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RESEARCH ARTICLE

# Applications of Equivalent Curves to Ruled Surfaces

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**ABSTRACT**

**In this paper, the characterization of equivalent curves in $\mathbb{E}^3$ is used to define ruled surfaces whose base curves are equivalent curves and to examine the relationships between them. At the same time, an equivalence relation for ruled surfaces is obtained. The equivalence classes resulting from this relation are studied. It is concluded that all ruled surfaces in the equivalence class of a developable surface are developable. Thus, a method is established to obtain an infinite ruled surface from a ruled surface. Finally, a new method is given to obtain a developable surface.**

***Keywords:*** *Equivalent curve; ruled surface; developable surface; equivalence class.*

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## 1. Introduction

The theory of surfaces has a significant place in differential geometry and there are many studies on this subject [3, 4, 9]. One of the most important surfaces, which has a major place not only in differential geometry but also in many different disciplines such as physics, engineering, computer graphics, and architecture, is the ruled surface, which was first introduced and studied by Gaspard Monge [11]. Generally, ruled surfaces are defined as surfaces obtained by moving a line along a curve. The theory of ruled surfaces was developed using the E. Study transformation, which allows studying the theory of surfaces and the geometry of ruled surfaces with a single real parameter [16]. Izumiya and Takeuchi discussed some special curves lying on ruled surfaces and explained their relationship with the Gaussian and mean curvatures of the surface [7]. Furthermore, the class of developable surfaces is one of the important study areas of classical differential geometry. Izumiya and Takeuchi also classified developable ruled surfaces in their article, which described slant helices [8]. Inspired by the concept of slant helix, Önder defined the concept of slant ruled surface and studied different types of these surfaces in both Euclidean and Minkowski space [12]. Önder and Kaya defined Darboux slant ruled surfaces as a new type of slant ruled surfaces and showed the relations of these surfaces with other slant ruled surfaces [13]. Önder and Kahraman has defined rectifying ruled surfaces in 3-dimensional Euclidean space and examined the relationships between these surfaces and slant ruled surfaces [14]. Karakaş and Gündoğan established the isomorphism between non-cylindrical ruled surfaces, $DS^2$ and $TS^2$ for the first time [10]. Later Hathout, Bekar and Yaylı have done studies on ruled surfaces and $TS^2$. They have also obtained results on the developability condition of ruled surfaces in Euclidean space [6].

In this study, we used a new characterization given by Camcı et al. for equivalent curves [1]. In their work, Camcı et al. obtained an equivalence relation for curves and analyzed the equivalence classes resulting from this relation for some special curves. At the heart of their work, they have developed a useful method for obtaining another curve from a given curve using the Combescure transformation. Some special curves can be obtained from any curve using this method. We defined ruled surfaces with equivalent base curves and the same direction vectors, examined the relationships between them, and obtained an equivalence relation for the ruled surfaces. At the same time, we have given a method by which we can obtain infinite ruled surfaces from a ruled surface. Finally, we gave a new method for developable surfaces.

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## 2. Equivalent Ruled Surfaces

Two curves are said to be related by a Combescure transformation if they correspond point by point and have parallel tangents at corresponding points [5, 15]. In this case, the curves share the same Frenet frames. Camcı et al. defined an equivalence relation based on the fact that these curves have a common Frenet frame and explored the equivalence classes that result from this relation for some special curves [1].

In this section, we obtained an equivalence relation for ruled surfaces by defining ruled surfaces whose base curves are equivalent curves, using the equivalent curve characterization given by Camcı et al.

The following theorem can be seen in [1].

**Theorem 2.1.** *Let $\alpha : I \subset \mathbb{R} \to \mathbb{E}^3$ and $\beta : I \subset \mathbb{R} \to \mathbb{E}^3$ be regular curves in Euclidean 3-space with Frenet apparatus $\{T, N, B, \kappa, \tau\}$ and $\{T^*, N^*, B^*, \kappa^*, \tau^*\}$, respectively. Then the tangent vector $T$ of $\alpha$ is equal to the tangent vector $T^*$ of $\beta$ if and only if*

$$\beta(\varphi(s)) = \varphi^{'}(s)\alpha(s) - \int \varphi^{''}(s)\alpha(s)ds \tag{2.1}$$

*for $\varphi^{'}(s) = \frac{ds^*}{ds}$ where $s$ and $s^*$ are arclength parameters of $\alpha$ and $\beta$, respectively.*

**Corollary 2.1.** *Let $\alpha : I \subset \mathbb{R} \to \mathbb{E}^3$ and $\beta : I \subset \mathbb{R} \to \mathbb{E}^3$ be regular curves in Euclidean 3-space with Frenet apparatus $\{T, N, B, \kappa, \tau\}$ and $\{T^*, N^*, B^*, \kappa^*, \tau^*\}$, respectively. Then the tangent vector $T$ of $\alpha$ is equal to the tangent vector $T^*$ of $\beta$ if and only if $N = N^*$ and $B = B^*$.*

**Definition 2.1.** Let $\alpha : I \subset \mathbb{R} \to \mathbb{E}^3$ and $\beta : J \subset \mathbb{R} \to \mathbb{E}^3$ be two regular curves with Frenet frames $\{T_1, N_1, B_1\}$ and $\{T_2, N_2, B_2\}$, respectively. If there is a diffeomorphism $h : I \to J$ such that the Frenet vectors $T_1, N_1, B_1$ are parallel to the Frenet vectors $T_2 \circ h$, $N_2 \circ h$, $B_2 \circ h$ respectively and $T_1 \circ h^{-1}$, $N_1 \circ h^{-1}$, $B_1 \circ h^{-1}$ are parallel to the Frenet vectors $T_2, N_2, B_2$ respectively, then $\alpha$ and $\beta$ are said to be "$equivalent$" [1].

"$Being\ equivalent\ curves$" is an equivalence relation. Here $J = I$ can be chosen. So we have $\alpha, \beta : I \subset \mathbb{R} \to \mathbb{E}^3$.

Let $\alpha : I \subset \mathbb{R} \to \mathbb{E}^3$ and $\beta : I \subset \mathbb{R} \to \mathbb{E}^3$ be regular curves in Euclidean 3-space with Frenet apparatus $\{T, N, B, \kappa, \tau\}$ and $\{T^*, N^*, B^*, \kappa^*, \tau^*\}$ respectively. "$Having\ same\ Frenet\ frame\ for\ curves$" is an equivalence relation. The equivalence class of a regular curve $\alpha : I \subset \mathbb{R} \to \mathbb{E}^3$ is given by

$$[\alpha] = \{\beta | \beta : I \subset \mathbb{R} \to \mathbb{E}^3, \beta(\varphi(s)) = \varphi^{'}(s)\alpha(s) - \int \varphi^{''}(s)\alpha(s)ds\},$$

where $\varphi : I \to \mathbb{R}$ is a non-constant differentiable function, and $\forall s \in I$, $\varphi^{'}(s) \neq 0$.

If $\alpha$ is equivalent $\beta$, let us denote it by "$\alpha \sim \beta$".

**Corollary 2.2.** *Let $\alpha : I \subset \mathbb{R} \to \mathbb{E}^3$ and $\beta : I \subset \mathbb{R} \to \mathbb{E}^3$ be regular curves in Euclidean 3-space with Frenet apparatus $\{T, N, B, \kappa, \tau\}$ and $\{T^*, N^*, B^*, \kappa^*, \tau^*\}$ respectively. If the tangent vector $T$ of $\alpha$ is equal to the tangent vector $T^*$ of $\beta$, then we get*

$$\varphi^{'}(s) = \frac{ds^*}{ds} = \frac{\kappa}{\kappa^*} = \frac{\tau}{\tau^*}$$

*[1].*

Let's define a ruled surface as

$$\Psi(s, \lambda) = \alpha(s) + \lambda e(s)$$

with the transformation $\Psi : I \times \mathbb{R} \to \mathbb{E}^3$. Here $\alpha(s)$ is the base curve, and $e(s)$ is the direction vector. Let another ruled surface with base curve $\beta(s^*)$ and direction vector $\widetilde{e}(s^*)$ be defined by the transformation $\widetilde{\Psi} : J \times \mathbb{R} \to \mathbb{E}^3$, given by

$$\widetilde{\Psi}(s^*, \widetilde{\lambda}) = \beta(s^*) + \widetilde{\lambda}\widetilde{e}(s^*) \tag{2.2}$$

where $\alpha \sim \beta$. $\varphi : I \to J$ is a diffeomorphism. So we can write $\widetilde{e}(s^*) = (e \circ \varphi^{-1})(s^*)$ or $e(s) = (\widetilde{e} \circ \varphi)(s)$. That is, the direction vectors of the surfaces are the same. We can choose $\widetilde{\lambda} = \lambda\varphi^{'}(s)$ and since $\varphi(s) = s^*$ we can write the surface in equation 2.2 as

$$\widetilde{\Psi}(s, \lambda) = \varphi^{'}(s)\Psi(s, \lambda) - \int \varphi''(s)\alpha(s)ds.$$

**Definition 2.2.** Let $M$ be the ruled surface defined by $\Psi(s, \lambda) = \alpha(s) + \lambda e(s)$ in $\mathbb{E}^3$ and $\widetilde{M}$ be the ruled surface defined by $\widetilde{\Psi}(s^*, \widetilde{\lambda}) = \beta(s^*) + \widetilde{\lambda}\widetilde{e}(s^*)$ in $\mathbb{E}^3$. If $\alpha \sim \beta$ and $\widetilde{e}(s^*) = (e \circ \varphi^{-1})(s^*)$, where $\varphi : I \to J$ is a diffeomorphism, then $M$ and $\widetilde{M}$ are said to be "$equivalent$".

**Remark 2.1.** *"Being equivalent surfaces" is an equivalence relation.*

If $M$ is equivalent $\widetilde{M}$, let us denote it by "$M \equiv \widetilde{M}$".

**Example 2.1.** Let us consider $\alpha : I \subset \mathbb{R} \to \mathbb{E}^3$ given by $\alpha(t) = (t, t^2, t^3)$. By taking $\varphi(t) = \frac{t^3}{3} + t$, we find

$$\beta(\varphi(t)) = (\frac{1}{3}t^3 + t, \frac{1}{2}t^4 + t^2, \frac{3}{5}t^5 + t^3).$$

Let's choose $e(t) = \widetilde{e}(t) = (1, t, t^2)$ as the direction vector. Then,

$$\Psi(t, \lambda) = (t + \lambda,\ t^2 + \lambda t,\ t^3 + \lambda t^2)$$

and

$$\widetilde{\Psi}(t, \widetilde{\lambda}) = (\frac{1}{3}t^3 + t + \widetilde{\lambda}, \frac{1}{2}t^4 + t^2 + \widetilde{\lambda}t, \frac{3}{5}t^5 + t^3 + \widetilde{\lambda}t^2)$$

is obtained. Let $M$ be the ruled surface defined by $\Psi(t, \lambda) = \alpha(t) + \lambda e(t)$ in $\mathbb{E}^3$ and $\widetilde{M}$ be the ruled surface defined by $\widetilde{\Psi}(t, \widetilde{\lambda}) = \beta(t) + \widetilde{\lambda}\widetilde{e}(t)$ in $\mathbb{E}^3$. $M$ and $\widetilde{M}$ are equivalent.

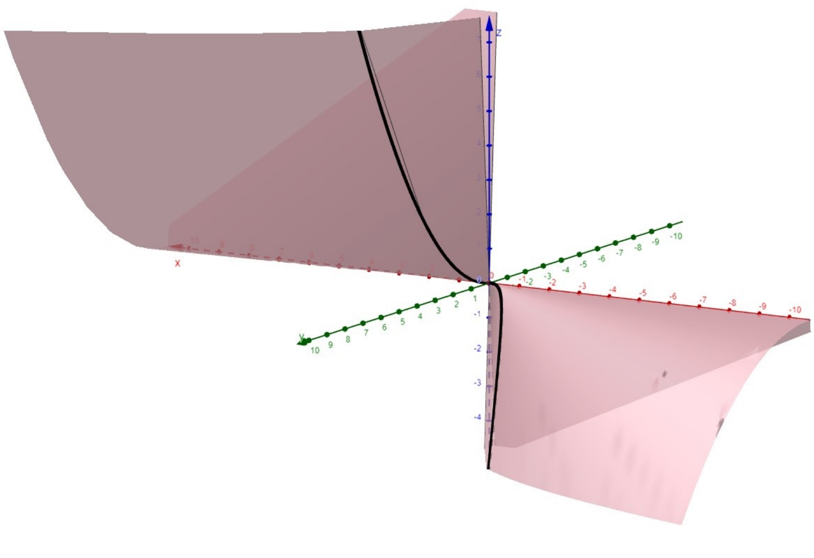

**Figure 1.** Surface $M$ with base curve $\alpha$

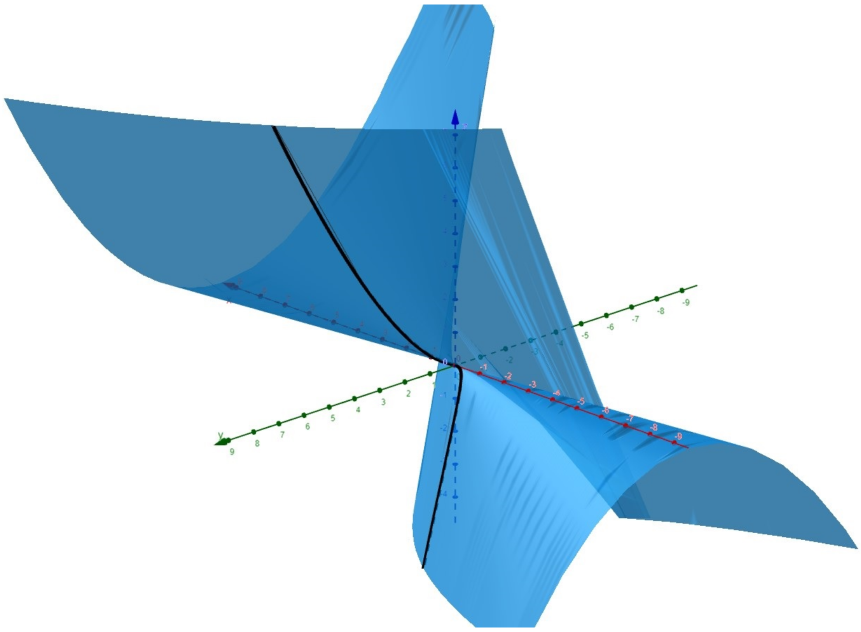

**Figure 2.** Surface $\widetilde{M}$ with base curve $\beta$

## 3. Relations Between Equivalent Ruled Surfaces

In this section, various properties of the equivalent surfaces $M$ and $\widetilde{M}$ have been examined, and the relationships between them have been established.

Let $M$ be the ruled surface defined by $\Psi(s,\lambda) = \alpha(s) + \lambda e(s)$ in $\mathbb{E}^3$ and $\widetilde{M}$ be the ruled surface defined by $\widetilde{\Psi}(s^*,\widetilde{\lambda}) = \beta(s^*) + \widetilde{\lambda}\widetilde{e}(s^*)$ in $\mathbb{E}^3$, here $s$ and $s^*$ are arclength parameters of $\alpha$ and $\beta$. Take $\{T, N, B\}$ and $\{T^*, N^*, B^*\}$ be the Frenet vectors of the curves $\alpha$ and $\beta$, respectively.

**Theorem 3.1.** *The base curve $\alpha$ of $M$ is perpendicular to the direction vector of $M$ at each point of the $\alpha$ if and only if the base curve $\beta$ of $\widetilde{M}$ is perpendicular to the direction vector of $\widetilde{M}$ at each point of the $\beta$.*

Let $e$ be a unit vector and $\langle T, e\rangle = 0$. Due to the theorem, choosing in this way does not result in losing anything general. If $Z = T \times e$, then the system $\{T, e, Z\}$ forms an orthonormal frame at the point $\alpha(s)$. Here, $Z = (Z \circ \alpha)(s)$ is the unit normal vector field of the $M$ at the point $\alpha(s)$. If

$$
\begin{aligned}
a &= \left\langle \frac{dT}{ds}, e \right\rangle \\
b &= \left\langle \frac{dT}{ds}, Z \right\rangle \\
c &= \left\langle \frac{de}{ds}, Z \right\rangle,
\end{aligned}
$$

then

$$
\begin{aligned}
T' &= \frac{dT}{ds} = ae{+}bZ \\
e' &= \frac{de}{ds} = -aT + cZ \\
Z' &= \frac{dZ}{ds} = -bT - ce.
\end{aligned}
$$

Similarly, $\{T^*, \widetilde{e}, \widetilde{Z}\}$ is the orthonormal basis at the point $\beta(s^*)$, where $\beta(s^*) = \beta(\varphi(s))$ is the point corresponding to $\alpha(s)$ and $\widetilde{Z}{=}\ (\widetilde{Z} \circ \beta)(s^*)$ is the unit normal vector field of the $\widetilde{M}$ at the point $\beta(s^*)$. Also

$\widetilde{Z}(s^*) = (Z \circ \varphi^{-1})(s^*)$. If

$$\begin{aligned} \widetilde{a} &= \left\langle \frac{dT^*}{ds^*}, \widetilde{e} \right\rangle \\ \widetilde{b} &= \left\langle \frac{dT^*}{ds^*}, \widetilde{Z} \right\rangle \\ \widetilde{c} &= \left\langle \frac{d(e \circ \varphi^{-1})}{ds^*}, \widetilde{Z} \right\rangle \end{aligned}$$

then

$$\begin{aligned} a &= \varphi'(s)\widetilde{a} \\ b &= \varphi'(s)\widetilde{b} \\ c &= \varphi'(s)\widetilde{c}. \end{aligned}$$

In this case we can give the following result.

**Corollary 3.1.** *$\widehat{\alpha}(s)$ and $\widehat{\beta}(s^*)$ be the striction curves of the surfaces $M$ and $\widetilde{M}$, respectively. If $M \equiv \widetilde{M}$, then*

$$\widehat{\beta}(\varphi(s)) = \varphi^{'}(s)\widehat{\alpha}(s) - \int \varphi^{''}(s)\alpha(s)ds.$$

**Corollary 3.2.** *Let $d$ denote the distribution parameter of the ruled surface $M$ and $\widetilde{d} \circ \varphi^{-1}$ denote the distribution parameter of the ruled surface $\widetilde{M}$. If $M \equiv \widetilde{M}$, then*

$$\widetilde{d} \circ \varphi^{-1} = \varphi'(s)d.$$

Let's say $\varphi'(s) = f(s)$ in equation 2.1. Here, $f : I \to \mathbb{R}$, for $\forall s \in I$, $f(s) > 0$. Then we can write

$$\beta(s) = f(s)\alpha(s) - \int f^{'}(s)\alpha(s)ds.$$

In this case, $\beta^{'}(s) = f(s)T$ is obtained. Also, we can choose $\widetilde{\lambda} = f\lambda$. The unit normal vector field $Z(s, \lambda)$ at a point $\Psi(s, \lambda)$ on the ruled surface $M$ is given by $Z(s, \lambda) = \frac{T \times e + \lambda(e^{'} \times e)}{\|T \times e + \lambda(e^{'} \times e)\|}$. Similarly, the unit normal vector field $\widetilde{Z}(s, \widetilde{\lambda})$ at a point $\widetilde{\Psi}(s, \widetilde{\lambda})$ on the ruled surface $\widetilde{M}$ is given by $\widetilde{Z}(s, \widetilde{\lambda}) = \frac{f\varphi_s \times e}{\|f\varphi_s \times e\|}$.

$$Z(s, \lambda) = \widetilde{Z}(s, \widetilde{\lambda})$$

is obtained as a result.

We can derive the following results.

**Corollary 3.3.** *$S$ and $\widetilde{S}$ are shape operators of surfaces $M$ and $\widetilde{M}$, respectively. If $M \equiv \widetilde{M}$, then*

$$\widetilde{S} = \frac{1}{f}S.$$

**Corollary 3.4.** *If $M \equiv \widetilde{M}$, then*

$$\begin{aligned} \widetilde{K} &= \frac{1}{f^2}K \\ \widetilde{H} &= \frac{1}{f}H \end{aligned}$$

*where $K$ and $H$ are the Gaussian and mean curvatures of the surface $M$, $\widetilde{K}$ and $\widetilde{H}$ are the Gaussian and mean curvatures of the surface $\widetilde{M}$.*

**Corollary 3.5.** *The necessary and sufficient condition for the $M$ ruled surface to be developable is that the $\widetilde{M}$ ruled surface be developable.*

From Corollary 3.5, if $M$ is a developable surface, we can say that,

$$[M] = \left\{ \widetilde{M} \in RS \ : M \equiv \widetilde{M} \right\}$$

is a family of developable surfaces. Here, $RS$ denotes the set of all ruled surfaces in $\mathbb{E}^3$.

**Corollary 3.6.** *The necessary and sufficient condition for the $M$ ruled surface to be minimal is that the $\widetilde{M}$ ruled surface be minimal.*

From Corollary 3.6, if $M$ is a minimal surface, we can say that

$$[M] = \left\{ \widetilde{M} \in RS \ : M \equiv \widetilde{M} \right\} \tag{3.1}$$

is a family of minimal surfaces.

**Remark 3.1.** *It is well known that Catalan proved in 1842 that the helicoid and the plane are the only ruled minimal surfaces [2]. The base curves of the helicoid and plane are lines. Since all curves that are equivalent to a line are also line, the family of minimal ruled surfaces in 3.1 is a set of helicoids or planes. Therefore, this does not contradict the results known in $\mathbb{E}^3$.*

**Corollary 3.7.** *The principal curvatures of the ruled surfaces $M$ and $\widetilde{M}$ are $k_1, k_2, \widetilde{k_1}, \widetilde{k_2}$ respectively. If $M \equiv \widetilde{M}$, then*

$$\begin{aligned} \widetilde{k_1} &= \frac{1}{f} k_1 \\ \widetilde{k_2} &= \frac{1}{f} k_2. \end{aligned}$$

**Corollary 3.8.** *If $M \equiv \widetilde{M}$, the characters of the corresponding points of these surfaces (parabolic, flat, hyperbolic, umbilical) are the same.*

*Proof.* It can be easily seen using Corollary 3.3, Corollary 3.7. □

## 4. Generating Developable Surface

It is known to the lines connecting the corresponding points of two curves obtained from each other by the Combescure transformation generate a developable surface [17]. In this section, a more general form of this theorem is given and a new proof is made using equivalent curves.

**Theorem 4.1.** *Let $\alpha, \beta, \gamma : I \subset \mathbb{R} \to \mathbb{E}^3$ be regular curves in Euclidean 3-space. If $[\alpha] = [\beta] = [\gamma]$, then*

$$\varphi(s, u) = \gamma(s) + u\overrightarrow{\alpha\beta}$$

*is a developable surface.*

*Proof.* Let $\alpha, \beta, \gamma : I \subset \mathbb{R} \to \mathbb{E}^3$ be regular curves in Euclidean 3-space. Assume that $[\alpha] = [\beta] = [\gamma]$ . In this case, the ruled surface with the base curve $\gamma$ and the direction vector $\overrightarrow{\alpha\beta}$ is denoted by

$$\varphi(s, u) = \gamma(s) + u\overrightarrow{\alpha\beta}.$$

If $\varphi^{'}(s) = f(s)$ is written in equation 2.1,

$$\beta(s) = f(s)\alpha(s) - \int f^{'}(s)\alpha(s)ds$$

is obtained, where $f : I \to \mathbb{R}$, $\forall s \in I$, $f(s) \neq 0$. When $\overrightarrow{X} = \overrightarrow{\alpha\beta}$, then $\varphi(s, u) = \gamma(s) + u\overrightarrow{X}$. To determine whether this ruled surface is developable, let's find the distribution parameter. The direction vector of the line connecting corresponding points is obtained as

$$X = (f(s) - 1)\alpha(s) - \int f^{'}(s)\alpha(s)ds$$

and

$$X^{'}(s) = (f(s) - 1)T_\alpha$$

where $T_\alpha$ is the unit tangent vector field of $\alpha$. Since $\alpha \sim \gamma$, $T_\alpha = T_\gamma$. Here let's say $(f(s) - 1) = a$. Then, since

$$\det(T_\gamma, X, X^{'}) = \det(T_\alpha, X, X^{'}) = a\det(T_\alpha, X, T_\alpha) = 0$$

the defined surface is developable. □

The theorem states that an infinite number of different developable surfaces can be produced using a unique curve.

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Not applicable.

## Competing interests

The authors declare that they have no competing interests.

## Author's contributions

This paper is derived from the first author's doctoral dissertation supervised by the second author.
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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