NÖHÜ Müh. Bilim. Derg. / NOHU J. Eng. Sci., 2025; 14(2), 751-759

Niğde Ömer Halisdemir Üniversitesi Mühendislik Bilimleri Dergisi
Niğde Ömer Halisdemir University Journal of Engineering Sciences

Araştırma makalesi / Research article

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Mixed model assembly line balancing with in-house parts supply considering the supermarket concept: A new mathematical model

Süpermarket konseptini dikkate alarak tesis içi parça tedariki ile karışık modelli montaj hattı dengeleme: Yeni bir matematiksel model

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Abstract

The supermarket concept has become popular recently, especially for in-house part logistics systems, which are suitable for the full-time production philosophy of assembly lines that provide mass and high-volume production. The supermarket location problem (SLP) is to determine the location and number of supermarkets for the part supply from supermarkets to assembly stations. Considering the assembly line balancing problem (ALBP) and SLP together offers an important perspective in the facility planning step. This paper proposes a new mixed-integer linear programming (MILP) model to solve mixed-model assembly line balancing problem (MMALBP) and SLP together. The aim is to minimize the total cost of workstation installation, supermarket installation, and transportation. To verify the effectiveness of the proposed MILP model, two different model structures that solve MMALBP and SLP, both separately and simultaneously, are considered. Test problems widely used in the ALBP literature are used for computational experiments. The results of computational experiments show that the proposed model performs quite well in terms of both total costs and model execution time.

Keywords: Supermarket concept, MMALBP, Part supply, Supermarket location, MILP

Öz

Süpermarket konsepti, son yıllarda seri ve yüksek hacimli üretim sağlayan montaj hatlarının tam zamanlı üretim felsefesine uygun olan özellikle tesis içi parça lojistik sistemleri için popüler hale gelmiştir. Süpermarket yerleşim problemi (SYP) süpermarketlerden montaj istasyonlarına parça tedariki için süpermarketlerin yerini ve sayısını belirleyen problemdir. Montaj hattı dengeleme problemini (MHDP) ve SYP'yi birlikte ele almak tesis planlama adımında önemli bir bakış açısı sunmaktadır. Bu çalışma, karışık modelli montaj hattı dengeleme problemini (KMMHDP) ve SYP'yi birlikte çözmek için yeni bir karma tam sayılı doğrusal programlama (KTDP) modeli önermektedir. Amaç, iş istasyonu kurulumu, süpermarket kurulumu ve nakliye toplam maliyetini en aza indirmektir. Önerilen MILP modelinin etkinliğini doğrulamak için KMMHDP'yi ve SYP'yi hem ayrı hem de birlikte çözen iki farklı model yapısı ele alınmaktadır. Hesaplamalı deneyler için MHDP literatüründe yaygın bir şekilde kullanılan test problemleri kullanılmaktadır. Hesaplamalı deneylere göre önerilen matematiksel model maliyet ve model çalışma süreleri açısından başarılı bir performans sergilemiştir.

Anahtar Kelimeler: Süpermarket konsepti, KMMHDP, Parça tedariki, Süpermarket yerleşimi, KTDP

1 Introduction

The basic principles of assembly lines (ALs) are based on the production of products by combining parts in a continuous flow-oriented production system. Although the AL was first used for meat packing operations in Cincinnati and Chicago slaughterhouses in the 19th century, significant development of the AL began with the production system designed by Henry Ford and his engineers for the Model-T automobile in Michigan (see., [1-4]). The production time of a Model-T was reduced from 12.5 hours to only 93 minutes through Henry Ford's ALs [3, 4]. The mass and high-volume production in the ALs led to a major manufacturing revolution all over the world.

There are several requirements and constraints for designing the AL. For example, the precedence relationships among tasks for the parts to be assembled should be considered. Moreover, the processing times of the tasks to be performed by workers in the stations should be balanced. The problem that basically addresses these requirements is called the assembly line balancing problem (ALBP).

ALs have tremendously evolved from the past to the present. According to the features of the problem, the ALs can be classified such as product diversity (single model, multi/mixed model), the certainty of the task times (deterministic, stochastic), the shape and layout of the line (straight-shaped, U-shaped), and assembly operations' location (one-sided, two-sided, parallel). There are many review papers that address detailed literature reviews of the ALs (see., [5-15]).

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Geliş / Received: 26.03.2025 Kabul / Accepted: 25.04.2025 Yayımlanma / Published: 15.04.2025
doi: 10.28948/ngumuh.1577703

In addition to mass and high-volume production on ALs, in recent years, the production concept has changed along with Just-In-Time (JIT) and lean manufacturing philosophies. Accordingly, establishing a well-organized in-house part supply logistic system is a major requirement in order to perform the JIT philosophy [16]. The parts must be delivered to the workstations of the AL on time to hinder part insufficiency or accumulation of the parts in the workstations. Accordingly, decentralized in-house logistics areas, called supermarkets, have recently begun to come to the fore. The supermarkets serve as intermediate stores for parts required by nearby AL stations [17]. According to the JIT philosophy, it is aimed that parts are transported in frequent small-volume (amount or lot). The delivery of parts to the supermarkets is carried out by the tow trains/ tuggers consisting of several wagons. The parts demanded by the workstations are filled/loaded from the supermarket into empty bins in the wagons of the tow train. The parts-loaded bins transported by tow train are put into part racks of the related workstation manually or through robotic systems. Then, the tow train that collects the empty bins on the part racks returns to the related supermarket. This process consistently continues without any failure/disruption. This in-logistic part supply system is called the 'supermarket concept'. The supermarket concept is shown in Figure 1.

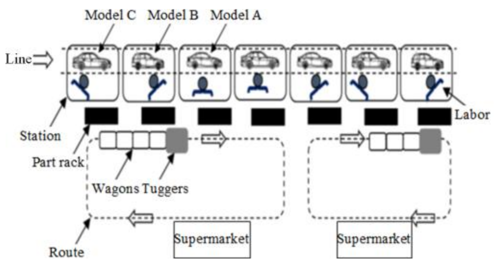

Figure 1. Supermarket concept for the MMALs

The supermarket concept includes four major problems from long-term strategy to short-term operation [18]. These are locating (locating and number of supermarkets), routing (number of tow trains), scheduling (number of tours), and loading (type and number of part bins) problems [16]. The location planning problem is the most basic problem of the supermarket concept and is called the supermarket location problem (SLP). However, the number of papers considering SLP on the ALs is quite limited. Battini et al. [19] aimed to optimize stocking policies using supermarket warehouses. Accordingly, they proposed a decision-making procedure (DMP) determining the location and number of supermarkets. Emde and Boysen [17] studied the SLP for the mixed-model assembly lines (MMALs). They proposed a mathematical model and an exact dynamic programming (DP) algorithm to optimize the part transportation and supermarket installation costs. Battini et al. [16] discussed in detail the supermarket concept for in-house logistics in the automotive industry. They focused on the basic structure and assumptions of the SLP and its usage in the automotive industry. Alnahhal and Noche [20] considered the SLP for the part feeding with the supermarket concept. They presented a mathematical model and a real genetic algorithm (GA) considering minimization transportation and inventory fixed costs. Nourmohammadi et al. [21] presented a mathematical model considering the SLP with stochastic part demands. Fathi et al. [22] proposed a simulated annealing (SA) algorithm to minimize shipment and installation costs in the SLP in terms of manufacturing sustainability through JIT-part supply on the ALs. Nourmohammadi et al. [23] presented a mixed-integer programming model that considered the transport vehicle selection problem along with SLP. In addition, they proposed a GA based on a variable neighborhood search (VNS) algorithm for large-sized problem sets.

The SLP, like the ALBP, is a long-term problem to be solved in the facility planning stage. In fact, SLP's solution is directly dependent on line balance in ALs. In order to solve SLP, AL information, i.e., line type, number of stations, cycle time, and line balance (tasks assigned to stations), must be available. Therefore, SLP should be considered with the ALBP.

The first paper that deals with the supermarket concept and ALs together was presented by Nourmohammadi and Eskandari [24]. They proposed a bi-level mathematical model to solve simple ALBP (SALBP) and SLP together. After they minimized the number of workstations in the upper level, they minimized the supermarket transportation and installation costs in the lower level. They utilized the mathematical model proposed by Alnahhal and Noche [20] for the SLP in the lower level. Finally, they used the well-known test problems in the ALBP literature to evaluate the performance of the proposed model. Nourmohammadi et al. [25] considered the same perspective in Nourmohammadi and Eskandari [24] for the stochastic nature of the task times and demands. Nourmohammadi et al. [26] proposed a cost-based binary linear programming (BLP) model to minimize the total of the workstations and supermarket installation costs. Chen et al. [27] presented a bi-level perspective to solve the ALBP and SLP together. At the upper level, they aimed to solve the number of workstations and workload smoothness. At the lower level, it is aimed to minimize the number of supermarkets corresponding to the number of workstations and workload smoothness in the upper level. Accordingly, they presented both a mathematical model for small-sized problems and a bi-level multi-objective GA for medium-sized and large-sized problems. Zangaro et al. [28] proposed a MILP model and adaptive large neighborhood search (ALNS) algorithm to jointly solve ALBP and SLP in the multi-manned ALs. They aimed to minimize the AL systems by considering supermarket, transportation, assembly operations, and investment costs.

To the best of the author's knowledge, the only study that considered the SLP and the mixed-model assembly line balancing problem (MMALBP) together was presented by Delice et al. [29]. They proposed a mixed-integer non-linear programming (MINLP) model due to the complexity of the problem. In addition, they improved the SA algorithm based on ant colony optimization (ACO) for large-sized problems.

Table 1. Comparison of available literature on ALBP and SLP

References	Product Model	ALBP	SLP		Problem Concept	Model	Analysis Data	
		WSC	PSC	SSC			IA	CE
[17]	Mixed		✓	✓	O	MINLP		✓
[20]	Mixed		✓	✓	O	MILP		✓
[24]	Single	✓	✓	✓	C	MILP		✓
[21]	Single		✓	✓	O	MILP		✓
[26]	Single	✓		✓	S	BLP	✓	✓
[22]	Single		✓	✓	O	MILP		✓
[23]	Single		✓	✓	S	MILP		✓
[25]	Single	✓	✓	✓		MILP	✓	✓
[27]	Single	✓	✓	✓		MILP		✓
[28]	Single	✓	✓	✓		MILP	✓	✓
[29]	Mixed	✓	✓	✓	S	MINLP		✓
This study	**Mixed**	**✓**	**✓**	**✓**	**S**	**MILP**	**✓**	**✓**

[1]SLP with stochastic demand is considered.
[2]They also consider transport vehicle selection problem.
[3]Stochastic ALBP and SLP are considered together.
[4]They also consider multi-manned workstations.

O: Only SLP is considered.
C: ALBP and SLP are consecutively considered.
S: ALBP and SLP are simultaneously considered.
IA: Industrial application
CE: Computational experiment

They used constraint programming to solve the proposed MINLP model. While the MINLP model can find solutions with high CPU times for small-sized problems, it cannot find solutions for medium and large-sized ones. A comparison of the studies on ALBP and SLP is presented in Table 1.

This study proposes a MILP model to solve MMALBP and SLP simultaneously. To analyze the effectiveness of the proposed model in terms of both cost and solution time, it is compared with two different models. The first model is the hierarchical MILP model that solves MMALP and SLP sequentially, and the second model is the MINLP model proposed by Delice et al. [29], which solves MMALP and SLP simultaneously. In the hierarchical MILP model, after the MMAL is balanced by minimizing the number of workstations in the line, the SLP is solved to minimize the sum of supermarket installation and part supply costs. The MINLP model of Delice et al. [29] and the proposed MILP model simultaneously consider the sum of workstation installation, supermarket installation, and part supply costs. For the three models, computational experiments are carried out using test problems presented by Delice et al. [29].

The remainder of the paper is organized as follows. The problem definition and the proposed mathematical model are presented in Section 2. Computational test results and analyses are presented in Section 3. Finally, concluding remarks and future research subjects are included in Section 4.

2 Problem definition

2.1 Integrated mixed-model assembly line balancing and supermarket location problems

In this study, a straight MMAL layout is focused on. The related AL type is suitable to produce more than one model $(m = 1, \dots, M)$. A set of tasks $(i = 1, \dots, I)$ is performed for the manufacture of each model. Each task i has a certain processing time (t_i). There are precedence relationships among the tasks $(P(r, s))$. The processing time t_i of each task i may vary for each model m. Not every task has to be performed for every model. For example, assume that two different automobile models are produced on the same line. While one automobile model may have a sunroof, the other model may not have a sunroof. Each task is assigned to the workstations $(k = 1, \dots, K_{max})$ according to a predetermined cycle time of each model m (CT_m). CT_m is valid for all product models. For example, any task that can be assigned to a station by considering any model may not be assigned to the same station for another model due to the CT_m constraint. Classical assumptions for the MMALBPs are given below:

- The processing time t_{mi} of task i on model m is deterministic and known beforehand.
- The precedence relationships $P(r, s)$ among the tasks for the models are available and known beforehand.
- Cycle time of each model is deterministic and known beforehand.
- The total processing time of the tasks assigned to the workstation k cannot exceed the cycle time.
- Each task i on model m is assigned to only one workstation.
- All tasks must be included in any workstation.

Some tasks do not require parts supply for workstations, while a certain amount of parts supply may be required to perform some tasks. The part supply of these tasks is provided by supermarkets. Each part required for each task i on model m should be delivered to the related workstation k on time. Each workstation demands parts from a supermarket according to the tasks assigned to it. If the tasks assigned to a workstation change, the number of parts needed by the related workstation also changes. In addition, assigning a task to another workstation affects not only the workstation's part demand but also transportation operations. Each tow train in every supermarket has a certain route. The transportation costs are calculated according to this route. An established supermarket increases the installation costs and reduces the transportation costs. Therefore, transportation and installation costs are important criteria. The assumptions for the SLPs are given below:

- The part supply of each task i on model m is provided by only one supermarket. That is, if any workstation is established, it is assigned to only one supermarket.
- The number of parts demanded by each task i on model m is deterministic and known beforehand.
- Each supermarket should serve a group of consecutive workstations [17]. That is, while workstations 1 and 4 can be served by one supermarket, workstations 2 and 3 cannot be served by another supermarket. Otherwise, transportation costs will be adversely affected.
- Each supermarket has its own tow train(s), and it/they cannot be used by other supermarkets.

The general solution structure of the problem is presented in Figure 2.

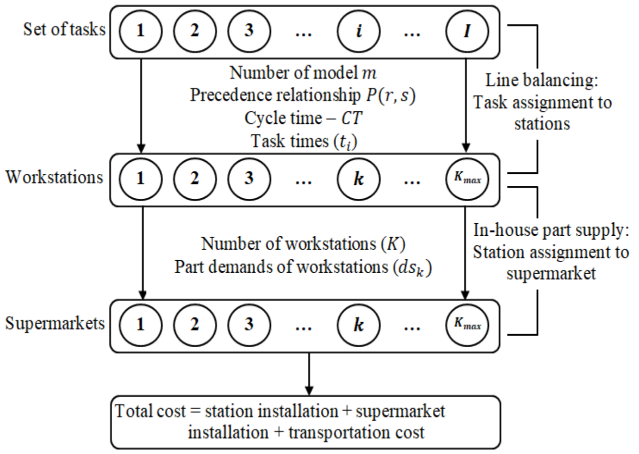

Figure 2. General solution structure of MMALBP and SLP

2.2 The proposed MILP model

This section defines the proposed MILP model. The notations in the mathematical formulations are presented in Table 2. The proposed MILP model is modified from the MINLP model of Delice et al. [29]. The proposed MILP model is as follows;

$$min\, TC = WSC + SSC + PSC \quad (1)$$

$$WSC = \sum_{k=1}^{K_{max}} (\lambda \cdot Z_k) \quad (1.1)$$

$$SSC = \sum_{n=1}^{N_{max}} (\sigma \cdot S_n) \quad (1.2)$$

$$PSC = \sum_{k=1}^{K_{max}} \sum_{l=1}^{K_{max}} \sum_{n=1}^{N_{max}} td_{kl}\, TTC_{kln} \quad (1.3)$$

Table 2. Notations of the mathematical formulations

Notations	Definitions
Indices:	
i, r, s	Task number
k, l, w	Workstation number
m	Model number
n	Supermarket number
Parameters:	
I	Number of tasks
K_{max}	Potential maximum number of workstations
M	Number of models
N_{max}	Potential maximum number of supermarkets
$P(r,s)$	The precedence relationship between the tasks r and s
CT_m	Cycle time of the model m
d_i	The part demand of the task i
t_{mi}	The processing time of the task i for model m
λ	Workstation installation cost
σ	Supermarket installation cost
td_{kl}	Total rectilinear distance from workstation k to workstation l for a tow train
lx	The x-axis length of the workstations and supermarkets
dx	x-axis distance among the workstations
dy	y-axis distance between workstations and supermarkets
LN	A large number
Decision variables:	
Z_k	$\begin{cases}1, \text{if workstation } k \text{ is installed;}\\ 0, \quad \text{otherwise}\end{cases}$
X_{ik}	$\begin{cases}1, \text{if task } i \text{ is assigned to workstation } k;\\ 0, \quad \text{otherwise}\end{cases}$
S_n	$\begin{cases}1, \text{if supermarket } n \text{ is installed;}\\ 0, \quad \text{otherwise}\end{cases}$
Y_{kn}	$\begin{cases}1, \text{if supermarket } n \text{ feeds workstation } k;\\ 0, \quad otherwise\end{cases}$
R_{kln}	$\begin{cases}1, \text{if supermarket } n \text{ feeds workstations between } k \text{ and } l;\\ 0, \quad otherwise\end{cases}$
V_{ikn}	$\begin{cases}1, \text{if supermarket } n \text{ feeds task } i \text{ assigned to workstations } k;\\ 0, \quad otherwise\end{cases}$
TTC_{kln}	The total transportation cost from workstation k to workstation l for supermarket n
D_n	Total part demand supplied by supermarket n

Subject to:

$$Z_k \geq Z_{k+1} \text{ for } \forall k = 1, \dots, K_{max} - 1 \quad (2)$$

$$\sum_{k=1}^{K_{max}} X_{ik} = 1 \text{ for } \forall i = 1, \dots, I \quad (3)$$

$$\sum_{i=1}^{I} t_{mi} \cdot X_{ik} \leq CT_m \cdot Z_k \quad \text{for } \forall k = 1, \dots, K_{max} \text{ and } \forall m = 1, \dots, M \quad (4)$$

$$\sum_{k=1}^{K_{max}} k \cdot (X_{rk} - X_{sk}) \leq 0 \text{ for } \forall (r,s) \in P(r,s) \quad (5)$$

$$\sum_{n=1}^{N_{max}} Y_{kn} = Z_k \text{ for } \forall k = 1, \dots, K_{max} \quad (6)$$

$$\sum_{k=1}^{K_{max}} Y_{kn} \leq LN \cdot S_n \text{ for } \forall n = 1, \dots, N_{max} \quad (7)$$

$$\sum_{k=1}^{K_{max}} \sum_{l=1}^{K_{max}} R_{kln} = S_n \text{ for } \forall n = 1, \dots, N_{max} \text{ and } k \leq l \tag{8}$$

$$\sum_{k=1}^{n} \sum_{l=n}^{K_{max}} R_{kln} = S_n \text{ for } \forall n = 1, \dots, N_{max} \text{ and } k \leq l \tag{9}$$

$$\sum_{l=1}^{k} \sum_{w=k}^{K_{max}} R_{lwn} = Y_{kn} \text{ for } \forall k = 1, \dots, K_{max}, \quad \forall n = 1, \dots, N_{max} \text{ and } l \leq w \tag{10}$$

$$X_{ik} + Y_{kn} - V_{ikn} \leq 1 \text{ for } \forall i = 1, \dots, I, \text{ for } \forall k = 1, \dots, K_{max} \text{ and } \forall n = 1, \dots, N_{max} \tag{11}$$

$$X_{ik} + Y_{kn} - 2 \cdot V_{ikn} \geq 0 \text{ for } \forall i = 1, \dots, I, \text{ for } \forall k = 1, \dots, K_{max} \text{ and } \forall n = 1, \dots, N_{max} \tag{12}$$

$$\sum_{i=1}^{I} \sum_{k=1}^{K_{max}} d_i \cdot V_{ikn} = D_n \text{ for } \forall n = 1, \dots, N_{max} \tag{13}$$

$$TTC_{kln} \leq LN \cdot R_{kln} \text{ for } \forall k, l = 1, \dots, K_{max}, \text{ for } \forall n = 1, \dots, N_{max} \text{ and } k \leq l \tag{14}$$

$$TTC_{kln} \leq D_n \text{ for } \forall k, l = 1, \dots, K_{max}, \text{ for } \forall n = 1, \dots, N_{max} \text{ and } k \leq l \tag{15}$$

$$TTC_{kln} \geq D_n - LN \cdot (1 - R_{kln}) \text{ for } \forall k, l = 1, \dots, K_{max}, \forall n = 1, \dots, N_{max} \text{ and } k \leq l \tag{16}$$

$$TTC_{kln} \leq LN \text{ for } \forall k, l = 1, \dots, K_{max}, \text{ for } \forall n = 1, \dots, N_{max} \text{ and } k \leq l \tag{17}$$

$$X_{ik}, Y_{kn}, Z_k, S_n, R_{kln}, V_{ikn} \in \{0, 1\} \text{ for } \forall i = 1, \dots, I, \ \forall k = 1, \dots, K_{max} \text{ and } \forall n = 1, \dots, N_{max} \tag{18}$$

$$TTC_{kln}, D_n \geq 0 \text{ for } \forall k, l = 1, \dots, K_{max} \text{ and } \forall n = 1, \dots, N_{max} \tag{19}$$

The objective function (1) consists of the workstation setup cost (*WSC*), supermarket setup cost (*SSC*), and part supply cost (*PSC*). The aim is to minimize the total cost (*TC*) by considering both MMALP and SLP simultaneously. Constraint (2) ensures that the next workstation is not established before a workstation is established. Although this constraint limits alternative solutions that can be obtained in a shorter time, it is required to reduce part transportation distances in the AL and supermarket layouts. Equation (3) ensures that each task is assigned to only one workstation. The total times of tasks assigned to workstation k cannot exceed the CT_m with constraint (4). Constraint (5) satisfies all precedence relationships in the set of $P(r,s)$. Equation (6) provides that the part supply of each workstation is ensured through only one supermarket. Constraint (7) ensures that for a supermarket to be established, at least one workstation must be assigned. Equation (8) satisfies that if any supermarket is established, it must have a single path. Equation (9) provides that if any supermarket is installed, it must provide the part supply to the workstation in front of it. Equation (10) guarantees that a tow train must provide the part supply to the workstations on a supermarket's route. Constraints (11) and (12) ensure that supermarket n assigned to workstation k meets the parts demand(s) of task i assigned to the same workstation. Equation (13) calculates the total part demand met by supermarket n. Constraints (14) – (17) consider the total part demand of workstations between k and l assigned to supermarket n. Constraints (18) and (19) restrict the decision variables.

The td_{kln} in the objective function (1) is calculated with equation (20).

$$2 \cdot (l - k) \cdot (dx + lx) + 2 \cdot dy = td_{kl} \text{ for } \forall k, l = 1, \dots, K_{max} \text{ and } k \leq l \tag{20}$$

3 Computational experiment results and discussions

Two different mathematical formulations that solve MMALBP and SLP sequentially and simultaneously are considered to verify the performance of the proposed MILP model. The first model is the hierarchical MILP model that solves MMALP and SLP sequentially, and the second model is the MINLP model proposed by Delice et al. [29], which solves MMALP and SLP simultaneously.

The hierarchical MILP model has two main steps: line balancing (top level) and in-house part supply (base level). In the hierarchical MILP model, the MMAL is firstly balanced by minimizing the number of workstations in the line, and then the SLP is solved to minimize the sum of supermarket installation and part supply costs. The number of workstations and the assignment of the tasks to workstations, which are the outputs of the MMALBP, are used as input parameters for the SLP in the part supply step. Here, since the tasks assigned to the workstations are determined at the top level, the total part demand of each station is known in advance at the base level. The outputs of the base level are the number of supermarkets and the part supply cost. Finally, the workstation installation cost, the supermarket installation cost, and the part supply cost are added up. The MINLP model of Delice et al. [29] and the proposed MILP model simultaneously consider the sum of workstation installation, supermarket installation, and the part supply costs. While the hierarchical model solves SLP by considering a single line balance for the MMALBP, the MINLP and proposed MILP models consider the total cost more comprehensively by also considering alternative line balances. For the two and three models, computational experiments are carried out using test problems presented by Delice et al. [29]. Three different test problems are utilized: *Mansoor*, *Mitchell*, and *Heskia*. In addition, a case study is performed using two-model assembly line data obtained from a furniture company. The data of the case study are given in Table 3. The test problems are summarized in Table 4. In MMALBP, not all tasks must be performed in each model type. In addition, some tasks may not need any part demand. In Table 3, for example, Tasks 4, 14, 28, and 30 are not performed in Model-1, and Tasks 5, 10, 19, and 26 are not performed in Model-2. In addition, Tasks 4, 5, 19, 26, 28, and 30 to be performed in the AL do not require any part demand.

Table 3. Data of the case study

Task number	Immediate Successor(s) Model-1	Immediate Successor(s) Model-2	Task Times Model-1	Task Times Model-2	Amount of part demand
1	2	2	23	19	8
2	3, 6	3, 4	2	5	2
3	5	7	10	12	2
4	--	6	--	9	--
5	7	--	11	--	--
6	7	7	20	16	4
7	8, 9, 10, 11	8, 9, 11, 16	7	4	3
8	16	16	10	10	1
9	21	21	7	3	4
10	16	--	17	--	3
11	12	12	15	9	2
12	13	13	15	19	3
13	15	14	19	25	5
14	--	15	--	11	4
15	17	17	19	18	6
16	18	18	13	12	1
17	19	20	7	9	1
18	9	9	15	12	3
19	20	--	10	--	--
20	21	21	15	16	3
21	22, 23, 24	22, 23, 24	20	19	3
22	25	25	15	11	2
23	32	32	16	18	3
24	32	32	14	12	1
25	26	27	15	12	2
26	27	--	11	--	--
27	29	28	18	23	3
28	--	29	--	24	--
29	31	30	16	12	9
30	--	31	--	8	--
31	32	32	6	6	4
32	-	-	8	8	1

Table 4. Summary of computational test problems

Problem Name	Task Size	t_{max}	*M*
Mansoor	11	45	2, 3
Mitchell	21	13	2, 3
Heskia	28	108	2, 3
Case study	32	25	2, 3

Two different CTs are considered for each test problem. In addition, two different supermarket installation costs, 500 and 700 units, are taken into account. The workstation installation cost is assumed as 400 units. The workstation and supermarket lengths are assumed as 10 units. Distances among station locations and among supermarket locations are assumed as 1 unit. Distances between stations and supermarket locations are assumed as 1 unit. In summary, 38 different problem tests are taken into account. The hierarchical MILP and the proposed MILP models are coded by the Linear programming in IBM® ILOG® CPLEX® Optimization Studio V12.8.0. On the other hand, the MINLP model is coded by the Constraint Programming in IBM® ILOG® CPLEX® Optimization Studio V12.8.0 due to its nature. A personal computer with 10th Gen Intel® Core™ i5, 2.3 GHz processor, and 8.00 GB memory is used. The time limit is 3600 seconds.

The computational results are presented in Table 5. The mark * represents the results obtained in the time limit. Appendix A shows that the proposed MILP model performs better than the hierarchical MILP model. The hierarchical MILP model has lower CPU times than the proposed MILP model. The main reason for this is that although the proposed MILP model simultaneously solves the MMALBP and SLP, the hierarchical MILP model consecutively solves them. The simultaneous model structure enlarges the solution space and increases the complexity of the problem. The MINLP and the proposed MILP models find better solutions than the hierarchical MILP model for the variants of the *Mansoor* problem. In *Mitchell*, *Heskia*, and case study problems, the proposed MILP model can find a better solution than the hierarchical MILP model. However, the MINLP model cannot even find solutions for most problem sets within the time limit. It is also seen that the proposed MILP model outperforms the MINLP model in terms of CPU times in general problem sets.

4 Conclusion

Today, manufacturing companies focus not only on improving their production systems but also on in-house part supply logistics, which increases production efficiency. The supermarket concept is preferable for in-house logistics systems in many sectors that adopt the JIT philosophy, such as the automotive industry. Therefore, considering SLP and ALBP, which are long-term decision-making problems, together provides an important perspective in the design stage. In this study, a new MILP model is presented in order to consider MMALBP and SLP together. In order to verify the effectiveness of the proposed model, the test problems generated by Delice et al. [29] are considered. According to the obtained computational results, the proposed model performs quite well.

ALBP and SLP are long-term strategic decision-making problems. To simultaneously optimize both production costs and in-house part supply costs, decision-makers should address them together. Solving these two problems separately may be misleading in terms of costs. For example, even if the production cost of the current assembly line is low, parts supply and supermarket costs may increase. Addressing the cost relationship between ALBP and SLP together will provide significant benefits for decision-makers in the medium and long term.

Table 5. The comparison of the computational results for the SLP + MMALP considering the central warehouse and decentralized supermarket concept (λ=400)

Problem	*M*	*CT*	*sigma*	**Hierarchical MILP (Linear programming)**					**MINLP of Delice et al. [29] (Constraint programming)**					**The proposed MILP (Linear programming)**				
				K	*N*	*PSC*	*TC*	*CPU time (sec.)*	*K*	*N*	*PSC*	*TC*	*CPU time (sec.)*	*K*	*N*	*PSC*	*TC*	*CPU time (sec.)*
Mansoor	2	48	500	7	4	754	5554	1.19	7	4	688	5488	261.38	7	4	688	5488	12.56
			700	7	3	1436	6336	1.04	7	3	1370	6270	1085.55	7	3	1370	6270	20.29
		62	500	5	4	402	4402	1.03	5	3	754	4254	19.2	5	3	754	4254	2.32
			700	5	3	908	5008	0.93	5	3	754	4854	22.03	5	3	754	4854	3.85
		94	500	3	3	94	2794	1.02	3	3	94	2794	11.68	3	3	94	2794	1.84
			700	3	2	622	3222	1	3	2	622	3222	10.91	3	2	622	3222	1.85
	3	48	500	7	4	798	5598	1.63	7	4	710	5510	669.57	7	4	710	5510	20.58
			700	7	3	1414	6314	1.58	7	4	710	6310	290.79	7	4	710	6310	20.05
		62	500	5	3	864	4364	1.58	5	3	754	4254	19.5	5	3	754	4254	4.64
			700	5	3	864	4964	1.56	5	3	754	4854	23.03	5	3	754	4854	3.42
		94	500	3	3	94	2794	0.86	3	3	94	2794	12.69	3	3	94	2794	1.75
			700	3	2	622	3222	0.84	3	2	622	3222	16.85	3	2	622	3222	1.59
Mitchell	2	14	500	17	7	2832	13132	1.32	17	8	2194	12994	7200*	17	8	2084	12884	4971.45
			700	17	7	2832	14532	1.23	18	7	2964	15064	7200*	17	7	2810	14510	4490.09
		21	500	9	7	764	7864	1.33	9	7	588	7688	7200*	9	7	588	7688	4369.21
			700	9	5	1798	8898	1.27	9	5	1622	8722	7200*	9	5	1622	8722	7200*
		26	500	7	6	456	6256	1	-	-	-	-	-	7	6	390	6190	399.73
			700	7	5	962	7262	0.95	-	-	-	-	-	7	5	896	7196	516.52
	3	14	500	18	9	2194	13894	2.01	19	8	2612	14212	7200*	18	8	2392	13592	5205.23
			700	18	7	3426	15526	1.94	-	-	-	-	-	18	7	2964	15064	4325.32
		21	500	9	8	324	7924	1.58	-	-	-	-	-	9	7	742	7842	4216.36
			700	9	5	1864	8964	1.6	-	-	-	-	-	9	5	1842	8942	3612.27
		26	500	8	6	808	7008	1.07	-	-	-	-	-	8	6	698	6898	2948.85
			700	8	5	1490	8190	1.01	-	-	-	-	-	8	5	1314	8014	7200*
Heskia	2	138	500	12	10	1112	10912	1.56	-	-	-	-	-	12	10	738	10538	7200*
			700	12	7	2696	12396	1.48	-	-	-	-	-	12	7	2498	12198	7200*
		216	500	8	8	298	7498	1.05	-	-	-	-	-	8	7	496	7196	7200*
			700	8	7	980	9080	1.07	-	-	-	-	-	8	6	1046	8446	7200*
		256	500	6	6	298	5698	2.76	-	-	-	-	-	6	6	298	5698	7200*
			700	6	6	298	6898	2.74	-	-	-	-	-	6	6	298	6898	7200*
	3	138	500	15	9	2696	13196	2.07	-	-	-	-	-	15	11	1266	12766	7200*
			700	15	8	3246	14846	2.09	-	-	-	-	-	15	8	2872	14472	7200*
		216	500	8	8	298	7498	1.71	-	-	-	-	-	8	7	672	7372	7200*
			700	8	7	870	8970	1.67	-	-	-	-	-	8	6	1310	8710	7200*
		256	500	7	7	298	6598	1.11	-	-	-	-	-	7	6	584	6384	7200*
			700	7	7	298	7998	1.08	-	-	-	-	-	7	6	584	7584	7200*
Case Study	2	36	500	12	6	1992	9792	1.64	-	-	-	-	-	12	6	1814	9614	7200*
			700	12	6	1992	10992	1.73	-	-	-	-	-	12	6	1814	10814	7200*

Different limitations can be considered when applying this model in real-life applications. One of these is the zone/area constraints for the locations of supermarkets and stations. The areas, where supermarkets and stations will be placed, can be limited according to the area and size of the facility. In addition, determining supermarket locations in u-type, two-sided, and parallel ALs can be more complex. The mathematical models proposed for these AL layouts can be developed for future studies. Another limitation is the size of the problem. As the size of the problem increases, the mathematical model may be unable to find a solution. Due to the NP-hard nature of the ALBPs, metaheuristic algorithms should be used for large-sized problems. Another issue to be addressed may be the variability of task durations. Accordingly, models addressing stochastic processes, fuzzy logic, gray number theory, or robust optimization can be developed for task processing times with a statistical distribution or a certain range.

Conflict of interest

The author declares that there is no conflict of interest.

Similarity rate (Ithenticate): %14

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