

Optimal PID Controller Design for Liquid Level Tank via Modified Artificial Hummingbird Algorithm

Erdal Eker¹ , Serdar Ekinci² , Davut Izci^{*2} 

¹Vocational School of Social Sciences, Muş Alparslan University, Muş, Turkey

²Department of Computer Engineering, Batman University, Batman, Turkey

(e.eker@alparslan.edu.tr, serdar.ekinci@batman.edu.tr, davut.izci@batman.edu.tr)

Received:Aug.19,2023

Accepted:Aug.26,2023

Published:Oct.18,2023

Abstract—To enhance controller performance, the optimization of control parameters has emerged as a critical research area. Among the array of optimization algorithms, the modified elite opposition-based artificial hummingbird algorithm (m-AHA) stands out for its ability to emulate behavioral strategies of hummingbirds and elite opposition-based technique. This paper, therefore, proposes m-AHA optimizer as a novel approach to optimize control parameters in a three-tanks liquid level system. By fine-tuning the parameters of proportional-integral-derivative (PID) controller, superior performance is achieved. Comparative evaluations with competitive algorithms, including the arithmetic optimization algorithm with Harris hawks optimization and covariance matrix adaptation evolution strategy, assess the m-AHA optimizer-based approach for three-tank liquid level system control. The ITAE (integral of time multiplied absolute error) performance index analyzes time domain and frequency metrics, revealing the outstanding performance of the m-AHA optimizer-based approach.

Keywords: *Artificial hummingbird algorithm, Liquid level system, Controller design, Optimization.*

1. Introduction

The effective control of dynamic systems plays a crucial role in achieving desired performance and stability. Consequently, the optimization of control parameters has emerged as a pivotal research area, aiming to enhance controller performance across diverse systems. The choice of an appropriate optimization algorithm is essential to ensure optimal tuning of controller parameters for improved system performance. Among the metaheuristic optimization algorithms, the artificial hummingbird algorithm (AHA) has gained popularity as a new technique (Ekinci, Izci, & Kayri, 2023; Kıymaç & Kaya, 2023; Yildiz et al., 2022). The AHA simulates a population of hummingbirds foraging for food in an environment (Zhao et al., 2022).

In this paper, we employ a modified AHA (m-AHA) optimizer, reported by (Abualigah et al., 2023), as a novel approach for controlling a three-tank liquid level system. The m-AHA optimizer is an improved version of the original AHA (Zhao et al., 2022) using a modified version of the elite opposition based learning (EOBL) technique (Ekinci, Izci, Eker, et al., 2023). Our focus lies in using the m-AHA optimizer, leveraging its inherent characteristics to fine-tune proportional-integral-derivative (PID) controller (Izci et al., 2023) for the stated application. Liquid level control is crucial in various industrial processes, such as chemical plants and water treatment systems (Amuthambigaiyin Sundari & Maruthupandi, 2022; Bhookya et al., 2022; Issa, 2022; Moharam et al., 2016; Stefanoiu & Culita, 2021). Extending the m-AHA optimizer-based approach to the control of a three-tank liquid level system, we propose it as an efficient alternative to previously reported methods. We evaluate the effectiveness of our approach by comparing it against other competitive algorithms such as arithmetic optimization algorithm with Harris hawks optimization and covariance matrix adaptation evolution strategy (Issa, 2023). These methods are used in this study as good performing competitive algorithms reported for liquid level system. The ITAE (integral of time multiplied absolute error) performance index serves as the cost function to assess time and frequency domains performance, and the results highlight the m-AHA optimizer-based approach's superiority in controlling three-tank liquid level systems.

In conclusion, this paper presents an impressive approach for optimizing control parameters in three-tank liquid level system using the m-AHA optimizer. Our proposed methodology showcases its efficacy in achieving superior control performance. Extensive comparisons with competitive algorithms provide compelling evidence of the m-

AHA optimizer-based approach's superiority, offering a reliable and efficient optimization methodology in the field of control systems.

2. Mathematical Model of Three Tanks Liquid Level System

The liquid level control system comprises three interconnected tanks, namely Tank 1, Tank 2, and Tank 3. Its objective is to regulate the liquid levels in each tank through a control system. In order to create a mathematical model for this system, we make the following assumptions and simplifications: The tanks have open tops, allowing the liquid surface to be exposed to the atmosphere, the liquid is incompressible and has a constant density, liquid flow between the tanks occurs in one direction, moving from higher-level tanks to lower-level tanks, the system does not have any leaks and the flow rate between the tanks is directly proportional to the difference in liquid level between the two tanks. Considering these assumptions, we can represent the dynamics of the liquid levels in each tank using a set of coupled differential equations. Let $H1$, $H2$, and $H3$ represent the liquid levels in Tank 1, Tank 2, and Tank 3, respectively. The system's dynamics can be described by the following differential equations:

$$dH1/dt = q_{in} - q_{12} - q_{13} \quad (1)$$

$$dH2/dt = q_{12} - q_{23} \quad (2)$$

$$dH3/dt = q_{13} + q_{23} - q_{out} \quad (3)$$

where q_{in} is the flow rate into Tank 1, q_{out} is the flow rate out of Tank 3, and q_{12} , q_{13} , and q_{23} are the flow rates between the tanks. The flow rates are proportional to the difference in liquid levels between the tanks, so the followings can be written:

$$q_{12} = k_{12}(H1 - H2) \quad (4)$$

$$q_{13} = k_{13}(H1 - H3) \quad (5)$$

$$q_{23} = k_{23}(H2 - H3) \quad (6)$$

where k_{12} , k_{13} , and k_{23} are the proportionality constants that depend on the geometry of the system and the properties of the liquid. With these equations, the behavior of the system for different flow rates and control strategies can be simulated. Considering the above explanation, the transfer function control theory techniques can also be used to design a control system that regulates the liquid levels in each tank by adjusting the flow rates. The design of such a control system will depend on the specific requirements and constraints of the application. Fig. 1 visualizes a simple structure of a tank that is used in a three-tank system.

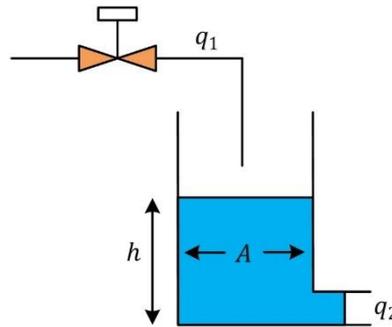


Fig. 1. Simple structure of a tank

In the simplified structure given by Fig. 1, q_1 and q_2 represent the liquid flow rates towards in and out of the tank, respectively. h represents the height and A is the cross-sectional area of the related tank. The following transfer function is used in this study for a three-tanks liquid level system (Issa, 2023).

$$G_{plant}(s) = \frac{1}{(4s+0.2)^3} = \frac{1}{64s^3+9.6s^2+0.48s+0.008} \quad (7)$$

3. m-AHA Optimizer

The design of efficient and robust control for a three-tank system is a critical task. A significant challenge lies in creating controllers that can ensure stable and accurate control of this system. One promising approach to tackle this challenge involves the utilization of advanced optimization techniques to design controllers capable of achieving optimal control performance. One such optimization technique is the m-AHA optimizer reported by Abualigah et al., (Abualigah et al., 2023). The m-AHA is an improved version of the original AHA optimizer reported by Zhao et al., (Zhao et al., 2022) which incorporates a novel version of the EOBL strategy to enhance the performance of the original AHA optimizer.

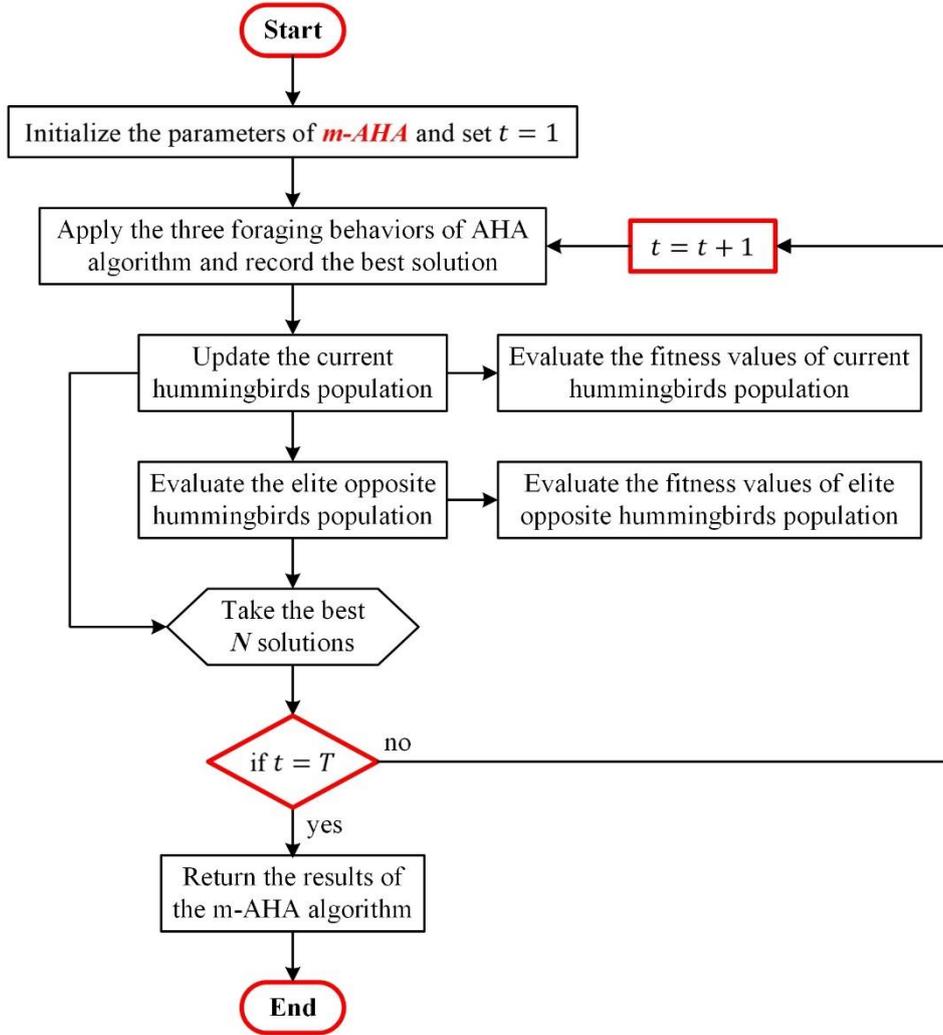


Fig. 2. Flowchart of recommended m-AHA optimizer

Standard EOBL is defined as $X^o = \langle x_1^o, x_2^o, \dots, x_k^o \rangle$ where $X = \langle x_1, x_2, \dots, x_k \rangle$ is an elite candidate solution with k decision variables. In the m-AHA optimizer, the EOBL is redefined as $x_i^o = \delta(a \cdot da_i + b \cdot db_i) - c \cdot x_i$ where δ is a parameter within $(0, 1)$, and a, b , and c are random variables within $[0, 1]$. The solution in the basic EOBL is kept within boundaries (lower, Lb_i , and upper, Ub_i) using the definition of $x_i^o = rand(Lb_i, Ub_i)$. However, in this study, if the solution exceeds the upper level, it is set to the upper boundary; otherwise, it is set to the lower boundary. The m-AHA optimizer starts with the initialization of its parameters. It then applies three foraging behaviors of the basic AHA while evaluating the current and elite candidate solutions and selecting the best N solutions. This process continues for a total of iterations (T).

Fig. 2 shows the flowchart of the m-AHA optimizer. Basically, the m-AHA optimizer works by simulating a population of hummingbirds foraging for food in an environment. These hummingbirds update their positions based on their experiences and the experiences of the best hummingbirds in the group. The algorithm also incorporates a modified EOBL strategy to further improve its performance. To enhance the control performance of three tank system, the m-AHA optimizer has been adapted and combined with the PID controller. Using m-AHA optimizer has shown promising results in enhancing the control performance of three tanks system through effective tuning of PID controller parameters.

4. PID controlled liquid level system and proposed design procedure

This paper adopts a PID controller for the control of a three-tanks system. A PID controller has the following form where the gains known as proportional, integral, and derivative are denoted by K_P , K_I , and K_D , respectively (Ekinici et al., 2022; Izci & Ekinici, 2021).

$$PID(s) = K_P + \frac{K_I}{s} + K_D s \quad (8)$$

The block diagram illustrated in Fig. 3 shows how a PID controller achieves the task for adopted plants in a feedback control system.

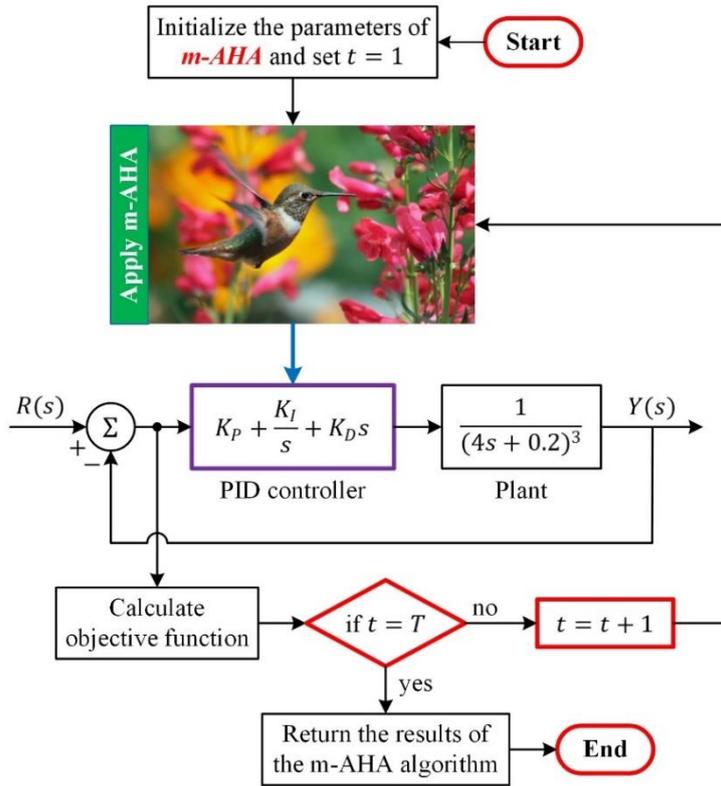


Fig. 3. Block diagram of m-AHA optimizer-based PID parameter estimation for liquid level system

Initially, the problem is represented as $\vec{X} = [x_1, x_2, x_3] = [K_P, K_I, K_D]$ and then the following integral of time multiplied absolute error (ITAE) cost function (Ekinçi et al., 2021; Snášel et al., 2023) is adopted for appropriate minimization via m-AHA optimizer.

$$ITAE(K_P, K_I, K_D) = \int_0^{1000} t|e(t)|dt \quad (9)$$

In here, $e(t)$ denotes the error signal and the minimization problem is subjected to the constraints of $10^{-3} \leq K_P \leq 20$, $10^{-3} \leq K_I \leq 20$ and $10^{-3} \leq K_D \leq 20$.

5. Simulation Results and Discussion

5.1. Statistical Performance of m-AHA Optimizer

The m-AHA optimizer is initially assessed for its performance of minimizing the ITAE cost function for the liquid level system. For the statistical evaluation the migration coefficient of the m-AHA optimizer is set to $2 \times n$ with a total iteration $T = 50$ and population size $n = 30$. Table 1 presents the statistical metrics obtained from the minimization of the ITAE cost function. The related statistical metrics are obtained after 25 individual runs. As seen from the data in the table, the m-AHA optimizer has a consistent minimization ability within a narrow band indicating its good performance characteristics.

Table 1. Statistical metrics of ITAE minimized by m-AHA optimizer

| Minimum | Maximum | Median | Average | Standard Deviation |
|----------|----------|----------|----------|--------------------|
| 324.0611 | 340.6854 | 329.8294 | 330.2053 | 4.1424 |

5.2. Compared Metaheuristic Algorithms

For comparisons with the m-AHA optimizer, arithmetic optimization algorithm with Harris hawks optimization (AOA-HHO) and covariance matrix adaptation evolution strategy (CMA-ES) are used in this study as good performing competitive algorithms reported for liquid level system (Issa, 2023). The best controller parameters obtained via m-AHA optimizer are: $K_P = 0.05149270$, $K_I = 0.00100636$ and $K_D = 1.39664147$. The transfer function obtained via those parameters is as follows.

$$TF_{m-AHA}(s) = \frac{1.397s^2 + 0.05149s + 0.001006}{64s^4 + 9.6s^3 + 1.877s^2 + 0.05949s + 0.001006} \quad (10)$$

The best controller parameters obtained via AOA-HHO algorithm are: $K_P = 0.040$, $K_I = 0.0005$ and $K_D = 0.4269$ (Issa, 2023). The transfer function obtained via those parameters is as follows.

$$TF_{AOA-HHO}(s) = \frac{0.4269s^2 + 0.04s + 0.0005}{64s^4 + 9.6s^3 + 0.9069s^2 + 0.048s + 0.0005} \quad (11)$$

Similarly, the best controller parameters obtained via CMA-ES are: $K_P = 0.051$, $K_I = 0.0013$ and $K_D = 0.3914$ (Issa, 2023). The transfer function obtained via those parameters is as follows.

$$TF_{CMA-ES}(s) = \frac{0.3914s^2 + 0.051s + 0.0013}{64s^4 + 9.6s^3 + 0.8714s^2 + 0.059s + 0.0013} \quad (12)$$

These transfer functions can be used to perform the comparative assessments provided in the following subsections.

5.3. Comparative Step Response Analysis

Fig. 4 displays the comparative step responses of m-AHA, AOA-HHO and CMA-ES approaches for the liquid level system. As seen from the respective plots, the m-AHA optimizer is capable of demonstrating a more desirable response in terms of overshoot, rise time, settling time and peak time, making it the best approach that can be used to reach more desirable time domain-based performance characteristics for a liquid level system. The related illustrations are also supported by the numerical values presented in Table 2.

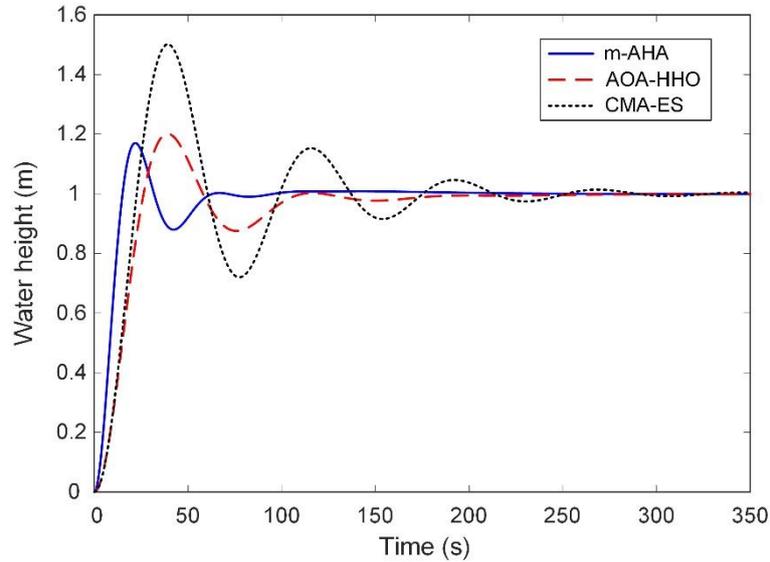


Fig. 4. Step response of different optimizers-based controller designs for water height

Table 2. Comparisons of step response characteristics

| Optimizer | Rise time (s) | Settling time (s) | Overshoot (%) | Peak time (s) |
|----------------------|---------------|-------------------|---------------|---------------|
| m-AHA | 10.1884 | 57.7791 | 16.9774 | 22.3629 |
| AOA-HHO (Issa, 2023) | 17.7926 | 160.1051 | 20.1160 | 39.2792 |
| CMA-ES (Issa, 2023) | 15.0133 | 238.6552 | 49.9912 | 38.2793 |

5.4. Comparative Frequency Response Analysis

Fig. 5 displays the comparative Bode plots of m-AHA, AOA-HHO and CMA-ES approaches for the liquid level system. As seen from the respective plots, the m-AHA optimizer is capable of demonstrating a more desirable response in terms of phase margin, bandwidth, making it the best approach that can be used to reach more desirable frequency domain-based performance characteristics for a liquid level system. The related illustrations are also supported by the numerical values presented in Table 3.

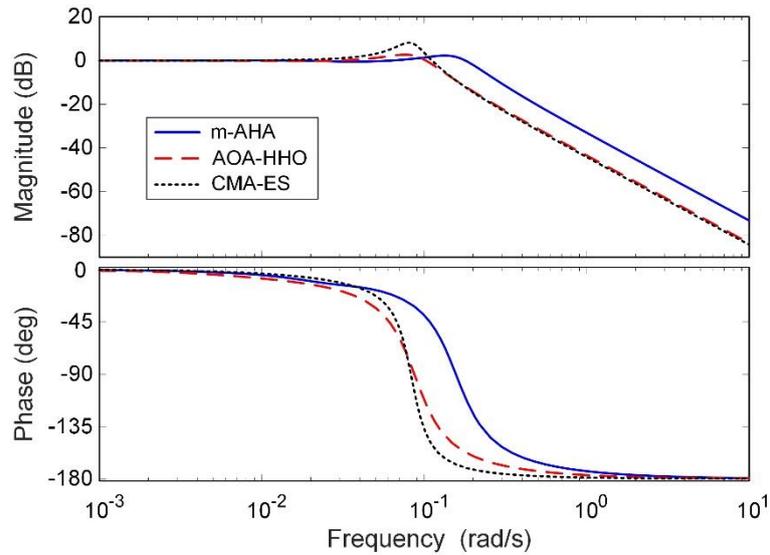


Fig. 5. Comparative Bode plot analysis

Table 3. Comparisons of frequency response characteristics

| Optimizer | Gain margin (dB) | Phase margin ($^{\circ}$) | Bandwidth (rad/s) |
|----------------------|------------------|-----------------------------|-------------------|
| m-AHA | Inf | 45.4407 | 0.2094 |
| AOA-HHO (Issa, 2023) | Inf | 43.1866 | 0.1183 |
| CMA-ES (Issa, 2023) | Inf | 22.6244 | 0.1232 |

6. Conclusion

This study delves deeper into the potential of the m-AHA optimizer for fine-tuning the parameters of a PID controller in a three-tank liquid level system. The m-AHA optimizer is designed by skillfully integrating the original form of the AHA with a novel modified EOBL strategy, enhancing its capabilities for this specific application. To demonstrate the efficacy of the m-AHA optimizer, we conduct comprehensive comparisons with more recent and best-performing approaches that also utilize the PID controller for controlling the three-tank liquid level system. By subjecting the system to rigorous evaluations based on statistical, transient, and frequency response characteristics, we aim to showcase the superior performance of the m-AHA optimizer in comparison to these competitive methods. Through statistical analysis, we assess various metrics to provide a good understanding of the optimizer's performance. The transient response analysis involves studying the system's behavior during the initial phase of control to evaluate the speed and stability of the m-AHA-optimized PID controller in achieving the desired liquid level setpoints. Furthermore, the frequency response analysis examines the controller's ability to respond to varying input frequencies, providing valuable insights into its performance under different dynamic conditions. By considering these diverse evaluation aspects, we aim to offer a comprehensive assessment of the m-AHA optimizer's efficiency in fine-tuning the PID controller for the three-tank liquid level system. The results of our evaluations consistently demonstrate the m-AHA optimizer's superior capacity in achieving optimal control performance for the liquid level system. Its ability to efficiently tune the PID controller parameters outperforms the other considered methods, highlighting its advantage in achieving stable and accurate liquid level control.

References

- Abualigah, L., Ekinci, S., Izci, D., & Abu Zitar, R. (2023). Modified Elite Opposition-Based Artificial Hummingbird Algorithm for Designing FOPID Controlled Cruise Control System. *Intelligent Automation & Soft Computing*. <https://doi.org/10.32604/iasc.2023.040291>
- Amuthambigaiyin Sundari, K., & Maruthupandi, P. (2022). Optimal Design of PID Controller for the analysis of Two TANK System Using Metaheuristic Optimization Algorithm. *Journal of Electrical Engineering & Technology*, 17(1), 627–640. <https://doi.org/10.1007/s42835-021-00891-6>
- Bhookya, J., Vijaya Kumar, M., Ravi Kumar, J., & Seshagiri Rao, A. (2022). Implementation of PID controller for liquid level system using mGWO and integration of IoT application. *Journal of Industrial Information Integration*, 28, 100368. <https://doi.org/10.1016/j.jii.2022.100368>
- Ekinci, S., Izci, D., Abu Zitar, R., Alsoud, A. R., & Abualigah, L. (2022). Development of Lévy flight-based reptile search algorithm with local search ability for power systems engineering design problems. *Neural Computing*

- and Applications*, 34(22), 20263–20283. <https://doi.org/10.1007/s00521-022-07575-w>
- Ekinci, S., Izci, D., Eker, E., Abualigah, L., Thanh, C.-L., & Khatir, S. (2023). Hunger games pattern search with elite opposite-based solution for solving complex engineering design problems. *Evolving Systems*. <https://doi.org/10.1007/s12530-023-09526-9>
- Ekinci, S., Izci, D., & Hekimoglu, B. (2021). Implementing the Henry Gas Solubility Optimization Algorithm for Optimal Power System Stabilizer Design. *Electrica*, 21(2), 250–258. <https://doi.org/10.5152/electrica.2021.20088>
- Ekinci, S., Izci, D., & Kayri, M. (2023). Artificial hummingbird optimizer as a novel adaptive algorithm for identifying optimal coefficients of digital IIR filtering systems. *International Journal of Modelling and Simulation*, 1–15. <https://doi.org/10.1080/02286203.2023.2240564>
- Issa, M. (2022). *Parameter Tuning of PID Controller Based on Arithmetic Optimization Algorithm in IOT Systems* (pp. 399–417). https://doi.org/10.1007/978-3-030-99079-4_15
- Issa, M. (2023). Enhanced Arithmetic Optimization Algorithm for Parameter Estimation of PID Controller. *Arabian Journal for Science and Engineering*, 48(2), 2191–2205. <https://doi.org/10.1007/s13369-022-07136-2>
- Izci, D., & Ekinci, S. (2021). Comparative Performance Analysis of Slime Mould Algorithm For Efficient Design of Proportional–Integral–Derivative Controller. *Electrica*, 21(1), 151–159. <https://doi.org/10.5152/electrica.2021.20077>
- Izci, D., Ekinci, S., & Hussien, A. G. (2023). Effective PID controller design using a novel hybrid algorithm for high order systems. *PLOS ONE*, 18(5), e0286060. <https://doi.org/10.1371/journal.pone.0286060>
- Kıymaç, E., & Kaya, Y. (2023). A novel automated CNN arrhythmia classifier with memory-enhanced artificial hummingbird algorithm. *Expert Systems with Applications*, 213, 119162. <https://doi.org/10.1016/j.eswa.2022.119162>
- Moharam, A., El-Hosseini, M. A., & Ali, H. A. (2016). Design of optimal PID controller using hybrid differential evolution and particle swarm optimization with an aging leader and challengers. *Applied Soft Computing*, 38, 727–737. <https://doi.org/10.1016/j.asoc.2015.10.041>
- Snaštel, V., Rizk-Allah, R. M., Izci, D., & Ekinci, S. (2023). Weighted mean of vectors optimization algorithm and its application in designing the power system stabilizer. *Applied Soft Computing*, 136, 110085. <https://doi.org/10.1016/j.asoc.2023.110085>
- Stefanoiu, D., & Culita, J. (2021). Optimal Identification and Metaheuristic PID Control of a Two-Tank System. *Electronics*, 10(9), 1101. <https://doi.org/10.3390/electronics10091101>
- Yildiz, B. S., Mehta, P., Sait, S. M., Panagant, N., Kumar, S., & Yildiz, A. R. (2022). A new hybrid artificial hummingbird-simulated annealing algorithm to solve constrained mechanical engineering problems. *Materials Testing*, 64(7), 1043–1050. <https://doi.org/10.1515/mt-2022-0123>
- Zhao, W., Wang, L., & Mirjalili, S. (2022). Artificial hummingbird algorithm: A new bio-inspired optimizer with its engineering applications. *Computer Methods in Applied Mechanics and Engineering*, 388, 114194. <https://doi.org/10.1016/j.cma.2021.114194>