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Research Article

Numerical Investigation of Flow Characteristics of Broad-Crested Weirs with Different Geometry

Abdulkadir Demirçelik¹, Mehmet Mustafa Duman ¹, Erdinç İkincioğulları ^{2*}

¹Bingol University, Institute of Science, 12000, Bingol, TÜRKİYE ²Bingol University, Department of Civil Engineering, 12000, Bingol, TÜRKİYE

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Abstract

Broad-crested weirs are important hydraulic structures used for water level control and flow regulation. In the literature, novel designs with upstream and downstream slope angles have been made to increase the discharge capacity of these structures, which are generally designed with vertical upstream and downstream faces. This study numerically investigates the effect on the flow characteristics of broad-crested weirs whose crest is inclined and whose upstream and downstream faces are stepped. The open-source software OpenFOAM and the k- ε turbulence model are used for the numerical analysis. The numerical results are validated with the literature study of a classical broad-crested weir (Imanian et al., 2021). According to the results, it is seen that the discharge capacity of the newly designed models is about 4% higher than the classical broad-crested weir, and the discharge capacity of the models with negative crest slope is higher. In addition, the discharge capacity is higher, and the flow separation zone is lower in the models designed with stepped upstream and downstream surfaces.

Keywords

"Broad-crested weir, discharge coefficient, OpenFOAM, CFD"

1. Introduction

Weirs are hydraulic structures used to increase water level, prevent flow, and manage water velocity (Samani, A. K., & Bagheri, 2014). These structures are usually sharp or broad-crested and can be categorized as normal or side weirs (Zahiri et al., 2013). Broad-crested weirs are structures where the streamlines on the weir are parallel to each other, and the weir crest is horizontal (Bos MG, 1976). These structures, used in open channels, are also used as flow control structures. Broad-crested weirs designed with inclined upstream and downstream surfaces have a higher discharge capacity than traditionally designed broad-crested weirs (Fritz and Hager, 1998).

Many researchers have conducted experimental studies to determine the formation criteria of broad-crested weir flow, investigate its hydraulics, calculate the discharge coefficient, and obtain empirical equations. Blanche et al. (1963) emphasized that the ratio of the total energy over the weir (H_1) to the weir crest length (L) is in the range of $0.1 < H_1/L < 0.4$ while classifying broad-crested weirs. Singer (1964) suggested the range of $0.08 < H_1/L < 5.6$ for this classification, while Bos MG (1976) stated that this range should be $0.08 \le H_1/L \le 0.50$. Rao and Shukla (1971) experimentally investigated the effect of the crest length on the upstream surface of broadcrested weirs depending on whether they have a vertical or elliptical upstream surface. Fritz and Hager (1998) conducted experimental studies on trapezoidal-shaped weirs with different crest lengths. They emphasized that the discharge coefficient for a broad-crested weir is almost 10% less than an embankment-type weir with the same crest length. Sargison and Percy (2009) conducted experimental studies investigating water flow over an embankment-type weir. They stated that decreasing the upstream surface slope increases the water height and the static pressure on the crest. They also emphasized that changing the downstream surface slope has little effect on the discharge coefficients. Goodarzi et al. (2012) conducted experimental studies by changing the upstream surface slope in broadcrested weirs. According to the results obtained, they stated that the flow discharge capacity increases with the decrease in the upstream surface slope. Maghsoodi et al. (2012) numerically investigated the flow in broad-crested weirs with different upstream and downstream surface slopes. The results showed that the k-ɛ turbulence model agreed well with the experimental data. Azimi et al. (2012), in their laboratory studies on different broad-crested weir geometries, stated that the discharge coefficient for broad-crested weirs with upstream and downstream surface slopes was higher than that of non-sloping weirs. Madadi et al. (2014) obtained results showing that reducing the upstream slope increases the discharge coefficient. Tenase et al. (2015) numerically investigated the flow over a rectangular broad-crested weir with the k-ɛ turbulence model. Al-Hashimi et al. (2017) investigated the ability of turbulence models to simulate water surface profiles for rectangular and stepped broad-crested weirs. They emphasized that the k-E turbulence model has the highest accuracy among all the tested models. Shaymaa et al. (2017) compared two- and three-dimensional numerical simulations to determine the water surface height in broad-crested weirs. The results obtained showed that these two methods provide similar accuracy. Jiang et al. (2018) numerically investigated the effect of the upstream surface slope of a trapezoidal broad-crested weir on the flow using OpenFOAM software. The researchers emphasized in their results that the sloped upstream surface positively affects the discharge capacity. Daneshfaraz et al. (2021) examined the flow of the gaps in different configurations left in the body of broad-crested weirs with different surface slopes using Flow-3D software. They showed that the new models cause an increase in the discharge coefficient and a decrease in the upstream water surface level. Zerihun (2020) examined the effect of the results on the flow characteristics by changing the upstream and downstream surface slopes in a trapezoidal cross-section weir. The researcher stated that the discharge capacity increased due to the decrease in the water separation zone with the decrease in the upstream surface slope. On the contrary, the discharge capacity decreased with the decrease in the downstream surface slope. Nourani et al. (2021) investigated the effects of positive and negative crest slope and upstream-downstream surface slope on hydraulic performance in broad-crested weirs using ANSYS FLUENT software. They confirmed that the RNG k-E turbulence model gave the most effective values. The researchers emphasized that the discharge capacity increased with the decrease in the upstream surface slope and that the negative crest slope increased the discharge coefficient even more than the positive crest slope. Imanian et al. (2021) investigated the separation zones of the flow over a broad-crested rectangular weir in detail using OpenFOAM software. Malekzadeh et al. (2022) numerically investigated the flow characteristics of broad-crested weirs with different geometric features using ANSYS FLUENT software. Researchers emphasized that while the downstream surface slope has little effect on the discharge capacity, the discharge capacity increases due to reducing the upstream surface slope.

This study investigated numerically the flow characteristics over broad-crested weirs with different geometric shapes. The literature studies where the upstream or downstream surface is inclined were mentioned above. In this study, the effect of the slope of the possible crest surface on the flow characteristics was investigated numerically in the case of a stepped design of the upstream or downstream surface. OpenFOAM, an open-source computational fluid dynamics software, was used for numerical analysis.

1.1. Hydraulics of Broad-Crested Weirs

Broad-crested weirs' starting and ending points are longer than other weirs. When the water level downstream of these weirs is sufficiently high, the flow over the weir is downstream controlled and in the subcritical flow regime. However, when the downstream water depth is low, the flow over the weir is in the critical flow regime (Özbek, 2009). According to the Bernoulli equation, if the control section is designed for no significant energy losses in the acceleration region upstream, Equation (1) is obtained (Bos MG, 1976).

$$H_{1} = h_{1} + \frac{\alpha V_{1}^{2}}{2g} = H = y + \frac{\alpha V^{2}}{2g}$$

$$V = \{2g \cdot (H_{1} - y)\}^{0.50} \cdot \alpha^{-0.50}$$
(1)
(2)

Here, H_1 represents the total energy in the upstream obtained by taking the weir crest as a reference, h_1 represents the upstream water height, V is the flow velocity, y is the flow depth, and α represents the kinetic energy correction factor (Figure 1). If both sides of Equation (2) are multiplied by the cross-sectional area (A) and α =1 is taken,

$$V \cdot A = A \cdot \{2g \cdot (H_1 - y)\}^{0.50}$$

$$Q = A \cdot \{2g \cdot (H_1 - y)\}^{0.50}$$
(3)
When the flow passes over the weir, a critical flow regime occurs, and if y_c is written instead of y,

$$Q = A_c \cdot \{2g \cdot (H_1 - y_c)\}^{0.50}$$
(5)

Equation (5) is rearranged, and Equality (6) is obtained according to $A_c = b \cdot y_c$ and $y_c = \frac{2}{3}H = \frac{2}{3}H_1$ for rectangular channels.

$$Q = \frac{2}{3} \cdot \left(\frac{2}{3}g\right)^{0.50} \cdot b \cdot H_1^{0.50}$$
(6)

This equation is the result of the idealization of some effects, such as centripetal forces, viscous effects, and turbulence. However, the existence of these effects can be explained by the addition of a discharge coefficient (C_d) (Bos MG, 1976).

$$Q = C_{d} \cdot \frac{2}{3} \cdot \left(\frac{2}{3}g\right)^{0.50} \cdot b \cdot H_{1}^{1.50}$$
(7)

Since it is impossible to measure the total energy (H1) directly, it is common to relate this equation to the upstream water level above the crest as follows. Here, C_v is a correction factor used due to the neglect of the velocity height in the approach channel $(\frac{\alpha V_1^2}{2g})$ and is calculated as in Equation (9) for rectangular cross-section channels (Bos MG, 1976).



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Broad-crested weir

Figure 1. Flow over a broad-crested weir (Bos MG, 1976)

2. Computational Fluid Dynamics

Reference level

 H_1

The Computational Fluid Dynamics (CFD) method solves flow motion using numerical methods (Nilay Sezer Uzol, 2021). This method, which covers the fields of computers, mathematics, and engineering, is based on the continuity and Navier-Stokes equations ("https://en.wikipedia.org/wiki/Computational_fluid_dynamics," 2022). The three-dimensional continuity for Navier-Stokes and incompressible flows is given below.

$$div(\vec{V}) = 0$$

$$\frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \operatorname{grad} \vec{V} = -\frac{1}{2} \operatorname{grad} p + \vec{F} + \nu \cdot \nabla^2 \vec{V}$$
(10)
(11)

Here, V is the velocity, ρ is the specific gravity of fluid, p is the pressure, v is the kinematic viscosity, and F is the gravitational acceleration force.

The CFD method is widely used in industry and academia, reducing production costs both economically and temporally. With the development of computer technology in recent years, this method offers users a virtual laboratory environment with a decrease in solution times. Simulations have three stages; the first stage is the pre-processing step, where the geometry, solution network, and boundary conditions are determined. The second stage is the process section, where the analyses are carried out. The last stage is the post-process stage, where the outputs are displayed (Nilay Sezer Uzol, 2021).

3. Material and Methods

3.1. Geometric Model

Within the scope of the study, the flow characteristics over broad-crested weirs with different geometric shapes were investigated numerically. In order to validate the numerical study, the results of the broad-crested weir placed in a rectangular channel in the literature (Imanian et al., 2021) were used. The channel is 3.335 m long, 0.40 m wide and 0.325 m high. The height and length of the thick-sided weir are 0.12 m and 0.45 m, respectively (Figure 2).



Figure 2. Validated experimental model (Imanian et al., 2021)

In this study, eight different geometries as an alternative to the classical broad-crested weir were investigated. In particular, the effect of the step lengths and the crest slope placed on the upstream or downstream surfaces of the weir on the flow was investigated. Without changing the channel dimensions and the location of the weir, numerical simulations were conducted for eight different broad-crested weirs, the dimensions of which are shown in Figure (3).



Figure 3. Designed models (all measurements are in cm)

3.2. Numerical Model

The open-source OpenFOAM (v7) software and the two-phase flow (air+water) solver interFoam were used in the simulations. In order to validate, a rectangular broad-crested weir was modeled with the dimensions specified above, and the upstream water level was fixed around 0.20 m as in the literature study (Imanian et al. 2021), and the flow rate was determined as Q = 13 L/s. The mesh grid was designed to be two-dimensional and more frequent around the weir (Figure 4a). For the boundary conditions, a constant flow rate (variableHeightFlowRateInletVelocity) was determined for the channel inlet and discharged from the outlet (zeroGradient). The standard wall function (noSlip) was defined for the channel bottom, and the atmospheric boundary condition was defined for the channel top. Since the model is two-dimensional, the front and back walls were not considered (empty) (Figure 4b)





Figure 4. Numerical model; a) mesh grid, and b) boundary conditions

Imanian et al. (2021) compared the results obtained with many turbulence methods with experimental results in their studies. The researchers stated that the k- ε turbulence method was compatible with the experimental results. For this reason, the k- ε turbulence method was used in this study. The standard $k-\epsilon$ model is a widely used in open channel flows. The equations of standard transport for this model are as follows (Imanian et al., 2021).

$$\frac{\partial k}{\partial t} + \frac{\partial k u_i}{\partial x_i} = \frac{\partial}{\partial x_i} \left[\left(v + \frac{v_T}{\sigma_k} \right) \frac{\partial k}{\partial x_i} \right] + \sigma_k - \varepsilon$$
(12)

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial \varepsilon u_i}{\partial x_i} = \frac{\partial}{\partial x_i} \left[\left(v + \frac{v_T}{\sigma_{\varepsilon}} \right) \frac{\partial \varepsilon}{\partial x_i} \right] + C_{1\varepsilon} \frac{\varepsilon}{k} \sigma_k - C_{2\varepsilon} \frac{\varepsilon^2}{k}$$
(13)
$$v_T = C_{u} \frac{\varepsilon^2}{k}$$
(14)

$$v_{\rm T} = C_{\mu} \frac{\varepsilon^2}{k}$$

Here, k is the turbulent kinetic energy; ε is the turbulent kinetic energy dissipation ratio; v is the kinematic viscosity; v_T is the turbulent kinematic viscosity and σ_k , σ_{ϵ} , $C_{1\epsilon}$, $C_{2\epsilon}$, C_{μ} values are constants.

4. Results and Discussion

Validation with an experimental study is an important stage in testing the reliability of the numerical results. In this process, both the shape of the solution grid and the number of cells used are quite effective. In Figure (5), as in the study of Imanian et al. (2021), the ratio of the dimensionless velocity value (U_x/U) to the dimensionless flow depth (y/h) is compared. Taking the starting point of the broad-crested weir as a reference (Figure 2), validation was performed for six different points (x=-0.04 m, -0.08 m, -0.14 m, -0.20 m, -0.40 m, and -1.10 m) taken from the upstream of the weir. In these graphs, h is the fixed water height at the point of flow inlet, and many trial analyses were carried out to obtain a 0.20 m water level as in the experimental study (Imanian et al., 2021). According to the results, it was concluded that the unit flow rate was $0.013 \text{ m}^3/\text{sm}$. U_x; x is the flow velocity in the direction, U; is the uniform flow velocity passing through the section, and y is the point flow depth. Since the uniform flow depth and flow rate are known, the classical flow formula was used to calculate the U value. Figure 5(a-b) shows three different solution grids with a total cell number of 16500-21800-30300 were used. Although time-unchanging flow conditions were reached as a result of 20-second solutions, the solutions were continued for 35 seconds (Figure 5g). According to the results, a slight change was observed in the results increasing the total cell number from 16500 to 21800, while no change was observed in the results when the total cell number was increased from 21800 to 30300. Although a slight deviation was observed in the middle region of the graph in Figure 5a, it was observed that the results did not change as a result of increasing the cell number, and the deviation in this region was also present in the experimental study (Imanian et al., 2021). When looking at the other graphs (Figure 5b-c-d-e-f), it was seen that the obtained results were compatible with the experimental results, and it was concluded that the 21800 cells used were sufficient for this study. For this reason, the results obtained using only 21800 solution cells for the remaining points are shown (Figure 5c-d-e-f). According to the results, it is seen that the numerical results and the experimental results (Imanian et al., 2021) are quite compatible.









(g)

Figure 5. Validation of numerical results: a) x=-0.04 m, b) x=-0.08 m, c) x=-0.14 m, d) x=-0.20 m, e) x=-0.40 m, f) x=-1.10 m and g) Variation of flow velocity versus analysis time

The streamlines formed on the broad-crested weirs in different models designed within the scope of the study are shown in Figure (6). The streamlines colored according to the same speed values are adjusted so that the maximum speed value is 1.50 m/s. The separation zones are seen in the upstream part of the classical rectangular weir and other models designed with a vertical upstream surface (Figure 6a-b-c-f-g). For this reason, it is important to investigate these regions since vibrations may occur in the flow over the weir (Imanian et al., 2021). The weirs designed with stepped upstream faces considerably reduce the separation zone. It is observed that there is a smoother transition, especially in Model-7, which has a large step width and a sloped crest (Figure 6a). In the weirs with a vertical downstream surface, the vortices formed in the downstream region are remarkable (Figure 6a-d-e-h-i). In models designed with stepped upstream surface and positively sloped crest, it was observed that the mentioned downstream vortex regions were smaller compared to other models (Figure 6h-i). It is thought that this vortex region decreased as a result of the decrease in flow velocity due to the increasing crest slope. These vortices may cause an increase in the probable scour depths downstream of the weir. The vortex region formed in

models designed with stepped downstream surface became considerably smaller (Figure 6b-c-f-g). However, the vortex regions observed downstream of Model 5 and Model 6 designed with a negatively sloped crest and stepped downstream surface, are smaller than other models (Figure 6f-g). For this reason, it is thought that the probable scour depths downstream of the mentioned models may be lower. When the velocity values are examined, the flow velocity over the negatively sloped weirs is higher than in other models, while the flow velocity is lower in models with positively sloped crests. When the velocities occurring in the downstream region are compared, the flow velocity occurring downstream of Model 3 is higher than the other models, while the flow velocity downstream of the models with positive crest slope is lower.





(h) (i) **Figure 6.** Streamlines obtained from numerical models: a) Rectangle, b) Model-1, c) Model-2, d) Model-3, e) Model-4, f) Model-5, g) Model-6, h) Model-7 and i) Model-8

Equation (8), given by Bos (1976), was used to compare the flow discharge capacities of the designed models. When the results were examined, it was observed that the discharge capacity was about 4% higher in the newly designed models. In particular, it was seen that the discharge capacity of Models 5 and 6, which were designed with a negative crest slope, was higher (Table 1). It was seen that the discharge capacity was higher, and the flow separation zone was lower in models designed with stepped upstream surfaces. Many researchers in the literature emphasized that the discharge capacity increased with the decrease in the upstream surface slope (Azimi et al., 2012; Goodarzi et al., 2012; Jiang et al., 2018; Madadi et al., 2014; Nourani et al., 2021; Zerihun, 2020). Therefore, the results are also compatible with the literature. In addition, while Zerihun (2020) emphasized a decrease in discharge capacity with the decrease in the downstream surface slope, this study observed that models designed with stepped downstream surfaces also increased the discharge capacity. It was also observed that the discharge capacity was not affected by the change in the step sizes used on the downstream surface, and the discharge capacity increased with the decrease in the upstream step sizes in models with positive crest slopes (Table 1).

Table 1. Discharge coefficients of compared models

Models	Nap height (m)	Discharge (m ³ /s)	Channel width (m)	Energy height on the weir (m)	C v (-)	Cd (-)
Rectangular	0.078	0.013	0.40	0.0865	1.178	0.636
Model-1	0.077	0.013	0.40	0.0862	1.180	0.638
Model-2	0.077	0.013	0.40	0.0860	1.183	0.639
Model-3	0.075	0.013	0.40	0.0846	1.197	0.647
Model-4	0.075	0.013	0.40	0.0848	1.195	0.646
Model-5	0.069	0.013	0.40	0.0802	1.258	0.668
Model-6	0.069	0.013	0.40	0.0802	1.258	0.668
Model-7	0.058	0.013	0.40	0.0739	1.448	0.656
Model-8	0.060	0.013	0.40	0.0750	1.395	0.666

5. Conclusion

In this study, the flow characteristics of broad-crested weirs designed with sloped crests and stepped upstream or downstream surfaces were numerically investigated using OpenFOAM software. In this study, eight broad-crested weir models were examined, and the numerical results were confirmed with the results of the classical broad-crested weir in the literature (Imanian et al., 2021). According to the results, it was observed that in models designed with stepped upstream surface and positively sloped crest, downstream vortex regions were smaller compared to other models, the vortex region formed downstream of models designed with the stepped downstream surface was considerably smaller, the discharge capacity of broad-crested weirs designed with stepped upstream or downstream surfaces was higher than classical broad-crested weirs, the discharge capacities of models designed with negatively sloped crest and stepped downstream surfaces were higher, the dimensions of the steps on the downstream surface in models with positively sloped crests.

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