KSÜ Mühendislik Bilimleri Dergisi, 21(1), 2018
Araştırma Makalesi

KSU J Eng Sci, 21(1), 2018
Research Article

Kahramanmaras Sutcu Imam University
Journal of Engineering Sciences

Geliş Tarihi : 12.04.2017
Kabul Tarihi : 01.06.2017

Received Date : 12.04.2017
Accepted Date : 01.06.2017

İki Enklüzyon ve Bir İç Çatlak İçeren Sonsuz Silindir

Infinite Cylinder with an Internal Crack and Two Inclusions

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ABSTRACT

An infinite cylinder, of isotropic and linearly elastic material, with an internal ring shaped crack and two penny shaped rigid inclusions was considered in this study. The considered cylinder was subjected to axial tensile forces from its two ends. The complex problem of the axially loaded infinite cylinder was solved by using the superposition of two problems including: (i) an infinite cylinder, without any cracks or inclusions, loaded at infinity and (ii) an infinite cylinder with an internal ring shaped crack and two penny shaped inclusions and free of loading. Associated Navier equations are solved with Fourier and Hankel transforms to obtain general expressions for the considered problem. Then, the considered problem is reduced to three singular integral equations and numerically solved by using Gauss-Lobatto integration formula with associated system of linear algebraic equations.

Keywords: Internal Crack, Infinite Cylinder, Stress Intensity Factor, Rigid Inclusion.

ÖZET

Bu çalışmada elastik ve izotropik malzemeden imal edilmiş, halka şeklinde bir iç çatlak ve daire şeklinde iki adet rijit enklüzyon içeren sonsuz bir silindir incelenmiştir. İncelenen silidir iki ucundan eksenel yüke tabi tutulmuştur. Eksenel yüklenmiş sonsuz silindiri içeren karmaşık problem iki problemin süperpoze edilmesiyle çözülmüştür. Bu problemler: (i) çatlak veya enklüzyon içermeyen sonsuzda yüklenmiş bir sonsuz silindir, (ii) yükleme altında olmayan, halka şeklinde bir iç çatlak ve iki daire şekilli enklüzyon içeren sonsuz silindirdir. İlgili Navier eşitlikleri, Hankel ve Fourier dönüşümleri kullanılarak çözülerek, ilgilenilen probleme yönelik genel ifadeler elde edilmiştir. Sonra, problem üç adet tekil integral denklemine indirgenmiş ve ilgili doğrusal cebrik denklem takımı Gauss- Lobatto integrasyon formülleri kullanılarak sayısal yöntemlerle çözülmüştür.

Anahtar Kelimeler: İç Çatlak, Sonsuz Silindir, Gerilme Yığılma Faktörü, Rijit Enklüzyon

1. INTRODUCTION

Various engineering branches use machine elements which have numerous discontinuities. These discontinuities may occur in the form of voids, cracks, inclusions etc. They are among the major factors affecting the load carrying capacities and influencing the stress distributions in the bodies. They must be carefully examined. Stress distributions become infinity in the vicinities of the inclusions and cracks as well as the corners of the elements. In these regions, stress distributions can be calculated in terms of the stress intensity factors. Stress intensity factors are dependent to the loading conditions and geometric properties of the bodies. Geometry and the locations of the corresponding cracks, inclusions, notches and holes as well as geometry of the body are some of the geometric properties affecting the related stress intensity factors.

Several solutions for infinite cylinder problems containing edge cracks and penny-shaped inclusions can be found in the literature (Kaman and Geçit, 2007). Erdol and Erdogan (1978) considered the problem of a long thick walled hollow cylinder containing ring shaped internal or edge crack which is subjected to uniform axial load and steady-state thermal stress. Artem and Gecit (2002) considered an infinite elastic hollow cylinder under axial tension containing a crack and two rigid inclusions of ring shape. Lee (2003) considered the singular stress problem of a peripheral edge crack in a long circular cylinder under torsion. Kadioglu (2005) obtained an analytical solution for the linear elastic, axisymmetric problem of edge cracks in an infinite hollow cylinder. Toygar and Gecit (2006) considered the problem of an axisymmetric infinite cylinder of linearly elastic and isotropic material containing a ring shaped crack and two ring-shaped rigid inclusions. Kaman and Geçit (2006) considered the problem of a cracked semi-infinite cylinder short end being fixed and an uncracked finite cylinder of linearly elastic and isotropic material. However, problem of the infinite cylinder containing a ring shaped edge crack and two penny shaped inclusions has not been solved by the method used in this research study. In this study, solution for the infinite cylinder having a ring-shaped crack as well as two penny-shaped rigid inclusions and loaded at infinity is obtained by superposition of the following two problems: (i) An infinite cylinder loaded at infinity without any cracks or inclusions, (ii) an infinite cylinder with a ring-shaped crack and two penny-shaped rigid inclusions with no load at infinity.

2. SOLUTION METHODOLOGY AND DEVELOPMENT OF THE GENERAL EXPRESSIONS

An axisymmetric infinite cylinder of radius A with a transverse ring-shaped crack of width $(b-a)$ located at $z=0$ plane and two penny shaped inclusions of radius c located at $z=\pm L$ planes is considered in this study. This cylinder is under the action of uniformly distributed tensile loads of intensity p_0 at infinity (Fig. 1).

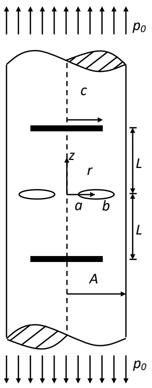

Figure 1. Geometry and loading of the infinite cylinder.

General expressions for the solution of the problem must contain sufficient number of unknowns in order to satisfy all of the necessary boundary conditions. For this purpose, the perturbation problem (problem II) is separated into three main sub-problems in terms of three infinite media; (II-i) an infinite medium containing a ring-shaped crack located at $z = 0$ plane, (II-ii) an infinite medium containing two penny-shaped rigid inclusions located at $z = \pm L$ planes and (II-iii) an infinite medium with no crack or inclusion (Fig. 2).

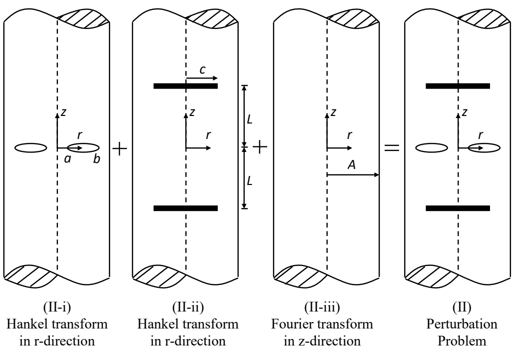

Figure 2. Addition of solutions for the perturbation problem.

General expressions of these sub-problems are obtained by applying Hankel transforms to the first and the second media, in r-direction, and by applying Fourier transform to the third medium, in z-direction, on Navier equations. However, the stresses should reduce to zero at the cylinder surfaces. In order to obtain the required zero stress condition, the derived general expressions are rearranged to calculate the stress and deformation expressions for an axially loaded infinite cylinder, with a ring shaped crack located at $z=0$ plane and two penny shaped inclusions of radius c located at $z=\pm L$ planes. The expressions for the displacements and the stresses may be written in the form

$$u(r,z) = -\frac{\nu}{1+\nu}\frac{p_0}{2\mu}r + \frac{1}{1-\nu}\left\{ \int_a^b d_{11}(r,z,t)g_1(t)tdt + \int_0^c \left[d_{12}(r,z,t)g_2(t) + d_{13}(r,z,t)g_3(t)\right]tdt \right\}$$

$$w(r,z) = \frac{1}{1+\nu}\frac{p_0}{2\mu}z + \frac{1}{1-\nu}\left\{ \int_a^b d_{21}(r,z,t)g_1(t)tdt + \int_0^c \left[d_{22}(r,z,t)g_2(t) + d_{23}(r,z,t)g_3(t)\right]tdt \right\} \tag{1a,b}$$

$$\sigma_r(r,z) = \frac{1}{1-\nu}\left\{ \int_a^b d_{31}(r,z,t)g_1(t)tdt + \int_0^c \left[d_{32}(r,z,t)g_2(t) + d_{33}(r,z,t)g_3(t)\right]tdt \right\}$$

$$\sigma_z(r,z) = p_0 + \frac{1}{1-\nu}\left\{ \int_a^b d_{41}(r,z,t)g_1(t)tdt + \int_0^c \left[d_{42}(r,z,t)g_2(t) + d_{43}(r,z,t)g_3(t)\right]tdt \right\}$$

$$\tau_{rz}(r,z)=\frac{1}{1-\nu}\left\{\int_a^b d_{51}(r,z,t)g_1(t)tdt+\int_0^c\left[d_{52}(r,z,t)g_2(t)+d_{53}(r,z,t)g_3(t)\right]tdt\right\} \tag{2a-c}$$

Where u and w are the displacements in r and z directions in polar coordinate system and σ and τ are the axial and shear stresses. Moreover, the unknown functions $g_i(r)$ (i = 1-3) are given as

$$\frac{\partial}{\partial r}\left[w(r,0^+)\right]-\frac{\partial}{\partial r}\left[w(r,0^-)\right]=2g_1(r), \quad (0\le r<\infty)$$

$$\tau_{rz}(r,L^+)-\tau_{rz}(r,L^-)=\mu g_2(r), \quad (0\le r<\infty)$$

$$\sigma_z(r,L^+)-\sigma_z(r,L^-)=\mu g_3(r), \quad (0\le r<\infty) \tag{3a-c}$$

such that $g_1(r) = 0$ when $0 \le r < a$ or $b < r < \infty$, $g_i(r) = 0$ (i = 2,3) when $c \le r < \infty$.

3. INTEGRAL EQUATIONS

By the use of combined general expressions for the stresses and the displacements, the boundary conditions at the lateral surface of the infinite cylinder and the boundary conditions on the crack and inclusion surfaces are satisfied. As a result, three singular integral equations are obtained. Then, the infinite cylinder problem is converted to the target problem, by letting the radius of the rigid inclusions approach the radius of the cylinder and letting the outer edge of the ring-shaped crack approach the lateral surface of the cylinder. The unknown function $g_1(r)$ is the crack surface displacement derivative in z-direction while $g_2(r)$ and $g_3(r)$ represent the jumps in shear and normal stresses through the rigid inclusions, respectively. These unknown functions will be determined by the use of the following conditions:

$$\sigma_z(r,0)=0, \quad (a<r<b)$$

$$u(r,L)=0, \quad (0<r<c)$$

$$w(r,L)=\text{constant}, \quad (0<r<c) \tag{4a-c}$$

Note that the condition (4a) is stress type while the conditions (4b,c) are displacement type and can be replaced by

$$\frac{1}{r}\frac{\partial}{\partial r}\left[ru(r,L)\right]=0, \quad (0<r<c)$$

$$\frac{\partial}{\partial r}\left[w(r,L)\right]=0, \quad (0<r<c) \tag{5a,b}$$

in order to obtain the same type conditions. Substituting Eqs. (1) and (2) in Eq. (4a) and Eqs. (5a,b) and noting that $g_i(r)$ (i = 1,2) are odd, $g_3(r)$ is even, one can obtain the following singular integral equations

$$\int_a^b g_1(t)\left[\frac{2}{t-r}+2M_1(r,t)+tN_{11}(r,t)\right]dt+\int_{-c}^{c}\{g_2(t)\left[tT_1(r,|t|)+|t|N_{12}(r,t)\right]$$
$$+g_3(t)|t|\left[T_2(r,|t|)+N_{13}(r,t)\right]\}dt=-2(1-\nu)\pi\frac{p_0}{\mu}, \quad (a<r<b)$$

$$\int_a^b g_1(t)\left[T_3(r,t)+N_{21}(r,t)\right]dt+\int_{-c}^{c}\left\{g_2(t)\left[tT_4(r,|t|)-\frac{3-4\nu}{2}\frac{1}{t-r}-\frac{3-4\nu}{2}M_2(r,t)+|t|N_{22}(r,t)\right]\right.$$
$$\left.+g_3(t)|t|\left[T_5(r,|t|)+N_{23}(r,t)\right]\right\}dt=\frac{4\nu(1-\nu)}{(\nu+1)}\pi\frac{p_0}{\mu}, \quad (-c<r<c)$$

$$\int_a^b g_1(t)\left[T_6(r,t)+N_{31}(r,t)\right]dt+\int_{-c}^{c}\left\{g_2(t)\left[tT_7(r,|t|)+|t|N_{32}(r,t)\right]\right.$$
$$\left.+g_3(t)\left[|t|T_8(r,|t|)-\frac{3-4\nu}{2}\frac{1}{t-r}-\frac{3-4\nu}{2}M_3(r,t)+|t|N_{33}(r,t)\right]\right\}dt=0, \qquad (-c<r<c)$$

(6a-c)

The singular integral equations (6a-c) must be solved subject to the single-valuedness condition for the crack and the equilibrium conditions for the inclusions written in the form

$$\int_a^b g_1(t)dt=0$$

(7a,b)

$$\int_{-c}^{c} g_i(t)tdt=0 \qquad (i=2,3)$$

The crack surface displacement derivative $g_1(r)$ and the stress jumps $g_2(r)$ and $g_3(r)$ through the rigid inclusions may have singularities at the ends $r = a$, b and $r = \pm c$, respectively. Their singular behavior may be determined by examining the singular integral equations (6a-c) around these end points using the complex function technique given in Muskhelishvili(1953). The singular behavior of gi(r) (i = 1-3) can be determined by first writing

$$g_1(r)=\begin{cases}\dfrac{g_1^*(r)}{[(r-a)(b-r)]^{\beta}}, & \text{for internal crack}\\[2ex] \dfrac{g_1^*(r)}{(r-a)^{\beta}(A-r)^{\theta}}, & \text{for edge crack}\end{cases} \qquad (0<\mathrm{Re}(\beta,\theta)<1)$$

$$g_i(r)=\left\{\frac{g_i^*(r)}{(c^2-r^2)^{\gamma}}\right., \qquad (i=2,3) \qquad (0<\mathrm{Re}(\gamma)<1)$$

(8a,b)

where β, θ and γ are unknown constants and g_i^*(r) (i = 1-3) are Hölder-continuous functions Durucan (2010) in the respective intervals (a, b) and (-c, c). Then, substituting Eqs. (8a,b) in Eqs. (6a-c), calculating the integrals around the end points in accordance with the technique presented in Cook & Erdogan (1972), the following characteristic equations are obtained:

$$\cot(\pi\beta)=0 \qquad (a,\, b<A)$$

$$\cos(\pi\theta)=2\theta(\theta-2)+1 \qquad (b=A)$$

$$\cot(\pi\gamma)=0 \qquad (c<A)$$

$$2(3-4\nu)\cos(\pi\gamma)=(3-4\nu)^2+1-4(\gamma-1)^2 \quad (c=A)$$

(9a-d)

Equations (9a,b) give 1/2 for β and θ which is the well known result for an embedded crack tip in a homogeneous medium (see, for example, Cook and Erdoğan(1972), Nied and Erdoğan (1983), Geçit (1987)). From Eq. (9c), it can be observed that the value for θ is zero which is obtained also in previous works Williams (1952), Geçit (1984), Geçit and Turgut (1988) indicating that the stresses at the apex of a 90^0 wedge with free sides are bounded. Equation (9d) gives 1/2 as the acceptable value for γ which is obtained also in previous works for an embedded inclusion tip in a homogeneous medium (Gupta, 1974; Yetmez and Geçit, 2005; Kaman and Geçit, 2006). When the penny-shaped inclusions spread out to the outer surface of the cylinder, the portion of the infinite cylinder between $z = \pm L$ planes becomes a finite cylinder of length $2L$ with rigid ends. Equation (9e) is in agreement with the results of previous works, Williams (1952), Gupta(1975), Geçit and Turgut (1988), which is used to calculate the power of stress singularity at the apex of a 90° wedge with one side fixed and the other side free.

4. SOLUTION OF INTEGRAL EQUATIONS AND STRESS INTENSITY FACTORS

Finally, the singular integral equations (Eqs. 6a-c) are converted to linear algebraic equations by using Gauss-Lobatto and Gauss-Jacobi integration formulas. Then, these linear algebraic equations are solved numerically to obtain the stress intensity factors at the edges of the internal crack, at the root of the edge crack in infinite cylinder and at the edge of the rigid inclusions in infinite cylinder.

Defining non-dimensional variables $\varnothing$ and ψ on the crack and η and ε on the inclusions by

$$(r,t)=\frac{b-a}{2}(\psi,\phi)+\frac{b+a}{2}, \quad (a<(r,t)<b), \ (-1<\psi,\phi<1)$$

$$(r,t)=c(\varepsilon,\eta), \quad (-c<(r,t)<c), \ (-1<\varepsilon,\eta<1) \tag{10a,b}$$

the singular integral equations, Eqs. (6a-c) and (7a,b), are expressed in terms of non-dimensional quantities.

The integrals are calculated by using the Gauss-Lobatto integration formula (Krenk, 1978), Artem and Geçit, 2002)) and the following system of $2n$ linear algebraic equations are obtained (see Durucan (2010) for details):

$$\sum_{i=1}^{n} C_i \bar{g}_1(\phi_i)\left[\bar{m}_1(\psi_j,\phi_i)+\bar{N}_{11}(\psi_j,\phi_i)\right]+2\sum_{i=1}^{n/2} C_i \bar{g}_2(\eta_i)\eta_i\left[\bar{T}_1(\psi_j,\eta_i)+\bar{N}_{12}(\psi_j,\eta_i)\right]+$$
$$2\sum_{i=1}^{n/2} C_i \bar{g}_3(\eta_i)\eta_i\left[\bar{T}_2(\psi_j,\eta_i)+\bar{N}_{13}(\psi_j,\eta_i)\right]=-(1-\nu), \qquad (j=1,....,n-1)$$

$$\sum_{i=1}^{n} C_i \bar{g}_1(\phi_i)\left[\bar{T}_3(\varepsilon_j,\phi_i)+\bar{N}_{21}(\varepsilon_j,\phi_i)\right]+2\sum_{i=1}^{n/2} C_i \bar{g}_2(\eta_i)\left[\eta_i\bar{T}_4(\varepsilon_j,\eta_i)+\bar{m}_2(\varepsilon_j,\eta_i)+\eta_i\bar{N}_{22}(\varepsilon_j,\eta_i)\right]+$$
$$2\sum_{i=1}^{n/2} C_i \bar{g}_3(\eta_i)\eta_i\left[\bar{T}_5(\varepsilon_j,\eta_i)+\bar{N}_{23}(\varepsilon_j,\eta_i)\right]=-\frac{2(1-\nu)\nu}{(\nu+1)(3-4\nu)}, \qquad (j=1,....,n/2)$$

$$\sum_{i=1}^{n} C_i \bar{g}_1(\phi_i)\left[\bar{T}_6(\varepsilon_j,\phi_i)+\bar{N}_{31}(\varepsilon_j,\phi_i)\right]+2\sum_{i=1}^{n/2} C_i \bar{g}_2(\eta_i)\eta_i\left[\bar{T}_7(\varepsilon_j,\eta_i)+\bar{N}_{32}(\varepsilon_j,\eta_i)\right]+$$
$$2\sum_{i=1}^{n/2} C_i \bar{g}_3(\eta_i)\left[\eta_i\bar{T}_8(\varepsilon_j,\eta_i)+\bar{m}_3(\varepsilon_j,\eta_i)+\eta_i\bar{N}_{33}(\varepsilon_j,\eta_i)\right]=0, \qquad (j=1,....,n/2-1) \tag{11a-c}$$

$$\sum_{i=1}^{n} C_i \bar{g}_1(\phi_i)=0,$$

$$\sum_{i=1}^{n/2} C_i \bar{g}_3(\eta_i)\eta_i=0, \tag{12a,b}$$

where

$$\bar{m}_1(\psi,\phi)=\frac{1}{\phi-\psi}m_1(\psi,\phi)$$

$$\bar{m}_2(\varepsilon,\eta)=\frac{\eta}{\eta^2-\varepsilon^2}m_2(|\varepsilon|,|\eta|)$$

$$\bar{m}_3(\varepsilon,\eta)=\frac{\varepsilon}{\eta^2-\varepsilon^2}m_3(|\varepsilon|,|\eta|) \tag{13a-c}$$

$$\phi_i,\eta_i=\cos[(i-1)\pi/(n-1)], \qquad (i=1,.....,n)$$

$$\psi_j,\varepsilon_j=\cos[(2j-1)\pi/(2n-2)], \qquad (j=1,.....,n-1)$$

$$C_1=C_n=1/(2n-2), C_i=1/(n-1), \quad (i=2,.....,n-1) \tag{14a-c}$$

Stresses become infinite in the vicinity of tips or edges of cracks and inclusions. These infinite stresses are represented by means of stress intensity factors. Mode I stress intensity factors at the edges of the internal ring-shaped crack are defined and calculated in the form

$$k_{1a}=\lim_{r\to a}\sqrt{2(a-r)}\sigma_z(r,0)=\frac{\sqrt{2}\mu}{(1-\nu)}\frac{g_1^*(a)}{\sqrt{b-a}}$$

$$k_{1b} = \lim_{r \to b} \sqrt{2(r-b)}\sigma_z(r,0) = -\frac{\sqrt{2}\mu}{(1-\nu)} \frac{g_1^*(b)}{\sqrt{b-a}} \tag{15a,b}$$

Mode I and II stress intensity factors at the edges of the internal rigid inclusion may be similarly calculated as

$$k_{1c} = \lim_{r \to c} \sqrt{2(r-c)}\sigma_z(r,L)$$

$$k_{2c} = \lim_{r \to c} \sqrt{2(r-c)}\tau_{rz}(r,L)$$

$$k_{1c} = \frac{\sqrt{2}}{2}\left[\frac{2\nu-1}{2(1-\nu)} g_2^*(1) - \frac{1}{\sqrt{2}} g_3^*(1)\right] p_0\sqrt{c}$$

$$k_{2c} = -\frac{\sqrt{2}}{2}\left[\frac{1}{\sqrt{2}} g_2^*(1) + \frac{2\nu-1}{2(1-\nu)} g_3^*(1)\right] p_0\sqrt{c} \tag{16a-d}$$

5. NUMERICAL RESULTS

The system of linear algebraic equations, Eqs. (11a-c) and (12a,b), are solved numerically and the unknown functions $g_1(\emptyset)$, $g_i(\eta)$ $(i = 2,3)$ are calculated at separate points. Then, the physically significant quantities, for example, the stress intensity factors at the edges of the crack, at the edges of the inclusion and around the clamped corners of the finite cylinder can be calculated.

Figures 3 to 4 present the variation of normalized stress intensity factors (Eqs. 17-18) vs. varying geometric and material properties. Normalized stress intensity factors may be defined and calculated as:

$$\bar{k}_{1a} = \frac{k_{1a}}{p_0\sqrt{(b-a)/2}} = \frac{\bar{g}_1(-1)}{4(1-\nu)}$$

$$\bar{k}_{1b} = \frac{k_{1b}}{p_0\sqrt{(b-a)/2}} = -\frac{\bar{g}_1(1)}{4(1-\nu)} \tag{17a,b}$$

for the crack and

$$\bar{k}_{ic} = \frac{k_{ic}}{p_0\sqrt{c}}, \qquad (\mathrm{i}=1,2) \tag{18}$$

for the rigid inclusions.

Figure 3 illustrates the variations of the Mode I normalized stress intensity factors $\bar{k}_{1a}$ and $\bar{k}_{1b}$ defined in Eqs. (17a,b) at the edges of the crack with relative crack width $(b\text{-}a)/A$ when $c = 0.5A$, $b+a = A$ and $\nu = 0.3$ for the case of an infinite cylinder with internal crack and inclusions. The center line of the ring-shaped crack is at $r = A/2$. Results are given for $L/A = 0.25$ and 0.5 noting that the numerical results for $\bar{k}_{1a}$ and $\bar{k}_{1b}$ remain unchanged for values of L/A greater than 0.5 which means that the effect of the rigid inclusions will fade away when $L/A < 0.5$. From the figure, it can be observed that $\bar{k}_{1a}$ is greater than $\bar{k}_{1b}$ for especially wider cracks. This may be due to the interaction of the inner edges of the crack around the axis of the cylinder. Both $\bar{k}_{1a}$ and $\bar{k}_{1b}$ increase as $(b\text{-}a)/A$ increases which is an expected observation.

Figure 4 shows variations of the normalized stress intensity factors $\bar{k}_{1c}$ and $\bar{k}_{2c}$ defined in Eqs. (18a,b) at the edge of the internal rigid inclusion with relative inclusion radius c/A when $(b\text{-}a)/A = L = 0.5A$ and again $b+a = A$. From the figure, it can be observed that $\bar{k}_{1c}$ and $\bar{k}_{2c}$ are heavily dependent on the Poisson's ratio ν. Here, it should be remembered that the rigid inclusions are effective on the deformation characteristics of the cylinder and therefore on the stress distributions for relatively large values of ν. In the limiting case of $\nu \to 0$, rigid inclusions will practically disappear. In particular, $\bar{k}_{2c}$ is larger for larger values of ν. $\bar{k}_{1c}$ is larger for larger values of ν and for relatively larger values of c/A.

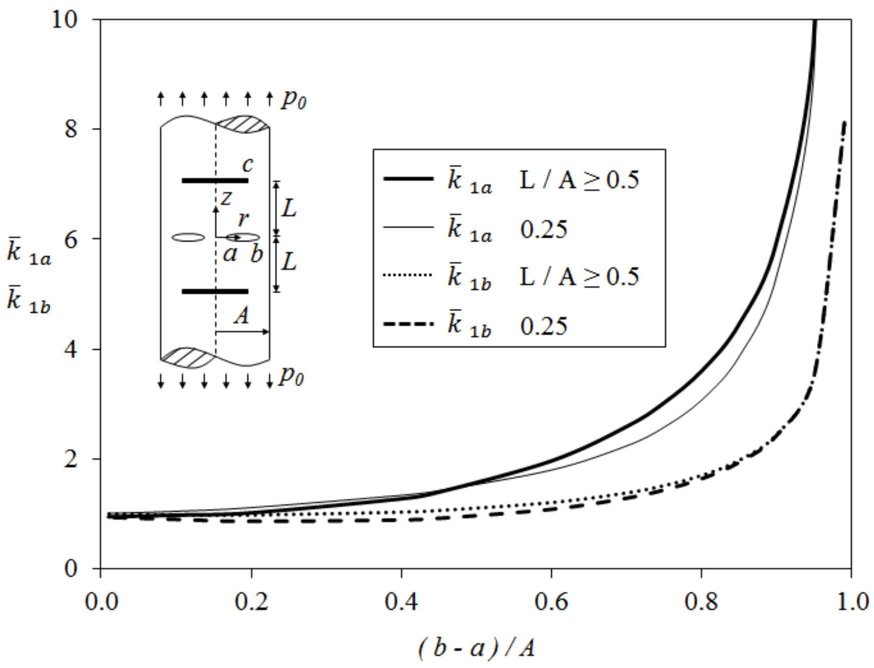

Figure 3. Normalized stress intensity factors $\bar{k}_{1a}$ and $\bar{k}_{1b}$ when c = 0.5A, b+a = A and ν = 0.3.

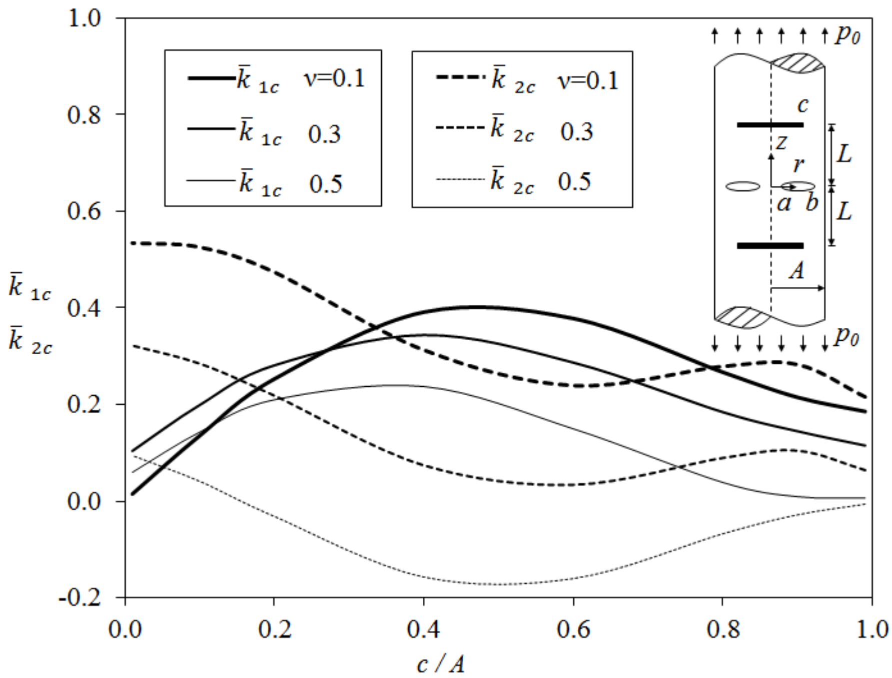

Figure 4. Normalized stress intensity factors $\bar{k}_{1c}$ and $\bar{k}_{2c}$ when b-a = L = 0.5A and b+a = A.

6. ACKNOWLEDGEMENT

The author is grateful to Prof. Dr. Mehmet Ruşen Geçit for his valueable information used during the preparation of this paper.

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Nomenclature

a, b	Inner and outer radii of ring-shaped crack
A	Radius of cylinder
c	Radius of penny-shaped inclusions
C_i	Weighting constants of the Gauss-Lobatto polynomials
d_{ij}	Coefficient functions
$g_1(r)$	Crack surface displacement derivative
$g_1^*(r)$	Hölder-continuous function on crack
$\overline{g_1}(r)$	Normalized bounded part of $g_1(r)$
$g_2(r)$, $g_3(r)$	Normal and shear stress jumps on rigid inclusions
$g_2^*(r)$, $g_3^*(r)$	Hölder-continuous functions on inclusions
$\overline{g_2}(r), \overline{g_3}(r)$	Normalized bounded parts of $g_2(r)$, $g_3(r)$

I_0, K_0, I_1, K_1	Modified Bessel functions of the 1st and 2nd kinds of order zero and one
J_0, J_1	Bessel functions of the 1st kind of order zero and one
k_{1a}, k_{1b}	Mode I stress intensity factors at the edges of crack
k_{1c}, k_{2c}	Stress intensity factors at the edge of internal rigid inclusions
$\bar{k}_{1a}, \bar{k}_{2b}$	Normalized stress intensity factors at the edges of crack
$\bar{k}_{1c}, \bar{k}_{2c}$	Normalized stress intensity factors at the edge of internal rigid inclusions
K, E	Complete elliptic integrals of the 1st and the 2nd kinds
L_{ij}	Integrands of the kernels N_{ij}
$L_{ij\infty}$	Dominants part of L_{ij} as $\alpha\to\infty$
L	Distance between crack and inclusions
m_i, M_i, N_{ij}, T_i	Kernels of the integral equations
N_{ijb}, N_{ijs}	Bounded and singular parts of N_{ij} as $\alpha\to\infty$
p_0	Intensity of the axial tensile load
$P_n(\alpha,\beta)$	Jacobi polynomials
r, z	Cylindrical coordinates
t	Integration variable
u, w	Displacement components in r- and z-directions
W_i	Weighting constants of the Jacobi polynomials
α	Fourier transform variable
β, θ, γ	Powers of singularity at the edges of the crack an inclusion
η, ε	Normalized variables on inclusions
ϕ, ψ	Normalized variables on crack
μ	Shear modulus of elasticity
ν	Poisson's ratio
σ, τ	Normal and shear stresses
σ_{zb}, σ_{zs}	Bounded and singular parts of σ_z at the edges of the crack and inclusions