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A New Soft Set Operation: Complementary Soft Binary Piecewise Lambda (λ) Operation

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Research Article

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Abstract

In 1999, Molodtsov introduced Soft Set Theory as a mathematical tool to deal with uncertainty. It has been applied to many fields both as theoretical and application aspects. Since 1999, different kinds of soft set operations have been defined and used in various types. In this paper, we define a new kind of soft set operation called, "complementary soft binary piecewise lambda operation" and we handle its basic algebraic properties. Also, it is intended to contribute to the literature of soft set by gaining the relationships between this new soft set operation and some other types of soft set operations via examining the distribution of complementary soft binary piecewise lambda operation over extended soft set operations, complementary extended soft set operations, soft binary piecewise operations, complementary soft binary piecewise operations and restricted soft set operations in order to inspire to obtain the algebraic structures of soft sets and some new decision making methods.

Keywords: Soft sets, soft set operations, conditional complements

Yeni Bir Esnek Küme İşlemi: Tümleyenli Esnek İkili Parçalı Lambda (λ) İşlemi

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Öz

Molodtsov, 1999 yılında Esnek Küme Teoriyi belirsizlikle başa çıkmak için bir matematiksel araç olarak ortaya koymuştur. Teori, hem teorik hem de uygulama yönüyle birçok alana uygulanmıştır. 1999 yılından bu yana, farklı çeşitlerde esnek küme işlemleri tanımlanmış ve çeşitli türlerde kullanılmıştır. Bu çalışmada, "tümleyenli esnek ikili parçalı lambda işlemi" adı verilen yeni bir esnek küme işlemi tanımlanmış ve temel cebirsel özellikleri araştırılmıştır. Ayrıca tümleyenli esnek ikili parçalı lambda işleminin genişletilmiş esnek küme işlemleri, tümleyenli genişletilmiş esnek küme işlemleri, esnek ikili parçalı işlemler, tümleyenli esnek ikili parçalı işlemler ve kısıtlanmış esnek küme işlemleri üzerine dağılması incelenerek bu yeni esnek küme işlemi ile diğer esnek küme işlemleri arasındaki ilişkiler elde edilerek esnek kümelerin cebirsel yapılarını ve bazı yeni karar verme yöntemlerini elde etmek için okuyuculara ilham vermek adına esnek küme literatürüne katkı sağlanması amaçlanmaktadır.

Anahtar Kelimeler: Esnek kümeler, esnek küme işlemleri, koşullu tümleyenler

Introduction

Molodtsov [1] introduced Soft Set Theory as a mathematical tool to overcome some types of uncertainty in many fields. There are three well-known basic theories that we can count as a mathematical tool to deal with uncertainties, which are Probability Theory, Fuzzy Set Theory and Interval Mathematics. But since all these theories have their own shortcomings, Soft Set Theory has seen many interest and it has been applied to many fields both as theoretically and application. Maji et. al. [2] and Pei and Miao [3] made first contributions as regards soft set operations. After then, several soft set operations (restricted and extended soft set operations) were introduced and examined in Ali et. al. [4]. Basic properties of soft set operations were discussed and the interconnections of soft set operations with each other were illustrated in Sezgin and Atagün [5]. They also defined the notion of restricted symmetric difference of soft sets and investigated its properties. A new soft set operation called extended difference of soft sets was defined in Sezgin et al. [6] and extended symmetric difference of soft sets was defined and its properties were investigated in Stojanovic [7]. When the studies are investigated, we see that the operations in soft set theory can be categorized in two main headings, as restricted soft set operations and extended soft set operations. Two conditional complements of sets as new concepts of set theory were proposed and the relationships between them were examined by Çağman [8]. Via the inspiration of this study, some new complements of sets were defined by Sezgin et al. [9]. They also transferred these complements to soft set theory, and some new restricted soft set operations and extended soft set operations were defined by Aybek [10]. Demirci [11], Sarıalioğlu [12], Akbulut [13] defined a new type of extended operation by changing the form of extended soft set operations and studied the basic properties of them in detail. Also, a new type of soft difference operation was defined in Eren [14] and by being inspired this study, Yavuz [15] defined some new soft set operations, which is called soft binary piecewise operations and they studied their basic properties in detail, too. Also, Sezgin and Sarıalioğlu [16], Sezgin and Demirci [17], Sezgin and Atagün [18], Sezgin and Çağman [19], Sezgin and Aybek [20] and Sezgin et al. [21, 22] continued their work on soft set operations by defining a new type of soft binary piecewise operation. They changed the form of soft binary piecewise operation by using the complement at the first row of the soft binary piecewise operations. The purpose of this study is to contribute to the soft set theory literature by describing a new soft set operation which is called "complementary soft binary piecewise lambda operation". For this intend, the definition of the operation and its example are given, the algebraic properties, such as closure, associative, unit and inverse element and abelian property of this new operation are examined in detail. Especially it is intended to contribute to the soft set literature by obtaining the distributions of the complementary soft binary piecewise lambda operation over extended soft set operations, complementary extended soft set operations, soft binary piecewise operations, complementary soft binary piecewise operations and restricted soft set operations.

Preliminaries

In this section, some basic concepts related to soft set theory are compiled.

Definition 1. Let U be the universal set, E be the parameter set, P(U) be the power set of U and $A \subseteq E$. A pair (F, A) is called a soft set over U where F is a set-valued function such that $F: A \rightarrow P(U)$ [1]. Throughout this paper, the set of all the soft sets over U (no matter what the parameter set is) is designated by $S_E(U)$. Let A be a fixed subset of E and $S_A(U)$ be the collection of all soft sets over U with the fixed parameters set A. Clearly, $S_A(U)$ is a subset of $S_E(U)$ and, in fact, all the soft sets are the elements of $S_E(U)$.

Definition 2. (Z, D) is called a relative null soft set (with respect to the parameter set D), denoted by $\emptyset_D$, if $Z(t) = \emptyset$ for all $t \in D$ and (Z, D) is called a relative whole soft set (with respect to the parameter set D), denoted by U_D if $Z(t) = U$ for all $t \in D$. The relative whole soft set U_E with respect to the universe set of parameters E is called the absolute soft set over U [4]. We shall denote by $\emptyset_\emptyset$ the unique soft set over U with an empty parameter set, which is called the empty soft set over U. Note that by $\emptyset_\emptyset$ and by $\emptyset_A$ are different soft sets over U [23].

Definition 3. For two soft sets (Z, D) and (R, J), we say that (Z, D) is a soft subset of (R, J) and it is denoted by $(Z, D) \tilde{\subseteq} (R, J)$, if $D \subseteq J$ and $Z(t) \subseteq R(t)$, $\forall t \in D$. Two soft sets (Z, D) and (R, J) are said to be soft equal if (Z, D) is a soft subset of (R, J) and (R, J) is a soft subset of (Z, D) [3].

Definition 4. The relative complement of a soft set (Z, D), denoted by $(Z, D)^r$, is defined by $(Z, D)^r = (Z, D)$, where $Z^r: D \rightarrow P(U)$ is a mapping given by $(Z, D)^r = U \setminus Z(t)$ for all $t \in D$ [4]. From now on, $U \setminus Z(t) = [Z(t)]'$ will be designated by Z'(t) for the sake of designation.

Two conditional complements of sets as a new concept of set theory were defined in [8]. For the sake of illustration, we show these complements as + and θ, respectively. These complements are binary operations and are defined as follows: Let D and J be two subsets of U. J-inclusive complement of D is defined by, D+J= D'∪J and J-Exlusive complement of D is defined by Dθ J = D'∩J'. Here, U refers to a universe, D' is the complement of D over U. For more information, we refer to [8]. The relations between these two complements were examined in detail by Sezgin et. al [9] and they also introduced such new three complements as binary operations of sets as follows: Let D and J be two subsets of U. Then, D*J=D'∪J', DγJ= D'∩J, D**λ**J=D∪J' [9]. These set operations were also conveyed to soft sets in [10] and they defined restricted and extended soft set operations and examined their properties. As a summary for soft set operations, we can categorize all types of soft set operations: Let "∇" be used to represent the set operations (i.e., here ∇ can be ∩, ∪,\, +,θ, *, λ,γ), then restricted operations, extended operations, complementary extended operations, soft binary piecewise operations, complementary soft binary piecewise operations are defined in soft set theory as follows:

Definition 5. [4, 5, 10] Let (Z, D) and (R, J) be soft sets over U. The restricted ∇ operation of (Z, D) and (R, J) is the soft set (S,T), denoted by, $(Z, D)\nabla_R(R, J) = (S, T)$, where $T = D \cap J \neq \emptyset$ and $\forall t \in T$, $S(t) = Z(t) \nabla R(t)$. Here note that if $D \cap J = \emptyset$, then $(Z, D)\nabla_R(R, J) = \emptyset_\emptyset$ [23].

Definition 6. [2, 4, 6, 7, 10] Let (Z, D) and (R, J) be soft sets over U. The extended ∇ operation of (Z, D) and (R, J) is the soft set (S,T), denoted by, $(Z, D)\nabla_\varepsilon(R, J) = (S, T)$, where $T = D \cup J$ and $\forall t \in T$,

$$S(t) = \begin{cases} Z(t), & t \in D\backslash J, \\ R(t), & t \in J\backslash D, \\ Z(t)\nabla R(t), & t \in D \cap J. \end{cases}$$

Definition 7. [11-13] Let (Z, D) and (R, J) be soft sets over U. The complementary extended ∇ operation of (Z, D) and (R, J) is the soft set (S,T), denoted by, $(Z, D) \overset{*}{\nabla_\varepsilon} (R, J) = (S, T)$, where $T = D \cup J$ and $\forall t \in T$

$$S(t) = \begin{cases} Z'(t), & t \in D\backslash J, \\ R'(t), & t \in J\backslash D, \\ Z(t)\nabla R(t), & t \in D \cap J. \end{cases}$$

Definition 8. [14, 15] Let (Z, D) and (R, J) be soft sets over U. The soft binary piecewise ∇ operation of (Z, D) and (R, J) is the soft set (S,D), denoted by $(Z, D)\underset{\nabla}{\sim}(R, J) = (S, D)$, where $\forall t \in D$,

$$S(t) = \begin{cases} Z(t), & t \in D\backslash J \\ Z(t)\nabla R(t), & t \in D \cap J \end{cases}$$

Definition 9. [16-22] Let (Z, D) and (R, J) be soft sets over U. The complementary soft binary piecewise ∇ operation of (Z, D) and (R, J) is the soft set (S,D), denoted by $(Z, D) \underset{\nabla}{\overset{*}{\sim}} (R, J) = (S, D)$, where $\forall t \in D$;

$$S(t) = \begin{cases} Z'(t), & t \in D\backslash J \\ Z(t)\nabla R(t), & t \in D \cap J \end{cases}$$

Let (S,•) be a groupoid. An element e∈S is called a left identity element if e•s=s for all s∈S. Similarly, e is a right identity element is s•e=x for all e∈S. An element which is both a left and a right identity is an identity element. A groupoid may have more than one left identify element: in fact the operation defined by x*y=y for all x, y∈S defines a groupoid (in fact, a semigroup) on any set S, and every element is a left identity. But as soon as a groupoid has both a left and a right identity, they are necessarily unique and equal. For if e is a left identity and f is a right identity, then f=e•f=e [24].

Let (S,•) be a groupoid. An absorbing (annihilating/zero) element is an element z such that for all s in S, z•s=s•z=z. This notion can be refined to the notions of left absorbing, where one requires only that z•s=z, and right absorbing, where s•z=z. As a similar case for identity element, if a groupoid has both a left absorbing element z and a right absorbing element z′, then it has an absorbing element, since z=z•z′ =z′ [25].

Complementary soft binary piecewise lambda (λ) operation and its properties

Definition 10. Let (F, Y) and (Z, L) be soft sets over U. The complementary soft binary lambda (λ) operation of (F, Y) and (Z, L) is the soft set (H,Y), denoted by, $(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,L)=(H,Y)$, where ∀l∈Y,

$$H(l)=\begin{cases} F'(l), & l\in Y\setminus L \\ F(l)\cup Z'(l), & l\in Y\cap L \end{cases}$$

Example 1. Let E={e_1,e_2,e_3,e_4} be the parameter set Y={e_1, e_3} and L={e_2, e_3 , e_4} be the subsets of E and U={h_1,h_2,h_3,h_4,h_5} be the initial universe set. Assume that (F,Y) and (Z,L) are the soft sets over U defined as follows:

(F,Y)={(e_1, {h_2,h_5}), (e_3,{h_1,h_2,h_5})}

(Z,L)={(e_2,{h_1 ,h_4,h_5}), (e_3,{h_2,h_3,h_4}),(e_4,{ h_3 ,h_5})}.

Let $(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,L)=(H,Y)$. Then,

$$H(l)=\begin{cases} F'(l), & l\in Y\setminus L \\ F(l)\cup Z'(l), & l\in Y\cap L \end{cases}$$

Since Y={e_1, e_3} and Y\L={e_1}, so H(e_1) =F'(e_1)={ h_1,h_3,h_4}. And since Y∩L={e_3} so H(e_3)=F(e_3) ∪Z'(e_3)={h_1, h_2,h_5}∪{h_1,h_5}= {h_1, h_2, h_5}.

Thus, $(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,L)$={(e_1,{h_1 ,h_3,h_4}), (e_3, {h_1, h_2, h_5})}

Theorem 1. (Algebraic properties of the operation)

1) The set $S_E(U)$ is closed under the operation $\underset{\lambda}{\overset{*}{\sim}}$. That is, when (F,Y) and (Z,L) are two soft sets over U, then so is $(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,L)$.

Proof. It is clear that $\underset{\lambda}{\overset{*}{\sim}}$ is a binary operation in $S_E(U)$. Hence, the set $S_E(U)$ is closed under the operation $\underset{\lambda}{\overset{*}{\sim}}$.

2) $[(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,Y)]\underset{\lambda}{\overset{*}{\sim}}(H,Y)\neq(F,Y)\underset{\lambda}{\overset{*}{\sim}}[(Z,Y)\underset{\lambda}{\overset{*}{\sim}}(H,Y)]$

Proof. Let $(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,Y)=(T,Y)$, where $\forall l\in Y$;

$$T(l)=\begin{cases} F'(l), & l\in Y\setminus Y=\emptyset \\ F(l)\cup Z'(l), & l\in Y\cap Y=Y \end{cases}$$

Let $(T,Y)\underset{\lambda}{\overset{*}{\sim}}(H,Y)=(M,Y)$, where $\forall l\in Y$;

$$M(l)=\begin{cases} T'(l), & l\in Y\setminus Y=\emptyset \\ T(l)\cup H'(l), & l\in Y\cap Y=Y \end{cases}$$

Thus,

$$M(l)=\begin{cases} T'(l), & l\in Y\setminus Y=\emptyset \\ [F(l)\cup Z'(l)]\cup H'(l), & l\in Y\cap Y=Y \end{cases}$$

Let $(Z,Y)\underset{\lambda}{\overset{*}{\sim}}(H,Y)=(L,Y)$, where $\forall l\in Y$;

$$L(l)=\begin{cases} Z'(l), & l\in Y\setminus Y=\emptyset \\ Z(l)\cup H'(l), & l\in Y\cap Y=Y \end{cases}$$

Let $(F,Y)\underset{\lambda}{\overset{*}{\sim}}(L,Y)=(N,Y)$, where $\forall l\in Y$;

$$N(l)=\begin{cases} F'(l), & l\in Y\setminus Y=\emptyset \\ F(l)\cup L'(l), & l\in Y\cap Y=Y \end{cases}$$

Thus,

$$N(l)=\begin{cases} F'(l), & l\in Y\setminus Y=\emptyset \\ F(l)\cup[Z'(l)\cap H(l)], & l\in Y\cap Y=Y \end{cases}$$

It is seen that $(M,Y)\neq(N,Y)$.

That is, for the soft sets whose parameter sets are the same, the operation $\underset{\lambda}{\overset{*}{\sim}}$ does not have associativity property. Moreover, we have the following:

3) $[(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,L)]\underset{\lambda}{\overset{*}{\sim}}(H,R)\neq(F,Y)\underset{\lambda}{\overset{*}{\sim}}[(Z,L)\underset{\lambda}{\overset{*}{\sim}}(H,R)]$

Proof. Let $(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,L)=(T,Y)$, where $\forall l\in Y$;

$$T(l)=\begin{cases} F'(l), & l\in Y\setminus L \\ F(l)\cup Z'(l), & l\in Y\cap L \end{cases}$$

Let $(T,Y)\underset{\lambda}{\overset{*}{\sim}}(H,R)=(M,Y)$, where $\forall l\in Y$;

$$M(l)=\begin{cases} T'(l), & l\in Y\setminus R \\ T(l)\cup H'(l), & l\in Y\cap R \end{cases}$$

$$M(l)=\begin{cases} F(l), & l\in(Y\setminus L)\setminus R=Y\cap L'\cap R' \\ F'(l)\cap Z(l), & l\in(Y\cap L)\setminus R=Y\cap L\cap R' \\ F'(l)\cup H'(l), & l\in(Y\setminus L)\cap R=Y\cap L'\cap R \\ [F(l)\cup Z'(l)]\cup H'(l), & l\in(Y\cap L)\cap R=Y\cap L\cap R \end{cases}$$

Let $(Z,L)\underset{\lambda}{\overset{*}{\sim}}(H,R)=(K,L)$, where $\forall l\in L$;

$$K(l)=\begin{cases} Z'(l), & l\in L\setminus R \\ Z(l)\cup H'(l), & l\in L\cap R \end{cases}$$

Let $(F,Y)\underset{\lambda}{\overset{*}{\sim}}(K,L)=(S,Y)$, where $\forall l\in Y$;

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$$S(l)=\begin{cases} F'(l), & l\in Y\backslash L \\ F(l)\cup K'(l), & l\in Y\cap L \end{cases}$$

Thus,

$$S(l)=\begin{cases} F'(l), & l\in Y\backslash L \\ F(l)\cup Z(l), & l\in Y\cap(L\backslash R)=Y\cap L\cap R' \\ F(l)\cup [Z'(l)\cap H(l)], & l\in Y\cap(L\cap R)=Y\cap L\cap R \end{cases}$$

Here let's handle $l\in Y\backslash L$ in the second equation of the first line. Since $Y\backslash L= Y\cap L'$, if $l\in L'$, then $l\in R\backslash L$ or $l\in(L\cup R)'$. Hence, if $l\in Y\backslash L$, then $l\in Y\cap L'\cap R'$ or $l\in Y\cap L'\cap R$. Thus, it is seen that $(M,Y) \neq (S,Y)$. That is, for the soft sets whose parameter sets are not the same, the operation $\underset{\lambda}{\overset{*}{\sim}}$ has not associativety property in the set $S_E(U)$.

4) $(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,L)\neq (Z,L)\underset{\lambda}{\overset{*}{\sim}}(F,Y)$

Proof. Here, while the parameter set of the soft set of the left hand side is Y; the parameter set of the soft set of the right hand side is L. Thus, by the definition of soft equality

$$(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,L)\neq(Z,L)\underset{\lambda}{\overset{*}{\sim}}(F,Y).$$

Hence, the operation $\underset{\lambda}{\overset{*}{\sim}}$ has not commutative property in the set $S_E(U)$,

5) $(F,Y)\underset{\lambda}{\overset{*}{\sim}}(F,Y)= U_Y$

Proof. Let $(F,Y)\underset{\lambda}{\overset{*}{\sim}}(F,Y)=(H,Y)$, where $\forall l\in Y$;

$$H(l)=\begin{cases} F'(l), & l\in Y\backslash Y=\emptyset \\ F(l)\cup F'(l), & l\in Y\cap Y=Y \end{cases}$$

Here $\forall l\in Y$; $H(l)= F(l)\cup F'(l)=U$, thus $(H,Y)= U_Y$.

That is, the operation $\underset{\lambda}{\overset{*}{\sim}}$ does not have idempotency property in the set $S_E(U)$.

6) $(F,Y)\underset{\lambda}{\overset{*}{\sim}}\emptyset_Y= U_Y$

Proof. Let $\emptyset_Y=(S,Y)$. Then, $\forall l\in Y$; $S(l)= \emptyset$. Let $(F,Y)\underset{\lambda}{\overset{*}{\sim}}(S,Y)=(H,Y)$, where $\forall l\in Y$,

$$H(l)=\begin{cases} F'(l), & l\in Y\setminus Y=\emptyset \\ F(l)\cup S'(l), & l\in Y\cap Y=Y \end{cases}$$

Hence, $\forall l\in Y$; $H(l)= F(l) \cup S'(l) = F(l) \cup U= U$. Thus, $(H,Y)= U_Y$

7) $\emptyset_Y\underset{\lambda}{\overset{*}{\sim}}(F,Y)= (F,Y)^r$.

Let $\emptyset_Y=(S,Y)$. Then, $\forall l\in Y$; $S(l)= \emptyset$. Let $(S,Y)\underset{\lambda}{\overset{*}{\sim}}(F,Y)=(H,Y)$, where $\forall l\in Y$;

$$H(l)=\begin{cases} S'(l), & l\in Y\setminus Y=\emptyset \\ S(l)\cup F'(l), & l\in Y\cap Y=Y \end{cases}$$

Thus, $\forall l\in Y$; $H(l)= S(l) \cup F'(l)= \emptyset\cup F'(l)= F'(l)$, hence $(H,Y)= (F,Y)^r$.

8) $(F,Y)\underset{\lambda}{\overset{*}{\sim}}\emptyset_E=U_Y$.

Proof. Let $\emptyset_E=(S,E)$. Hence $\forall l\in E$; $S(l)=\emptyset$. Let $(F,Y)\underset{\lambda}{\overset{*}{\sim}}(S,E)=(H,Y)$. Thus, $\forall l\in Y$,

$$H(l)=\begin{cases} F'(l), & l\in Y\setminus E=\emptyset \\ F(l)\cup S'(l), & l\in Y\cap E=Y \end{cases}$$

Hence, $\forall l\in Y$, $H(l)= F(l) \cup S'(l)= F(l) \cup U=U$, so $(H,Y)= U_Y$.

9) $(F,Y)\underset{\lambda}{\overset{*}{\sim}}U_Y=(F,Y)$.

Proof. Let $U_Y = (T,Y)$. Then, $\forall l\in Y$; $T(l)=U$. Let $(F,Y)\underset{\lambda}{\overset{*}{\sim}}(T,Y)=(H,Y)$, where $\forall l\in Y$;

$$H(l)=\begin{cases} F'(l), & l\in Y\setminus Y=\emptyset \\ F(l)\cup T'(l), & l\in Y\cap Y=Y \end{cases}$$

Thus, $\forall l\in Y$; $H(l)= F(l) \cup T'(l)= F(l) \cup\emptyset=F(l)$, hence $(H,Y)=(F,Y)$.

Note that, for the soft sets whose parameter set is Y, U_Y is the right-identity element for the operation $\underset{\lambda}{\overset{*}{\sim}}$.

10) $U_Y \underset{\lambda}{\overset{*}{\sim}} (F,Y) = U_Y$.

Proof. Let $U_Y = (T,Y)$. Then, $\forall l\in Y$; $T(l)=U$. Assume that $(T,Y) \underset{\lambda}{\overset{*}{\sim}} (F,Y)=(H,Y)$, where $\forall l\in Y$;

$$H(l)=\begin{cases} T'(l) & l\in Y\setminus Y=\emptyset \\ T(l)\cup F'(l) & l\in Y\cap Y=Y \end{cases}$$

Hence, $\forall l\in Y$; $H(l)= T(l) \cup F'(l)= U \cup F'(l)= U$. Thus, $(T,Y)= U_Y$

Note that for the soft sets whose parameter set is Y, U_Y is the left-absorbing element for the operation $\underset{\lambda}{\overset{*}{\sim}}$.

11) $(F,Y) \underset{\lambda}{\overset{*}{\sim}} U_E=(F,Y)$.

Proof. Let $U_E=(T,E)$. Hence, $\forall l\in E$, $T(l)=U$. Let $(F,Y) \underset{\lambda}{\overset{*}{\sim}} (T,E)=(H,Y)$, then $\forall l\in Y$,

$$H(l)=\begin{cases} F'(l), & l\in Y\setminus E=\emptyset \\ F(l)\cup T'(l), & l\in Y\cap E=Y \end{cases}$$

Hence, $\forall l\in Y$, $H(l)= F(l) \cup T'(l)= F(l) \cup\emptyset = F(l)$, so $(H,Y)= (F,Y)$.

Note that, for the soft sets (no matter what the parameter set is), U_E is the right-identity element for the operation $\underset{\lambda}{\overset{*}{\sim}}$ in the set $S_E(U)$.

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12) $(F,Y)\underset{\lambda}{\overset{*}{\sim}}(F,Y)^r=(F,Y)$.

Proof. Let $(F,Y)^r=(H,Y)$. Hence, $\forall l\in Y$; $H(l)=F'(l)$. Let $(F,Y)\underset{\lambda}{\overset{*}{\sim}}(H,Y)=(T,Y)$, where $\forall l\in Y$,

$$T(l)=\begin{cases} F'(l), & l\in Y\setminus Y=\emptyset \\ F(l)\cup H'(l), & l\in Y\cap Y=Y \end{cases}$$

Hence, $\forall l\in Y$; $T(l)= F(l)\cup H'(l)=F(l)\cup F(l)= F(l)$, thus $(T,Y)=(F,Y)$.

Note that, relative complement of every soft set is its own right-identity element for the operation $\underset{\lambda}{\overset{*}{\sim}}$ in the set $S_E(U)$.

13) $(F,Y)^r\underset{\lambda}{\overset{*}{\sim}}(F,Y)= (F,Y)^r$.

Proof. Let $(F,Y)^r=(H,Y)$. Hence, $\forall l\in Y$; $H(l)=F'(l)$. Let $(H,Y)\underset{\lambda}{\overset{*}{\sim}}(F,Y)=(T,Y)$, where $\forall l\in Y$;

$$T(l)=\begin{cases} H'(l), & l\in Y\setminus Y=\emptyset \\ H(l)\cup F'(l), & l\in Y\cap Y=Y \end{cases}$$

Hence, $\forall l\in Y$; $T(l)= H(l)\cup F'(l)= F'(l)\cup F'(l)= F'(l)$, thus $(T,Y)= (F,Y)^r$.

Note that, relative complement of a soft set is the left-absorbing element of its own soft set for the operation $\underset{\lambda}{\overset{*}{\sim}}$ in the set $S_E(U)$.

14) $[(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,L)]^r=(F,Y)\tilde{\gamma}(Z,L)$.

Proof. Let $(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,L)=(H,Y)$. Then, $\forall l\in Y$,

$$H(l)=\begin{cases} F'(l), & l\in Y\setminus L \\ F(l)\cup Z'(l), & l\in Y\cap L \end{cases}$$

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Let $(H,Y)^r=(T,Y)$, so $\forall l\in Y$,

$$T(l)=\begin{cases} F(l), & l\in Y\setminus L \\ F'(l)\cap Z(l), & l\in Y\cap L \end{cases}$$

Thus, $(T,Y)=(F,Y)\tilde{\gamma}\,(Z,L)$.

In classical theory, $A\cup B=\emptyset \Leftrightarrow A=\emptyset$ and $B=\emptyset$. Now, we have the following:

15) $(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,Y)=\emptyset_Y \Leftrightarrow (F,Y)=\emptyset_Y$ and $(Z,Y)=U_Y$.

Proof. Let $(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,Y)=(T,Y)$. Hence, $\forall l\in Y$,

$$T(l)=\begin{cases} F'(l), & l\in Y\setminus Y=\emptyset \\ F(l)\cup Z'(l), & l\in Y\cap Y=Y \end{cases}$$

Since $(T,Y)=\emptyset_Y$, $\forall l\in Y$, $T(l)=\emptyset$. Hence, $\forall l\in Y$, $T(l)=F(l)\cup Z'(l)=\emptyset \Leftrightarrow \forall l\in Y$, $F(l)=\emptyset$ and $Z'(l)=\emptyset$ $\Leftrightarrow \forall l\in Y$, $F(l)=\emptyset$ and $Z(l)=U \Leftrightarrow (F,Y)=\emptyset_Y$ and $(Z,Y)=U_Y$.

In classical theory, for all A, $\emptyset\subseteq A$. Now, we have the following:

16) $\emptyset_Y\,\tilde{\subseteq}\,(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,L)$ and $\emptyset_L\,\tilde{\subseteq}\,(Z,L)\underset{\lambda}{\overset{*}{\sim}}(F,Y)$.

In classical theory, for all A, $A\subseteq U$. Now, we have the following:

17) $(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,L)\,\tilde{\subseteq}\,U_Y$ and $(Z,L)\underset{\lambda}{\overset{*}{\sim}}(F,Y)\,\tilde{\subseteq}\,U_L$.

In classical theory, $A\subseteq A\cup B$ and $A\subseteq A\cup B$. Now, we have the following analogy:

18) $(F,Y)\,\tilde{\subseteq}\,(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,Y)$ and $(Z,Y)^r\,\tilde{\subseteq}\,(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,Y)$.

Proof. Let $(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,Y)=(H,Y)$. First of all, $Y\subseteq Y$. Moreover, $\forall l\in Y$,

$$H(l)=\begin{cases} F'(l), & l\in Y\setminus Y=\emptyset \\ F(l)\cup Z'(l), & l\in Y\cap Y=Y \end{cases}$$

Since $\forall l\in Y$, $H(l)= F(l) \subseteq F(l) \cup Z'(l)$ and $Z'(l) \subseteq F(l) \cup Z'(l)$, the rest of the proof is obvious.

Distribution Rules

In this section, distribution of complementary soft binary piecewise lambda (λ) operation over other soft set operations such as extended soft set operations, complementary extended soft set operations, soft binary piecewise operations, complementary soft binary piecewise operations and restricted soft set operations are examined in detail and many interesting results are obtained.

Distribution of complementary soft binary piecewise lambda (λ) operation over extended soft set operations:

i) Left-distribution of complementary soft binary piecewise lambda (λ) operation over extended soft set operations

1) $(F,Y)\underset{\lambda}{\overset{*}{\sim}}[(Z,L)\cap_{\varepsilon}(H,R)]=[(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,L)]\ \tilde{\cup}\ [(H,R)\underset{+}{\overset{*}{\sim}}(F,Y)]$, where $Y\cap L'\cap R=\emptyset$

Proof. Let's first handle the left hand side of the equality and let $(Z,L)\cap_{\varepsilon}((H,R)=(M,L\cup R)$ where $\forall l\in L\cup R$;

$$M(l)=\begin{cases} Z(l), & l\in L\setminus R \\ H(l), & l\in R\setminus L \\ Z(l)\cap H(l), & l\in L\cap R \end{cases}$$

Assume that $(F,Y)\underset{\lambda}{\overset{*}{\sim}}(M,L\cup R)=(N,Y)$, where $\forall l\in Y$;

$$N(l)=\begin{cases} F'(l), & l\in Y\setminus (L\cup R) \\ F(l)\cup M'(l), & l\in Y\cap(L\cup R) \end{cases}$$

Hence

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$$N(l)=\begin{cases} F'(l), & l\in Y\setminus(L\cup R)=Y\cap L'\cap R' \\ F(l)\cup Z'(l), & l\in Y\cap(L\setminus R)=Y\cap L\cap R' \\ F(l)\cup H'(l), & l\in Y\cap(R\setminus L)=Y\cap L'\cap R \\ F(l)\cup[(Z'(l)\cup H'(l)], & l\in Y\cap L\cap R=Y\cap L\cap R \end{cases}$$

Now let's handle the right hand side of the equality $[(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,L)]\,\tilde{\cup}[(H,R)\underset{+}{\overset{*}{\sim}}(F,Y)]$. Assume that

$(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,L)=(V,Y)$, where $\forall l\in Y$;

$$V(l)=\begin{cases} F'(l), & l\in Y\setminus L \\ F(l)\cup Z'(l), & l\in Y\cap L \end{cases}$$

Let $(H,R)\underset{+}{\overset{*}{\sim}}(F,Y)=(W,Y)$, where $\forall l\in R$;

$$W(l)=\begin{cases} H'(l), & l\in R\setminus Y \\ H'(l)\cup F(l), & l\in R\cap Y \end{cases}$$

Let $(V,Y)\tilde{\cup}(W,R)=(T,Y)$, where $\forall l\in Y$;

$$T(l)=\begin{cases} V(l), & l\in Y\setminus R \\ V(l)\cup W(l), & l\in Y\cap R \end{cases}$$

Thus,

$$T(l)=\begin{cases} F'(l), & l\in(Y\setminus L)\setminus R=Y\cap L'\cap R' \\ F(l)\cup Z'(l), & l\in(Y\cap L)\setminus R=Y\cap L\cap R' \\ F'(l)\cup H'(l), & l\in(Y\setminus L)\cap(R\setminus Y)=\emptyset \\ F'(l)\cup[H'(l)\cup F(l)], & l\in(Y\setminus L)\cap(R\cap Y)=Y\cap L'\cap R \\ [F(l)\cup Z'(l)]\cup H'(l), & l\in(Y\cap L)\cap(R\setminus Y)=\emptyset \\ [F(l)\cup Z'(l)]\cup[H'(l)\cup F(l)], & \in(Y\cap L)\cap(R\cap Y)=Y\cap L\cap R \end{cases}$$

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It is seen that (N,Y)=(T,Y).

2) $(F,Y)\underset{\lambda}{\overset{*}{\sim}}[(Z,L)\cup_{\varepsilon}(H,R)]=[(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,L)]\ \tilde{\cap}\ [(H,R)\underset{+}{\overset{*}{\sim}}(F,Y)]$, where $Y\cap L'\cap R=\emptyset$

3) $(F,Y)\underset{\lambda}{\overset{*}{\sim}}[(Z,L)\lambda_{\varepsilon}(H,R)]=[(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,L)]\ \tilde{\cap}\ [(H,R)\underset{\cup}{\overset{*}{\sim}}(F,Y)]$, where $Y\cap L'\cap R=\emptyset$.

4) $(F,Y)\underset{\lambda}{\overset{*}{\sim}}[(Z,L)\setminus_{\varepsilon}(H,R)]=[(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,L)]\ \tilde{\cup}\ [(H,R)\underset{\cup}{\overset{*}{\sim}}(F,Y)]$ where $Y\cap L'\cap R=\emptyset$.

ii) Right-distribution of complementary soft binary piecewise lambda (λ) operation over extended soft set operations

1) $[(F,Y)\cup_{\varepsilon}(Z,L)]\underset{\lambda}{\overset{*}{\sim}}(H,R)=[(F,Y)\underset{\lambda}{\overset{*}{\sim}}(H,R)]\cup_{\varepsilon}[(Z,L)\underset{\lambda}{\overset{*}{\sim}}(H,R)]$, where $Y\cap L\cap R'=\emptyset$.

Moreover, $[(F,Y)\cup_{\varepsilon}(Z,L)]\underset{\lambda}{\overset{*}{\sim}}(H,R)=[(F,Y)\underset{\lambda}{\overset{*}{\sim}}(H,R)]\cap_{\varepsilon}[(Z,L)\underset{\lambda}{\overset{*}{\sim}}(H,R)]$, where $Y\cap L\cap R=\emptyset$.

Proof. Let's first handle the left hand side of the equality and let $(F,Y)\cup_{\varepsilon}(Z,L)=(M,Y\cup L)$, where $\forall l\in Y\cup L$;

$$M(l)=\begin{cases}F(l), & l\in Y\setminus L\\ Z(l), & l\in L\setminus Y\\ F(l)\cup Z(l), & l\in Y\cap L\end{cases}$$

Suppose that $(M,Y\cup L)\underset{\lambda}{\overset{*}{\sim}}(H,R)=(N,Y\cup L)$, where $\forall l\in Y\cup L$;

$$N(l)=\begin{cases}M'(l), & l\in(Y\cup L)\setminus R\\ M(l)\cup H'(l), & l\in(Y\cup L)\cap R\end{cases}$$

Thus,

$$N(l)=\begin{cases} F'(l), & l\in(Y\setminus L)\setminus R=Y\cap L'\cap R' \\ Z'(l), & l\in(L\setminus Y)\setminus R=Y'\cap L\cap R' \\ F'(l)\cap Z'(l), & l\in(Y\cap L)\setminus R=Y\cap L\cap R' \\ F(l)\cup H'(l), & l\in(Y\setminus L)\cap R=Y\cap L'\cap R \\ Z(l)\cup H'(l), & l\in(L\setminus Y)\cap R=Y'\cap L\cap R \\ [F(l)\cup Z(l)]\cup H'(l), & l\in(Y\cap L)\cap R=Y\cap L\cap R \end{cases}$$

Now let's handle the right hand side of the equality: $[(F,Y)\underset{\lambda}{\overset{*}{\sim}}(H,R)]\cup_{\varepsilon}[(Z,L)\underset{\lambda}{\overset{*}{\sim}}(H,R)]$. Let

$(F,Y)\underset{\lambda}{\overset{*}{\sim}}(H,R)=(V,Y)$, where $\forall l\in Y$;

$$V(l)=\begin{cases} F'(l), & l\in Y\setminus R \\ F(l)\cup H'(l), & l\in Y\cap R \end{cases}$$

Let $(Z,L)\underset{\lambda}{\overset{*}{\sim}}(H,R)=(W,L)$, where $\forall l\in L$;

$$W(l)=\begin{cases} Z'(l), & l\in L\setminus R \\ Z(l)\cup H'(l), & l\in L\cap R \end{cases}$$

Assume that $(V,Y)\cup_{\varepsilon}(W,L)=(T,Y\cup L)$, where $\forall l\in Y\cup L$;

$$T(l)=\begin{cases} V(l), & l\in Y\setminus L \\ W(l), & l\in L\setminus Y \\ V(l)\cup W(l), & l\in Y\cap L \end{cases}$$

Hence,

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$$T(l)=\begin{cases} F'(l), & l\in(Y\backslash R)\backslash L=Y\cap L'\cap R' \\ F(l)\cup H'(l), & l\in(Y\cap R)\backslash L=Y\cap L'\cap R \\ Z'(l), & l\in(L\backslash R)\backslash Y=Y'\cap L\cap R' \\ Z(l)\cup H'(l), & l\in(L\cap R)\backslash Y=Y'\cap L\cap R \\ F'(l)\cup Z'(l), & l\in(Y\backslash R)\cap(L\backslash R)=Y\cap L\cap R' \\ F'(l)\cup [Z(l)\cup H'(l)] & l\in(Y\backslash R)\cap(L\cap R)=\emptyset \\ [F(l)\cup H'(l)]\cup Z'(l), & l\in(Y\cap R)\cap(L\backslash R)=\emptyset \\ [F(l)\cup H'(l)]\cup[Z(l)\cup H'(l)], & l\in(Y\cap R)\cap(L\cap R)=Y\cap L\cap R \end{cases}$$

It is seen that $(N,Y\cup L)=(T,Y\cup L)$.

$[(F,Y)\cup_\varepsilon (Z,L)] \overset{*}{\underset{\lambda}{\sim}} (H,R)= [(F,Y) \overset{*}{\underset{\lambda}{\sim}} (H,R)] \cap_\varepsilon [(Z,L) \overset{*}{\underset{\lambda}{\sim}} (H,R)]$, where $Y\cap L\cap R=\emptyset$ can be shown similarly.

2) $[(F,Y)\cap_\varepsilon (Z,L)] \overset{*}{\underset{\lambda}{\sim}} (H,R)= [(F,Y) \overset{*}{\underset{\lambda}{\sim}} (H,R)] \cap_\varepsilon [(Z,L) \overset{*}{\underset{\lambda}{\sim}} (H,R)]$, where $Y\cap L\cap R'=\emptyset$.

Moreover, $[(F,Y)\cap_\varepsilon (Z,L)] \overset{*}{\underset{\lambda}{\sim}} (H,R)= [(F,Y) \overset{*}{\underset{\lambda}{\sim}} (H,R)] \cup_\varepsilon [(Z,L) \overset{*}{\underset{\lambda}{\sim}} (H,R)]$, where $Y\cap L\cap R=\emptyset$.

3) $[(F,Y)\backslash_\varepsilon (Z,L)] \overset{*}{\underset{\lambda}{\sim}} (H,R)= [(F,Y) \overset{*}{\underset{\lambda}{\sim}} (H,R)] \cap_\varepsilon [(Z,L) \overset{*}{\underset{*}{\sim}} (H,R)]$, where $Y\cap L\cap R'= Y'\cap L\cap R=\emptyset$.

4) $[(F,Y)\lambda_\varepsilon (Z,L)] \overset{*}{\underset{\lambda}{\sim}} (H,R)= [(F,Y) \overset{*}{\underset{\lambda}{\sim}} (H,R)] \cup_\varepsilon [(Z,L) \overset{*}{\underset{*}{\sim}} (H,R)]$, where $Y\cap L\cap R'= Y'\cap L\cap R =\emptyset$.

Distribution of complementary soft binary piecewise lambda (λ) operation over complementary extended soft set operations:

i) Left-distribution of complementary soft binary piecewise lambda (λ) operation over complementary extended soft set operations

1) $(F,Y) \overset{*}{\underset{\lambda}{\sim}} [(Z,L) \overset{*}{\theta_\varepsilon} (H,R)] = [(F,Y) \overset{*}{\underset{\cup}{\sim}} (Z,L)] \tilde{\cup} [(H,R) \overset{*}{\underset{\cup}{\sim}} (F,Y)]$, where $Y\cap L'\cap R=\emptyset$

Proof. Let's first handle the left hand side of the equality. Assume $(Z,L)\overset{*}{\theta_{\varepsilon}}(H,R)=(M,L\cup R)$, so $\forall l\in L\cup R$,

$$M(l)=\begin{cases} Z'(l), & l\in L\setminus R \\ H'(l), & l\in R\setminus L \\ Z'(l)\cap H'(l), & l\in L\cap R \end{cases}$$

Let $(F,Y)\underset{\lambda}{\overset{*}{\sim}}(M,L\cup R)=(N,Y)$, then $\forall l\in Y$,

$$N(l)=\begin{cases} F'(l), & l\in Y\setminus(L\cup R) \\ F(l)\cup M'(l), & l\in Y\cap(L\cup R) \end{cases}$$

Hence,

$$N(l)=\begin{cases} F'(l), & l\in Y\setminus(L\cup R)=Y\cap L'\cap R' \\ F(l)\cup Z(l), & l\in Y\cap(L\setminus R)=Y\cap L\cap R' \\ F(l)\cup H(l), & l\in Y\cap(R\setminus L)=Y\cap L'\cap R \\ F(l)\cup[(Z(l)\cup H(l)], & l\in Y\cap L\cap R=Y\cap L\cap R \end{cases}$$

Now let's handle the right hand side of the equality $[(F,Y)\underset{\cup}{\overset{*}{\sim}}(Z,L)]\,\tilde{\cup}\,[(H,R)\underset{\cup}{\overset{*}{\sim}}(F,Y)]$. Let $(F,Y)\underset{\cup}{\overset{*}{\sim}}(Z,L)=(V,Y)$, so $\forall l\in Y$,

$$V(l)=\begin{cases} F'(l), & l\in Y\setminus L \\ F(l)\cup Z(l), & l\in Y\cap L \end{cases}$$

Let $(H,R)\underset{\cup}{\overset{*}{\sim}}(F,Y)=(W,R)$, hence $\forall l\in R$,

$$W(l)=\begin{cases} H'(l), & l\in Y\setminus R \\ H(l)\cup F(l), & l\in R\cap Y \end{cases}$$

Assume that $(V,Y)\tilde{\cup}(W,R)=(T,Y)$, hence $\forall l\in Y$,

$$T(l)=\begin{cases} V(l), & l\in Y\setminus R \\ V(l)\cup W(l), & l\in Y\cap R \end{cases}$$

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Hence,

$$T(l)=\begin{cases} F'(l), & l\in(Y\setminus L)\setminus R=Y\cap L'\cap R' \\ F(l)\cup Z(l), & l\in(Y\cap L)\setminus R=Y\cap L\cap R' \\ F'(l)\cup H'(l), & l\in(Y\setminus L)\cap(R\setminus Y)=\emptyset \\ F'(l)\cup[H(l)\cup F(l)], & l\in(Y\setminus L)\cap(R\cap Y)=Y\cap L'\cap R \\ [F(l)\cup Z(l)]\cup H'(l), & l\in(Y\cap L)\cap(R\setminus Y)=\emptyset \\ [F(l)\cup Z(l)]\cup[H(l)\cup F(l)], & l\in(Y\cap L)\cap(R\cap Y)=Y\cap L\cap R \end{cases}$$

It is seen that (N,Y)=(T,Y).

2) $(F,Y)\underset{\lambda}{\overset{*}{\sim}}[(Z,L)\overset{*}{*_{\varepsilon}}(H,R)]=[(F,Y)\underset{\cup}{\overset{*}{\sim}}(Z,L)]\;\tilde{\cap}\;[(H,R)\underset{\cup}{\overset{*}{\sim}}(F,Y)]$, where $Y\cap L'\cap R=\emptyset$

3) $(F,Y)\underset{\lambda}{\overset{*}{\sim}}[(Z,L)\overset{*}{+_{\varepsilon}}(H,R)]=[(F,Y)\underset{\cup}{\overset{*}{\sim}}(Z,L)]\;\tilde{\cap}\;[(H,R)\underset{+}{\overset{*}{\sim}}(F,Y)]$, where $Y\cap L'\cap R=\emptyset$.

4) $(F,Y)\underset{\lambda}{\overset{*}{\sim}}[(Z,L)\overset{*}{\gamma_{\varepsilon}}(H,R)]=[(F,Y)\underset{\cup}{\overset{*}{\sim}}(Z,L)]\;\tilde{\cup}\;[(H,R)\underset{+}{\overset{*}{\sim}}(F,Y)]$, where $Y\cap L'\cap R=\emptyset$.

ii) Right-distribution of complementary soft binary piecewise lambda (λ) operation over complementary extended soft set operations

1) $[(F,Y)\overset{*}{*_{\varepsilon}}(Z,L)]\underset{\lambda}{\overset{*}{\sim}}(H,R)=[(F,Y)\overset{\sim}{*}(H,R)]\cup_{\varepsilon}[(Z,L)\overset{\sim}{*}(H,R)]$, where $Y\cap L\cap R'=\emptyset$

Proof. Let's first handle the left hand side of the equality, let $(F,Y)\overset{*}{*_{\varepsilon}}(Z,L)=(M,Y\cup L)$, where $\forall l\in Y\cup L$;

$$M(l)=\begin{cases} F'(l), & l\in Y\setminus L \\ Z'(l), & l\in L\setminus Y \\ F'(l)\cup Z'(l), & l\in Y\cap L \end{cases}$$

Let $(M,Y\cup L)\underset{\lambda}{\overset{*}{\sim}}(H,R)=(N,Y\cup L)$, where $\forall l\in Y\cup L$;

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$$N(l)=\begin{cases} M'(l), & l\in(Y\cup L)\setminus R \\ M(l)\cup H'(l), & l\in(Y\cup L)\cap R \end{cases}$$

Thus,

$$N(l)=\begin{cases} F(l), & l\in(Y\setminus L)\setminus R=Y\cap L'\cap R' \\ Z(l), & l\in(L\setminus Y)\setminus R=Y'\cap L\cap R' \\ F(l)\cap Z(l), & l\in(Y\cap L)\setminus R=Y\cap L\cap R' \\ F'(l)\cup H'(l), & l\in(Y\setminus L)\cap R=Y\cap L'\cap R \\ Z'(l)\cup H'(l), & l\in(L\setminus Y)\cap R=Y'\cap L\cap R \\ [F'(l)\cup Z'(l)]\cup H'(l), & l\in(Y\cap L)\cap R=Y\cap L\cap R \end{cases}$$

Now let's handle the right hand side of the equality: $[(F,Y)\tilde{*}(H,R)]\ \cup_{\varepsilon}\ [(Z,L)\tilde{*}(H,R)]$. Assume that $(F,Y)\tilde{*}(H,R)=(V,Y)$, where $\forall l\in Y$;

$$V(l)=\begin{cases} F(l), & l\in Y\setminus R \\ F'(l)\cup H'(l), & l\in Y\cap R \end{cases}$$

Let $(Z,L)\tilde{*}(H,R)=(W,L)$, where $\forall l\in L$;

$$W(l)=\begin{cases} Z(l), & l\in L\setminus R \\ Z'(l)\cup H'(l), & l\in L\cap R \end{cases}$$

Assume that $(V,Y)\cup_{\varepsilon}(W,L)=(T,Y\cup L)$, where $\forall l\in Y\cup L$;

$$T(l)=\begin{cases} V(l), & l\in Y\setminus L \\ W(l), & l\in L\setminus Y \\ V(l)\cup W(l), & l\in Y\cap L \end{cases}$$

Thus,

$$T(l)=\begin{cases} F(l), & l\in(Y\setminus R)\setminus L=Y\cap L'\cap R' \\ F'(l)\cup H'(l), & l\in(Y\cap R)\setminus L=Y\cap L'\cap R \\ Z(l), & l\in(L\setminus R)\setminus Y=Y'\cap L\cap R' \\ Z'(l)\cup H'(l), & l\in(L\cap R)\setminus Y=Y'\cap L\cap R \\ F(l)\cup Z(l), & l\in(Y\setminus R)\cap(L\setminus R)=Y\cap L\cap R' \\ F(l)\cup[Z'(l)\cup H'(l)], & l\in(Y\setminus R)\cap(L\cap R)=\emptyset \\ [F'(l)\cup H'(l)]\cup Z(l), & l\in(Y\cap R)\cap(L\setminus R)=\emptyset \\ [F'(l)\cup H'(l)]\cup[Z'(l)\cup H'(l)], & l\in(Y\cap R)\cap(L\cap R)=Y\cap L\cap R \end{cases}$$

It is seen that (N,Y∪L)=(T,Y∪L).

2) $[(F,Y)\overset{*}{\theta_\varepsilon}(Z,L)]\ \underset{\lambda}{\overset{*}{\sim}}\ (H,R)=[(F,Y)\underset{*}{\tilde{\ }}(H,R)]\cap_\varepsilon[(Z,L)\underset{*}{\tilde{\ }}(H,R)]$, where Y∩L∩R'=∅

3) $[(F,Y)\overset{*}{\gamma_\varepsilon}(Z,L)]\ \underset{\lambda}{\overset{*}{\sim}}\ (H,R)=[(F,Y)\underset{*}{\tilde{\ }}(H,R)]\cap_\varepsilon[(Z,L)\underset{\lambda}{\tilde{\ }}(H,R)]$, where Y∩L∩R'= Y'∩L∩R =∅

4) $[(F,Y)\overset{*}{+_\varepsilon}(Z,L)]\ \underset{\lambda}{\overset{*}{\sim}}\ (H,R)=[(F,Y)\underset{*}{\tilde{\ }}(H,R)]\cup_\varepsilon[(Z,L)\underset{\lambda}{\tilde{\ }}(H,R)]$, where Y∩L∩R'= Y'∩L∩R =∅.

Distribution of complementary soft binary piecewise lambda (λ) operation over soft binary piecewise operations:

i) Left-distribution of complementary soft binary piecewise lambda (λ) operation over soft binary piecewise operations

The followings are held when Y∩L'∩R =∅ .

1) $(F,Y)\underset{\lambda}{\overset{*}{\sim}}[(Z,L)\,\tilde{\cup}\,(H,R)]=[(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,L)]\,\tilde{\cap}\,[(H,R)\underset{+}{\overset{*}{\sim}}(F,Y)]$.

Proof. Let's first handle the left hand side of the equality, let (Z,L) $\tilde{\cup}$ (H,R)=(M,L), where ∀l∈L;

$$M(l)=\begin{cases} Z(l), & l\in L\setminus R \\ Z(l)\cup H(l), & l\in L\cap R \end{cases}$$

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Let $(F,Y)\underset{\lambda}{\overset{*}{\sim}}(M,L)=(N,Y)$, where $\forall l\in Y$;

$$N(l)=\begin{cases} F'(l), & l\in Y\setminus L \\ F(l)\cup M'(l), & l\in Y\cap L \end{cases}$$

Thus,

$$N(l)=\begin{cases} F'(l), & l\in Y\setminus L \\ F(l)\cup Z'(l), & l\in Y\cap(L\setminus R)=Y\cap L\cap R' \\ F(l)\cup [Z'(l)\cap H'(l)] & l\in Y\cap L\cap R=Y\cap L\cap R \end{cases}$$

Now let's handle the right hand side of the equality: $[(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,L)]\ \tilde{\cap}\ [(H,R)\underset{+}{\overset{*}{\sim}}(F,Y)]$. Let $(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,L)=(V,Y)$, where $\forall l\in Y$;

$$V(l)=\begin{cases} F'(l), & l\in Y\setminus L \\ F(l)\cup Z'(l), & l\in Y\cap L \end{cases}$$

Let $(H,R)\underset{+}{\overset{*}{\sim}}(F,Y)=(W,R)$, where $\forall l\in R$;

$$W(l)=\begin{cases} H'(l), & l\in R\setminus Y \\ H'(l)\cup F(l), & l\in R\cap Y \end{cases}$$

Suppose $(V,Y)\tilde{\cap}(W,R)=(T,Y)$, where $\forall l\in Y$;

$$T(l)=\begin{cases} V(l), & l\in Y\setminus R \\ V(l)\cap W(l), & l\in Y\cap R \end{cases}$$

Therefore,

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$$T(l)=\begin{cases} F'(l), & l\in(Y\backslash L)\backslash R = Y\cap L'\cap R' \\ F(l)\cup Z'(l), & l\in(Y\cap L)\backslash R= Y\cap L\cap R' \\ F'(l)\cap H'(l), & l\in(Y\backslash L)\cap(R\backslash Y)=\emptyset \\ F'(l)\cap[H'(l)\cup F(l)], & l\in(Y\backslash L)\cap(R\cap Y)= Y\cap L'\cap R \\ [F(l)\cup Z'(l)]\cap H'(l), & l\in(Y\cap L)\cap(R\backslash Y)=\emptyset \\ [F(l)\cup Z'(l)]\cap[H'(l)\cup F(l)], & l\in(Y\cap L)\cap(R\cap Y)=Y\cap L\cap R \end{cases}$$

Here let's handle $l\in Y\backslash L$ in the first equation of the first line. Since $Y\backslash L= Y\cap L'$, if $l\in L'$, then $l\in R\backslash L$ or $l\in(L\cup R)'$. Hence, if $l\in Y\backslash L$, then $l\in Y\cap L'\cap R'$ or $l\in Y\cap L'\cap R$. Thus, it is seen that $(N,Y)=(T,Y)$.

2) $(F,Y)\underset{\lambda}{\overset{*}{\sim}}[(Z,L)\,\tilde{\cap}\,(H,R)] = [(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,L)]\,\tilde{\cup}\,[(H,R)\underset{+}{\overset{*}{\sim}}(F,Y)]$.

3) $(F,Y)\underset{\lambda}{\overset{*}{\sim}}[(Z,L)\,\tilde{\backslash}\,(H,R)] = [(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,L)]\,\tilde{\cup}\,[(H,R)\underset{\cup}{\overset{*}{\sim}}(F,Y)]$.

4) $(F,Y)\underset{\lambda}{\overset{*}{\sim}}[(Z,L)\,\tilde{\lambda}\,(H,R)] = [(F,Y)\underset{\lambda}{\overset{*}{\sim}}(Z,L)]\,\tilde{\cap}\,[(H,R)\underset{\cup}{\overset{*}{\sim}}(F,Y)]$.

ii)Right-distribution of complementary soft binary piecewise lambda (λ) operation over soft binary piecewise operations

1) $[(F,A)\,\tilde{\cap}\,(Z,L)]\underset{\lambda}{\overset{*}{\sim}}(H,R)= [(F,Y)\underset{\lambda}{\overset{*}{\sim}}(H,R)]\,\tilde{\cap}\,[(Z,L)\underset{\lambda}{\overset{*}{\sim}}(H,R)]$

Proof. Let's first handle the left hand side of the equality. Suppose $(F,Y)\tilde{\cap}(Z,L)=(M,Y)$, where $\forall l\in Y$,

$$M(l)=\begin{cases} F(l), & l\in Y\backslash L \\ F(l)\cap Z(l), & l\in Y\cap L \end{cases}$$

Let $(M,Y)\underset{\lambda}{\overset{*}{\sim}}(H,R)=(N,Y)$, where $\forall l\in Y$,

$$N(l)=\begin{cases} M'(l), & l\in Y\backslash R \\ M(l)\cup H'(l), & l\in Y\cap R \end{cases}$$

Thus,

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$$N(l)=\begin{cases} F'(l), & l\in(Y\setminus L)\setminus R = Y\cap L'\cap R' \\ F'(l)\cup Z'(l), & l\in(Y\cap L)\setminus R= Y\cap L\cap R' \\ F(l)\cup H'(l), & l\in(Y\setminus L)\cap R= Y\cap L'\cap R \\ [F(l)\cap Z(l)]\cup H'(l), & l\in(Y\cap L)\cap R= Y\cap L\cap R \end{cases}$$

Now let's handle the right hand side of the equality: $[(F,Y)\underset{\lambda}{\overset{*}{\sim}}(H,R)]\ \tilde{\cap}\ [(Z,L)\underset{\lambda}{\overset{*}{\sim}}(H,R)]$. Let

$(F,Y)\underset{\lambda}{\overset{*}{\sim}}(H,R)=(V,Y)$, where $\forall l\in Y$;

$$V(l)=\begin{cases} F'(l), & l\in Y\setminus R \\ F(l)\cup H'(l), & l\in Y\cap R \end{cases}$$

Let $(Z,L)\underset{\lambda}{\overset{*}{\sim}}(H,R)=(W,L)$, where $\forall l\in L$;

$$W(l)=\begin{cases} Z'(l), & l\in L\setminus R \\ Z(l)\cup H'(l), & l\in L\cap R \end{cases}$$

Suppose that $(V,Y)\tilde{\cap}(W,L)=(T,Y)$, where $\forall l\in Y$;

$$T(l)=\begin{cases} V(l), & l\in Y\setminus L \\ V(l)\cap W(l), & l\in Y\cap L \end{cases}$$

Hence,

$$T(l)=\begin{cases} F'(l), & l\in (Y\setminus R)\setminus L=Y\cap L'\cap R' \\ F(l)\cup H'(l), & l\in(Y\cap R)\setminus L=Y\cap L'\cap R \\ F'(l)\cap Z'(l), & l\in(Y\setminus R)\cap(L\setminus R)=Y\cap L\cap R' \\ F'(l)\cap [Z(l)\cup H'(l)], & l\in(Y\setminus R)\cap(L\cap R)=\emptyset \\ [F(l)\cup H'(l)]\cap Z'(l), & l\in(Y\cap R)\cap(L\setminus R)=\emptyset \\ [F(l)\cup H'(l)]\cap[Z(l)\cup H'(l)], & l\in(Y\cap R)\cap(L\cap R)=Y\cap L\cap R \end{cases}$$

It is seen that (N,Y)=(T,Y).

2) $[(F, A) \widetilde{\cup} (Z,L)] \overset{*}{\underset{\lambda}{\sim}} (H,R) = [(F,Y) \overset{*}{\underset{\lambda}{\sim}} (H,R)] \widetilde{\cup} [(Z,L) \overset{*}{\underset{\lambda}{\sim}} (H,R)]$

3) $[(F, A) \tilde{\lambda} (Z,L)] \overset{*}{\underset{\lambda}{\sim}} (H,R) = [(F,Y) \overset{*}{\underset{\lambda}{\sim}} (H,R)] \widetilde{\cup} [(Z,L) \overset{*}{\underset{*}{\sim}} (H,R)]$

4) $[(F, A) \widetilde{\setminus} (Z,L)] \overset{*}{\underset{\lambda}{\sim}} (H,R) = [(F,Y) \overset{*}{\underset{\lambda}{\sim}} (H,R)] \widetilde{\cap} [(Z,L) \overset{*}{\underset{*}{\sim}} (H,R)]$

Distribution of complementary soft binary piecewise lambda (λ) operation over complementary soft binary piecewise operations:

i) Left-distribution of complementarysoft binary piecewise lambda (λ) operation over complementary soft binary piecewise operations

The followings are held where $Y \cap L' \cap R = \emptyset$.

1) $(F,Y) \overset{*}{\underset{\lambda}{\sim}} [(Z,L) \overset{*}{\underset{*}{\sim}} (H,R)] = [(F,Y) \overset{*}{\underset{\cup}{\sim}} (Z,L)] \widetilde{\cap} [(H,R) \overset{*}{\underset{\cup}{\sim}} (F,Y)]$.

Proof. Let's first handle the left hand side of the equality, let $(Z,L) \overset{*}{\underset{*}{\sim}} (H,R) = (M,L)$, where $\forall e \in L$;

$$M(l) = \begin{cases} Z'(l), & l \in L \setminus R \\ Z'(l) \cup H'(l), & l \in L \cap R \end{cases}$$

Let $(F,Y) \overset{*}{\underset{\lambda}{\sim}} (M,L) = (N,Y)$, where $\forall l \in Y$;

$$N(l) = \begin{cases} F'(l), & l \in Y \setminus L \\ F(l) \cup M'(l), & l \in Y \cap L \end{cases}$$

Therefore,

$$N(l) = \begin{cases} F'(l), & l \in Y \setminus L \\ F(l) \cup Z(l), & l \in Y \cap (L \setminus R) = Y \cap L \cap R' \\ F(l) \cup [(Z(l) \cap H(l)], & l \in Y \cap L \cap R = Y \cap L \cap R \end{cases}$$

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Now let's handle the right hand side of the equality: $[(F,Y)\underset{\cup}{\overset{*}{\sim}}(Z,L)]\ \tilde{\cap}\ [(H,R)\underset{\cup}{\overset{*}{\sim}}(F,Y)]$ Let

$(F,Y)\underset{\cup}{\overset{*}{\sim}}(Z,L)=(V,Y)$, where $\forall l\in Y$;

$$V(l)=\begin{cases} F'(l), & l\in Y\setminus L \\ F(l)\cup Z(l), & l\in Y\cap L \end{cases}$$

Suppose that $(H,R)\underset{\cup}{\overset{*}{\sim}}(F,Y)=(W,R)$, where $\forall l\in R$;

$$W(l)=\begin{cases} H'(l), & l\in R\setminus Y \\ H(l)\cup F(l), & l\in R\cap Y \end{cases}$$

Let $(V,Y)\tilde{\cap}(W,R)=(T,Y)$, where $\forall l\in Y$;

$$T(l)=\begin{cases} V(l), & l\in Y\setminus R \\ V(l)\cap W(l), & l\in Y\cap R \end{cases}$$

Hence,

$$T(l)=\begin{cases} F'(l), & l\in (Y\setminus L)\setminus R=Y\cap L'\cap R' \\ F(l)\cup Z(l), & l\in (Y\cap L)\setminus R=Y\cap L\cap R' \\ F'(l)\cap H'(l), & l\in (Y\setminus L)\cap(R\setminus Y)=\emptyset \\ F'(l)\cap[H(l)\cup F(l)], & l\in (Y\setminus L)\cap(R\cap Y)=Y\cap L'\cap R \\ [F(l)\cup Z(l)]\cap H'(l), & l\in (Y\cap L)\cap(R\setminus Y)=\emptyset \\ [F(l)\cup Z(l)]\cap[H(l)\cup F(l)], & l\in (Y\cap L)\cap(R\cap Y)=Y\cap L\cap R \end{cases}$$

Take care that since $Y\setminus L=Y\cap L'$, if $l\in L'$, then $l\in R\setminus L$ or $l\in (L\cup R)'$. Hence, if $l\in Y\setminus L$, $l\in Y\cap L'\cap R'$ or $l\in Y\cap L'\cap R$. Thus, it is seen that $(N,Y)=(T,Y)$.

2) $(F,Y)\underset{\lambda}{\overset{*}{\sim}}[(Z,L)\underset{\theta}{\overset{*}{\sim}}(H,R)] = [(F,Y)\underset{\cup}{\overset{*}{\sim}}(Z,L)]\ \tilde{\cup}\ [(H,R)\underset{\cup}{\overset{*}{\sim}}(F,Y)]$

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3) $(F,Y)\underset{\lambda}{\overset{*}{\sim}}[(Z,L)\underset{\gamma}{\overset{*}{\sim}}(H,R)] = [(F,Y)\underset{\cup}{\overset{*}{\sim}}(Z,L)]\ \tilde{\cup}\ [(H,R)\underset{+}{\overset{*}{\sim}}(F,Y)]$

4) $(F,Y)\underset{\lambda}{\overset{*}{\sim}}[(Z,L)\underset{+}{\overset{*}{\sim}}(H,R)] = [(F,Y)\underset{\cup}{\overset{*}{\sim}}(Z,L)]\ \tilde{\cap}\ [(H,R)\underset{+}{\overset{*}{\sim}}(F,Y)]$

ii) Right-distribution of complementary soft binary piecewise lambda (λ) operation over complementary soft binary piecewise operations

The followings are held when $Y\cap L\cap R' = \emptyset$.

1) $[(F, A)\underset{\theta}{\overset{*}{\sim}}(Z,L)]\underset{\lambda}{\overset{*}{\sim}}(H,R) = [(F,Y)\overset{\sim}{*}(H,R)]\ \tilde{\cap}\ [(Z,L)\overset{\sim}{*}(H,R)]$.

Proof. Let's first handle the left hand side of the equality, let $(F,Y)\underset{\theta}{\overset{*}{\sim}}(Z,L)=(M,Y)$, where $\forall l\in Y$,

$$M(l)=\begin{cases} F'(l), & l\in Y\setminus L \\ F'(l)\cap Z'(l), & l\in Y\cap L \end{cases}$$

Let $(M,Y)\underset{\lambda}{\overset{*}{\sim}}(H,R)=(N,Y)$, where $l\in Y$,

$$N(l)=\begin{cases} M'(l), & l\in Y\setminus R \\ M(l)\cup H'(l), & l\in Y\cap R \end{cases}$$

Hence,

$$N(l)=\begin{cases} F(l), & l\in (Y\setminus L)\setminus R = Y\cap L'\cap R' \\ F(l)\cup Z(l) & l\in (Y\cap L)\setminus R = Y\cap L\cap R' \\ F'(l)\cup H'(l) & l\in (Y\setminus L)\cap L = Y\cap L'\cap R \\ [F'(l)\cap Z'(l)]\cup H'(l) & l\in (Y\cap L)\cap R = Y\cap L\cap R \end{cases}$$

Now let's handle the right hand side of the equality: $[(F,Y)\overset{\sim}{*}(H,R)]\ \tilde{\cap}\ [(Z,L)\overset{\sim}{*}(H,R)]$. Let

$(F,Y)\overset{\sim}{*}(H,R)=(V,Y)$, where $\forall l\in Y$;

$$V(l)=\begin{cases} F(l), & l\in Y\setminus R \\ F'(l)\cup H'(l), & l\in Y\cap R \end{cases}$$

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Assume that (Z,L) $\overset{\sim}{*}$ (H,R)=(W,L), where ∀l∈L;

$$W(l)=\begin{cases} Z(l), & l\in L\setminus R \\ Z'(l) \cup H'(l), & l\in L\cap R \end{cases}$$

Let (V,Y)$\tilde{\cap}$ (W,L)=(T,Y), where ∀l∈Y;

$$T(l)=\begin{cases} V(l) & l\in Y\setminus L \\ V(l)\cap W(l) & l\in Y\cap L \end{cases}$$

Therefore,

$$T(l)=\begin{cases} F(l), & l\in (Y\setminus R)\setminus L=Y\cap L'\cap R' \\ F'(l) \cup H'(l), & l\in (Y\cap R)\setminus L=Y\cap L'\cap R \\ F(l)\cap Z(l), & l\in (Y\setminus R)\cap(L\setminus R)=Y\cap L\cap R' \\ F(l)\cap[Z'(l) \cup H'(l)], & l\in (Y\setminus R)\cap(L\cap R)=\emptyset \\ [F'(l) \cup H'(l)] \cap Z(l), & l\in (Y\cap R)\cap(L\setminus R)=\emptyset \\ [F'(l) \cup H'(l)] \cap [Z'(l) \cup H'(l)], & l\in (Y\cap R)\cap(L\cap R)=Y\cap L\cap R \end{cases}$$

It is seen that (N,)=(T,Y).

2) [(F, A) $\underset{*}{\overset{*}{\sim}}$ (Z,L)] $\underset{\lambda}{\overset{*}{\sim}}$(H,R)= [(F,Y) $\overset{\sim}{*}$ (H,R)] $\tilde{\cup}$ [(Z,L) $\overset{\sim}{*}$ (H,R)]

3) [(F, A) $\underset{+}{\overset{*}{\sim}}$ (Z,L)] $\underset{\lambda}{\overset{*}{\sim}}$ (H,R)= [(F,Y) $\overset{\sim}{*}$ (H,R)] $\tilde{\cup}$ [(Z,L) $\overset{\sim}{\lambda}$ (H,R)]

4) [(F, A) $\underset{\gamma}{\overset{*}{\sim}}$ (Z,L)] $\underset{\lambda}{\overset{*}{\sim}}$ (H,R)= [(F,Y) $\overset{\sim}{*}$ (H,R)] $\tilde{\cap}$ [(Z,L) $\overset{\sim}{\lambda}$ (H,R)]

Distribution of complementary soft binary piecewise lambda (λ) operation over restricted soft set operations:

The followings are held when Y∩L∩R =∅.

1) $(F,Y)\underset{\lambda}{\overset{*}{\sim}}[(Z,L)\cap_R(H,R)] = [(F,Y)\underset{*}{\overset{*}{\sim}}(Z,L)]\cap_R[(F,Y)\underset{*}{\overset{*}{\sim}}(H,R)]$.

Proof. Let's first handle the left hand side of the equality, suppose $(Z,L)\cap_R(H,R)=(M,L\cap R)$ and so $\forall l\in L\cap R$, $M(l)=Z(l)\cap H(l)$. Let $(F,Y)\underset{\lambda}{\overset{*}{\sim}}(M,L\cap R)=(N,Y)$, so $\forall l\in Y$,

$$N(l)=\begin{cases} F'(l), & l\in Y\setminus(L\cap R) \\ F(l)\cup M'(l), & l\in Y\cap(L\cap R) \end{cases}$$

Thus,

$$N(l)=\begin{cases} F'(l), & l\in Y\setminus(L\cap R) \\ F(l)\cup[Z'(l)\cup H'(l)], & l\in Y\cap(L\cap R) \end{cases}$$

Now let's handle the right hand side of the equality: $[(F,Y)\underset{*}{\overset{*}{\sim}}(Z,L)]\cap_R[(F,Y)\underset{*}{\overset{*}{\sim}}(H,R)]$, Let

$(F,Y)\underset{*}{\overset{*}{\sim}}(Z,L)=(V,Y)$, and $\forall l\in Y$,

$$V(l)=\begin{cases} F'(l), & l\in Y\setminus L \\ F'(l)\cup Z'(l), & l\in Y\cap L \end{cases}$$

Let $(F,Y)\underset{*}{\overset{*}{\sim}}(H,R)=(W,Y)$and $\forall l\in Y$,

$$W(l)=\begin{cases} F'(l), & l\in Y\setminus R \\ F'(l)\cup H'(l), & l\in Y\cap R \end{cases}$$

Assume that $(V,Y)\cap_R(W,Y)=(T,Y)$, so $\forall l\in T(l)=V(l)\cup W(l)$,

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$$T(l)=\begin{cases} F'(l) \cap F'(l), & l\in (Y\setminus L)\cap(Y\setminus R) \\ F'(l) \cap [F'(l) \cup H'(l)], & l\in(Y\setminus L)\cap(Y\cap R) \\ [F'(l) \cup Z'(l)] \cap F'(l), & l\in(Y\cap L)\cap(Y\setminus R) \\ [F'(l) \cup Z'(l)] \cap [F'(l) \cup H'(l)], & l\in(Y\cap L)\cap(Y\cap R) \end{cases}$$

Thus,

$$T(l)=\begin{cases} F'(l), & l\in Y\cap L'\cap R' \\ F'(l), & l\in Y\cap L'\cap R \\ F'(l), & l\in Y\cap L\cap R' \\ [F'(l) \cup Z'(l)] \cap [F'(l) \cup H'(l)], & l\in Y\cap L\cap R \end{cases}$$

Considering the parameter set of the first equation of the first row, that is, $Y\setminus(L\cap R)$; since $Y\setminus(L\cap R) = Y\cap(L\cap R)'$, an element in $(L\cap R)'$ may be in $L\setminus R$, in $R\setminus L$ or $(L\cup R)$. Then, $Y\setminus(L\cap R)$ is equivalent to the following 3 states: $Y\cap(L\cap R')$, $Y\cap(L'\cap R)$ and $Y\cap(L'\cap R')$. Hence, $(N,Y)=(T,Y)$.

2) $(F,Y)\overset{*}{\underset{\lambda}{\sim}}[(Z,L)\cup_R(H,R)]=[(F,Y)\overset{*}{\underset{*}{\sim}}(Z,L)]\cap_R[(F,Y)\overset{*}{\underset{*}{\sim}}(H,R)]$.

3) $(F,Y)\overset{*}{\underset{\lambda}{\sim}}[(Z,L)\,\theta_R\,(H,R)]=(F,Y)\overset{*}{\underset{\gamma}{\sim}}(Z,L)]\cup_R[(F,Y)\overset{*}{\underset{\gamma}{\sim}}(H,R)]$.

4) $(F,Y)\overset{*}{\underset{\lambda}{\sim}}[(Z,L)\ast_R(H,R)]=[(F,Y)\overset{*}{\underset{+}{\sim}}(Z,L)]\cap_R[(F,Y)\overset{*}{\underset{+}{\sim}}(H,R)]$.

5) $(F,Y)\overset{*}{\underset{\lambda}{\sim}}[(Z,L)\,\gamma_R(H,R)]=[(F,Y)\overset{*}{\underset{*}{\sim}}(Z,L)]\cap_R[(F,Y)\overset{*}{\underset{+}{\sim}}(H,R)]$.

6) $(F,Y)\overset{*}{\underset{\lambda}{\sim}}[(Z,L)\,\lambda_R(H,R)]=[(F,Y)\overset{*}{\underset{\theta}{\sim}}(Z,L)]\cup[(F,Y)\overset{*}{\underset{\gamma}{\sim}}(H,R)]$.

7) $[(F,Y)\overset{*}{\underset{\lambda}{\sim}}[(Z,L)\setminus_R(H,R)]=[(F,Y)\overset{*}{\underset{\gamma}{\sim}}(Z,L)]\cap_R[(F,Y)\overset{*}{\underset{\theta}{\sim}}(H,R)]$.

8) $(F,Y)\underset{\lambda}{\overset{*}{\sim}}[(Z,L)+_R(H,R)]=[(F,Y)\underset{+}{\overset{*}{\sim}}(Z,L)]\cap_R[(F,Y)\underset{*}{\overset{*}{\sim}}(H,R)]$.

Conclusion

In this paper, we have contributed to the soft set literature by defining a new kind of soft set operation which we call complementary soft binary piecewise lambda operation. The basic algebraic properties of the operations have been investigated. Moreover by examining the distribution rules, we have obtained the relationships between this new soft set operation and other types of soft set operations such extended soft set operations, complementary extended soft set operations, soft binary piecewise operations, complementary soft binary piecewise operations and restricted soft set operations. This paper can be regarded as a theoretical study for soft sets and some future studies may continue by defining some new decision making methods by using this new operation and algebraic structures of soft sets can be handled again with the help o this new operation.

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