For convenience, let $\mu_{\tilde{A}}(x) = [\mu_{\tilde{A}}^-(x), \mu_{\tilde{A}}^+(x)] = [\mu_{\tilde{A}}^-, \mu_{\tilde{A}}^+]$, $\nu_{\tilde{A}}(x) = [\nu_{\tilde{A}}^-(x), \nu_{\tilde{A}}^+(x)] = [\nu_{\tilde{A}}^-, \nu_{\tilde{A}}^+]$, so $\tilde{A} = ([\mu_{\tilde{A}}^-, \mu_{\tilde{A}}^+], [\nu_{\tilde{A}}^-, \nu_{\tilde{A}}^+])$.

Some arithmetic operations are given in the following considering interval-valued intuitionistic fuzzy numbers and $\lambda \geq 0$. Let $\tilde{A} = ([\mu_{\tilde{A}}^-, \mu_{\tilde{A}}^+], [\nu_{\tilde{A}}^-, \nu_{\tilde{A}}^+])$ and $\tilde{B} = ([\mu_{\tilde{B}}^-, \mu_{\tilde{B}}^+], [\nu_{\tilde{B}}^-, \nu_{\tilde{B}}^+])$ be two interval-valued intuitionistic fuzzy numbers. Then (Zavadkas et al., 2014, Kahraman et al. 2016, Bolturk and Kahraman, 2019),

$$\tilde{A} \oplus \tilde{B} = ([\mu_{\tilde{A}}^- + \mu_{\tilde{B}}^- - \mu_{\tilde{A}}^-\mu_{\tilde{B}}^-, \mu_{\tilde{A}}^+ + \mu_{\tilde{B}}^+ - \mu_{\tilde{A}}^+\mu_{\tilde{B}}^+], [\nu_{\tilde{A}}^-\nu_{\tilde{B}}^-, \nu_{\tilde{A}}^+\nu_{\tilde{B}}^+]) \quad (9)$$

$$\tilde{A} \otimes \tilde{B} = ([\mu_{\tilde{A}}^{-}\mu_{\tilde{B}}^{-},\ \mu_{\tilde{A}}^{+}\mu_{\tilde{B}}^{+}],[\nu_{\tilde{A}}^{-} + \nu_{\tilde{B}}^{-} - \nu_{\tilde{A}}^{-}\nu_{\tilde{B}}^{-},\ \nu_{\tilde{A}}^{+} + \nu_{\tilde{B}}^{+} - \nu_{\tilde{A}}^{+}\nu_{\tilde{B}}^{+}]) \quad (10)$$

$$\tilde{A} \ominus \tilde{B} = \{(z, [max_{z=x-y}min\{\mu_{\tilde{A}}^{-}(x), \mu_{\tilde{B}}^{-}(y)\}, max_{z=x-y}min\{\mu_{\tilde{A}}^{+}(x), \mu_{\tilde{B}}^{+}(y)\}],$$
$$[min_{z=x-y}max\{\nu_{\tilde{A}}^{-}(x), \nu_{\tilde{B}}^{-}(y)\}, min_{z=x-y}max\{\nu_{\tilde{A}}^{+}(x), \nu_{\tilde{B}}^{+}(y)\}]) \mid (x,y) \in X\ x\ Y\} \quad (11)$$

$$(\tilde{A})^{\tilde{B}} = ([min\ (\mu_{\tilde{A}}^{-}, \mu_{\tilde{B}}^{-}), min\ (\mu_{\tilde{A}}^{+}, \mu_{\tilde{B}}^{+})], [max\ (\nu_{\tilde{A}}^{-}, \nu_{\tilde{B}}^{-}), max\ (\nu_{\tilde{A}}^{+}, \nu_{\tilde{B}}^{+})]) \quad (12)$$

$$\tilde{A} \oslash \tilde{B} = ([\min(\mu_{\tilde{A}}^{-}, \mu_{\tilde{B}}^{-}), \min(\mu_{\tilde{A}}^{+}, \mu_{\tilde{B}}^{+})],\ [\max(\nu_{\tilde{A}}^{-}, \nu_{\tilde{B}}^{-}), \max(\nu_{\tilde{A}}^{+}, \nu_{\tilde{B}}^{+})]) \quad (13)$$

$$\lambda\tilde{A} = ([1 - (1 - \mu_{\tilde{A}}^{-})^{\lambda},\ 1 - (1 - \mu_{\tilde{A}}^{+})^{\lambda}], [(\nu_{\tilde{A}}^{-})^{\lambda},\ (\nu_{\tilde{A}}^{+})^{\lambda}]),\ \lambda > 0 \quad (14)$$

$$\tilde{A}^{\lambda} = ([(\mu_{\tilde{A}}^{-})^{\lambda},\ (\mu_{\tilde{A}}^{+})^{\lambda}], [1 - (1 - \nu_{\tilde{A}}^{-})^{\lambda},\ 1 - (1 - \nu_{\tilde{A}}^{+})^{\lambda}]),\ \lambda > 0 \quad (15)$$

Aggregation operators are given in the following Eqs. (16-19). Let $\tilde{\alpha_j} = ([\mu_j^{-}, \mu_j^{+}][\nu_j^{-}, \nu_j^{+}])$ $(j = 1,2,\ldots,n)$ be a collection of interval-valued intuitionistic fuzzy numbers and let IIFWA: Qn → Q, if

$$IIFWA_w(\tilde{\alpha_1}, \tilde{\alpha_2}, \ldots, \tilde{\alpha_n}) = w_1\tilde{\alpha_1} \oplus w_2\tilde{\alpha_2} \oplus \ldots \oplus w_n\tilde{\alpha_n} \quad (16)$$

then IIFWA is called an interval-valued intuitionistic fuzzy weighted averaging (IIFWA) operator, where Q is the set of all IVIFNs, $w = (w_1, w_2, \ldots, w_n)$ is the weight vector of the IVIFNs $\tilde{\alpha_j}$ $(j = 1,2,\ldots,n)$, and $w_j > 0$, $\sum_{j=1}^{n} w_j = 1$. The $IIFWA$ operator can be further transformed in to the following form (Xu, 2007):

$$IIFWA_w(\tilde{\alpha_1}, \tilde{\alpha_2}, \ldots, \tilde{\alpha_n}) = (1 - (\prod_{j=1}^{n}(1 - \mu_j^{-}))^{w_j}, 1 - (\prod_{j=1}^{n}(1 - \mu_j^{+}))^{w_j},$$
$$(\prod_{j=1}^{n} \nu_j^{-})^{w_j}, (\prod_{j=1}^{n} \nu_j^{+})^{w_j}) \quad (17)$$

Especially if $w = (1/n, 1/n, \ldots, 1/n)$, then the IIFWA operator reduces to an IIFA (interval-valued intuitionistic fuzzy averaging) operator, where

$$IIFA(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \frac{1}{n}(\tilde{a}_1 \oplus \tilde{a}_2 \oplus \ldots \oplus \tilde{a}_n)$$
$$= \left(\left[1 - (\prod_{j=1}^{n}(1 - \mu_j^{-}))^{\frac{1}{n}}, 1 - (\prod_{j=1}^{n}(1 - \mu_j^{+}))^{\frac{1}{n}}\right], \left[(\prod_{j=1}^{n} \nu_j^{-})^{\frac{1}{n}}, (\prod_{j=1}^{n} \nu_j^{+})^{\frac{1}{n}}\right]\right) \quad (18)$$

Let $\tilde{\alpha_j} = ([\mu_j^{-}, \mu_j^{+}][\nu_j^{-}, \nu_j^{+}])$ $(j = 1,2,\ldots,n)$ be a collection of interval-valued intuitionistic fuzzy numbers and let IIFOWG: Qn → Q, if

$$IIFOWG_w(\tilde{\alpha_1}, \tilde{\alpha_2}, \ldots, \tilde{\alpha_n}) = \tilde{\alpha_1}^{w_1} \otimes \tilde{\alpha_2}^{w_2} \otimes \ldots \otimes \tilde{\alpha_2}^{w_n} \quad (19)$$

then $IIFOWG_w$ is called an interval-valued intuitionistic fuzzy ordered weighted geometric averaging operator, where Q is the set of all IVIFNs, $w = (w_1, w_2, \ldots, w_n)$ is the weight vector of the IVIFNs $\tilde{\alpha_j}$ $(j = 1,2,\ldots,n)$, and $w_j > 0$, $\sum_{j=1}^{n} w_j = 1$. The $IIFOWG$ operator can be further transformed in to the following form (Xu&Chen, 2007):

$$IIFOWG(\tilde{\alpha_1}, \tilde{\alpha_2}, \ldots, \tilde{\alpha_n}) = ([\prod_{j=1}^{n}(\mu_j^{-})^{w_j}, \prod_{j=1}^{n}(\mu_j^{+})^{w_j}],\ [1 - \prod_{j=1}^{n}(1 - \nu_j^{-})^{w_j}, 1 -$$
$$\prod_{j=1}^{n}(1 - \nu_j^{+})^{w_j}] \quad (20)$$

To convert an interval valued intuitionistic fuzzy number $\tilde{\alpha} = ([\mu^{-}, \mu^{+}][\nu^{-}, \nu^{+}])$ to its crisp equivalent, Eq. (21) can be used for defuzzification (Kahraman et al., 2016)

$$S(\tilde{\alpha}) = \left(\frac{\mu^{-} + \mu^{+} + (1 - \nu^{-}) + (1 - \nu^{+}) + \mu^{-} \times \mu^{+} - \sqrt{(1 - \nu^{-}) \times (1 - \nu^{+})}}{4}\right) \quad (21)$$

Interval Valued Intuitionistic Fuzzy Z-Numbers (IVIF-Z)

Let $\tilde{A}_I = ([\mu_{\tilde{A}_I}{}^-, \mu_{\tilde{A}_I}{}^+], [\nu_{\tilde{A}_I}{}^-, \nu_{\tilde{A}_I}{}^+])$ and $\tilde{B}_I = ([\mu_{\tilde{B}_I}{}^-, \mu_{\tilde{B}_I}{}^+], [\nu_{\tilde{B}_I}{}^-, \nu_{\tilde{B}_I}{}^+])$ are two interval-valued intuitionistic fuzzy numbers. $z(\tilde{A}_I, \tilde{B}_I)$ is an interval valued intuitionistic fuzzy-Z (IVIF-Z) number where $\tilde{A}_I$ represents a fuzzy restriction of the values of X variable and $\tilde{B}_I$ is referred to as fuzzy reliability of the fuzzy restriction.

To convert an IVIF-Z number into an IVIF number, first, the reliability can be converted into a crisp number using Eq. (22):

$$\alpha_I = \left(\frac{\mu_{\tilde{B}_I}{}^- + \mu_{\tilde{B}_I}{}^+ + (1-\nu_{\tilde{B}_I}{}^-) + (1-\nu_{\tilde{B}_I}{}^+) + \mu_{\tilde{B}_I}{}^- \times \mu_{\tilde{B}_I}{}^+ - \sqrt{(1-\nu_{\tilde{B}_I}{}^-) \times (1-\nu_{\tilde{B}_I}{}^+)}}{4}\right) \quad (22)$$

The weighted IVIF- Z number can be denoted as $\tilde{Z}^{\alpha_I}$ by adding the weight of the reliability to the restriction:

$$\tilde{Z}^{\alpha_I} = \left\{\langle x, \left(\mu_{\tilde{A}_I{}^{\alpha_I}}(x), \nu_{\tilde{A}_I{}^{\alpha_I}}(x)\right)\rangle \,\middle|\, \mu_{\tilde{A}_I{}^{\alpha_I}}(x) = \alpha_I \mu_{\tilde{A}_I}{}_{(x)}, \nu_{\tilde{A}_I{}^{\alpha_I}}(x) = \alpha_I \nu_{\tilde{A}_I}{}_{(x)}, \mu(x), \nu(x) \in [0,1]\right\} \quad (23)$$

The weighted Z number, in other words, weighted restriction can be converted to an ordinary fuzzy number $\tilde{Z}'$ using Eq. (24) and Eq. (25):

$$\tilde{Z}' = \left\{\langle x, \mu_{Z'}(x)\rangle \,\middle|\, \mu_{Z'}(x) = \left(\mu_{\tilde{A}_I}\left(\frac{x}{\sqrt{\alpha_I}}\right), \nu_{\tilde{A}_I}\left(\frac{x}{\sqrt{\alpha_I}}\right)\right), \mu(x), \nu(x) \in [0,1]\right\} \quad (24)$$

$$\tilde{Z}' = \left(\left[\left(1-(1-\mu_{\tilde{A}}{}^-)^{\sqrt{\alpha_I}}\right), \left(1-(1-\mu_{\tilde{A}}{}^+)^{\sqrt{\alpha_I}}\right)\right], \left[\left((\nu_{\tilde{A}}{}^-)^{\sqrt{\alpha_I}}\right), \left((\nu_{\tilde{A}}{}^+)^{\sqrt{\alpha_I}}\right)\right]\right) \quad (25)$$

IVIF-Z AHP Method

Saaty (1980) proposed Analytic Hierarchy Process to determine the criteria weights for a determined goal and since then it became one of the most used multi-criteria decision-making methods. Many fuzzy extensions of AHP have been proposed by various authors for different levels of uncertainty in the evaluation environment in scaling linguistic assessments (Figure 3).

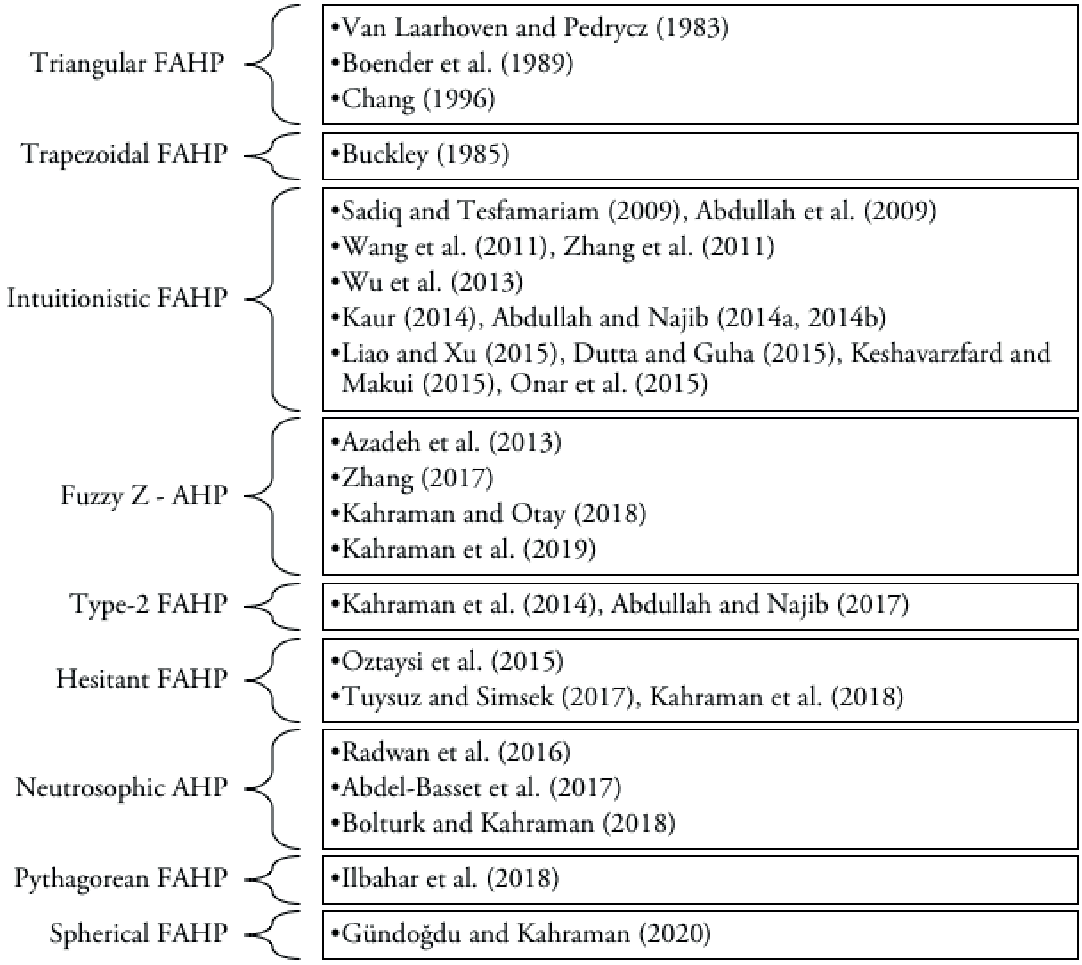

FIgure 3. The timeline of Fuzzy AHP extensions (Ucal Sari and Kahraman, 2020)

In this paper, a new extension of the AHP method that is IVIF-Z AHP is proposed and its steps are determined as follows:

Step 1. Define the problem and construct the hierarchical structure of the problem.

Step 2. Use the scale of linguistic restriction given in Table 3 and the scale of linguistic reliability function given in Table 4 for the pairwise comparisons.

Table 3. Linguistic scale for fuzzy restriction function and corresponding IVIF scales (Dogan et al., 2019)

Linguistic Restriction	IVIF scale of restriction function
Absolutely Less Important (ALI)	([0.1, 0.25], [0.65, 0.75])
Greatly Less Important (GLI)	([0.15, 0.3], [0.6, 0.7])
Moderately Less Important (MLI)	([0.2, 0.35], [0.55, 0.65])
Weakly Less Important (WLI)	([0.25, 0.4], [0.5, 0.6])
Equally Important (EI)	([0.5, 0.5], [0.5, 0.5])
Weakly More Important (WMI)	([0.5, 0.6], [0.25, 0.4])
Moderately More Important (MMI)	([0.55, 0.65], [0.2, 0.35])
Greatly More Important (GMI)	([0.6, 0.7], [0.15, 0.3])
Absolutely More Important (AMI)	([0.65, 0.75], [0.1, 0.25])

Table 4. Linguistic scale for fuzzy reliability function and corresponding IVIF scales

Linguistic Reliability	IVIF scale of reliability function
Absolutely Reliable (AR)	([1, 1], [0, 0])
Strongly Reliable (SR)	([0.7, 0.9], [0, 0.1])
Very Highly Reliable (VHR)	([0.6, 0.8], [0.05, 0.2])
Highly Reliable (HR)	([0.5, 0.7], [0.15, 0.3])
Fairly Reliable (FR)	([0.4, 0.6], [0.25, 0.4])
Weakly Reliable (WR)	([0.3, 0.5], [0.35, 0.5])
Very Weakly Reliable (VWR)	([0.2, 0.4], [0.45, 0.6])
Strongly Unreliable (SU)	([0.1, 0.3], [0.55, 0.7})
Absolutely Unreliable (AU)	([0, 0.2], [0.65, 0.8])

Step 3. Construct the pairwise comparison matrices and fill in them with IVIF-Z numbers using the linguistic terms determined in Step 2 and their corresponding linguistic scales that are determined in Table 3 and Table 4.

Step 4. Transform IVIF-Z numbers to their corresponding equivalent interval valued intuitionistic fuzzy numbers using Eqs. (22-25) and construct transformed interval-valued intuitionistic judgement matrix as given in Eq. (26):

$$\tilde{R} = \begin{bmatrix} ([\mu_{11}^-, \mu_{11}^+], [\nu_{11}^-, \nu_{11}^+]) & ([\mu_{12}^-, \mu_{12}^+], [\nu_{12}^-, \nu_{12}^+]) & \cdots & ([\mu_{1n}^-, \mu_{1n}^+], [\nu_{1n}^-, \nu_{1n}^+]) \\ ([\mu_{21}^-, \mu_{21}^+], [\nu_{21}^-, \nu_{21}^+]) & ([\mu_{22}^-, \mu_{22}^+], [\nu_{22}^-, \nu_{22}^+]) & \vdots & ([\mu_{2n}^-, \mu_{2n}^+], [\nu_{2n}^-, \nu_{2n}^+]) \\ \vdots & \vdots & \ddots & \vdots \\ ([\mu_{n1}^-, \mu_{n1}^+], [\nu_{n1}^-, \nu_{n1}^+]) & ([\mu_{n2}^-, \mu_{n2}^+], [\nu_{n2}^-, \nu_{n2}^+]) & \cdots & ([\mu_{nn}^-, \mu_{nn}^+], [\nu_{nn}^-, \nu_{nn}^+]) \end{bmatrix} \quad (26)$$

Step 5. Check the consistency of each fuzzy pairwise comparison matrix. Assume $\tilde{A} = [\tilde{a}_{ij}]$ is a fuzzy positive pairwise comparison matrix and $A = [a_{ij}]$ is its defuzzified positive pairwise comparison matrix. If the result of the comparisons of $A = [a_{ij}]$ is consistent, then it can imply that the result of the comparisons of $\tilde{A} = [\tilde{a}_{ij}]$ is also consistent.

Step 6. Construct the aggregated interval-valued intuitionistic judgement matrix using $IIFOWG_w$ operator using Eq. (20).

Step 7. Apply steps of IVIF AHP method proposed by Kahraman et al. (2020):

Step 7.1. Calculate the score judgement matrix using Eq. (27) :

$$\tilde{S} = \begin{bmatrix} [\mu_{11}^- - \nu_{11}^+, \mu_{11}^+ - \nu_{11}^-] & \cdots & [\mu_{1n}^- - \nu_{1n}^+, \mu_{1n}^+ - \nu_{1n}^-] \\ \vdots & \ddots & \vdots \\ [\mu_{n1}^- - \nu_{n1}^+, \mu_{n1}^+ - \nu_{n1}^-] & \cdots & [\mu_{nn}^- - \nu_{nn}^+, \mu_{nn}^+ - \nu_{nn}^-] \end{bmatrix} \quad (27)$$

Step 7.2. Calculate the interval multiplicative matrix using Eq. (28):

$$\tilde{A} = (\tilde{a}_{ij})_{nxn} = \begin{bmatrix} [10^{(\mu_{11}^- - \nu_{11}^+)}, 10^{(\mu_{11}^+ - \nu_{11}^-)}] & \cdots & [10^{(\mu_{1n}^- - \nu_{1n}^+)}, 10^{(\mu_{1n}^+ - \nu_{1n}^-)}] \\ \vdots & \ddots & \vdots \\ [10^{(\mu_{n1}^- - \nu_{n1}^+)}, 10^{(\mu_{n1}^+ - \nu_{n1}^-)}] & \cdots & [10^{(\mu_{nn}^- - \nu_{nn}^+)}, 10^{(\mu_{nn}^+ - \nu_{nn}^-)}] \end{bmatrix} \quad (28)$$

Step 7.3. Determine the priority vector of the interval multiplicative matrix by calculating the $\widetilde{w}_i$ interval for each criteria using Eq. (29):

$$\widetilde{w}_i = \left[\frac{\sum_{j=1}^{n} \tilde{a}_{ij}^{-}}{\sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{a}_{ij}^{+}}, \frac{\sum_{j=1}^{n} \tilde{a}_{ij}^{+}}{\sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{a}_{ij}^{-}}\right] \tag{29}$$

Step 7.4. Construct the possibility degree matrix $P = \left(p_{ij}\right)_{mxn}$ using Eq. (30):

$$P\left(w_i \geq w_j\right) = \frac{\min\left\{L_{w_i} + L_{w_j}, \max\left(w_i^{+} - w_j^{-}, 0\right)\right\}}{L_{w_i} + L_{w_j}} \tag{30}$$

where $L_{w_i} = w_i^{+} - w_i^{-}$ and $L_{w_j} = w_j^{+} - w_j^{-}$ and $p_{ij} \geq 0, p_{ij} + p_{ji} = 1, p_{ii} = 0.5$.

Step 7.5. Prioritize the possibility degree matrix $P = \left(p_{ij}\right)_{mxn}$ by Eq. (31)

$$w_i = \frac{1}{n}\left[\sum_{j=1}^{n} p_{ij} + \frac{n}{2} - 1\right] \tag{31}$$

Step 7.6. Normalize the weights vector obtained in Step 7.5.

Interval Valued Intuionistic Fuzzy Z-WASPAS

The WASPAS method was first introduced into the literature in 2012 by Zavadskas et al. (Zavadskas et al., 2012). Combining the weighted sum model and the weighted product model to increase the order accuracy, this method is widely used as an effective decision-making tool due to its simplicity and increased accuracy in ranking alternatives (Sergi and Sari, 2021). Timeline of the fuzzy extensions of WASPAS method is shown in Figure 4.

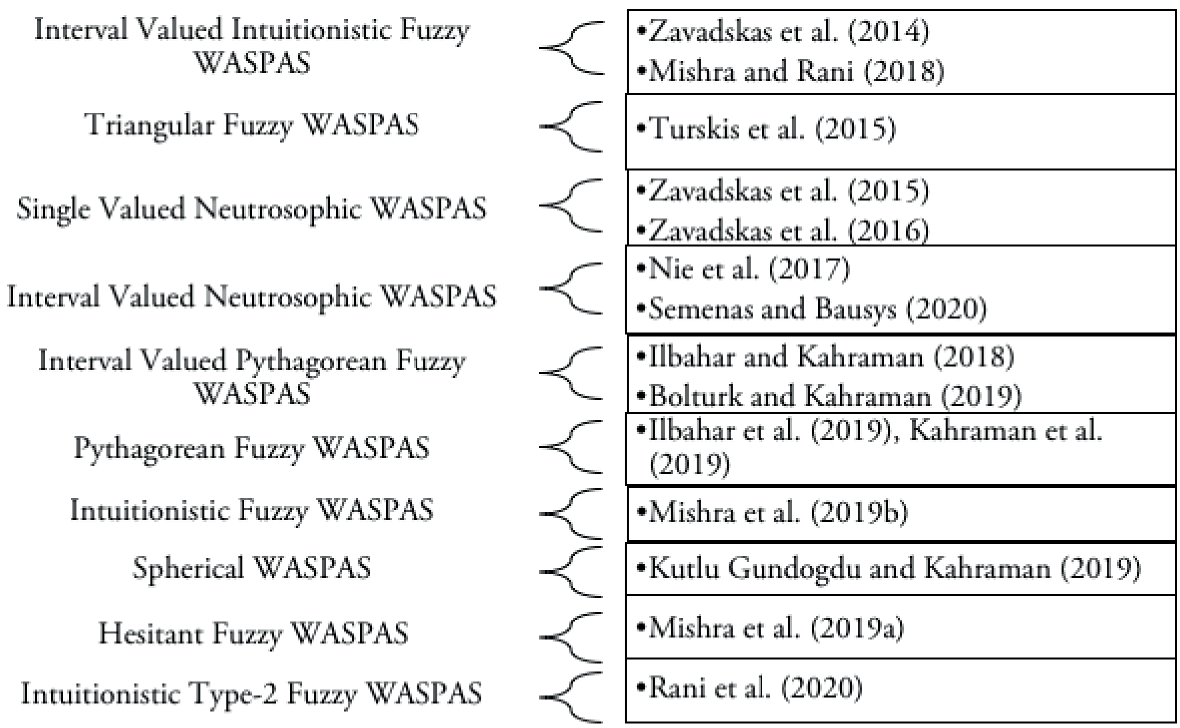

FIgure 4. The timeline of Fuzzy WASPAS extensions

In the following, the steps of proposed IVIF- Z WASPAS method are given step by step:

Step 1. Determine the decision matrix. Use the scale of linguistic restriction function and the scale of linguistic reliability function presented in Table 3 and Table 4.

Step 2. Transform IVIF-Z numbers to their corresponding equivalent interval valued intuitionistic fuzzy numbers.

Step 3. Normalize the decision matrix. For the decisions in which the highest score is preferred or in other words for positive criteria Eq. (32) is used for the normalization:

$$\tilde{\tilde{x}}_{ij} = \frac{\tilde{x}_{ij}}{\max \tilde{x}_{ij}} \tag{32}$$

For the decisions in which the lowest score is preferred or in other words for negative criteria Eq. (33) is used for the normalization:

$$\tilde{\tilde{x}}_{ij} = \frac{\min \tilde{x}_{ij}}{\tilde{x}_{ij}} \tag{33}$$

Score values of $\tilde{x}_{ij}$ can be used to determine the minimum and maximum values using Eq. (21).

Step 4. Apply the methods of weighted sum model and weighted product model.

Step 4.1. Construct weighted normalized decision matrix for weighted sum model (WSM) using Eq. (34) where $\widetilde{w}_j$ is the interval valued intuitionistic fuzzy weight of criterion j:

$$\tilde{\tilde{x}}_{ijWSM} = \tilde{\tilde{x}}_{ij}\widetilde{w}_j \tag{34}$$

Step 4.2. Construct weighted normalized decision matrix for weighted product model (WPM) using Eq. (35):

$$\tilde{\tilde{x}}_{ijWPM} = \tilde{\tilde{x}}_{ij}^{\widetilde{w}_j} \tag{35}$$

Step 5. Calculate the combined utility function values of the WASPAS method for each alternative as in Eq. (36):

$$U_i = \lambda \sum_{j=1}^{n} \tilde{\tilde{x}}_{ij}\widetilde{w}_j + (1-\lambda) \prod_{j=1}^{n} \tilde{\tilde{x}}_{ij}^{\widetilde{w}_j} \tag{36}$$

where λ is determined by the decision maker and belongs to the interval of [0,1].

Step 6. Calculate the score of each alternative by defuzzifying combined utility function values using the score function determined in Eq. (21).

Step 7. Rank the alternatives starting from the highest value of obtained defuzzified values.

APPLICATION: COMPARE AND SELECT THE BEST LMS, HIGHER EDUCATION SYSTEM IN TURKIYE

Learning Management Systems can be used in three different ways, as commercial products (e.g. Blackboard), free open source products (e.g. Moodle) and customized software systems that serve the educational purposes of specific organizations.

In this study, use LMS of higher education institutions in Turkiye are examined, some of the most popular LMS are listed along with universities that are used in Table 5:

Table 5. List of Learning Management Systems in Turkiye

LMS Platform	University	City
Moodle (A1)	Koc University	Istanbul
	Bogazici University	Istanbul
	Ozyegin University	Istanbul
	Bilkent University	Ankara
	Kocaeli University	Kocaeli
	Ege University	Izmir
	Karadeniz Technical University	Trabzon
Canvas (A2)	Eskisehir Technical University	Eskisehir
	Eskisehir Osmangazi University	Eskisehir
	Anadolu University	Eskisehir
	Alanya Alaaddin Keykubat University	Antalya
	Abdullah Gul University	Kayseri
ALMS – Advancity (A3)	Marmara University	Istanbul
	Istanbul Gelisim University	Istanbul
	Gazi University	Ankara
	Akdeniz University	Antalya
	Uludag University	Bursa
	Inonu University	Malatya
Blackboard (A4)	MEF University	Istanbul
	Istanbul Bilgi University	Istanbul
	Koc University	Istanbul
	Hacettepe University	Ankara
	Izmir University of Economics	Izmir
Microsoft Office 365 Teams (A5)	Galatasaray University	Istanbul
	Fenerbahce University	Istanbul
	Istanbul Medipol University	Istanbul
	Dogus University	Istanbul
	Canakkale Onsekiz Mart University	Canakkale
Customized Enterprise LMS (A6)	Ninova – Istanbul Technical University	Istanbul
	CATS – Istanbul Kultur University	Istanbul
	SAUPORT – Sakarya University	Sakarya
	Olives – Cukurova University	Adana

Moodle, which stands for Modular-Object-Oriented-Dynamic-Learning-Environment, is a free and open-source learning management system designed to help educators create online courses. The software can work in any environment under MySQL and PostgreSQL database systems and supporting PHP language such as Linux, Windows, etc. It is used by approximately 246,000,000 users in 235 countries and is available in 82 languages. It has a user-friendly interface and can be used comfortably from both computers and mobile devices. There are an online demo version and supporting system, and its different modules can be easily accessed online.

Canvas, whose open-source version is free, is a learning management system that also offers many paid and closed source services. Canvas LMS has a responsive design, so learners can access them from all operating systems, browsers and even mobile devices. Canvas LMS contains many tools and facilities for e-learning activities.

Academic LMS, namely ALMS, is a completely domestic academic learning management system developed by Advancity that meets all communication and sharing needs of academic staff and students for formal and distance education. 120 institutions including Turkiye's nearly 60 higher education institutions prefer the ALMS that is one of the most used academic learning management systems in Turkiye with an active user base of 800,000. Although it is asynchronous software, it has integration with synchronous virtual classroom applications. It works easily on any mobile device with an internet connection without requiring any extra software.

Blackboard Learn is a virtual learning environment and commercial learning management system that enables online lecturing, learning, community building and knowledge sharing. It has a scalable design that allows course management, customizable open architecture and integration with student information systems and authentication protocols.

Google Classroom is a flexible, secure and easy to use the platform offered by Google as an alternative to Blackboard and Moodle, which can be used by universities as well as non-governmental organizations and all users who have a personal Google Account. This platform offers educators the opportunity to create and upload free online learning resources, to send homework to learners, to collect and evaluate them.

These solutions, which enable all training processes to be easily managed at a single point, create efficiency and savings in educational processes while facilitating the work of training units and providing automation and digitalization. Institutions consider several criteria to choose the most suitable system for them. From a broader perspective, it can be said that the factors affecting the choice of learning management system are usability, integration, support services, accessibility, security, reduced cost/fee and personalization. Since these factors generally determine the characteristics of the system to be selected, these criteria were selected for the evaluation of the most appropriate LMS (Table 6).

Table 6. LMS criteria and descriptions

Criteria	Reference	Description
Usability (C1)	(Unal and Unal, 2011)	Easy to use the system
Integration (C2)	(Bilgic and Tuzun, 2020)	Easy integration and compatibility with different add-ons and platforms
Support Services (C3)	(Mtebe, 2015)	Assistance support for students and instructors through phone, email, online FAQ, user community, live chat, training videos etc.
Accessibility (C4)	(Chaubey and Bhattacharya, 2015)	Accessible for everyone from any device or browser
Security (C5)	(Muhammad and Cavus, 2017)	Ensuring user authentication and data integrity
Reduced Costs / Fee (C6)	(Kaya, 2012)	Includes common and setup fees and some other charges
Personalization (C7)	(Petrova, 2019)	Personal assigments, ability of grouping people, special assignments to groups

Generally, it was not possible to reach users who knew all the systems examined in the evaluation in detail, as the users were familiar with only some of the alternatives investigated in the study. In order to overcome this situation, which can be stated as the biggest limitation of the study, the decision makers were selected from among the professors who actively used at least three of these systems and had administrative duties.

After conducting the literature research and asking the expert's opinion, seven criteria as usability, integration, support services, accessibility, security, reduced costs / fee, personalization were selected for evaluation of LMS alternatives. Opinions on the determined criteria were received by a group of experts who use at least one of the LMS alternatives and they were asked to make a pairwise comparison through survey questions. The evaluations of the experts for pairwise comparisons are collected individually. Besides that, to see the difference between the outcomes of the aggregation of several evaluations and evaluations done in a focus group with consensus technique, the same experts agreed on a common comparison matrix as a result of a meeting. In the pairwise comparison matrices linguistic scales in Table 3 and 4 are used. The pairwise comparison results for restriction and reliability functions obtained from the individual evaluations and group evaluation are shown in Table 7.

Table 7. Pairwise comparison matrix - restriction and reliability of decision criteria

		Restriction Evaluations							Reliability Evaluations						
		C1	C2	C3	C4	C5	C6	C7	C1	C2	C3	C4	C5	C6	C7
EXPERT 1	C1	EI	MMI	AMI	GMI	WLI	MMI	EI	AR	SR	HR	SR	VHR	SR	SR
	C2	MLI	EI	EI	ALI	ALI	WLI	MLI	SR	AR	VHR	VHR	VHR	VHR	VHR
	C3	ALI	EI	EI	MLI	ALI	EI	MLI	HR	VHR	AR	AR	FR	FR	FR
	C4	GLI	AMI	MMI	EI	MLI	WMI	WLI	SR	VHR	AR	AR	SR	SR	FR
	C5	WMI	AMI	AMI	MMI	EI	AMI	WMI	VHR	VHR	FR	SR	AR	SR	AR
	C6	MLI	EMI	EI	WLI	ALI	EI	GLI	SR	VHR	FR	SR	SR	AR	HR
	C7	EI	MMI	MMI	WMI	WLI	GMI	EI	SR	VHR	FR	FR	AR	HR	AR
EXPERT 2	C1	EI	GMI	EI	EI	GMI	EI	GMI	AR	VHR	SR	AR	HR	HR	FR
	C2	GLI	EI	WLI	WLI	EI	WLI	EI	VHR	AR	SR	FR	SR	FR	FR
	C3	EI	WMI	EI	EI	WMI	EI	WMI	SR	SR	AR	AR	HR	FR	VHR
	C4	EI	WMI	EI	EI	WMI	EI	GMI	AR	FR	AR	AR	SR	AR	SR
	C5	GLI	EI	WLI	WLI	EI	WLI	EI	HR	SR	HR	SR	AR	FR	AR
	C6	EI	WMI	EI	EI	WMI	EI	WMI	HR	FR	FR	AR	FR	AR	AR
	C7	GLI	EI	WLI	GLI	EI	WLI	EI	FR	FR	VHR	SR	AR	AR	AR
EXPERT 3	C1	EI	GLI	GMI	WMI	WLI	WMI	WMI	AR	AR	VHR	SR	FR	HR	SR
	C2	GMI	EI	AMI	GMI	WMI	MMI	MMI	AR	AR	VHR	SR	HR	FR	AR
	C3	GLI	ALI	EI	WLI	ALI	WLI	WLI	VHR	VHR	AR	FR	HR	AR	VHR
	C4	WLI	GLI	WMI	EI	MLI	WMI	WLI	SR	SR	FR	AR	VHR	FR	FR
	C5	WMI	WLI	AMI	MMI	EI	AMI	MMI	FR	HR	HR	VHR	AR	SR	FR
	C6	WLI	MLI	WMI	WLI	ALI	EI	WLI	HR	FR	AR	FR	SR	AR	FR
	C7	WLI	MLI	WMI	WMI	MLI	WMI	EI	SR	AR	VHR	FR	FR	FR	AR
EXPERT 4	C1	EI	WLI	GLI	WLI	GLI	GLI	MLI	AR	AR	SR	VHR	VHR	FR	FR
	C2	WMI	EI	WLI	WLI	MLI	MLI	MLI	AR	AR	AR	SR	VHR	SR	FR
	C3	GMI	WMI	EI	MMI	MLI	EI	WLI	SR	AR	AR	HR	VHR	HR	FR
	C4	WMI	WMI	MLI	EI	GLI	WLI	WLI	VHR	SR	HR	AR	VHR	SR	AR
	C5	GMI	MMI	MMI	GMI	EI	WMI	MMI	VHR	VHR	VHR	VHR	AR	FR	HR
	C6	GMI	MMI	EI	WMI	WLI	EI	EI	FR	SR	HR	SR	FR	AR	SR
	C7	MMI	MMI	WMI	WMI	WML	EI	EI	FR	FR	FR	AR	HR	SR	AR
EXPERT 5	C1	EI	WLI	EI	MLI	GLI	MLI	MLI	AR	VHR	FR	VHR	AR	HR	AR
	C2	WMI	EI	MMI	WLI	MLI	WLI	WLI	VHR	AR	HR	FR	SR	FR	AR
	C3	EI	MLI	EI	MLI	GLI	MLI	MLI	FR	HR	AR	AR	FR	AR	VHR
	C4	MMI	WMI	MMI	EI	GLI	WLI	WLI	VHR	FR	AR	AR	SR	AR	AR
	C5	GMI	MMI	GMI	GMI	EI	WMI	MMI	AR	SR	FR	SR	AR	AR	VHR
	C6	MMI	WMI	MMI	WMI	WLI	EI	WMI	HR	FR	AR	AR	AR	AR	HR
	C7	MMI	WMI	MMI	WMI	MLI	WLI	EI	AR	AR	VHR	AR	VHR	HR	AR
GROUP OF EXPERTS	C1	EI	WLI	EI	MLI	GLI	MLI	MLI	AR	FR	WR	HR	SR	HR	VHR
	C2	WMI	EI	MMI	WLI	MLI	WLI	WLI	FR	AR	VHR	FR	VHR	FR	HR
	C3	EI	MLI	EI	MLI	GLI	MLI	MLI	WR	VHR	AR	HR	SR	HR	HR
	C4	MMI	WMI	MMI	EI	GLI	WLI	WLI	HR	FR	HR	AR	SR	FR	HR
	C5	GMI	MMI	GMI	GMI	EI	WMI	MMI	SR	VHR	SR	SR	AR	VHR	VHR
	C6	MMI	WMI	MMI	WMI	WLI	EI	WMI	HR	FR	HR	FR	VHR	AR	VHR
	C7	MMI	WMI	MMI	WMI	MLI	WLI	EI	VHR	HR	HR	HR	VHR	VHR	AR

All consistency ratios for the pairwise matrices are calculated less than 0.1, so comparisons are consistent. The linguistic statements in pairwise comparison matrices are converted to interval valued intuitionistic fuzzy reliability and restriction matrices using the scales in Table 2 and Table 3. Then, IVIF-Z evaluations for each comparison are transformed to IVIF numbers using Eqs. (22-25) and transformed interval-valued intuitionistic judgement matrices are obtained. For the aggregated analysis, individual evaluations of 5 experts are aggregated using Eq. (20) and the aggregated interval-valued intuitionistic judgement matrice is obtained and given in Table 8.

Table 8. Aggregated interval-valued intuitionistic judgement matrice

	C1	C2	C3	C4	C5	C6	C7
C1	([0.5,0.5], [0.5,0.5])	([0.281,0.415], [0.367,0.472])	([0.328,0.4], [0.299,0.355])	([0.335,0.44], [0.363,0.449])	([0.202,0.331], [0.404,0.494])	([0.297,0.41], [0.391,0.478])	([0.321,0.429], [0.373,0.46])
C2	([0.303,0.431], [0.341,0.455])	([0.5,0.5], [0.5,0.5])	([0.338,0.434], [0.313,0.392])	([0.206,0.338], [0.396,0.486])	([0.209,0.325], [0.425,0.5])	([0.232,0.356], [0.381,0.471])	([0.255,0.363], [0.39,0.464])
C3	([0.225,0.321], [0.379,0.433])	([0.251,0.362], [0.38,0.462])	([0.5,0.5], [0.5,0.5])	([0.308,0.437], [0.473,0.563])	([0.119,0.229], [0.382,0.457])	([0.251,0.308], [0.353,0.383])	([0.182,0.285], [0.33,0.405])
C4	([0.311,0.421], [0.381,0.467])	([0.344,0.454], [0.255,0.371])	([0.432,0.536], [0.359,0.464])	([0.5,0.5], [0.5,0.5])	([0.191,0.327], [0.466,0.562])	([0.338,0.439], [0.366,0.452])	([0.206,0.31], [0.304,0.379])
C5	([0.351,0.461], [0.247,0.362])	([0.396,0.478], [0.263,0.349])	([0.354,0.439], [0.153,0.248])	([0.434,0.542], [0.224,0.348])	([0.5,0.5], [0.5,0.5])	([0.431,0.539], [0.226,0.35])	([0.529,0.607], [0.281,0.393])
C6	([0.342,0.448], [0.355,0.441])	([0.352,0.455], [0.258,0.373])	([0.353,0.379], [0.276,0.313])	([0.338,0.439], [0.366,0.452])	([0.178,0.321], [0.469,0.567])	([0.5,0.5], [0.5,0.5])	([0.261,0.354], [0.329,0.404])
C7	([0.317,0.428], [0.374,0.46])	([0.359,0.445], [0.294,0.382])	([0.313,0.396], [0.198,0.294])	([0.272,0.362], [0.229,0.324])	([0.251,0.386], [0.531,0.614])	([0.3,0.387], [0.299,0.372])	([0.5,0.5], [0.5,0.5])

Normalized weights of the criteria are calculated by using Eqs. (27-31), and these weights are shown in Table 9.

Table 9. Normalized weights of decision criteria based on aggregated evaluations

	$\widetilde{w}_i$	w_i	Normalized weights	Rank
Usability	([0.091,0.191])	0.814	0.136	5
Integration	([0.083,0.178])	0.764	0.127	7
Support Services	([0.079,0.157])	0.689	0.115	6
Accessibility	([0.094,0.203])	0.852	0.142	4
Security	([0.141,0.313])	1.131	0.188	1
Reduced Costs / Fee	([0.099,0.201])	0.861	0.143	3
Personalization	([0.101,0.211])	0.889	0.148	2

The same procedure is followed for the pairwise comparison matrix that is constructed using consensus method and the normalized weights of the criteria are calculated as shown in Table 10.

Table 10. Normalized weights of decision criteria for the group evaluations

	$\widetilde{w}_i$	w_i	Normalized weights	Rank
Usability	([0.062,0.118])	0.566	0.094	5
Integration	([0.085,0.18])	0.811	0.135	7
Support Services	([0.06,0.115])	0.551	0.092	6
Accessibility	([0.095,0.203])	0.889	0.148	4
Security	([0.157,0.366])	1.183	0.197	1
Reduced Costs / Fee	([0.116,0.252])	1.028	0.171	2
Personalization	([0.106,0.233])	0.972	0.162	3

When the results obtained with the combined individual evaluations and the results of the analysis using group evaluation are compared, it is seen that the order of criterion weights is close to each other. Only the order of importance of reduced cost and personalization criteria has shifted.

Since the results obtained by the aggregation of individual assessments use more information in expressing uncertainty, the weights obtained in Table 9 will be used in the continuation of the study. According to fuzzy IVIF-Z-AHP results, security is determined as the most important criterion where integration is determined as the least important criterion. After obtaining the criterion weights with fuzzy IVIF-Z-AHP, the next step is to evaluate the alternatives by using the fuzzy IVIF-Z-WASPAS method.

The biggest limitation experienced during the alternative evaluation was the inability to find instructors who are familiar with all the alternatives. For this reason, it was decided that it would be more appropriate to take joint decisions in the focus group meeting, which was formed by the experts involved in the evaluation of the alternatives. The decision matrix which is shown in Table 11, is determined using the scale of linguistic restriction function and reliability function, according to Step 1 of the proposed fuzzy IVIF-Z WASPAS.

Table 11. Decision matrix with linguistic terms for restriction and reliability function

	Restriction Evaluations						Reliability Evaluations					
	A1	A2	A3	A4	A5	A6	A1	A2	A3	A4	A5	A6
C1	WMI	MMI	MMI	WMI	GMI	GMI	FR	HR	VHR	HR	SR	HR
C2	GMI	AMI	WMI	MMI	MMI	MLI	VHR	SR	FR	HR	HR	HR
C3	WMI	WLI	MLI	WLI	WMI	WMI	WR	FR	FR	WR	FR	FR
C4	GMI	MMI	MMI	GMI	MMI	MMI	HR	HR	HR	HR	VHR	VHR
C5	MMI	GMI	WMI	MMI	GMI	WMI	FR	HR	FR	VHR	WR	FR
C6	GMI	MMI	GMI	GMI	WMI	GMI	HR	HR	FR	FR	HR	HR
C7	AMI	MMI	WMI	GMI	GMI	WLI	SR	VHR	HR	HR	FR	VHR

Linguistic terms are converted to their corresponding IVIF-Z numbers by using IVIF scales in Table 3 and 4. Then, IVIF-Z numbers are transformed to their corresponding equivalent interval valued intuitionistic fuzzy numbers by using Eqs. (22-25) and the initial decision matrix is constructed as shown in Table 12.

Table 12. Initial decision matrix with IVIF numbers

	A1	A2	A3	A4	A5	A6
C1	([0.346, 0.416], [0.173, 0.277])	([0.420, 0.496], [0.153, 0.267])	([0.457, 0.540], [0.166, 0.291])	([0.381, 0.458], [0.191, 0.305])	([0.535, 0.624], [0.134, 0.268])	([0.458, 0.534], [0.114, 0.229])
C2	([0.498, 0.581], [0.125, 0.249])	([0.580, 0.669], [0.089, 0.223])	([0.346, 0.416], [0.173, 0.277])	([0.420, 0.496], [0.153, 0.267])	([0.420, 0.496], [0.153, 0.267])	([0.153, 0.267], [0.420, 0.496])
C3	([0.309, 0.371], [0.155, 0.247])	([0.173, 0.277], [0.346, 0.416])	([0.139, 0.242], [0.381, 0.450])	([0.155, 0.247], [0.309, 0.371])	([0.346, 0.416], [0.173, 0.277])	([0.346, 0.416], [0.173, 0.277])
C4	([0.458, 0.534], [0.114, 0.229])	([0.420, 0.496], [0.153, 0.267])	([0.420, 0.496], [0.153, 0.267])	([0.458, 0.534], [0.114, 0.229])	([0.457, 0.540], [0.166, 0.291])	([0.457, 0.540], [0.166, 0.291])
C5	([0.381, 0.450], [0.139, 0.242])	([0.458, 0.534], [0.114, 0.229])	([0.346, 0.416], [0.173, 0.277])	([0.457, 0.540], [0.166, 0.291])	([0.371, 0.433], [0.093, 0.186])	([0.346, 0.416], [0.173, 0.277])
C6	([0.458, 0.534], [0.114, 0.229])	([0.420, 0.496], [0.153, 0.267])	([0.416, 0.485], [0.104, 0.208])	([0.416, 0.485], [0.104, 0.208])	([0.381, 0.458], [0.191, 0.305])	([0.458, 0.534], [0.114, 0.229])
C7	([0.580, 0.669], [0.089, 0.223])	([0.457, 0.540], [0.166, 0.291])	([0.381, 0.458], [0.191, 0.305])	([0.458, 0.534], [0.114, 0.229])	([0.416, 0.485], [0.104, 0.208])	([0.208, 0.332], [0.415, 0.498])

Since all criteria in the initial decision matrix are benefit criteria, the maximum of the alternative scores for each criterion to be maximum is taken as the reference value. Then, the normalized decision matrix is obtained by using Eqs. (32-33) for normalization. The weighted normalized decision matrices for the weighted sum model and weighted product model are constructed using Eqs. (34-35). Then, the combined utility function values for each alternative are calculated using Eq. (36) as shown in Table 13 depending on WSM and WPM values, where λ is determined by the decision-maker to be 0.5.

Table 13. WSM, WPM and combined utility function values

	WSM	WPM	
A1	([0.441, 0.519] , [0.128, 0.244])	([0.426, 0.501] , [0.132, 0.245])	([0.434, 0.51] , [0.13, 0.245])
A2	([0.433, 0.516] , [0.15, 0.27])	([0.407, 0.495] , [0.165, 0.277])	([0.42, 0.506] , [0.157, 0.274])
A3	([0.369, 0.446] , [0.177, 0.29])	([0.346, 0.431] , [0.191, 0.297])	([0.358, 0.438] , [0.184, 0.293])
A4	([0.408, 0.485] , [0.153, 0.268])	([0.385, 0.468] , [0.164, 0.274])	([0.396, 0.476] , [0.158, 0.271])
A5	([0.42, 0.495] , [0.142, 0.262])	([0.413, 0.487] , [0.146, 0.264])	([0.417, 0.491] , [0.144, 0.263])
A6	([0.357, 0.442] , [0.199, 0.317])	([0.325, 0.422] , [0.236, 0.34])	([0.342, 0.432] , [0.217, 0.329])

Finally, the score of each alternative is determined by defuzzifying the values of the combined utility function with Eq (21) and the alternatives are ranked starting from the highest value to the lowest one. The score values and ranks of the alternatives are listed in Table 14.

Table 14. Ranking of the LMS alternatives

	Score values	Ranking
Moodle	0.4948	1
Canvas	0.4811	2
ALMS	0.4290	5
Blackboard	0.4622	4
Microsoft Office	0.4777	3
Customized LMS	0.4128	6

The results obtained in Table 14 indicate that Moodle is the most appropriate platform among the LMS platforms compared in this study for the universities in Turkiye. Also, it is observed that the second and third ranked alternatives, Canvas and MS Office, have very close score values.

To validate the proposed method, ordinary fuzzy AHP and WASPAS methods are performed using same evaluations.The results of the ordinary fuzzy AHP and IVIF-Z AHP are compared in Table 15.

Table 15. Comparison of the IVIF-Z AHP and Fuzzy AHP results

	IVIF-Z AHP		Fuzzy AHP	
	$\tilde{w}_i$	Normalized weights	Fuzzy Weights	Defuzzified Normalized Weights
Usability	([0.091,0.191])	0.136	([0.057, 0.116, 0.251])	0.116
Integration	([0.083,0.178])	0.127	([0.043, 0.091, 0.222])	0.096
Support Services	([0.079,0.157])	0.115	([0.036, 0.071, 0.152])	0.071
Accessibility	([0.094,0.203])	0.142	([0.05, 0.118, 0.281])	0.121
Security	([0.141,0.313])	0.189	([0.144, 0.324, 0.673])	0.314
Reduced Costs / Fee	([0.099,0.201])	0.143	([0.056, 0.125, 0.279])	0.125
Personalization	([0.101,0.211])	0.148	([0.065, 0.157, 0.358])	0.158

Although the ranking of the criteria according to their importance remained the same with the proposed method and ordinary fuzzy AHP, the difference between the importance weights of the criteria has decreased with the effect of the reliability of the evaluators in the proposed method.

The results of the ordinary fuzzy WASPAS and IVIF-Z WASPAS are compared in Table 16.

Table 16. Comparison of the IVIF-Z WASPAS and Fuzzy WASPAS results

	IVIF-Z WASPAS		Fuzzy WASPAS	
	Normalized Score values	Ranking	Normalized Score Values	Ranking
Moodle	0.1794381	1	0.204513	2
Canvas	0.1744789	2	0.183033	3
ALMS	0.1555592	5	0.122557	5
Blackboard	0.1676134	4	0.170527	4
Microsoft Office	0.173225	3	0.204745	1
Customized LMS	0.1496854	6	0.114625	6

According to the results given in Table 16, the relative ranking of the alternatives remains same except "Microsoft Office" alternative. Again IVIF-Z WASPAS results in closer score values between alternatives than the ordinary fuzzy WASPAS method because of the reliabilities of the linguistic evaluations. The results of the comparison showed that, as expected, rankings are formed that are close to each other but differ under the effect of the additional uncertainty taken into account.

SENSITIVITY ANALYSIS

One-at-a time sensitivity analysis has been performed for investigating the robustness and validation of the proposed IVIF-Z CODAS methodology. When the weight of the "usability" criterion is taken into account and the weights of the other criteria are updated according to their relative importance, the first change in the rankings of alternatives occurs after a 24% increase or 89% decrease, where the first alternative remains the same. The first alternative changes only after the weight of the "usability" criterion is increased by 65%. The results of the sensitivity analysis for each criterion are given in Table 17.

Table 17. Sensitiviy Analysis Results

	Q1	Rank	Q2	Rank	Q3	Rank	Q4	Rank	Q5	Rank	Q6	Rank	Q7	Rank
Decrease in the weight of the criterion that first affects the alternative ranking	89%	1 2 5 4 3 6	18%	1 3 5 4 2 6	73%	1 2 5 3 4 6	never		23%	1 3 5 4 2 6	61%	1 3 5 4 2 6	67%	1 2 6 4 3 5
Decrease in the weight of the criterion that first affects the first-ranked alternative	never		never		85%	2 1 5 3 4 6	never		never		never		80%	2 1 6 4 3 5
The increase in the weight of the criterion that first affects the alternative ranking	24%	1 3 5 4 2 6	11%	2 1 5 4 3 6	16%	1 3 5 4 2 6	78%	1 3 5 4 2 6	77%	2 1 5 4 3 6	165%	1 2 5 3 4 6	197%	1 2 5 3 4 6
The increase in the weight of the criterion that first affects the alternative in the first place	65%	2 3 5 4 1 6	110%	2 1 5 4 3 6	279%	2 4 6 5 1 3	605%	2 6 5 1 4 3	77%	2 1 5 4 3 6	600%	2 5 3 4 6 1		never

According to these results, due to the fact that the weights of the alternatives are close to each other, the decrease or increase in the weights of the alternatives affects the selected alternative only with very large percentage changes. It has also been observed that the reduction of the alternative weights hardly changes the first-order alternative. This shows that the results of the study are robust.

CONCLUSION AND FURTHER RESEARCH

With the changing dynamics, online education has become more common. The effect of the tools used, especially the LMS platforms, on the quality of education cannot be denied. Therefore, in this study, it is aimed to examine the features expected from LMS platforms and to compare existing LMS platforms in line with these features. For this purpose, IVIF-Z numbers are defined for the first time in this study, and AHP and WASPAS methods are adapted as the proposed new fuzzy extension. The results of the proposed methods are compared with the ordinary fuzzy methods for validation. Additionally, one-at-a time sensitivity analysis has been performed for investigating the robustness and validation of the proposed methodology.

The most important limitation of this study is the inability to reach an expert who uses all alternatives in his/her lectures. In order to minimize the effect of this limitation, the group decision making process was preferred for the evaluation of alternatives.

For further researches, it is suggested to apply IVIF-Z number scales to the other multicriteria decision-making methods. It is also suggested to compare several fuzzy extensions of the same methods to determine the effects of the amount of uncertain information considered in the analysis.

Authors' Note: This research is a part of PhD study of Duygu Sergi, doctoral candidate in the Department of Industrial Engineering. The research is funded by Istanbul Technical University, grant number 44406.

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