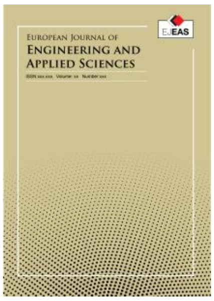
European Journal of Engineering and Applied Sciences

ISSN: 2651-3412 (Print) & 2667-8454 (Online)
Journal homepage: http://dergipark.gov.tr/EJEAS
Published by Çorlu Faculty of Engineering, Tekirdağ Namık Kemal University

European J. Eng. App. Sci. 6(2), 98-101, 2023



Research Article

A Variational Problem for the Power Factor in Capacitance Charging Process

Hafız ALİSOY[1,*], Gülizar ALİSOY[2], Mahir YAŞAR[1,3], Selcen ÇALIŞKAN[3], Gizem Büşra GÖK[3], Mohammadreza MASOUMISOUREH[3],

[1] Department of Electronics and Telecommunication Engineering, Corlu Engineering Faculty, Tekirdağ Namık Kemal University, Tekirdag, Türkiye, 59860
[2] Department of Arts and Science,Tekirdağ Namık Kemal University, Tekirdag, Türkiye, 59860
3 Technocast Otomotiv San. Tic. A.Ş. Arge Department, Tekirdag, Türkiye, 59500
[1*]halisoy@nku.edu.tr, [2*]e-mail, galisoy@nku.edu.tr, [3]mahir.yasar@technocast.com.tr, [3]selcen.caliskan@technocast.com, [3] gizembusra.gok@technocast.com.tr, reza.masoumi@technocast.com.tr

Received: 12.10.2023 *Accepted: 21.11.2023* *DOI: 10.55581/ejeas.1375193*

Abstract: In this study, the solution of a variational problem that allows determining the input voltage form that provides the maximum power factor of the battery charging process in the RLC circuit and the change character of the charging current for constant R, L, C and charging time has been examined. The defined problem is obtained and solved in the form of a linear integro-differential equation. A quadratic voltage criterion is proposed to analyze the convergence of the power factor to its maximum value for various modes of charging the battery.

Keywords: *Charging efficiency, Maksimum power factor, Variational problem.*

Kapasitans Şarj İşleminde Güç Faktörü İçin Varyasyonel Problem

Özet: Bu çalışmada, sabit R, L, C ve şarj süresi için, R L C devresindeki batarya şarj işleminin maksimum güç faktörünü sağlayan giriş voltajı formunu ve şarj akımının değişim karakterini belirlemeye olanak sağlayan bir varyasyonel problemin çözümü incelenmiştir. Tanımlanan problem, lineer bir integro-diferansiyel denklem biçiminde elde edilir ve çözülür. Bataryanın şarj etmenin çeşitli modları için, güç faktörünün maksimum değerine yakınsamasının analizi için ikinci dereceden bir voltaj kriteri önerilmiştir.

Anahtar kelimeler: *Maksimum güç faktörü, Şarj verimliliği, Varyasyonel problem*

1. Introduction

In connection with the ever-increasing technological applications of capacitive energy storage devices with charge-discharge cycle features, analyzing the power factor of the charging process of devices in these device circuits attracts great attention as important problems. There is a lack of data in the literature about the extremal operating conditions of circuits that provide maximum power factor at non-sinusoidal currents and voltages.

As it is known, battery packs consisting of series or parallel connected cells are generally used as energy storage in BMSs [1,2]. Series connections control voltage, while parallel connections determine current and capacity. The charging of these capacitive energy storage devices is generally provided through either a current limiting resistor or a current limiting inductance, depending on the operating conditions of the device and the place of use [3-6].

This study aims to analyze the solution of a variational problem regarding the determination of the optimal form of the

[*] Corresponding author
E-mail adress: halisoy@nku.edu.tr (H. Alisoy)

input voltage $U_{in}(t)$ and the change character of the charging current $i(t)$ when charging the capacitive energy storage device C, corresponding to the upper limits of the power factor in an $R-L-C$ circuit. General expressions for power factor are found at optimum $U_{opt}(t)$ voltage and $i_{opt}(t)$ current values for various charging modes. When solving the problem, it is assumed that R, L, C circuit parameters and T charging time are given as initial data

Since a circular wire of radius R_0 has a potential $V > 0$ and a grounded flat ideally conducting electrode has zero potential, the maximum electric field strength on the surface of the cylinder will be achieved at its point P which is closest to the grounded electrode (see Fig. 1)

2. Theory

The charging efficiency is equal to:

$$\eta = \frac{1}{1 + \frac{2I^2RT}{CU_c{}^2}} \quad (1)$$

where I- is the effective value of the charging current.

According to (1), with the accepted initial parameters, the efficiency is determined only by the value of the charging current. The problem of finding the maximum efficiency is a variational problem on a conditional extremum and reduces to finding the extremum of some functional.

$$L = \int_0^T i^2(t)dt \quad (2)$$

Provided that the required function $i(t)$ must satisfy the additional condition:

$$\int_0^T i(t)dt = CU_c(T) \neq 0 \quad (3)$$

By using condition (3), as will be shown below, we determine the constant in solving the problem posed, without resorting to the initial conditions.

The formulated isoperimetric problem on a conditional extremum is reduced to the usual problem of the calculus of variations without additional conditions using Euler's theorem [4-8]. Euler's theorem as applied to (2) leads to varying the functional

$$L = \int_0^T [i^2(t) + \theta \cdot i(t)]dt \quad (4)$$

Let us establish the first variation of the functional (4) δL, while considering the extremum condition $\delta L = 0$, wich relies on the first lemma of the calculus of variations [7-11] we have:

$$2i + \theta = 0, \qquad i = -\frac{\theta}{2} \quad (5)$$

Substituting (5) into (3), we find:

$$\theta = -\frac{2CU_c}{T} \quad (6)$$

The law of change in current when charging a capacitive storage device in the R-L-C circuit, corresponding to the minimum losses in ohmic resistance

$$i_{opt} = \frac{CU_c}{T} = const \quad (7)$$

On the other hand, if the charging circuit consists of passive RLC elements connected in series, the input voltage will be determined by the following integro-differential equation.

$$U_{in}(t) = L\frac{di(t)}{dt} + R \cdot i(t) + \frac{1}{C}\int_0^T i(t)dt \quad (8)$$

and satisfying condition (7a), then we get:

$$U_{in.opt}(t) = \frac{U_c}{T}(RC + t) \quad (9)$$

The efficiency of the charging process with the taken initial data and the optimal charging mode:

$$\eta = \frac{1}{1+\frac{2RC}{T}}\,. \quad (10)$$

3. Results and Discussion

Thus, to obtain maximum efficiency when charging a capacitance in the $R-L-C$ circuit (see Figure 1), it is necessary to maintain the charging current constant during the charging cycle (7), and the shape of the input voltage must increase according to a linear law (9)

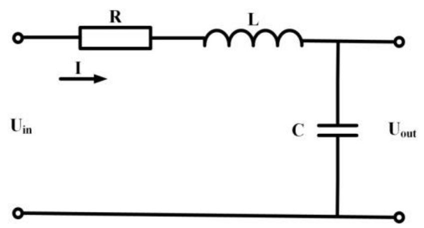


Figure 1. *Equivalent charging circuit of series RLC elements*

To analyze this case, consider a series-connected RLC circuit. The change in voltage across the capacitor is described by the equation [9].

$$LCU_c'' + RCU_c' + U_c = u \quad (11)$$

and the magnitude of the current in this case is equal to:

$$i(t) = CU_c'(t) \quad (12)$$

taking into account expressions (11) and (12), the functional takes the form:

$$\int_0^T u(t)i(t)\,dt = C\int_0^T \left[LCU_c''U_c' + RCU_c'^2 + U_cU_c'\right]dt \quad (13)$$

The minimum of functional (13) is provided by the function $u(t)$ satisfying the Euler equation, which in this case has the form:

$$U_c''(t) = 0\,. \quad (14)$$

Considering the law of switching in an electrical circuit, according to which the current value $i(t) = CU_c'(t)$, at the initial moment should be equal to zero. Therefore, Euler's equation (14) must satisfy the conditions:

$$\begin{cases} U_c(0) = 0 \\ U_c(T) = U \\ U_c'(0) = 0 \end{cases} \quad (15)$$

Since the general solution of the second-order equation (14) depends only on two arbitrary constants. Therefore, it is impossible to subject it to the three specified conditions. It follows that if there is inductance in the circuit, it is impossible to change the voltage on the capacitor, giving maximum efficiency

To clarify the physical meaning of the result obtained, let us consider the following law of voltage change on the capacitor:

$$U_c(t) = \frac{U \cdot (exp(-\alpha t) + \alpha t - 1)}{exp(-\alpha T) + \alpha T - 1} \quad (16)$$

where α is some parameter. Note that for any value of the parameter $\alpha \neq 0$, conditions (15) are satisfied. Based on this formula, taking into account the values of the parameters included in it, for two different values of α, the calculated values of U_c are shown in the Figure 2.

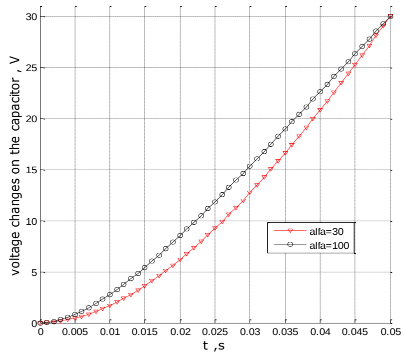


Figure 2. *Change in voltage on the capacitor at different values of the alpha parameter*

Obviously, function (16) is a solution to equation (11) if its right-hand side is defined as:

$$u(t) = \frac{U \cdot [(LC\alpha^2 - RC\alpha + 1)exp(-\alpha t) + \alpha t + RC\alpha - 1]}{exp(-\alpha T) + \alpha T - 1} \quad (17)$$

As can be seen from figure 2, as the parameter α increases, the function $u_c(t)$ approaches the linear function $U_c(t) \approx \frac{U}{T} t$, satisfying conditions (16).

But with increasing α, the initial value of the function u(0) increases without limit (see Fig. 3), i.e. in the limit as α→∞ the initial value of the function u(0)→∞. This explains the impossibility of creating a corresponding law for changing the voltage of the current source.

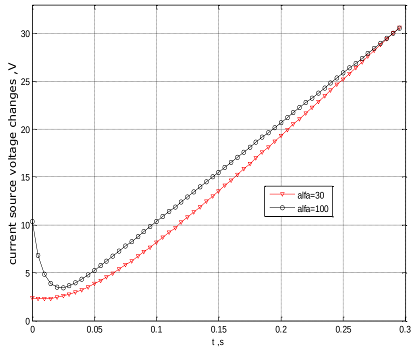


Figure 3. *Changes in current source voltage at different values of the alpha parameter*

The found solution to the problem is valid for $i_{opt} = const$ at all points, including for $t = 0$. However, under zero initial conditions $i(t)_{|t=0} = 0$, due to the presence of inductance in the charging circuit, the solution to the problem can be obtained only in the class of generalized functions. To determine the input voltage shape that provides maximum efficiency at zero initial conditions, we use the passage to the limit. Let the current be described by the expression:

$$i(t) = i_{opt}(1 - exp(-\beta t)) \quad (18)$$

In the limit as $\beta \to \infty$ (18) becomes a singular function

$$i(t) = \begin{cases} i_{opt}, & t > 0 \\ 0, & t = 0 \end{cases} \quad (19)$$

The input voltage shape corresponding to (19) under condition (8) has the form:

$$U_{in}(t) = \begin{cases} U_{in.opt}, & t > 0 \\ \infty, & t = 0 \end{cases} \quad (20)$$

In the practice of using capacitive storage devices, the law of change in the charging current and the shape of the input voltage are different from the obtained optimal ones (7), (19), (9), (20) and are adjustable. The shape of the input voltage is determined by the rectification circuit adopted, and the shape of the charging current, in addition, depends on the parameters R, L, C and charging modes. It is known [7-11] that if F(t) is a controlled variable, then the integral:

$$\int_0^\infty |F(t)|^2 dt$$

It is a so-called quadratic criterion for the quality of regulation. One of the optimization methods in control technology is to give this criterion a minimum value. In our case, to assess the degree of optimization of the capacity charging process in the R-L-C boiler, we have a quadratic criterion for current

$$\gamma^2 = \int_0^T \left| \left[i(t) - i_{opt}(t) \right]^2 \right| dt \quad (21)$$

where $i_{opt}(t)$ is the change in current according to (7), $i(t)$ is the law of change in current for one charging cycle in the circuit at a given mode.

For charging modes of capacitor banks, in which the value of the charging current $i_{max}(t)$ is less than the value $i_{opt}(t)$, it is appropriate to estimate γ using the formula

$$\gamma = \left| CU_c - \int_0^T i(t)\,dt \right| \tag{22}$$

We analyzed the process of charging the capacitance in the circuit for various forms of input voltage and circuit parameters. Efficiency rating and γ for different modes of charging the capacity was carried out subject to condition (3). Equation (8) was taken as the initial equation, which was replaced during the analysis by a finite-difference equation of the form:

$$i(t_{n+1}) = i(t_n) + \frac{\Delta t}{L}\left[U_{in}(t) - R \cdot i(t_n) - \frac{1}{C}\sum_{k=0}^{n} i(t_k)\Delta t\right] \tag{23}$$

Based on this formula, the calculated results of the charging current for given parameters are presented in Figure 4.

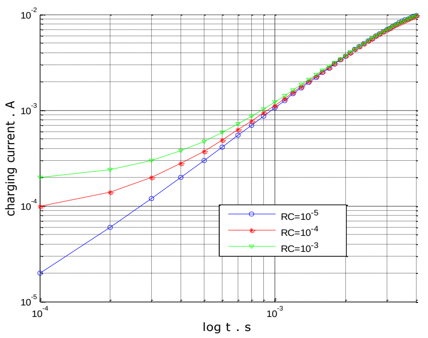


Figure 4. *Charging current variation at different RC values*

Summarizing the above, we can draw the following conclusions.

4. Conclusions

1. Let's find the optimal charging mode for a capacitive energy storage device in the R-L-C circuit. From the analysis of (7), (19), (9), (20) it is clear that in order to obtain the maximum possible efficiency for given R, L, C, T, Uc, it is necessary to have an initial current i(t) in the circuit $i(t)_{|t=0} = \frac{CU_c}{T}$ and the shape of the input voltage should change according to (9).

2. A quadratic criterion for current has been obtained, which makes it possible to compare different modes of charging the capacity.

Author Contribution

Conceive-G.A.,H.A.; Design-H.A.,M.Y.,S.A; Supervision-HA; Experimental Performance, Data Collection and/or Processing-G.B.,M.M.,H.A; Analysis and/or Interpretation-G.A., H.A; Literature Review-M.Y.,S.A.,G.B.,H.A; Writer-S.A. G.B.; Critical Reviews –G.A., M.M.,H.A.

Declaration of Conflict of Interest

The authors have declared no conflicts of interest.

References

[1] Jiang, C., Wang, S., Wu, B., Fernandez, C., Xiong, X., & Coffie-Ken, J. (2021). A state-of-charge estimation method of the power lithium-ion battery in complex conditions based on adaptive square root extended Kalman filter. *Energy*, 219, 119603.

[2] Perez, H. E., Hu, X., Dey, S., & Moura, S. J. (2017). Optimal charging of Li-ion batteries with coupled electro-thermal-aging dynamics. *IEEE Transactions on Vehicular Technology*, 66 (9), 7761-7770.

[3] Pontryagin L.S., Boltianski V., Gamkrelidze R., and. Mitchtchenko E. (1962). The Mathematical Theory of Optimal Processes, John Wiley and Sons, New York.

[4] Mumford, D. B., & Shah, J. (1989). Optimal approximations by piecewise smooth functions and associated variational problems. Communications on pure and applied mathematics.

[5] Sethi S. P., and Thompson G. L. (2000). Optimal Control Theory: Applications to Management Science and Economics, Kluwer, Boston, 2nd edition.

[6] Sotskov A.I., Kolesnik G.V. (2002). Optimal Control: Problems and Solutions. Moscow, New Econ.School,58 p.

[7] Jost, J.; Li-Jost, X. (2003) Calculus of Variations. Cambridge University Press.

[8] Troutman, J. L. (2012). Variational calculus with elementary convexity. Springer Science & Business Media.

[9] Alisoy, H. Z., Alisoy, G. T., Keleş, C., Deniz, F. N., Işık, İ., & İçel, Y. (2013).Darbe generatörünün maksimum enerjisi hakkında varyasyonel problem. İnönü Üniversitesi Malatya.TOK.

[10] Clarke F. (2013) Functional Analysis, Calculus of Variations and Optimal Control. Springer.

[11] Noton, A. R. M. (2013). Introduction to variational methods in control engineering. Elsevier.