Communications in Advanced Mathematical Sciences
Vol. 6, No. 1, 60-66, 2023
Research Article
e-ISSN: 2651-4001
DOI: 10.33434/cams.1223523

Some Relations between Stieltjes Transform and Hankel Transform with Applications

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Abstract
In the present paper four theorems connecting Stieltjes transform and Hankel transform are established. The theorems are general in nature. Four integral formulae involving special functions are obtained with the help of these theorems. Otherwise it is very difficult to evaluate such type of integrals. Other several integrals may be evaluated with the help of these theorems.

Keywords: Bessel functions, Hankel transform, Stieltjes transform, Struve's functions
2010 AMS: 44A05

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Received: 23 December 2022, **Accepted:** 29 March 2023, **Available online:** 31 March 2023
How to cite this article: TV. Kumar, *Some Relations between Stieltjes Transform and Hankel Transform with Applications*, Commun. Adv. Math. Sci., (6)1 (2023) 60-66.

1. Introduction

Several authors have made significant contributions for the development of integral transforms through a series of papers. Among other eminent authors, Bhonsle [1, 2], Sharma [5] Gupta and Agrawal [6], Goyal and Vasishta [7], Goyal and Jain [8], Saxena [14], Srivastava [15, 16, 18], Srivastava and Vyas [17], Srivastava and Tuan [19], Srivastava and Y*ü*rekli [20] and Yakubovich and Martins [21] have studied and explored Laplace, Meijer, Stieltjes, $H-$ function, Kontorovitch-Lebdev and Hankel transforms at large in the form of generalizations, convolution and interconnecting theorems.
Bhonsle [1, 2], Sharma [5], Saxena [14], Srivastava [15, 16], Srivastava and Vyas [17] have obtained integral formulae involving Legendre functions of the first kind, Bessel functions of the first kind and modified Bessel functions of the second kind.
In the present paper we have obtained four integral formulae involving Bessel functions of the first kind and second kind, modified Bessel functions of the first kind and second kind, Struve's functions and Anger functions.
Now, we define the Stieltjes transform and Hankel transform.

Definition 1.1. *The Stieltjes transform [4, 8, 19] of a function $f(x) \in L(0,\infty)$ is defined in the following manner.*

$$G(f;\, y) = \int_0^\infty (x+y)^{-1}\, f(x)dx,$$

where y is a complex variable.

Definition 1.2. *The Hankel transform [4, 5, 16] of order v of a function $f(x) \in L(0,\infty)$ is defined in the following manner.*

$$h_v(f;\, \zeta) = \int_0^\infty (\zeta x)^{1/2}\, J_v(\zeta x)\, f(x)dx, \quad \zeta > 0,$$

where $J_v(z)$ stands for the Bessel function of the first kind ([3], Page 4, Equation (2)).

2. Main Theorems

In this section we establish four theorems connecting Stieltjes transform and Hankel transform.

Theorem 2.1. *If $\zeta > 0$, $-1 <Re(v) < 1/2$ and $|arg\ y| < \pi$, then*

$$G\{x^{v+1/2}\ f(x);\ y\} = \int_0^\infty K(y,\ \zeta)\ h_v(f;\ \zeta)d\zeta, \tag{2.1}$$

where

$$K(y,\ \zeta) = 2^v \pi^{-1/2}\ \zeta^{-v-1/2}\ \Gamma(v+1/2) + \zeta^{1/2}\ 2^{-1}\pi\ y^{v+1} sec(v\pi)\ [Y_{-v}(\zeta y) - H_{-v}(\zeta y)],$$

where $Y_{-v}(z)$ and $H_{-v}(z)$ stand for the Bessel function of the second kind ([3], Page 4, Equation (4)) and Struve's function ([3], Page 38, Equation (55)) respectively.

Proof. We have by the Hankel inversion theorem [13] that

$$f(x) = \int_0^\infty (\zeta x)^{1/2}\ h_v(f;\ \zeta) J_v(\zeta x) d\zeta. \tag{2.2}$$

Hence

$$G\{x^{v+1/2}\ f(x);\ y\} = \int_0^\infty \zeta^{1/2}\ h_v(f;\ \zeta)\ G\{x^{v+1}\ J_v(\zeta x);\ y\} d\zeta. \tag{2.3}$$

The change of order of integration is justified because $\zeta > 0$, $-1 <$Re$(v) < 1/2$ and $J_v(\zeta x)$ is a bounded function for both the variables for Landau's bounds [9] (see also [10]) i.e

$$|J_v(x)| \le b_L v^{-1/3}, \quad b_L := 2^{1/3} \sup_{x\in R_+} (\mathbf{Ai}(x)) \tag{2.4}$$

and

$$|J_v(x)| \le c_L |x|^{-1/3}, \quad c_L := \sup_{x\in R_+} (J_0(x)), \tag{2.5}$$

where $\mathbf{Ai}(x)$ stands for the familiar Airy function.
Now, using the following result ([4], Page 224, Equation (4)) in (2.3)

$$G\{x^{v+1}\ J_v(ax);\ y\} = 2^v \pi^{-1/2} a^{-v-1}\ \Gamma(v+1/2) + 2^{-1}\pi\ y^{v+1} sec(v\pi)[Y_{-v}(ay) - H_{-v}(ay)], \tag{2.6}$$

provided that $a > 0$, $-1 <$Re$(v) < 1/2$ and $|arg\ y| < \pi$ we arrive at the desired result (2.1), where $\zeta > 0$, $-1 <$Re$(v) < 1/2$ and $|arg\ y| < \pi$. □

Theorem 2.2. *If $\zeta > 0$, $Re(v) > -1$ and $|arg\ y| < \pi$, then*

$$G\{x^{-1/2}\ f(x);\ y\} = \int_0^\infty K(y,\ \zeta)\ h_v(f;\ \zeta) d\zeta, \tag{2.7}$$

where

$$K(y,\ \zeta) = \zeta^{1/2}\ \pi\ cosec(v\pi)\ [\boldsymbol{J}_v(\zeta y) - J_v(\zeta y)],$$

where $\boldsymbol{J}_v(z)$ and $J_v(z)$ stand for the Anger's function ([3], Page 35, Equation (33)) and Bessel function of the first kind ([3], Page 4, Equation (2)) respectively.

Proof. Again, by (2.2) we have that

$$G\{x^{-1/2}\ f(x);\ y\} = \int_0^\infty \zeta^{1/2}\ h_v(f;\ \zeta)\ G\{J_v(\zeta x);\ y\} d\zeta. \tag{2.8}$$

The change of order of integration is justified because $\zeta > 0$, Re$(v) > -1$ and $J_v(\zeta x)$ is a bounded function for both the variables for Landau's bounds [9, 10] (see (2.4) and (2.5)).
Now, using the following result ([4], Page 224, Eq. (2)) in (2.8)

$$G\{J_v(ax);\ y\} = \pi\ cosec(v\pi)\ [\mathbf{J}_v(ay) - J_v(ay)],$$

provided that $a > 0$, Re$(v) > -1$ and $|arg\ y| < \pi$ we arrive at the desired result (2.7), where $\zeta > 0$, Re$(v) > -1$ and $|arg\ y| < \pi$. □

Theorem 2.3. *If $0 < a < \zeta$, $-1 <Re(v) < 3/2$ and $|arg\ y| < \pi$, then*

$$G\{x^{v/2-3/4}\ sin(ax^{1/2})\ f(x^{1/2});\ y\} = \int_0^\infty K(y,\ \zeta)\ h_v(f;\ \zeta)d\zeta, \tag{2.9}$$

where

$$K(y,\ \zeta) = 2\ \zeta^{1/2}\ y^{v/2-1/2}\ sinh(ay^{1/2})\ K_v(\zeta y^{1/2}),$$

where $K_v(z)$ stands for the modified Bessel function of the second kind or Basset's function ([3], Page 5, Equation (13)).

Proof. Again, by (2.2) we have that

$$G\{x^{v/2-3/4}\ sin(ax^{1/2})\ f(x^{1/2});\ y\} = \int_0^\infty \zeta^{1/2}\ h_v(f;\ \zeta)\ G\{x^{v/2-1/2}\ sin(ax^{1/2})\ J_v(\zeta x^{1/2});\ y\}d\zeta. \tag{2.10}$$

The change of order of integration is justified because $0 < a < \zeta$, $-1 <\text{Re}(v) < 3/2$ and $J_v(\zeta x)$ is a bounded function for both the variables for Landau's bounds [9, 10] (see (2.4) and (2.5)).
Now, using the following result ([4], Page 226, Equation (18)) in (2.10)

$$G\{x^{v/2-1/2}\ sin(ax^{1/2})\ J_v(bx^{1/2});\ y\} = 2\ y^{v/2-1/2}\ sinh(ay^{1/2})K_v(by^{1/2}), \tag{2.11}$$

provided that $0 < a < b$, $-1 <\text{Re}(v) < 3/2$ and $|arg\ y| < \pi$ we arrive at the desired result (2.9), where $0 < a < \zeta$, $-1 <\text{Re}(v) < 3/2$ and $|arg\ y| < \pi$. □

Theorem 2.4. *If $0 < \zeta < a$, $Re(v) > -1/2$ and $|arg\ y| < \pi$, then*

$$G\{x^{-v/2-1/4}\ sin(ax^{1/2})\ f(x^{1/2});\ y\} = \int_0^\infty K(y,\ \zeta)\ h_v(f;\ \zeta)d\zeta, \tag{2.12}$$

where

$$K(y,\ \zeta) = \zeta^{1/2}\ \pi y^{-v/2}\ exp(-ay^{1/2})\ I_v(\zeta y^{1/2}),$$

where $I_v(z)$ stands for the modified Bessel function of the first kind ([3], Page 5, Equation (12)).

Proof. Again, by (2.2) we have that

$$G\{x^{-v/2-1/4}\ sin(ax^{1/2})\ f(x^{1/2});\ y\} = \int_0^\infty \zeta^{1/2}\ h_v(f;\ \zeta)\ G\{x^{-v/2}\ sin(ax^{1/2})\ J_v(\zeta x^{1/2});\ y\}d\zeta. \tag{2.13}$$

The change of order of integration is justified because $0 < \zeta < a$, $\text{Re}(v) > -1/2$ and $J_v(\zeta x)$ is a bounded function for both the variables for Landau's bounds [9, 10] (see (2.4) and (2.5)).
Now, using the following result ([4], Page 226, Equation (19)) in (2.13)

$$G\{x^{-v/2}\ sin(ax^{1/2})\ J_v(bx^{1/2});\ y\} = \pi\ y^{-v/2}\ \exp(-ay^{1/2})\ I_v(by^{1/2}), \tag{2.14}$$

provided that $0 < b < a$, $\text{Re}(v) > -1/2$ and $|arg\ y| < \pi$ we arrive at the desired result (2.12), where $0 < \zeta < a$, $\text{Re}(v) > -1/2$ and $|arg\ y| < \pi$. □

3. Applications

In this section we make applications of our theorems to obtain integral formulae.

Example 3.1. *Let $f(x) = x^{\mu-v+1/2}J_\mu(ax)$, $[a > 0,\ Re(v) > Re(\mu) > -1]$. Then*

$$G\{x^{v+1/2}\ f(x);\ y\} = G\{x^{\mu+1}\ J_\mu(ax);\ y\}. \tag{3.1}$$

Using the result (2.6) in (3.1), we get

$$G\{x^{v+1/2}\ f(x);\ y\} = 2^\mu\ \pi^{-1/2}\ a^{-\mu-1}\ \Gamma(\mu+1/2) + 2^{-1}\pi\ y^{\mu+1}sec(\mu\pi)\ [Y_{-\mu}(ay) - H_{-\mu}(ay)], \tag{3.2}$$

where $a > 0$, $-1 < Re(\mu) < 1/2$ and $|arg\ y| < \pi$.
Now, we have

$$h_\nu(f;\ \zeta) = h_\nu\{x^{\mu-\nu+1/2}\ J_\mu(ax);\ \zeta\}. \tag{3.3}$$

Using the following result ([4], Page 48, Equation (8)) in (3.3)

$$h_\nu\{x^{\mu-\nu+1/2}\ J_\mu(ax);\ y\} = \frac{2^{\mu-\nu+1}a^\mu}{\Gamma(\nu-\mu)y^{\nu-1/2}}(y^2-a^2)^{\nu-\mu-1}, \tag{3.4}$$

provided that $Re(\nu) > Re(\mu) > -1$ and $0 < a < y < \infty$ we get

$$h_\nu(f;\ \zeta) = \frac{2^{\mu-\nu+1}a^\mu}{\Gamma(\nu-\mu)\zeta^{\nu-1/2}}(\zeta^2-a^2)^{\nu-\mu-1}, \tag{3.5}$$

where $Re(\nu) > Re(\mu) > -1$ and $0 < a < \zeta < \infty$.
Now, using the results (3.2) and (3.5) in (2.1), we get

$$\begin{aligned}\int_a^\infty [2^\nu\ \pi^{-1/2}\ \zeta^{-\nu-1/2}\ \Gamma(\nu+1/2) + \zeta^{1/2}\ 2^{-1}\pi\ \boldsymbol{sec}(\nu\pi)\ y^{\nu+1}\{Y_{-\nu}(\zeta y) - H_{-\nu}(\zeta y)\}]\ \zeta^{1/2-\nu}(\zeta^2-a^2)^{\nu-\mu-1}d\zeta \\ = 2^{\nu-1}\ \pi^{-1/2}\ a^{-2\mu-1}\ \Gamma(\nu-\mu) + \pi\ y^{\mu+1}\ 2^{\nu-\mu-2}a^{-\mu}\Gamma(\nu-\mu)\ sec(\mu\pi)\left[Y_{-\mu}(ay) - H_{-\mu}(ay)\right],\end{aligned} \tag{3.6}$$

where $a > 0$, $Re(\nu) > Re(\mu) > -1$, $Re(\nu-\mu) > 0$ and $|arg\ y| < \pi$.

Example 3.2. *Let $f(x) = x^{\nu+1/2}$, $[0 < x < 1,\ Re(\nu) > -1]$. Then*

$$G\{x^{-1/2}\ f(x);\ y\} = G\{x^\nu;\ y\}. \tag{3.7}$$

Using the following result ([4], Page 216, Equation (5)) in (3.7)

$$G\{x^\nu;\ y\} = -\pi\ y^\nu\ cosec(\pi\nu),$$

where $-1 < Re(\nu) < 0$ and $|arg\ y| < \pi$, we get

$$G\{x^{-1/2}\ f(x);\ y\} = -\pi\ y^\nu\ cosec(\pi\nu), \tag{3.8}$$

where $-1 < Re(\nu) < 0$ and $|arg\ y| < \pi$.
Now, we have

$$h_\nu(f;\ \zeta) = h_\nu\{x^{\nu+1/2};\ \zeta\}. \tag{3.9}$$

Using the following result ([4], Page 22, Equation (6)) in (3.9)

$$h_\nu\{x^{\nu+1/2};\ y\} = y^{-1/2}\ J_{\nu+1}(y),$$

where $0 < x < 1$, $Re(\nu) > -1$ and $y > 0$, we get

$$h_\nu(f;\ \zeta) = \zeta^{-1/2}\ J_{\nu+1}(\zeta), \tag{3.10}$$

where $0 < x < 1$, $Re(\nu) > -1$ and $\zeta > 0$.
Now, using the results (3.8) and (3.10) in (2.7), we get

$$\int_0^\infty [\boldsymbol{J}_\nu(\zeta y) - J_\nu(\zeta y)]J_{\nu+1}(\zeta)d\zeta = -y^\nu, \tag{3.11}$$

where $-1 < Re(\nu)$ and $|arg\ y| < \pi$.

Example 3.3. *Let $f(x) = x^{\mu-\nu+1/2}J_\mu(bx)$, $[b > 0,\ Re(\nu) > Re(\mu) > -1]$. Then*

$$f(x^{1/2}) = x^{\mu/2-\nu/2+1/4}J_\mu(bx^{1/2})$$

and

$$G\{x^{\nu/2-3/4}\ sin(ax^{1/2})f(x^{1/2});\ y\} = G\{x^{\mu/2-1/2}\ sin(ax^{1/2})\ J_\mu(bx^{1/2});\ y\}. \tag{3.12}$$

Using the result (2.11) in (3.12), we get

$$G\{x^{\nu/2-3/4}\ sin(ax^{1/2})f(x^{1/2});\ y\} = 2\ y^{\mu/2-1/2}\ sinh(ay^{1/2})\ K_\mu(by^{1/2}), \tag{3.13}$$

where $0 < a < b$, $-1 <Re(\mu) < 3/2$ *and* $|arg\ y| < \pi$.
Now, we have

$$h_\nu(f;\ \zeta) = h_\nu\{x^{\mu-\nu+1/2}\ J_\mu(bx);\ \zeta\}. \tag{3.14}$$

Using the result (3.4) in (3.14), we get

$$h_\nu(f;\ \zeta) = \frac{2^{\mu-\nu+1}b^\mu}{\Gamma(\nu-\mu)\zeta^{\nu-1/2}}(\zeta^2-b^2)^{\nu-\mu-1}, \tag{3.15}$$

where $Re(\nu) >Re(\mu) > -1$ *and* $0 < b < \zeta < \infty$.
Now, using the results (3.13) and (3.15) in (2.9), we get

$$\int_b^\infty \zeta^{1-\nu}(\zeta^2-b^2)^{\nu-\mu-1}K_\nu(\zeta y^{1/2})d\zeta = 2^{\nu-\mu-1}b^{-\mu}y^{\mu/2-\nu/2}\ \Gamma(\nu-\mu)K_\mu(by^{1/2}), \tag{3.16}$$

where $Re(\nu) > Re(\mu) > -1$, $Re(\nu-\mu) > 0$ *and* $|arg\ y| < \pi$.

Example 3.4. *Let* $f(x) = x^{\nu-\mu+1/2}J_\mu(bx)$, $[b > 0, -1 < Re(\nu) < Re(\mu)]$. *Then*

$$f(x^{1/2}) = x^{\nu/2-\mu/2+1/4}J_\mu(bx^{1/2})$$

and

$$G\{x^{-\nu/2-1/4}\ sin(ax^{1/2})f(x^{1/2});\ y\} = G\{x^{-\mu/2}\ sin(ax^{1/2})\ J_\mu(bx^{1/2});\ y\}. \tag{3.17}$$

Using the result (2.14) in (3.17), we get

$$G\{x^{-\nu/2-1/4}\ sin(ax^{1/2})f(x^{1/2});\ y\} = \pi\ y^{-\mu/2}\ \exp(-ay^{1/2})\ I_\mu(by^{1/2}), \tag{3.18}$$

where $0 < b < a$, $Re(\mu) > -1/2$ *and* $|arg\ y| < \pi$.
Now, we have

$$h_\nu(f;\ \zeta) = h_\nu\{x^{\nu-\mu+1/2}\ J_\mu(bx);\ \zeta\}. \tag{3.19}$$

Using the following result ([4], Page 48, Equation (7)) in (3.19)

$$h_\nu\{x^{\nu-\mu+1/2}\ J_\mu(ax);\ y\} = \frac{2^{\nu-\mu+1}\ y^{\nu+1/2}}{\Gamma(\mu-\nu)\ a^\mu}\ (a^2-y^2)^{\mu-\nu-1},$$

provided that $a > 0$, $-1 <Re(\nu) <Re(\mu)$ *and* $0 < y < a$ *we get*

$$h_\nu(f;\ \zeta) = \frac{2^{\nu-\mu+1}\zeta^{\nu+1/2}}{\Gamma(\mu-\nu)\ b^\mu}(b^2-\zeta^2)^{\mu-\nu-1}, \tag{3.20}$$

where $b > 0$, $-1 <Re(\nu) <Re(\mu)$ *and* $0 < \zeta < b$.
Now, using the results (3.18) and (3.20) in (2.12), we get

$$\int_0^b \zeta^{\nu+1}(b^2-\zeta^2)^{\mu-\nu-1}I_\nu(\zeta y^{1/2})d\zeta = 2^{\mu-\nu-1}b^\mu y^{-\mu/2+\nu/2}\Gamma(\mu-\nu)I_\mu(by^{1/2}), \tag{3.21}$$

where $b > 0, -1 < Re(\nu) < Re(\mu)$, $Re(\mu-\nu) > 0$ *and* $|arg\ y| < \pi$.

4. Conclusion

Four integral formulae (3.6), (3.11), (3.16) and (3.21) involving special functions have been obtained with the help of the theorems established in this paper. Several other integral formulae extending the results given in [11, 12] may be obtained with the help of the theorems established in this paper and Stieltjes transforms available in [4].

Article Information

Acknowledgements: The authors would like to express their sincere thanks to the editor and the anonymous reviewers for their helpful comments and suggestions.

Author's contributions: All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

Conflict of Interest Disclosure: No potential conflict of interest was declared by the author.

Supporting/Supporting Organizations: No grants were received from any public, private or non-profit organizations for this research.

Ethical Approval and Participant Consent: It is declared that during the preparation process of this study, scientific and ethical principles were followed and all the studies benefited from are stated in the bibliography.

Plagiarism Statement: This article was scanned by the plagiarism program. No plagiarism detected.

Availability of data and materials: Not applicable.

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